MEASUREMENTS OF CKM ELEMENTS
AND
THE UNITARITY TRIANGLE

Helmut Marsiske

Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309

ABSTRACT

In this presentation, I review the status of selected Cabibbo-Kobayashi-Maskawa (CKM) matrix elements and their role in the Unitarity Triangle (UT). Since this conference concluded, many new results have been finalized and are included in the new world averages (WAs) in the 2002 edition of the PDG [1, 2, 3]. I will focus here on some outstanding issues in the measurements of the CKM elements in the third row and column.

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

1 CKM, UTs, and Theoretical Tools

In the Standard Model (SM) [4], the charged-current electro-weak interactions amongst three generations of quarks involve a complex, unitary 3×3 matrix, V, known as the CKM matrix [1]. Its elements, V_{ij}, determine the relative (weak) couplings of up-type (*i.e.*, $i = u, c, t$) and down-type (*i.e.*, $j = d, s, b$) quarks to the W boson. The matrix V is determined by four independent parameters, one of which can be used to describe CP violation (CPV). A parameterization developed by Wolfenstein emphasizes the hierarchy of elements by expanding them in powers of the sine of the Cabibbo angle: $\lambda = |V_{us}| = \sin \theta_C \approx 0.22$. The other three parameters are labelled A, ρ, and η, with η describing the level of CPV. In the improved Wolfenstein parameterization [5], using $\bar{\rho} = \rho(1 - \frac{\lambda^2}{2})$ and $\bar{\eta} = \eta(1 - \frac{\lambda^2}{2})$, the matrix reads:

$$
V = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda - iA^2\lambda^5\eta & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \bar{\rho} - i\bar{\eta}) & -A\lambda^2 + A\lambda^4(\frac{1}{2} - \rho) - iA\lambda^4\eta & 1 - \frac{1}{2}A^2\lambda^4 \end{pmatrix} + \mathcal{O}(\lambda^6).
$$

(1)

The unitarity constraint results in six orthogonality equations which can be expressed geometrically by six UTs in the complex plane. All triangles have the same area, Δ, which determines the level of CPV: $2\Delta = J_{\mathrm{CP}} \approx A^2\lambda^6\eta$. Most triangles are squashed, *i.e.*, one side is very much smaller than the others. However, there is one "golden triangle" with all sides of approximately equal size (*i.e.*, $\mathcal{O}(\lambda^3)$), determined by products of CKM elements that are experimentally most accessible:

$$
V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0
$$

(2)

It is customary to re-scale its sides by $|V_{cd}V_{cb}^*| = A\lambda^3$, resulting in a triangle in the complex plane with a unit-length baseline on the real axis. The UT in the $\bar{\rho}$, $\bar{\eta}$ plane has sides of length:

$$
\begin{aligned}
R_u &= \sqrt{\bar{\rho}^2 + \bar{\eta}^2} = (1 - \frac{\lambda^2}{2})\frac{1}{\lambda}\frac{|V_{ub}|}{|V_{cb}|} = \frac{|V_{ud}V_{ub}^*|}{|V_{cd}V_{cb}^*|}, \\
R_t &= \sqrt{(1 - \bar{\rho})^2 + \bar{\eta}^2} = \frac{1}{\lambda}\frac{|V_{td}|}{|V_{cb}|} = \frac{|V_{td}V_{tb}^*|}{|V_{cd}V_{cb}^*|}.
\end{aligned}
$$

(3)

In order to over-constrain the UT as a test of the SM, it is thus imperative to measure as many of the angles as possible, and improve the measurements of the relatively poorly known elements V_{ub}, V_{cb}, and V_{td} which determine the sides. (Note that V_{ud} and V_{us} are know to about 0.1% and 1%, respectively.)

Six CKM elements, V_{uq} and V_{cq}, $q = d, s, b$, can currently be determined directly from tree-level processes. The remaining three elements, V_{tq}, involve the top quark and are currently accessible only via loop processes in the K and B system (*i.e.*, rare K and B decay, and $K^0/B_d^0/B_s^0$ mixing). In the LHC/LC future, V_{tq} and V_{cq} will be measured at tree level from top and W decay, respectively, possibly with very high precision [6].

A general problem in determining CKM matrix elements arises from the fact that they enter into the charged currents between *quarks*, whereas experiments observe initial and final states containing *hadrons* and/or leptons. The "dressing" of quarks makes for strong-interaction effects that are difficult to deal with, in particular since one is often in the non-perturbative regime of Quantum Chromodynamics (QCD). The problem is somewhat simplified for semi-leptonic decays, because the (leptonic) W decay is well-understood and can be factored out.

The formalism to evaluate (weak) decay amplitudes of hadrons is that of the Operator Product Expansion (OPE) and Renormalization Group Evolution. Using a scale, μ, long- and short-distance contributions are separated into perturbative (and thus calculable) Wilson coefficients, $C_i(\mu)$, and non-perturbative hadronic matrix elements, $\langle F|Q_i(\mu)|M\rangle$, written in terms of local operators generated by QCD and electroweak interactions:

$$A(M \to F) = \frac{G_F}{\sqrt{2}} \sum_i V_{CKM}^i C_i(\mu) \langle F|Q_i(\mu)|M\rangle. \tag{4}$$

The tool of choice to get at hadronic matrix elements, in particular for *exclusive* final states, is Lattice QCD (LQCD), which has recently made great strides towards unquenched calculations, which could eventually yield quantifiable errors at the $\mathcal{O}(1\%)$ level. Until then, other methods (*e.g.*, $1/n_f$ expansion, QCD sum rules, chiral perturbation theory, etc.) have to be used, the results of which are usually formulated in terms of (meson) decay constants and form factors (FFs).

In case of the heavy-to-heavy $b \to c$ transition, Heavy Quark Effective Theory (HQET) can provide an absolute normalization for FFs at $q^2 = (p_\ell + p_\nu)^2 = q_{max}^2$ and in the infinite quark mass limit, with corrections of the order of only 10%. Unfortunately, no such luck for the heavy-to-light $b \to u$ transition: the FF absolute normalization (and possibly the q^2 dependence) must be calculated from the ground up.

Inclusive (semi-leptonic) decays of B mesons turn out to be particularly favorable, for their treatment in the framework of Heavy Quark Expansion (HQE) allows a well-defined expansion of the decay rate in powers of α_s (perturbative part) and Λ_{QCD}/m_b (non-perturbative part):

$$\Gamma(B \to X\ell\nu) = \Gamma(b \to q\ell\nu) + \mathcal{O}\left[\alpha_s, (\Lambda_{\mathrm{QCD}}/m_b)^2\right] , \tag{5}$$

where the first (and dominant) term represents the well-known quark spectator model rate (containing $|V_{qb}|^2$). Non-perturbative corrections are suppressed by at least two powers of $\Lambda_{\mathrm{QCD}}/m_b$ and can be expressed in terms of a small number of measurable parameters. This approach is valid only under the *ab initio* assumption of quark-hadron duality, which needs experimental verification. As has been advocated in Ref. [7], the semi-leptonic data itself can be used to test for duality violations by checking consistency among a variety of OPE-calculated variables.

2 V_{cb} [2]

$|V_{cb}|$ normalizes the baseline of the UT and determines the Wolfenstein parameter A (together with $\lambda = |V_{us}|$, which is very precisely known).

2.1   From Exclusive Semi-leptonic B Decays

$|V_{cb}|$ can be determined from the decay $B \to D^*\ell\nu$ by measuring the differential decay rate:

$$\frac{d\Gamma}{dw} \propto (F(w)|V_{cb}|)^2 , \text{ with } w = \frac{m_B^2 + m_{D^*}^2 - (p_\ell + p_\nu)^2}{2m_B m_{D^*}} , \tag{6}$$

and extrapolating the FF $F(w)$ to the kinematic limit $w = 1$, where HQET provides an absolute normalization in the form of the Isgur-Wise function: $F(w) = \eta_A \xi(w)$. In the limit of infinite quark masses, $\xi(1) \equiv 1$. Finite-mass and perturbative QED/QCD corrections are subsumed into the η_A factor, yielding $F(1) = \eta_A = 0.91 \pm 0.04$. This value for $F(1)$ results from a combination of quark model, OPE sum rule, and (quenched) LQCD calculations. It is promising to note that the LQCD calculation can provide a rather detailed breakdown of the errors, with the single largest one being statistical in nature.

Issues affecting this type of measurement are the assumed shape of the FF, parameterized in terms of its slope, ρ^2, at $w = 1$, the relative vector and axial vector contributions to the FF, and the treatment of feed down background from hadronic systems heavier than the D^*. A particular problem for experiments running on the $\Upsilon(4S)$ resonance lies in understanding the detection efficiency for the slow charged pion in the $D^{*\pm} \to \pi^\pm D^0$ decay. An important cross check is a simultaneous measurement of the $D^{*0} \to \pi^0 D^0$ decay, so far performed only by CLEO.

All these issues need to be tackled in order to improve the measurement beyond the current 5.2% error (which is, however, dominated by the theoretical error on $F(1)$). Moreover, the $B \to D^*\ell\nu$ and $B \to D\ell\nu$ data should be fit together to reduce the error on the FF slope, since the slopes in both channels are related to each other in a calculable way.

The LEP V_{cb} Working Group (WG) [8] has combined the Belle, CLEO, and LEP measurements of $F(1)|V_{cb}|$ *vs* ρ^2 – note that those two quantities are highly correlated – after adjusting for common inputs. Most measurements and their errors are changed only slightly by this procedure, except for the ALEPH result for which $F(1)|V_{cb}|$ and ρ^2 increase by 1.5 σ_{syst}^{old}, and σ_{syst}^{new} is four times as large as the old one. Unfortunately, the cited Ref. [8] does not provide quite enough information to reconstruct these calculations. Despite these changes, the ALEPH and CLEO measurements remain rather far apart, resulting in a somewhat low confidence level (CL) of only 5% for the new WA. One should also point out that the $|V_{cb}|$ value extracted from the exclusive CLEO measurement by dividing out the above $F(1)$ comes out rather high compared with the (presumably correlated) inclusive CLEO measurement in Ref. [9].

2.2 From Inclusive Semi-leptonic B Decays

$|V_{cb}|$ can also be extracted from the *inclusive* semi-leptonic branching fraction (BF) $\mathcal{B}(B \to X_c\ell\nu)$. In order to distinguish prompt leptons (in $B \to X_c\ell\nu$) from cascade leptons (in $B \to X_c \to X_{s/d}\,\ell\nu$), $\Upsilon(4S)$ experiments use a double-lepton technique: a high-momentum lepton tags the opposite B, allowing to measure a signal-lepton down to center-of-mass (COM) momenta of $p^* \approx 0.6\,\text{GeV}$ by exploiting charge and angular correlations. Table 1 shows the most current set of $\Upsilon(4S)$ measurements, which are consistent with the LEP average.

After subtracting the (small) $B \to X_u\ell\nu$ contribution, these BFs can be used to determine $|V_{cb}|$, albeit with a 5.9% theoretical error[1]. As indicated in connection with Equation 5, the theoretical error can be reduced by expressing the non-perturbative contributions to the semi-leptonic width in terms of three parameters (at order $(\Lambda_{\text{QCD}}/m_b)^2$): $\bar{\Lambda} = m_B - m_b + \cdots$, λ_1, and λ_2, all of which are *measurable* up to uncertainties of order $(\Lambda_{\text{QCD}}/m_b)^3$. The parameter λ_2 is readily determined from the $B^* - B$ mass splitting; the other two require more elaborate measurements.

---

[1]Note that the CL of this error is not particularly well defined: (not) following a suggestion in Ref. [10] to combine theoretical errors linearly, the LEP V_{cb} WG decided to first scale them by a factor two before combining them quadratically [11].

Table 1: *Inclusive B semi-leptonic BFs. In calculating the $\Upsilon(4S)$ average, the systematic errors have (wrongly) been assumed to be uncorrelated among experiments.*

Experiment	$\mathcal{B}(B \to X\ell\nu)$ (%)
BABAR prel.	$10.87 \pm 0.18 \pm 0.30$
BELLE prel.	$10.90 \pm 0.12 \pm 0.49$
CLEO 96	$10.49 \pm 0.17 \pm 0.43$
CLEO 92	$10.8 \pm 0.2 \pm 0.56$
ARGUS	$9.7 \pm 0.5 \pm 0.4$
$\Upsilon(4S)$ average	$10.66 \pm 0.08 \pm 0.18$
LEP average	$10.59 \pm 0.09 \pm 0.15 \pm 0.26$

Several experiments have embarked on a program to measure $\bar{\Lambda}$ (and thus m_b) and λ_1 from a variety of moments in semi-leptonic and radiative B decay data. For example, CLEO has measured the first moment of the photon energy in $B \to X_s\gamma$ to determine $\bar{\Lambda}$, and the first moment of the hadronic mass-squared, $\langle M_X^2 \rangle$, recoiling against the di-leptons in $B \to X_c\ell\nu$ to determine λ_1. With these parameters as input, the above-mentioned (inclusive) semi-leptonic BFs have been used to determine $|V_{cb}|$ with a 2.6% total error (and consistent with the exclusive result).

This all looks very promising and should improve even further, with preliminary CLEO [12] and DELPHI [13] measurements of lepton energy moments in $B \to X_c\ell\nu$ on their way. One trouble spot has emerged, though, highlighted by the preliminary *BABAR* measurement of $\langle M_X^2 \rangle$ *vs* minimum lepton COM momentum, $p_{\ell,min}^*$ [14]: while they obtain a result consistent with CLEO for the same $p_{\ell,min}^*$ cut of $1.5 \, \text{GeV}$, they find a $p_{\ell,min}^*$ dependence of $\langle M_X^2 \rangle$ that cannot be described by the CLEO values for $\bar{\Lambda}$ and λ_1 (but *can* be described by OPE using a different set of parameter values). This inconsistency suggests that some of the underlying assumptions HQE is based on require further scrutiny.

3 V_{ub} [3]

The second side of the UT, R_u, is determined by $|V_{ub}|$; see Equation 3.

3.1 From Inclusive Semi-leptonic B Decays

Using the OPE, the inclusive semi-leptonic BF $\mathcal{B}(B \to X_u\ell\nu)$ can be related to $|V_{ub}|^2$ with an accuracy of $5 - 10\%$. Unfortunately, experiments can only observe a

small portion of this decay: the overwhelming $B \to X_c \ell \nu$ background forces them to impose stringent analysis cuts, selecting a very restricted region of phase space near kinematic boundaries (*i.e.*, high lepton momentum, and/or low hadronic mass). Under those circumstances, OPE cannot reliably predict the (partial) decay rate due to non-convergence. The theorists way out of that is a "twist expansion" with the complete series of terms re-summed into an incalculable structure function (light-cone distribution function), describing the Fermi motion of the b quark inside the the B meson. This structure/distribution function determines the decay rate at leading order, with sub-leading twist corrections suppressed by (higher) powers of $\Lambda_{\mathrm{QCD}}/m_b$. Historically, it has been modeled in a somewhat *ad hoc* way, with the theoretical uncertainty in the fraction of events passing the analysis cuts estimated by varying the shape parameters. This uncertainty can be removed in principle, though, because the structure/distribution function is a property of the B meson itself and can thus be determined (to leading order) in other processes, for example, $B \to X_s \gamma$. It has been argued in Ref. [15, 16] that there is an uncontrollable uncertainty for $|V_{ub}|$, estimated at $\sim 15\%$, from transferring the $B \to X_s \gamma$ structure/distribution function to $B \to X_u \ell \nu$ due to the sub-leading $\mathcal{O}(\Lambda_{\mathrm{QCD}}/m_b)$ corrections. In contrast, Ref. [17] presents an actual calculation of the dominant source of power corrections, verifying the size of the effect, with a residual uncertainty for $|V_{ub}|$ of only a few percent.

The issue of sub-leading corrections needs to be resolved among the theorists, because it affects the recent lepton-endpoint $|V_{ub}|$ measurements from CLEO [18] and *BABAR* [19] in one of two ways: either the theoretical error associated with this needs to more than double (dominating then any other single error), or the value of $|V_{ub}|$ needs to change by $10 - 15\%$ (corresponding to $2 - 3\sigma$ of the theoretical error associated with this previously). On top of all of this, one has to worry about the restrictive analyses introducing quark-hadron duality violations, for which no estimate whatsoever exists currently.

One can nevertheless hold out hope for the future of (inclusive) $|V_{ub}|$ measurements because of the enormous data samples becoming available at the $\Upsilon(4S)$ B factories. The most promising analysis technique with hundreds of millions of Bs is that of fully reconstructing one of the two Bs in the event, thus gaining complete control over the kinematics of the semi-leptonically decaying signal B. This allows rather precise neutrino reconstruction, and charm suppression without kinematic cuts using particle identification (*i.e.*, no Kaon on the signal side). Furthermore, one can hope to use lepton momentum, hadronic mass, and di-lepton mass simultaneously to isolate $B \to X_u \ell \nu$ while retaining sufficient inclusiveness for OPE convergence and quark-hadron duality.

3.2 From Exclusive Semi-leptonic B Decays

The very same B reconstruction (Breco) technique will work beautifully for exclusive decays, *i.e.*, $B \rightarrow \pi/\eta/\rho/\omega \, \ell\nu$, providing very clean signals, with signal-to-background (S/B) ratios $>> 1$. Already with the data samples Belle and $BABAR$ have in hand, the Breco technique is favored over the conventional analysis pioneered by CLEO.

The main problem for extracting $|V_{ub}|$ from these B decay channels is again theoretical in nature: a transition FF is needed, which can currently be calculated only with uncertainties in the $15 - 20\%$ range. All hopes rest, again, on LQCD to eventually provide unquenched calculations with much improved precision for channels with stable particles (*i.e.*, $B \rightarrow \pi\ell\nu$) or *narrow* resonances (*i.e.*, $B \rightarrow \eta/\omega \, \ell\nu$).

3.3 From Exclusive Fully-leptonic B Decays

Fully-leptonic B decay provide the theoretically cleanest way to determine $|V_{ub}|$:

$$\Gamma(B^- \rightarrow \ell^- \nu) = \frac{G_F^2}{8\pi} m_B^3 f_B^2 |V_{ub}|^2 x_\ell (1 - x_\ell)^2, \text{ with } x_\ell = m_\ell/m_B. \tag{7}$$

Unfortunately, the current best LQCD determination of the B decay constant comes with a 15% error: $f_B = (200 \pm 30) \, \text{MeV}$. With a SM expectation of $B(B^- \rightarrow \tau^- \nu) \simeq 1.1 \times 10^{-4}$, observation of this decay is just around the corner, exploiting again the Breco technique. Hopefully, LQCD will manage to push down their error as fast as the B factories accumulate data.

4 V_{td}, V_{ts}

The third side of the UT, R_t, is determined by $|V_{td}|$. It can be extracted from Δm_d, the $B_d^0 - \bar{B}_d^0$ mass difference measured in mixing:

$$\Delta m_d \propto (\sqrt{B_{B_d}} f_{B_d})^2 |V_{td}|^2 \propto R_t^2 = \frac{1}{\lambda^2} \frac{|V_{td}|^2}{|V_{cb}|^2} = (1 - \bar{\rho})^2 + \bar{\eta}^2, \tag{8}$$

where LQCD provides [21] $(\sqrt{B_{B_d}} f_{B_d}) = (230 \pm 40) \, \text{MeV}$ with a 17% uncertainty.

It has been customary to use the *ratio* $\Delta m_d/\Delta m_s$, where Δm_s is the corresponding mass difference from $B_s^0 - \bar{B}_s^0$ mixing, because the *ratio* $\xi = (\sqrt{B_{B_d}} f_{B_d})/(\sqrt{B_{B_s}} f_{B_s})$ was believed to be better known (and most other factors cancel). Using $\xi = 1.16 \pm$

0.05 together with the LEP/SLD/CDF mixing amplitude analysis, $\Delta m_s > 14.9\,\mathrm{ps}^{-1}$ at 95% CL, one can get a significant constraint on R_t:

$$\frac{\Delta m_d}{\Delta m_s} = \frac{m_{B_d}}{m_{B_s}}\xi\frac{|V_{td}|^2}{|V_{ts}|^2} \propto \frac{1}{\xi^2}\lambda^2\left[(1-\bar{\rho})^2 + \bar{\eta}^2\right] = \frac{1}{\xi^2}\lambda^2 R_t^2 \tag{9}$$

However, it was recently discovered [20] that ξ might have to change by $4\sigma^{old}$ and assume an error twice as large as currently, resulting in a much relaxed constraint on R_t.

5   Conclusions

(Inclusive) $|V_{cb}|$ is becoming a precision quantity with an uncertainty of less than 3%; it could become even more precise with better measurements of HQE non-perturbative parameters – barring quark-hadron duality violations.

$|V_{ub}|$ is far from that, with an uncertainty of about 15%. There are good prospects, though, for significant improvements over the next few years: new analysis approaches can significantly reduce both the experimental *and* theoretical errors of the inclusive measurements. The exclusive measurements will equally benefit, contingent upon sufficient progress in LQCD.

$|V_{td}|$ and $|V_{ts}|$ are awaiting a more precise ξ ratio from LQCD and, more importantly, a *measurement* of (not a *limit* on) Δm_s.

$|V_{ud}|$ will benefit from consistent neutron β-decay data on $\lambda = g_A/g_V$ and from better pion β-decay data.

$|V_{us}|$ is awaiting a (theoretical) conclusion on the right value and error for the $K \to \pi$ transition FF.

6   Acknowledgements

References

1. Particle Data Group, K. Hagiwara*et al*, Phys. Rev. **D66**, 010001-113 (2002) and references therein.

2. Particle Data Group, K. Hagiwara*et al*, Phys. Rev. **D66**, 010001-701 (2002) and references therein.

3. Particle Data Group, K. Hagiwara*et al*, Phys. Rev. **D66**, 010001-706 (2002) and references therein.

4. Particle Data Group, K. Hagiwara*et al*, Phys. Rev. **D66**, 010001-98 (2002) and references therein.

5. A.J. Buras, R. Fleischer, Quark Mixing, CP Violation And Rare Decays After The Top Quark Discovery, in: Heavy Flavour II, World Scientific (1997).

6. J. Letts, P. Mättig, Eur. Phys. J. **C21**, 211 (2001).

7. C. Bauer, hep-ph/0112243.

8. LEP V_{cb} Working Group, Internal Note, `http://lepvcb.web.cern.ch/LEPVCB/`.

9. D. Cronin-Hennessy *et al*, Phys. Rev. Lett. **87**, 251808 (2001).

10. I. Bigi, hep-ph/9907270.

11. D. Abbaneo *et al.*, CERN-EP/2001-050 and hep-ex/0112028.

12. R.A. Briere *et al.*, hep-ex/0209024.

13. M. Battaglia, M. Calvi, L. Salmi, DELPHI 2002-071-CONF-605.

14. B. Aubert *et al.*, hep-ex/0207084.

15. A.K. Leibovich, Z. Ligeti, M.B. Wise, hep-ph/0205148.

16. C. Bauer, M. Luke, T. Mannel, hep-ph/0205150.

17. M. Neubert, hep-ph/0207002.

18. A. Bornheim *et al.*, Phys. Rev. Lett. **88**, 231803 (2002).

19. B. Aubert *et al.*, hep-ex/0207081.

20. A. Kronfeld, these proceedings and hep-ph/0209231.

21. Particle Data Group, K. Hagiwara*et al*, Phys. Rev. **D66**, 010001-670 (2002) and references therein.