Loop-Effects in Pseudo-Supersymmetry

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ABSTRACT

We analyze the transmission of supersymmetry breaking in brane-world models of pseudo-supersymmetry. In these models two branes preserve different halves of the bulk supersymmetry. Thus supersymmetry is broken although each sector of the model is supersymmetric when considered separately. The world-volume theory on one brane feels the breakdown of supersymmetry only through two-loop interactions involving a coupling to fields from the other brane. In a 5D toy model with bulk vectors, we compute the diagrams that contribute to scalar masses on one brane and find that the masses are proportional to the compactification scale up to logarithmic corrections, $m^2 \propto (2\pi R)^{-2}(\ln(2\pi R\, m_{\mathrm{S}}) - 1.1)$, where m_{S} is an ultraviolet cutoff. Thus, for large compactification radii, where this result is valid, the brane scalars acquire a positive mass squared. We also compute the three-loop diagrams relevant to the Casimir energy between the two branes and find $E \propto (2\pi R)^{-4}\big((\ln(2\pi R\, m_{\mathrm{S}}) - 1.7)^2 + 0.2\big)$. For large radii, this yields a repulsive Casimir force.

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1 Introduction

Supersymmetry breaking in brane-world models has several very attractive features. The possibility of breaking supersymmetry on a distant brane offers a geometric realization of the idea of hidden sectors. Supersymmetry can be completely broken in a non-local way by partially breaking supersymmetry on different branes in such a way that each brane preserves a different fraction of the extended bulk supersymmetry, e.g., [1, 2, 3, 4]. It has been argued that such a framework could help to tackle the cosmological constant problem [5]. In 5D orbifold models which realize this supersymmetry breaking scenario, quantitative results can be obtained for the radiatively generated mass splittings [6]. The exciting result is that the one-loop contribution to the mass squared of a scalar that has Yukawa couplings to quarks and leptons and vanishing tree-level mass is finite and negative.[1] A similar calculation has been performed for a non-supersymmetric type I string model [7].

In this article, we would like to study a brane-world model where supersymmetry is not broken by some orbifold projection but rather by the branes themselves. This is the way supersymmetry is broken in almost all of the realistic D-brane models of string theory. A D-brane breaks half of the bulk supersymmetry by its mere presence. The fields of the effective world-volume theory only fill multiplets of the smaller supersymmetry algebra. Supersymmetry can be completely broken either by adding anti-D-branes, which break the half of supersymmetry that is preserved by the D-branes, or by considering configurations of intersecting D-branes where different intersections preserve different fractions of the bulk supersymmetry. Such a scenario has been called pseudo-supersymmetry [9].

An effective four-dimensional description of a broad class of models where supersymmetry is partially broken by branes was recently discussed in [10] (see also [11]). By considering only the minimal field content, consisting of the $\mathcal{N} = 2$ gravity multiplet, the Goldstone fields and their superpartners, and requiring the broken supersymmetry to be non-linearly realized, the effective action could be uniquely determined. Although the supergravity approach is the appropriate framework to analyze supersymmetry breaking in brane worlds, it turns out that many important results can already be obtained by just considering the situation in global supersymmetry (see, e.g., [12]). Building upon the pioneering work of [13], the explicit 4D effective Lagrangian of global pseudo-supersymmetry was determined in [9]. This is an important result because it applies to any string model realizing the pseudo-supersymmetry scenario.

The aim of this work is to obtain quantitative results for the radiatively generated mass-splittings in pseudo-supersymmetry. We consider a toy model of two 3-branes located at $x^5 = 0$ and $x^5 = \pi R$ in $M^{3,1} \times S^1$. There are an $\mathcal{N}_4 = 2$ vector corresponding to the gauge symmetry G in the bulk and $\mathcal{N}_4 = 1$ chiral multiplets charged under the gauge symmetry on the 3-branes. The chiral multiplets from the two 3-branes couple to different bulk gauginos and thus preserve different halves of the bulk supersymmetry. Brane scalar masses are generated through Feynman diagrams involving a loop of fields from the distant brane. Such diagrams arise at the two-loop level. The explicit computation shows that the expected quadratic cutoff dependence is regulated by the finite brane separation. Only logarithmic

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[1]See however [8], where it was shown that quadratic divergences arise if the gauge group is Abelian.

divergences arise. They are due to wave-function renormalization of the brane fields. More precisely, we find that the brane scalar mass squared is positive and, for R much larger than the inverse cutoff scale $m_{\rm S}^{-1}$, proportional to $(2\pi R)^{-2}\ln(2\pi R\, m_{\rm S})$. Similarly, we find that the Casimir energy depends quartically on the inverse of the brane separation but only logarithmically on the cutoff scale. Thus supersymmetry breaking is soft in this class of models.

These results are particularly interesting in view of the recent D-brane constructions which represent embeddings of the standard model in string theory [1, 4]. The knowledge of the precise expression for the mass splittings and their dependence on the interbrane distance is is an important first step towards a phenomenological analysis of those models. Although many of the explicit models have additional features, the toy model of this article captures their main supersymmetry breaking mechanism.

The paper is organized as follows. In the next section, we generalize the results of [9] to five-dimensional models. Using this result, we compute one-loop corrections to the bulk Lagrangian, two-loop corrections to the brane Lagrangians and three-loop corrections to the vacuum energy density. In three appendices, we explain our notation, give the explicit component Lagrangians of our model and list the Feynman rules and some useful formulae to do the loop integrations.

2 Pseudo-Supersymmetry in D=5

Consider two 3-branes[2] that break different halves of the bulk supersymmetry. For concreteness, we concentrate on a flat 5-dimensional bulk space of the form $M^{3,1}\times S^1$. The 3-branes are extended along the 4-dimensional Minkowski space-time $M^{3,1}$ and are separated by a distance l on the circle S^1 of radius R. For simplicity, we assume $l=\pi R$ throughout this article. The simplest model of pseudo-supersymmetry consists of an $\mathcal{N}_5=1$, $D=5$ vector multiplet in the bulk and $\mathcal{N}_4=1$, $D=4$ chiral multiplets on the 3-branes. The chiral matter on each of the two branes couples to the bulk vector and to one of the bulk gauginos via the usual supersymmetric gauge couplings. The crucial point, however, is that the fields on the first brane couple only to the first bulk gaugino while the fields on the second brane couple only to the second bulk gaugino. The Lagrangian for the dimensional reduction of this model to $D=4$ has been given in [9]. Here, we are interested in the situation where the radius R of the fifth dimension is much larger than the inverse supersymmetry breaking scale[3] $m_{\rm S}^{-1}$. The 5-dimensional generalization of [9] is easily found using the formalism of [15]. To fix our notation, we review the Lagrangian for the 5D bulk vector multiplet, its reduction to $D=4$ and its form in terms of 4D superfields. Then we discuss the couplings of the brane fields to the bulk vector.

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[2]In string theory, these are two stacks of branes. Whenever writing 'brane' in this article, we understand that it could either be a single brane or a stack of branes.

[3]This is the scale where the bulk supersymmetry is partially broken on the branes. In string models, $m_{\rm S}$ is related to the the string scale by $m_{\rm S}=(2\pi^2 g_{\rm str})^{-1/4}M_{\rm str}$, as has been shown in [14]. See also the discussion at the end of section 4.

The bulk vector multiplet contains the component fields

$$A_M, \quad \lambda^{(5)i}, \quad \phi^{(5)}, \quad X^a, \qquad M = 0, \ldots, 3, 5, \; i = 1, 2, \; a = 1, 2, 3, \qquad (2.1)$$

where A_M is a 5D vector, $\lambda^{(5)i}$ is a 5D symplectic Majorana spinor, $\phi^{(5)}$ is a real scalar and X^a is an $SU(2)_R$ triplet of real auxiliary fields. Our conventions for spinors and γ-matrices are explained in appendix A.

The Lagrangian for this multiplet is

$$\mathcal{L}_5 = \frac{2}{g_{(5)}^2} \operatorname{tr} \left(-\frac{1}{4} F^{MN} F_{MN} - \frac{1}{2} \mathcal{D}^M \phi^{(5)} \mathcal{D}_M \phi^{(5)} - \frac{i}{2} \overline{\lambda}_i^{(5)} \gamma^M \mathcal{D}_M \lambda^{(5)i} \right.$$

$$\left. + \frac{1}{2} X^a X^a - \frac{1}{2} \overline{\lambda}_i^{(5)} \left[\phi^{(5)}, \lambda^{(5)i} \right] \right). \qquad (2.2)$$

The components of the $\mathcal{N}_5 = 1$, $D = 5$ vector multiplet can be rearranged to fit into an $\mathcal{N}_4 = 2$, $D = 4$ vector multiplet

$$A_m, \quad \lambda_i, \quad \phi, \quad D, \quad F, \qquad i = 1, 2, \qquad (2.3)$$

where A_m is a 4D vector, λ_i are two 4D Weyl spinors, ϕ is a complex scalar, D is a real auxiliary field and F is a complex auxiliary field. The precise mapping is

$$\lambda^{(5)1} = \begin{pmatrix} \lambda_1 \\ -\overline{\lambda}_2 \end{pmatrix}, \quad \phi = \frac{1}{\sqrt{2}} \left(A_5 + i\, \phi^{(5)} \right), \quad D = X^3, \quad F = \frac{i}{\sqrt{2}} \left(X^1 + i\, X^2 \right). \qquad (2.4)$$

The $\mathcal{N}_4 = 2$ vector can be split into an $\mathcal{N}_4 = 1$ vector multiplet $V = (A_m, \lambda_1, D)$ and an $\mathcal{N}_4 = 1$ chiral multiplet $\Phi = (\phi, \lambda_2, F)$. These transform irreducibly under the first supersymmetry. Their superspace expansion reads[4]

$$V = -\theta \sigma^m \bar{\theta} A_m + i\theta\theta\bar{\theta}\bar{\lambda}_1 - i\bar{\theta}\bar{\theta}\theta\lambda_1 + \frac{1}{2} \theta\theta\bar{\theta}\bar{\theta} D,$$

$$\Phi = \phi + \sqrt{2}\,\theta\lambda_2 + \theta\theta F. \qquad (2.5)$$

Alternatively, one can split the $\mathcal{N}_4 = 2$ vector into an $\mathcal{N}_4 = 1$ vector multiplet $V' = (A_m, \lambda_2, D)$ and an $\mathcal{N}_4 = 1$ chiral multiplet $\Phi' = (\phi, \lambda_1, F)$ transforming irreducibly under the second supersymmetry. Their superspace expansion reads

$$V' = -\tilde{\theta} \sigma^m \bar{\tilde{\theta}} A_m + i\tilde{\theta}\tilde{\theta}\bar{\tilde{\theta}}\bar{\lambda}_2 - i\bar{\tilde{\theta}}\bar{\tilde{\theta}}\tilde{\theta}\lambda_2 - \frac{1}{2} \tilde{\theta}\tilde{\theta}\bar{\tilde{\theta}}\bar{\tilde{\theta}} D,$$

$$\Phi' = \phi - \sqrt{2}\,\tilde{\theta}\lambda_1 + \tilde{\theta}\tilde{\theta} F^\dagger. \qquad (2.6)$$

The interesting observation of [15] is that the complete 5D Lagrangian (2.2) can be written in terms of 4D superfields. Indeed, one can verify that

$$\mathcal{L}_{\text{bulk}} = \frac{2}{4\,g_{(5)}^2} \int d^2\theta \; \operatorname{tr}(W^\alpha W_\alpha) + \frac{2}{4\,g_{(5)}^2} \int d^2\bar{\theta} \; \operatorname{tr}\left(\bar{W}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}} \right) \qquad (2.7)$$

$$+ \frac{2}{g_{(5)}^2} \int d^2\theta d^2\bar{\theta} \; \operatorname{tr} \left[\left(\frac{i}{\sqrt{2}} \partial_5 - \Phi^\dagger \right) e^{2V} \left(\frac{i}{\sqrt{2}} \partial_5 - \Phi \right) e^{-2V} + \frac{1}{4} \partial_5 e^{2V} \partial_5 e^{-2V} \right]$$

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[4] V is in Wess-Zumino gauge and Φ in the y-basis, see [16].

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coincides with (2.2) after integrating out the auxiliary fields. All fields may depend on the coordinate x^5 and it is understood that the action is to be integrated over all five dimensions, $S_{\text{bulk}} = \int d^5x \, \mathcal{L}_{\text{bulk}}$.

Now, consider an $\mathcal{N}_4 = 1$, $D = 4$ chiral multiplet $\Phi^{(1)}$ which is charged under the bulk gauge symmetry and localized on the first 3-brane. For simplicity, we take $\Phi^{(1)}$ to transform in the fundamental representation of the gauge group. The coupling of the bulk vector to brane matter is determined by gauge symmetry and supersymmetry.

$$\mathcal{L}_{(1)}^0 = \delta(x^5) \int d^2\theta d^2\bar{\theta} \, \Phi^{(1)\dagger} e^{2V} \Phi^{(1)}, \tag{2.8}$$

where we assumed that the first 3-brane is located at $x^5 = 0$. The authors of [12] pointed out that there is an additional interaction between the bulk scalar and the brane scalars of the form $\partial_5 \text{Im}(\phi) \, \phi^{(1)\dagger} \phi^{(1)} \delta(x^5)$. In the approach of writing the 5D Lagrangian in terms of 4D superfields, precisely the same interaction arises upon integrating out the auxiliary fields from $\mathcal{L}_{\text{bulk}} + \mathcal{L}_{(1)}^0$.

One of the main points of [9] was to show how the second supersymmetry that is broken by the above brane-bulk coupling can still be non-linearly realized. This is possible if the model under consideration contains an $\mathcal{N}_4 = 1$, $D = 4$ goldstino superfield Λ_g[5] that is localized on the world-volume of the brane and transforms under the second supersymmetry as

$$\delta_{\xi_2} \Lambda_g = m_{\text{S}}^2 \, \xi_2 - m_{\text{S}}^{-2} V_{\xi_2}^m \partial_m \Lambda_g, \qquad \text{where} \quad V_{\xi_2}^m = i\Lambda_g \sigma^m \bar{\xi}_2 - i\xi_2 \sigma^m \bar{\Lambda}_g. \tag{2.9}$$

The Lagrangian (2.8) is rendered invariant under the second supersymmetry by assigning the transformation law

$$\delta_{\xi_2} \Phi^{(1)} = -m_{\text{S}}^{-2} V_{\xi_2}^m \partial_m \Phi^{(1)} \tag{2.10}$$

to the brane chiral multiplets and by adding appropriate couplings involving the goldstino to the Lagrangian. To find these goldstino couplings, first note that there exists a density superfield

$$\hat{E} = 1 + \frac{1}{8 \, m_{\text{S}}^4} \bar{D}^2 \bar{\Lambda}_g^2 + \frac{1}{8 \, m_{\text{S}}^4} D^2 \Lambda_g^2 + \mathcal{O}\left(m_{\text{S}}^{-8}\right) \tag{2.11}$$

which has the property that $\int d^2\theta d^2\bar{\theta} \, \hat{E} \, \Phi^{\dagger} \Phi$ is invariant under the second supersymmetry. In this expression, D_α is the supercovariant derivative $\partial_\alpha + i \, (\sigma^m \bar{\theta})_\alpha \partial_m$. Then, one replaces V by

$$\hat{V} = V + i m_{\text{S}}^{-2} \, \theta \sigma^m \bar{\theta} \, (\bar{\Lambda}_g \bar{\sigma}_m D\Phi + \Lambda_g \sigma_m \bar{D}\Phi^{\dagger}) + \mathcal{O}\left(m_{\text{S}}^{-4}\right) \tag{2.12}$$

and finds [9] that

$$\mathcal{L}_{(1)} = \delta(x^5) \int d^2\theta d^2\bar{\theta} \, \hat{E} \, \Phi^{(1)\dagger} e^{2\hat{V}} \Phi^{(1)} \tag{2.13}$$

is invariant under both supersymmetries.

Next, consider an $\mathcal{N}_4 = 1$, $D = 4$ chiral multiplet $\Phi^{(2)}$ localized on the second 3-brane and transforming irreducibly under the second supersymmetry. Again, the coupling of the bulk vector to brane matter is determined by gauge symmetry and supersymmetry. But we

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[5] Λ_g is a Weyl spinor under 4D Lorentz transformations but we mostly suppress its spinor index. The lowest component of Λ_g is the Goldstone fermion λ_g.

need to split the bulk vector vector multiplet according to (2.6). The Lagrangian for this brane-bulk coupling is given by

$$\mathcal{L}^0_{(2)} = \delta(x^5 - l) \int d^2\tilde{\theta} d^2\bar{\tilde{\theta}} \, \Phi^{(2)\dagger} e^{2V'} \Phi^{(2)}. \tag{2.14}$$

This interaction is manifestly invariant under the second supersymmetry. It can be rendered invariant under both supersymmetries by adding appropriate goldstino interactions. Of course, there need to be a second goldstino superfield Λ'_g which is localized on the second 3-brane. In complete analogy to the above definitions (2.11) and (2.12), one defines $\hat{E}'$ and $\hat{V}'$ from Λ'_g and V'. The invariant Lagrangian is given by

$$\mathcal{L}_{(2)} = \delta(x^5 - l) \int d^2\tilde{\theta} d^2\bar{\tilde{\theta}} \, \hat{E}' \, \Phi^{(2)\dagger} e^{2\hat{V}'} \Phi^{(2)}. \tag{2.15}$$

Finally, the total Lagrangian for our pseudo-supersymmetry toy model is the sum of the bulk Lagrangian (2.7) and the two bulk-brane interactions (2.13) and (2.15):

$$\begin{aligned}
\mathcal{L} &= \mathcal{L}_{\text{bulk}} + \mathcal{L}_{(1)} + \mathcal{L}_{(2)} \\
&= \frac{2}{4 \, g^2_{(5)}} \int d^2\theta \, \text{tr}\,(W^\alpha W_\alpha) + \frac{2}{4 g^2_{(5)}} \int d^2\bar{\theta} \, \text{tr}\left(\bar{W}_{\dot\alpha} \bar{W}^{\dot\alpha}\right) \\
&\quad + \frac{2}{g^2_{(5)}} \int d^2\theta d^2\bar{\theta} \, \text{tr}\left[\left(\frac{i}{\sqrt{2}} \partial_5 - \Phi^\dagger\right) e^{2V} \left(\frac{i}{\sqrt{2}} \partial_5 - \Phi\right) e^{-2V} + \frac{1}{4} \partial_5 e^{2V} \partial_5 e^{-2V}\right] \\
&\quad + \delta(x^5) \int d^2\theta d^2\bar{\theta} \, \hat{E} \, \Phi^{(1)\dagger} e^{2\hat{V}} \Phi^{(1)} + \delta(x^5 - l) \int d^2\tilde{\theta} d^2\bar{\tilde{\theta}} \, \hat{E}' \, \Phi^{(2)\dagger} e^{2\hat{V}'} \Phi^{(2)}.
\end{aligned} \tag{2.16}$$

The expansion of this Lagrangian into component fields is given in appendix B.

3 Loop corrections to the bulk Lagrangian

The $\mathcal{N}_4 = 2$ supersymmetry of the tree-level bulk Lagrangian (2.7) is broken at one-loop by the brane-bulk interactions. Interestingly, one-loop corrections induce kinetic terms for the bulk vector and gauginos localized on the 3-branes (for a recent discussion of this phenomenon, see [17]). The relevant diagrams contributing to the one-loop self-energy of the bulk fields are of the form shown in fig. 1. Note that the goldstino couples to bulk fields only through the brane-bulk interactions (B.2), (B.3). As a consequence, there is no one-loop contribution to the bulk Lagrangian involving the goldstino. Taking into account the residual symmetry in each subsector, one finds that the one-loop corrected bulk Lagrangian is of the form

$$\begin{aligned}
\frac{1}{2} \mathcal{L}^{(1\text{-loop})}_{\text{bulk}} &= \frac{M_c}{4 \, g^2_0} \int d^2\theta \, \text{tr}\,(W^\alpha W_\alpha) + \frac{M_c}{4 g^2_0} \int d^2\bar{\theta} \, \text{tr}\left(\bar{W}_{\dot\alpha} \bar{W}^{\dot\alpha}\right) \\
&\quad + \frac{M_c}{g^2_0} \int d^2\theta d^2\bar{\theta} \, \text{tr}\left[\left(\frac{i}{\sqrt{2}} \partial_5 - \Phi^\dagger\right) e^{2V} \left(\frac{i}{\sqrt{2}} \partial_5 - \Phi\right) e^{-2V} + \frac{1}{4} \partial_5 e^{-2V} \partial_5 e^{-2V}\right]
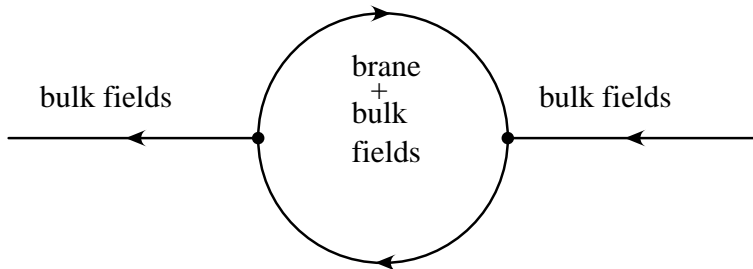\end{aligned}$$

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Figure 1: One-loop Feynman diagrams contributing to the self-energy of the bulk fields.

$$+\frac{\delta(x^5)}{4\,g_1^2}\int d^2\theta\,\,\text{tr}\,(W^\alpha W_\alpha)+\frac{\delta(x^5)}{4\,g_1^2}\int d^2\bar\theta\,\,\text{tr}\left(\bar W_{\dot\alpha}\bar W^{\dot\alpha}\right) \qquad (3.1)$$

$$+\frac{\delta(x^5-l)}{4\,g_2^2}\int d^2\tilde\theta\,\,\text{tr}\,(W'^\alpha W'_\alpha)+\frac{\delta(x^5-l)}{4\,g_2^2}\int d^2\bar{\tilde\theta}\,\,\text{tr}\left(\bar W'_{\dot\alpha}\bar W'^{\dot\alpha}\right),$$

where $M_c=(2\pi R)^{-1}$ and W'_α is the field strength superfield associated to the $\mathcal{N}_4=1$ vector multiplet V' defined in (2.6). The gauge couplings g_0, g_1, g_2 are given by

$$\frac{M_c}{g_0^2} \;=\; \frac{1}{g_{(5)}^2}+\Delta_{\text{bulk}}, \qquad \frac{1}{g_i^2} \;=\; \frac{b_i}{8\pi^2}\ln\left(\frac{m}{\Lambda}\right), \qquad (3.2)$$

where Δ_{bulk} is the contribution from the bulk fields running in the loop, b_1, b_2 are the beta-function coefficients of the matter fields living on the two 3-branes, m is an IR cutoff and Λ is a UV cutoff. We are not interested in the precise form of Δ_{bulk} but content ourselves to note that it depends linearly on the cutoff and has been computed in [18]. The problems related to the non-renormalizability of the 5D Super-Yang-Mills theory can be avoided by embedding it into a $D=5$, $\mathcal{N}_5=2$ theory, i.e., by adding a bulk hyper multiplet in the adjoint representation. That theory is free of ultra-violet divergences and, in particular, Δ_{bulk} vanishes.

The logarithmic divergences localized on the world-volume of the 3-branes can be eliminated through standard four-dimensional renormalization. Requiring the brane-localized contributions to the bulk gauge kinetic terms to vanish at the scale of supersymmetry breaking, m_S, one has the running couplings

$$\frac{1}{g_i^2(\mu)} \;=\; \frac{b_i}{8\pi^2}\ln\left(\frac{\mu}{m_S}\right). \qquad (3.3)$$

It is interesting that the counterterm necessary to cancel the logarithmic divergences is absent at tree-level and only arises through quantum corrections.[6]

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[6]A model where such counterterms are already present at tree-level, has recently been discussed in [19]. In orbifold compactifications of type I string theory, there is a tree-level contribution to brane localized gauge-kinetic terms proportional to the expectation value of a linear combination of twisted moduli fields [20].

Supersymmetry breaking manifests itself by the different $1/g^2$ prefactors of the various terms in the bulk Lagrangian. This is most clearly seen in the component field expansion of (3.1). We find

$$
\begin{aligned}
\frac{1}{2}\mathcal{L}_{\text{bulk}}^{(\text{1-loop})} \;=\; & \left(\frac{M_{\text{c}}}{g_0^2} + \frac{\delta(x^5)}{g_1^2} + \frac{\delta(x^5-l)}{g_2^2}\right) \text{tr}\left(-\frac{1}{4}F^{mn}F_{mn}\right) \\
& + \left(\frac{M_{\text{c}}}{g_0^2} + \frac{\delta(x^5)}{g_1^2}\right)\text{tr}\left(-\frac{i}{2}\lambda_1\sigma^m\mathcal{D}_m\bar\lambda_1 - \frac{i}{2}\bar\lambda_1\bar\sigma^m\mathcal{D}_m\lambda_1\right) \\
& + \left(\frac{M_{\text{c}}}{g_0^2} + \frac{\delta(x^5-l)}{g_2^2}\right)\text{tr}\left(-\frac{i}{2}\lambda_2\sigma^m\mathcal{D}_m\bar\lambda_2 - \frac{i}{2}\bar\lambda_2\bar\sigma^m\mathcal{D}_m\lambda_2\right) \\
& + \frac{M_{\text{c}}}{g_0^2}\text{tr}\left(-\mathcal{D}^m\phi^\dagger\mathcal{D}_m\phi + F^\dagger F\right. \\
& \left. -\frac{i}{\sqrt2}\epsilon^{ij}\left(\lambda_i[\phi^\dagger,\lambda_j]+\bar\lambda_i[\phi,\bar\lambda_j]\right) - \frac{1}{2}\epsilon^{ij}\left(\lambda_i\partial_5\lambda_j+\bar\lambda_i\partial_5\bar\lambda_j\right) + [\phi,\phi^\dagger]D + \frac{1}{2}D^2\right. \\
& \left. -\frac{1}{2}\partial_5 A^m\partial_5 A_m + \frac{1}{\sqrt2}\partial_5 A^m\mathcal{D}_m\left(\phi+\phi^\dagger\right) - \frac{i}{\sqrt2}\left(\phi-\phi^\dagger\right)\partial_5 D\right).
\end{aligned}
\tag{3.4}
$$

The effective four-dimensional gauge coupling constant is the coefficient of $-\frac{1}{4}F^{mn}F_{mn}$ after having integrated over the fifth dimension:

$$
\frac{1}{(g_{(4),\text{1-loop}})^2} \;=\; \frac{1}{g_0^2} + \frac{1}{g_1^2} + \frac{1}{g_2^2}.
\tag{3.5}
$$

Similarly, we define

$$
\frac{1}{(\rho_{i,\text{1-loop}})^2} \;=\; \frac{1}{g_0^2} + \frac{1}{g_i^2}.
\tag{3.6}
$$

In order to have canonically normalized kinetic terms for the bulk field zero modes after compactification to four dimensions, we have to replace

$$
A_m \;\to\; g_{(4),\text{1-loop}}\,A_m, \quad \lambda_i \;\to\; \rho_{i,\text{1-loop}}\,\lambda_i, \quad \phi \;\to\; g_0\,\phi.
\tag{3.7}
$$

This breaks supersymmetry in the brane-bulk Lagrangians (2.13), (2.15), as can be seen from the component Lagrangians (B.2), (B.3).

Although the bulk supersymmetry is broken at one-loop, there are no mass splittings at this order in perturbation theory. This is because each of the two subsectors $\mathcal{L}_{\text{bulk}}+\mathcal{L}_{(1)}$ and $\mathcal{L}_{\text{bulk}}+\mathcal{L}_{(2)}$ preserves $\mathcal{N}_4=1$ at tree-level. Therefore the supersymmetric non-renormalization theorems are still valid for all diagrams involving only fields from the bulk and one of the branes. Self-energy diagrams for bulk fields involving couplings to fields from both branes are only possible at three-loop. This can also be seen by considering one-loop corrections to bulk scalar masses, which arise from diagrams of the form $\phi \to \lambda_1\lambda_2 \to \phi$. If λ_1, λ_2 were taken to be the tree-level fields, then the bulk $\mathcal{N}_4=2$ supersymmetry would guarantee that the contribution of this diagram to the scalar mass is cancelled by the remaining one-loop diagrams. Only if both λ_i propagators are replaced by their one-loop corrected values as

in (3.7), is supersymmetry broken and no cancellation in the mass contribution takes place. Thus, the bulk scalar acquires a mass a three-loop.

The situation is more complicated for the fermions. Contributions to bulk gaugino self-energy are of the form $\lambda_i \to \lambda_i A_m \to \lambda_i$, where the A_m and λ_i propagators have one-loop insertions of brane fields. This implies that bulk fermion masses can only arise at three-loop. However, it is easy to see that it is not possible to obtain a non-vanishing contribution to gaugino masses if all brane fermions are massless. In the Weyl fermion basis, there are two types of propagators for a fermion χ of mass m; one takes $\chi \to \bar{\chi}$ and is proportional to $k^m \bar{\sigma}_m$, the other takes $\chi \to \chi$ and is proportional to the mass m. It turns out that there is no Feynman diagram containing only Kaluza-Klein zero modes that generates a non-vanishing gaugino mass. All such self-energy diagrams only represent a wave function renormalization. This result is the consequence of a $\mathbb{Z}_4$ symmetry [9] which forbids terms quadratic in λ_i in the Lagrangian. Under this symmetry, λ_1, λ_2, ϕ, A_m have charges 1, -1, 0, 0, respectively. However, this symmetry does not exclude non-diagonal mass terms of the form $\lambda_1 \lambda_2$. Indeed the excited Kaluza-Klein modes of the bulk gauginos have such non-diagonal masses, as follows from the $\lambda_1 \partial_5 \lambda_2$ term in (B.7). It seems probable that Feynman diagrams with massive Kaluza-Klein modes propagating on internal lines can generate non-diagonal masses for the gaugino zero-modes. But an explicit calculation of such diagrams is beyond the scope of this article.

We would like to mention one other possibility how bulk gauginos can acquire a mass. Gaugino masses can arise through anomaly-mediation [22]. According to [22, 23], anomaly mediated gaugino masses appear whenever the auxiliary fields of the gravity multiplet acquire a vacuum expectation value (vev) and the the gauge theory has a non-vanishing beta-function. In section 5, we will see that the vacuum energy receives a positive contribution at three-loop. To cancel this vacuum energy when the model is coupled to supergravity the auxiliary fields in the gravity multiplet acquire a vev.[7] This vev breaks the above-mentioned $\mathbb{Z}_4$ symmetry and gives mass to the bulk gauginos of order $(g_{(4)}^2/4\pi)^2 M_c^2/M_{\mathrm{Pl}}$.

4   Loop corrections to the brane Lagrangian

The $\mathcal{N}_4 = 1$ brane supersymmetry is broken at one-loop by the corrections to the gauge coupling constant. After the replacement (3.7), the $\psi^{(1)}\sigma^m \bar{\psi}^{(1)} A_m$ coupling is governed by $g_{(4),1\text{-loop}}$, eq. (3.5), while the $\phi^{(1)\dagger}\lambda_1 \psi^{(1)}$ coupling is governed by $\rho_{1,1\text{-loop}}$, eq. (3.6). However, mass splittings only arise at two-loop. We would like to compute the two-loop contribution to scalar masses assuming that all fields are massless to start with. It is instructive to first consider the case where both branes preserve the same supersymmetry, i.e., V', $\hat{E}'$, $\tilde{\theta}$, are replaced by V, $\hat{E}$, θ, respectively, in (2.16). Of course the brane scalars stay massless to all orders in perturbation theory in this case. The relevant diagrams contributing to the mass of the brane scalars are shown in fig. 2. Supersymmetry guarantees that the sum of all these diagrams vanishes in the limit of vanishing external momenta. This has been explicitly

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[7]We do not address the question of how this vev arises dynamically. Here, we just assume that there exists a mechanism to cancel the cosmological constant.
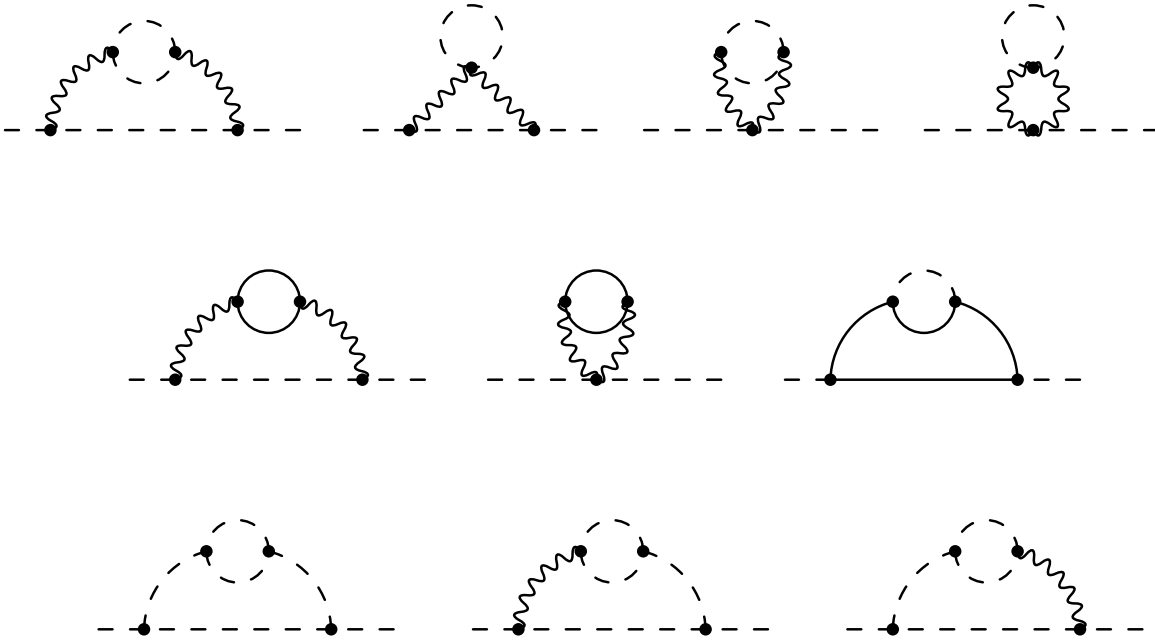
Figure 2: Two-loop Feynman diagrams contributing to the self-energy of the brane scalars in the case where both branes preserve the same supersymmetry. Dashed lines are scalars, plain lines are fermions and wavy lines are vectors. The lines in the lower level of each diagram correspond to fields from the first brane, the lines going vertically correspond to bulk fields and the loop in the upper level contains fields from the second brane. If the two branes preserve different supersymmetries, the last diagram in the second row is absent while all other diagrams remain unchanged.

verified in [12].

Let us now draw the two-loop Feynman diagrams contributing to brane scalar masses in the case of our toy model described by the Lagrangian (2.16). Interestingly, we find exactly the same diagrams as in fig. 2 except for the last diagram in the second row, which is absent because λ_1 does not couple to the fields on the second brane.[8]

Let us compare in more detail the two scenarios: (i) both branes preserve the same supersymmetry, (ii) they preserve different supersymmetries. The bulk vector fields couple to brane fields in exactly the same way in both cases. The coupling of the bulk scalar to the brane fields only differs by a sign in the two cases, as can be seen from (B.7). One can verify that the last two diagrams in fig. 2 vanish identically [12]. Thus, we find that the analytic expressions for all the Feynman diagrams in fig. 2 except for the diagram involving λ_1 (the last diagram in the second row) are independent of whether both branes preserve the same or a different supersymmetry. This observation, together with the fact that the sum of all

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[8]There are several additional two-loop diagrams involving the goldstino. But they do not contain fields from the second brane. As a consequence of the supersymmetry preserved on the first brane, their contribution to the scalar mass cancels.
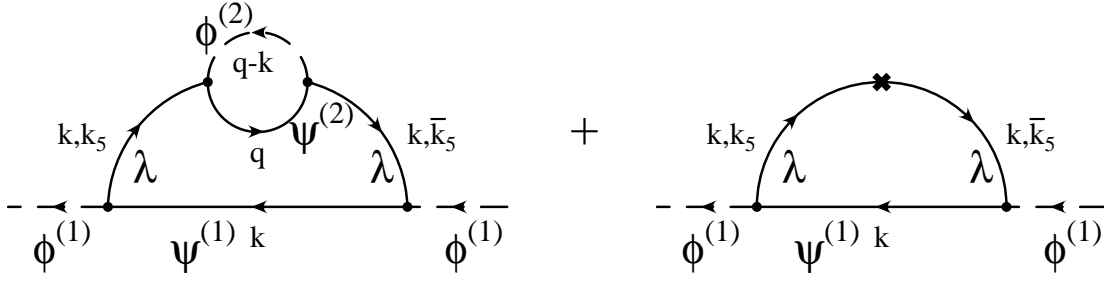
Figure 3: The leading order Feynman diagram giving rise to brane scalar masses. This diagram does not really exist in pseudo-supersymmetry but rather represents a shortcut to compute the sum of the nine two-loop diagrams that do exist. The counterterm is fixed by the condition that the one-loop corrected bulk gauge coupling constant receives no contributions from brane fields at the UV cutoff scale m_S.

diagrams cancels in the limit of vanishing external momenta in the first scenario, enables us to compute the sum of the nine diagrams present in the second scenario by just computing the single diagram diagram involving λ_1, which, of course, only exists in the first scenario. We conclude that the computation of the single diagram involving λ_1 in the case where both branes preserve the same supersymmetry yields the two-loop contribution to minus the mass squared of the brane scalars in pseudo-supersymmetry.

To do the calculation, we canonically normalize the bulk vector fields according to the tree-level four-dimensional gauge coupling constant $g_{(4)} = g_{(5)}(2\pi R)^{-1/2}$,

$$ A_m \;\to\; g_{(4)}\, A_m, \quad \lambda_i \;\to\; g_{(4)}, \lambda_i, \quad \phi \;\to\; g_{(4)}\, \phi. \tag{4.1}$$

This yields the component Lagrangian (B.7). Then, we transform the terms in this Lagrangian to the conventions where the Minkowski metric is $\eta_{mn} = \mathrm{diag}(1, -1, -1, -1)$. The Feynman rules for our toy model are shown in appendix C. We assume that the fields on the first (second) brane transform according to a representation r (r') of the bulk gauge group. It turns out that the loop containing the bulk fields in the Feynman diagram fig. 3 is regularized by the finite distance between the two branes. The loop of brane fields gives rise to a logarithmic UV divergence which is eliminated by adding an appropriate counterterm. This counterterm cannot be freely chosen but is already fixed by the condition imposed on the bulk gauge coupling. We required that the the contribution from the brane fields to the one-loop correction to the bulk gauge coupling constant should vanish at the UV cutoff scale m_S. This resulted in the brane contribution to bulk coupling constant of the form (3.3) but it also fixes the precise form of the counterterm.

To determine the precise expression of the one-loop counterterm, let us compute the one-loop correction to the bulk gaugino self-energy from fields on the second brane, fig. 4. The amplitude for the amputated one-loop diagram is

$$ -i\,\Sigma_\lambda(k) \;=\; (g_{(4)}\sqrt{2})^2\, d^2(r') \int \frac{d^4q}{(2\pi)^4} \frac{iq^m \sigma_m}{q^2} \frac{i}{(q-k)^2}, \tag{4.2}$$

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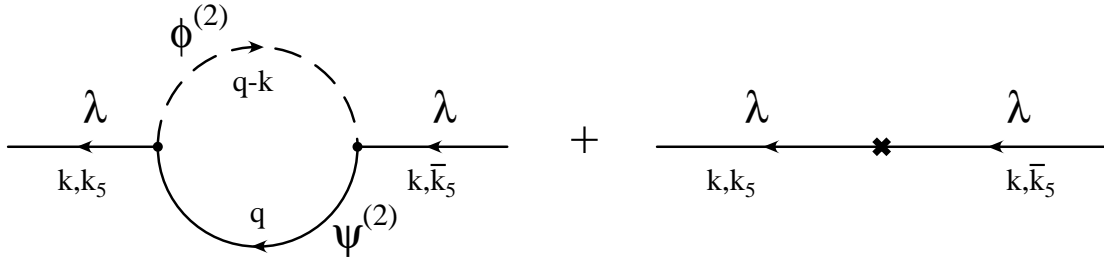
Figure 4: One-loop correction to the bulk gaugino self-energy from fields on the second brane

where $d^2(r')\,\delta^{ab} = \text{tr}(t^a_{r'} t^b_{r'})$. To evaluate the q-integral, we use dimensional regularization. With the help of eqs. (C.9), (C.11) and (C.12), we find

$$- i\,\Sigma_\lambda(k) \;=\; -i\,g^2_{(4)}\, d^2(r')\, k^m \sigma_m\, \frac{k^{-\epsilon}(4\pi)^{\epsilon/2}\Gamma(\epsilon/2)}{(4\pi)^2} \int_0^1 dx\,[x(1-x)]^{-\epsilon/2}, \qquad (4.3)$$

where $\epsilon = 4 - d$. The counterterm which cancels this contribution at an energy scale $k = m_S$ is given by

$$+ i\,g^2_{(4)}\, d^2(r')\, k^m \sigma_m\, \frac{m_S^{-\epsilon}(4\pi)^{\epsilon/2}\Gamma(\epsilon/2)}{(4\pi)^2} \int_0^1 dx\,[x(1-x)]^{-\epsilon/2}. \qquad (4.4)$$

For the renormalized self-energy, this yields

$$- i\,\Sigma_{\lambda,\mathrm{R}} \;=\; \lim_{\epsilon \to 0} (-i\,\Sigma_\lambda + \text{counterterm}) \;=\; i\,k^m \sigma_m\, \frac{g^2_{(4)}\, d^2(r')}{8\pi^2} \ln\!\left(\frac{k}{m_S}\right). \qquad (4.5)$$

This confirms our result for the running coupling, eq. (3.3), of the previous section. Replacing $\lambda_2 \to \lambda_2/g_{(4)}$ — i.e., dividing $-i\,\Sigma_{\lambda,\mathrm{R}}$ by $g^2_{(4)}$ — and using $b_2 = -d^2(r')$, (4.5) yields the expected correction to the kinetic terms of λ_2 as given in (3.4).

Let us now return to the computation of the mass squared of the scalars on the first brane arising through two-loop interactions with fields from the second brane. The precise expression for the Feynman diagram shown in fig. 3 is[9]

$$- i\,m^2 \;=\; (-1)(-1)(g_{(4)}\sqrt{2})^4\, C_2(r) d^2(r') \int_{k,q} \frac{i\,\text{tr}\,(ik^m\bar\sigma_m ik^n\sigma_n iq^p\bar\sigma_p ik^q\sigma_q)}{(q-k)^2 k^2 (k^2 - (k_5)^2) q^2 (k^2 - (\bar k_5)^2)}$$

$$+ \,(\text{counterterm contribution}), \qquad (4.6)$$

where the first (-1) factor is due to the fact that the diagram we are computing is the negative of the sum of the nine two-loop diagrams that contribute to scalar masses and the second (-1) factor is due to the fermion loop. We have written the 5D momenta of the bulk fields in terms of their 4D components (k^m, k_5). Note that k_5 need not be conserved in a

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[9]A priori, there is an additional contribution with $-i\,k_5\bar k_5\,\text{tr}(ik^m\bar\sigma_m iq^p\sigma_p)$ in the denominator. But this contribution vanishes after performing the sum over Kaluza-Klein modes because it is odd in k_5, $\bar k_5$.

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brane-bulk interaction since the branes break translational invariance in the fifth direction. The integration measure is given by[10]

$$\int_{k,q} = \sum_{k_5=\frac{n}{R}} 2(-1)^n \sum_{k_5=\frac{\bar{n}}{R}} 2(-1)^{\bar{n}} \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4q}{(2\pi)^4}. \tag{4.7}$$

To compute the sum over the Kaluza-Klein modes, we first perform a Wick rotation and then transform the infinite sum into a contour integral in the complex k_5 plane [12, 21]. One finds

$$\sum_{k_5=\frac{n}{R}} \frac{(-1)^n}{k^2 + (k_5)^2} f(k) = \oint \frac{dk_5}{2\pi i} \frac{\pi R}{\sin(\pi R k_5)} \frac{1}{k^2 + (k_5)^2} f(k) = -\frac{\pi R}{k \sinh(\pi R k)} f(k), \tag{4.8}$$

where $f(k)$ is an analytic function with no poles for finite k. The denominator of (4.6) can be evaluated to give

$$i \operatorname{tr}\left(ik^m \bar{\sigma}_m ik^n \sigma_n iq^p \bar{\sigma}_p ik^q \sigma_q\right) = 2i\, k^2\, k \cdot q. \tag{4.9}$$

To simplify the q-integral, we use the identity (C.10)

$$\int d^4q \frac{k \cdot q}{q^2(q-k)^2} = \frac{1}{2}k^2 \int d^4q \frac{1}{q^2(q-k)^2}. \tag{4.10}$$

Putting these results together, we find

$$-i\, m^2 = \frac{4i\, g_{(4)}^4}{8\pi^2} C_2(r) d^2(r')(2\pi R)^2 \int_0^\infty dk \frac{i\, k^3}{\sinh(\pi R k)^2} \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2(q-k)^2}$$

$$+ \text{(counterterm contrib.)}, \tag{4.11}$$

where the additional factor i in the k-integral is from the Wick rotation.

The q-integral can be computed in the dimensional regularization scheme using eqs. (C.8) and (C.11). Defining $\epsilon = 4-d$ and adding the explicit expression for the counterterm, which is easily derived from the Feynman diagram in fig. 3 and the result (4.4), the expression for the scalar mass squared reads

$$-i\, m^2 = \frac{-i\, g_{(4)}^4}{2\pi^2 (4\pi)^2} C_2(r) d^2(r')(2\pi R)^2 \int_0^\infty dk \frac{k^{d-1} - k^3 m_S^{-\epsilon}}{\sinh(\pi R k)^2}$$

$$\int_0^1 dx\, [x(1-x)]^{-\epsilon/2} (4\pi)^{\epsilon/2} \Gamma(\epsilon/2). \tag{4.12}$$

Finally, using the identity

$$\int_0^\infty dx \frac{x^{d-1}}{\sinh(ax)^2} = \frac{4}{(2a)^d} \Gamma(d)\, \zeta(d-1) \tag{4.13}$$

[10]See [12] for a derivation and note that the additional factors of $1/(2\pi R)$ which multiply each of the two Kaluza-Klein sums in [12] are absent in (4.7) because we prefer to write all interactions in terms of the 4D gauge coupling $g_{(4)}$. The factor $2(-1)^n$ is from the the two ways of propagating from one brane to the other, $e^{ik_5(x^5-y^5)} + e^{ik_5(x^5+y^5)}$, where $x^5 = 0$, $y^5 = \pi R$ are the positions of the two branes.

and taking the limit $\epsilon \to 0$, we find

$$
\begin{aligned}
m^2 &= \frac{g_{(4)}^4}{8\pi^4} C_2(r) d^2(r') \frac{\Gamma(4)\zeta(3)}{(2\pi R)^2} \left(2\gamma - 2\frac{\zeta'(3)}{\zeta(3)} - \frac{11}{3} + 2\ln(2\pi R m_{\mathrm{S}}) \right) \\
&= \left(\frac{g_{(4)}^2}{4\pi} \right)^2 C_2(r) d^2(r') \frac{24\,\zeta(3)}{\pi^2\,(2\pi R)^2} \left(\ln(2\pi R m_{\mathrm{S}}) - 1.091\ldots \right), \qquad (4.14)
\end{aligned}
$$

where γ is the Euler constant and $\zeta'(x) = \frac{d}{dx}\zeta(x)$. For large compactification radii, this yields a positive mass squared for the brane scalars. For R smaller than a critical radius $R_\star \approx 0.474\, m_{\mathrm{S}}^{-1}$, the potential for the brane scalars seems to develop a tachyonic instability. But this is just an artefact of our renormalization condition. The condition we chose only makes sense if many Kaluza-Klein modes have masses below m_{S}, i.e., $R \gg m_{\mathrm{S}}^{-1}$.

It is interesting that the scalar masses depend only logarithmically on the cutoff. The expected quadratic divergence is regulated by the finite brane distance. A very similar effect has first been found in an $S^1/(\mathbb{Z}_2 \times \mathbb{Z}_2')$ orbifold model [6].[11]

In string theory, the UV cutoff m_{S} is naturally identified with the string scale. Indeed, it has been shown in [14] that in string models where supersymmetry is broken at the scale m_{S} in the effective world-volume theory on D3-branes, m_{S} is related to the D3-brane tension τ_3 by $\frac{1}{2}m_{\mathrm{S}}^4 = \tau_3 \equiv M_{\mathrm{str}}^4/(4\pi^2 g_{\mathrm{str}})$, where g_{str} is the string coupling. The result (4.14) for the scalar masses, implies that the mass splittings in pseudo-supersymmetric D-brane models are mainly determined by the compactification scale and depend only weakly on the string scale. In particular, it this could open the possibility to have models of intersecting D-branes where supersymmetry is broken at a high scale but mass splittings are only of the order of the much smaller compactification scale.[12] However, it is not clear whether the 5D gauge theory is consistent at very high energy scales. The scalar masses as a function of the compactification scale M_{c} for some specific choices the cutoff scale m_{S} are shown in table 1 at the end of the next section.

5   Casimir energy

Since supersymmetry is broken, the vacuum energy receives non-vanishing contributions from the quantum fluctuations of the various fields. The dependence of this vacuum energy on the distance between the two branes leads to a Casimir force. It is very interesting to compute the Casimir energy because we would like to know (i) whether the Casimir force is attractive or repulsive and (ii) how the vacuum energy scales with the cutoff m_{S}.

The computation of the Casimir energy is very similar to the computation of the mass squared of the brane scalars. It is easy to convince oneself that Feynman diagrams with-

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[11]The authors of [8] have shown that for an Abelian gauge group quadratic divergences arise in this type of orbifold models through radiatively generated Fayet-Iliopoulos terms. However, the analysis of [8] does not apply to pseudo-supersymmetry since there is no orbifold projection involved in the supersymmetry breaking mechanism.

[12]Here we assume an asymmetric compactification with one large and five small compact dimensions, which are only slightly larger than the string length and whose effect is neglected.
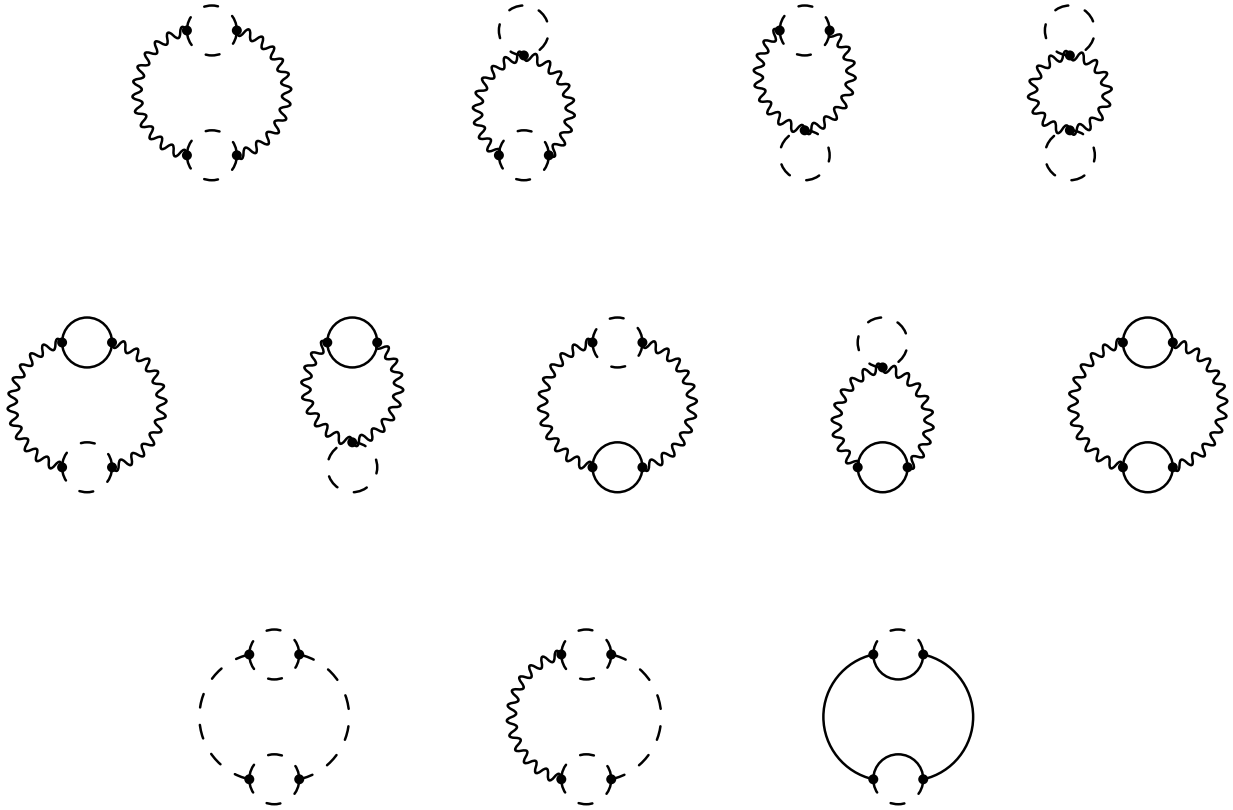
Figure 5: Three-loop Feynman diagrams contributing to the vacuum energy in the case where both branes preserve the same supersymmetry. Dashed lines are scalars, plain lines are fermions and wavy lines are vectors. The lines in the lower level of each diagram correspond to fields from the first brane, the lines going vertically correspond to bulk fields and the loop in the upper level contains fields from the second brane. If the two branes preserve different supersymmetries, the last diagram in the last row is absent while all other diagrams remain unchanged.

out external legs involving fields from both branes are only possible at three-loop. For all diagrams that contain only fields from one brane and/or the bulk, the supersymmetric non-renormalization theorems still apply; as a consequence their contribution to the vacuum energy cancels.

Like in the case of the two-loop correction to the brane scalar mass, it is useful to first consider the scenario where both branes preserve the same supersymmetry. All three-loop diagrams without external legs involving fields from both branes are shown in fig. 5.

Again, one finds that only the single diagram involving λ_1 is absent if both branes preserve different supersymmetries. A reasoning very similar to the one of the previous section leads to the conclusion that the sum of the eleven diagrams present in pseudo-supersymmetry can be computed by just computing the single diagram diagram involving λ_1, which, of course, only exists in the scenario where both branes preserve the same supersymmetry.
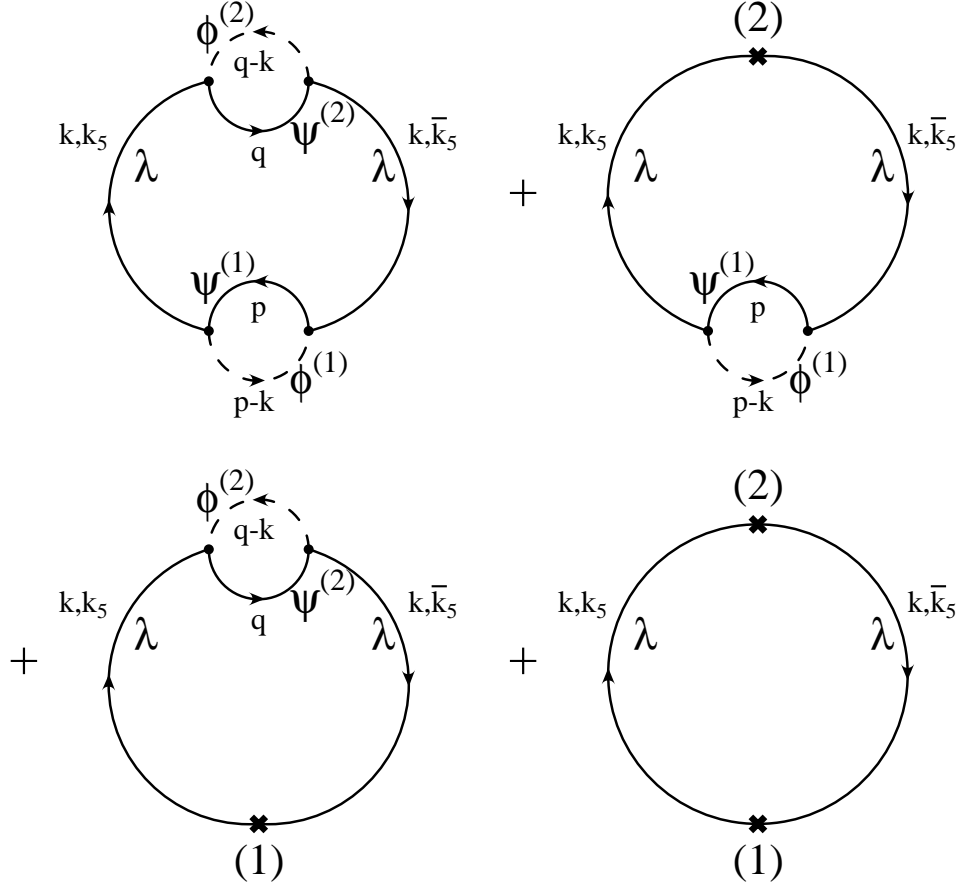
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Figure 6: The leading order Feynman diagram giving rise to a vacuum energy. This diagram does not really exist in pseudo-supersymmetry but rather represents a shortcut to compute the sum of the eleven three-loop diagrams that do exist. The counterterms are fixed by the condition that the one-loop corrected bulk gauge coupling constant receives no contributions from brane fields at the UV cutoff scale m_{S}.

In the same way as in the brane scalar mass computation, the loop-integration over the momenta of the bulk fields in the Feynman diagram of fig. 6 is regularized by the finite distance between the two branes. The loops of brane fields give rise to logarithmic UV divergences which are eliminated by adding appropriate counterterms. Again, the counterterms are dictated by renormalization condition imposed on the bulk gauge coupling.

The precise expression for the Feynman diagram shown in fig. 6 is

$$
\begin{aligned}
i\,E \;=\;\; & (-1)(-1)(g_{(4)}\sqrt{2})^4\,d^2(r)d^2(r')\dim(G) \\[4pt]
& \int_{k,q,p}\frac{(i)^2\,\mathrm{tr}\,(ip^m\bar{\sigma}_m ik^n\sigma_n iq^p\bar{\sigma}_p ik^q\sigma_q)}{(q-k)^2(p-k)^2p^2(k^2-(k_5)^2)q^2(k^2-(\bar{k}_5)^2)} \\[4pt]
& +\ (\text{counterterm contributions}),
\end{aligned}
\tag{5.1}
$$

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where the first (-1) factor is due to the fact that the diagram we are computing is the negative of the sum of the eleven three-loop diagrams that contribute to the vacuum energy and the second (-1) factor is due to the fermion loop. We have written the 5D momenta of the bulk fields in terms of their 4D components (k^m, k_5). The integration measure is given by

$$\int_{k,q,p} = \sum_{k_5=\frac{n}{R}} 2(-1)^n \sum_{k_5=\frac{\bar{n}}{R}} 2(-1)^{\bar{n}} \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4q}{(2\pi)^4} \int \frac{d^4p}{(2\pi)^4}. \tag{5.2}$$

To compute the sum over the Kaluza-Klein modes, we first perform a Wick rotation and then transform the infinite sum into a contour integral in the complex k_5 plane [12, 21], as we did in the previous section, eq. (4.8). The denominator of (5.1) can be evaluated to give

$$(i)^2 \, \mathrm{tr} \, (ip^m \bar{\sigma}_m ik^n \sigma_n iq^p \bar{\sigma}_p ik^q \sigma_q) = -2 \, (p \cdot k) \, (q \cdot k) + 4 \, (p \cdot q) \, k^2. \tag{5.3}$$

The explicit expressions for the counterterms are easily derived from their Feynman diagrams in fig. 6 and the result (4.4). Putting all this together and using the formulae of appendix C, we find

$$i \, E = \frac{i \, g_{(4)}^4}{4\pi^2 \, (4\pi)^4} \, d^2(r)d^2(r') \, \dim(G)(2\pi R)^2 \int_0^\infty dk \frac{k^{5-2\epsilon} - 2 \, k^{5-\epsilon} m_{\mathrm{S}}^{-\epsilon} + k^5 \, m_{\mathrm{S}}^{-2\epsilon}}{\sinh(\pi Rk)^2}$$

$$\left[\int_0^1 dx \, [x(1-x)]^{-\epsilon/2} \, (4\pi)^{\epsilon/2} \, \Gamma(\epsilon/2) \right]^2. \tag{5.4}$$

Finally, using the identity (4.13) and taking the limit $\epsilon \to 0$, we find the vacuum energy density

$$E = \frac{4 \, g_{(4)}^4}{\pi^2 \, (4\pi)^4} \, d^2(r)d^2(r') \, \dim(G) \frac{\Gamma(6)\zeta(5)}{(2\pi R)^4} \left(\left(\ln(2\pi R m_{\mathrm{S}}) + A \right)^2 + B \right), \tag{5.5}$$

where

$$A = \gamma - \frac{\zeta'(5)}{\zeta(5)} - \frac{274}{120} \approx -1.679$$

$$B = \frac{\pi^2}{6} + \frac{15}{4} - \left(\frac{137}{60} \right)^2 - \left(\frac{\zeta'(5)}{\zeta(5)} \right)^2 + \frac{\zeta''(5)}{\zeta(5)} \approx 0.203 \tag{5.6}$$

For large compactification radii, where our calculation is valid, this yields a repulsive Casimir force. This result could have been expected from the fact that the brane scalar masses decrease with increasing radius; as a consequence the brane contribution to the vacuum energy decreases with increasing radius. For infinite brane separation, one recovers the supersymmetric configuration with vanishing vacuum energy and mass splittings.

The non-vanishing vacuum energy for finite brane separation has two important implications. First, when coupled to supergravity, it needs to be cancelled by a non-vanishing expectation value of the auxiliary fields in the gravity multiplet (see, e.g., [23]). Dynamically, this vev may arise from appropriate additional terms in the Lagrangian; but here we content

ourselves to assume that such a vev does appear. We expect that it implies gravitino masses of the form

$$m_{3/2} = \frac{\sqrt{E}}{\sqrt{3}\,M_{\mathrm{Pl}}} = \frac{g_{(4)}^2}{4\pi}\frac{M_{\mathrm{c}}^2}{M_{\mathrm{Pl}}}\frac{\sqrt{d^2(r)d^2(r')\dim(G)\,10\,\zeta(5)}}{\pi^2}\sqrt{\left(\ln\left(\frac{m_{\mathrm{S}}}{M_{\mathrm{c}}}\right)+A\right)^2+B} \quad (5.7)$$

The first equality looks very similar to the standard formula for the gravitino mass in spontaneously broken $\mathcal{N}_4 = 1$, $D = 4$ supergravity. The two differences are: (i) the supersymmetry breaking scale is set by the radiatively generated vacuum energy density, not by some non-vanishing F-term, (ii) there are two bulk gravitinos. We expect that the general formula (5.7) is still valid, yielding the same non-vanishing mass for both gravitinos. The result (5.7) is similar to the situation in $D = 4$ gauge-mediated scenarios [24] in that the gravitino masses are suppressed with respect to the scalar masses. This is not surprising since supersymmetry breaking is transmitted via the bulk gauge fields. However, in pseudo-supersymmetry, the scalar soft masses depend only on the compactification scale whereas in gauge-mediated scenarios, the scalar masses depend on ratio of the supersymmetry breaking vev F to the messenger mass scale M.

Second, if the branes are D-branes, then the Casimir energy may be important to obtain a stable configuration. In non-supersymmetric D-brane models, the RR-force is generically attractive between the branes that preserve different supersymmetries. The repulsive Casimir force could possibly balance the RR-force. To understand this effect in a little more detail, consider two D3-branes of opposite RR-charges μ_3 and $-\mu_3$. Assume that five of the six internal dimensions are compactified at the string scale M_{str} and one dimension is compactified at M_{c}. At energies well below M_{str} but above M_{c} the D3-branes are of codimension 1. The RR-force between them is constant and attractive. The force per unit brane-volume is given by $\kappa_{10}^2\mu_3^2 M_{\mathrm{str}}^5 = \pi M_{\mathrm{str}}^5$, since $\mu_3 = \sqrt{\pi}\kappa_{10}$. Thus, the configuration is stabilized at

$$\frac{M_{\mathrm{str}}}{M_{\mathrm{c}}} \approx \left(\left(\frac{g_{(4)}^2}{4\pi}\right)^2 d^2(r)d^2(r')\dim(G)\frac{120\,\zeta(5)}{\pi^4}\ln\left(\frac{M_{\mathrm{str}}}{M_{\mathrm{c}}}\right)^2\right)^{1/5}. \quad (5.8)$$

A large ratio $M_{\mathrm{str}}/M_{\mathrm{c}}$ is only obtained for very large numbers of brane fields and/or bulk gauge fields. Thus, in our simple toy model, the Casimir force seems to be too weak to stabilize the system at large radii where our approximation is valid. Of course, a more careful treatment needs to take into account gravity and, in particular, address the question how the cosmological constant is cancelled. Here, we only wanted to show that the Casimir effect might give a relevant contribution.

It is interesting that the vacuum energy depends only logarithmically on the cutoff. But it depends quartically on the compactification scale which has to be chosen close to the weak scale to avoid a hierarchy problem for scalar masses. Thus, there seems to be no natural mechanism to suppress the cosmological constant in this model.

In table 1, we give the explicit values of scalar masses and Casimir energy density for some specific choices of the cutoff scale m_{S}.

$\frac{m_{\mathrm{S}}}{M_{\mathrm{c}}}$	10^2	10^5	10^8	10^{11}	10^{14}	10^{17}
m_{scalar}	3.205	5.519	7.117	8.417	9.541	10.55
$E^{1/4}$	1.294	2.359	3.076	3.656	4.156	4.601

Table 1: Numeric values for brane scalar masses in units of $g_{(4)}^2/(4\pi)\sqrt{C_2(r)d^2(r')}\,M_{\mathrm{c}}$ and the Casimir energy density $E^{1/4}$ in units of $g_{(4)}/\sqrt{4\pi}\,(d^2(r)d^2(r')\dim(G))^{1/4}\,M_{\mathrm{c}}$.

6 Conclusions and outlook

We have computed the scalar soft masses and the Casimir energy in a 5D brane-world model where two branes preserve different halves of the bulk supersymmetry. In this scenario, supersymmetry breaking involves no orbifold projection. The fields on the world-volume of the branes simply lack the superpartners corresponding to the extended bulk supersymmetry. In our toy model, a 5D vector multiplet propagates in the bulk and 4D chiral multiplets charged under the bulk gauge symmetry are confined to the branes. Supersymmetry is broken by the fact that the chiral multiplets from the two branes couple to different bulk gauginos.

At one-loop, the gauge kinetic terms acquire contributions which are localized on the branes. These contributions show logarithmic divergences in the ultraviolet, which can be eliminated through standard 4D renormalization. Brane scalar masses arise at the two-loop level. It is instructive to see in detail how radiative corrections generate a positive mass-squared for the brane scalars while all tree-level masses are vanishing. Interestingly, the masses are finite, once the brane contribution to the bulk gauge coupling constant has been renormalized by adding the appropriate counterterm. With our choice of the renormalization condition and for large brane separations πR, we find $m^2 \propto (2\pi R)^{-2}(\ln(2\pi R\,m_{\mathrm{S}}) - 1.1)$, where m_{S} is an ultraviolet cutoff. Luckily, the three-loop diagrams that contribute to the vacuum energy density can also be calculated without too much effort. The result, $E \propto (2\pi R)^{-4}\big((\ln(2\pi R\,m_{\mathrm{S}}) - 1.7)^2 + 0.2\big)$, shows that the Casimir force is repulsive at large distances.

The main motivation for this work was the fact that there are D-brane constructions of the standard model [4] where supersymmetry is broken in the way discussed in this article. It would be very interesting to apply the techniques and results of this work to an explicit intersecting brane construction of the standard model in order to derive precise predictions for the soft masses. However, our methods are only valid if the string scale is at least a few orders of magnitude larger than the compactification scale. In that case, massive string excitations can be neglected.

The computation of the soft masses enables us to compare supersymmetry breaking in intersecting brane models (with a high enough string scale for our approximations to be valid) to standard 4D gauge-mediation models. In the former, the mass splittings only depend on the distance πR between the intersections where the chiral multiplets are localized, up to

logarithmic corrections involving the string scale. The product of the compactification scale $M_c = (2\pi R)^{-1}$ and the gauge coupling constant g is the order parameter of supersymmetry breaking. In the limit $g\,M_c \to 0$, supersymmetry is restored; for $M_c > m_S$, string effects become relevant and the field theory approximation breaks down. Models of gauge-mediation, in contrast, have two mass parameters: the vev of an auxiliary field F, which is the order parameter of supersymmetry breaking, and the mass scale M of the messenger sector. Scalar masses squared are of order $g^4\,F^2/M^2$, as compared to $g^4\,M_c^2$ in pseudo-supersymmetry. Gravitino masses are of comparable order in both scenarios, $F/M_{\rm Pl}$ in gauge-mediation and $g^2\,M_c^2/M_{\rm Pl}$ in pseudo-supersymmetry. In gauge-mediated models, there is a clear asymmetry between the hidden sector, where supersymmetry is broken, and the visible sector that learns about this breaking only from interactions with the messenger sector. In pseudo-supersymmetry, hidden and visible sector appear on equal footing. Each sector sees its own supersymmetry and learns from interactions with bulk fields that the other sector does not preserve the same supersymmetry. Actually, realistic intersecting brane constructions do not have a hidden sector in the usual sense. The supersymmetry breaking mechanism of these models only requires the known fields of the standard model and their superpartners.

The authors of [25] have developed a superfield formalism to determine supersymmetry breaking effects. In this formalism, soft masses arise through spurion superfields whose auxiliary components acquire vacuum expectation values $F \ll M^2$. Although this method is not applicable to pseudo-supersymmetry since the dimensionless small number F/M^2 has no obvious counterpart in the scenario discussed in this article, it is very interesting to see whether there is some effective 4D superfield formalism that captures the radiative generation of soft masses in pseudo-supersymmetry.

Another direction for future work is the analysis of brane constructions containing branes and antibranes. In those cases, only the gravity multiplet and the moduli fields propagate in the bulk. A simple toy model consists of an $\mathcal{N} = 2$ hyper multiplet in the bulk coupling with gravitational strength to $\mathcal{N} = 1$ chiral multiplets on the branes. The computation of the soft masses in that case is probably more involved because gravity effects have to be included. However, we can get a rough estimate of the soft terms by replacing the gauge coupling $g_{(4)}$ with the gravitational coupling $M_c/M_{\rm Pl}$ in the expression for the brane scalar mass squared, eq. (4.14). Thus, we expect brane scalar masses of order $m^2 \propto M_c^6/M_{\rm Pl}^4$. Requiring the soft masses to be of the order of 1 TeV, implies $M_c \sim 10^{13}$ GeV. This favors a high string scale around the GUT scale.

A Notation and conventions

We use the metric $\eta_{MN} = \mathrm{diag}(-1, 1, 1, 1, 1)$ and a set of γ-matrices that satisfies

$$\{\gamma^M, \gamma^N\} \;=\; -2\,\eta^{MN}. \tag{A.1}$$

Specifically, we take

$$\gamma^m \;=\; \begin{pmatrix} 0 & \sigma^m \\ \bar\sigma^m & 0 \end{pmatrix} \quad \text{for } m = 0, \ldots 3 \quad \text{and} \quad \gamma^5 \;=\; \begin{pmatrix} -i\,\mathbb{I}_2 & 0 \\ 0 & i\,\mathbb{I}_2 \end{pmatrix}, \tag{A.2}$$

where $\sigma^m = (-\mathbb{I}_2, \sigma^1, \sigma^2, \sigma^3)$, $\bar\sigma^m = (-\mathbb{I}_2, -\sigma^1, -\sigma^2, -\sigma^3)$ and

$$\sigma^1 \;=\; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 \;=\; \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 \;=\; \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{A.3}$$

A 5D symplectic Majorana spinor Ψ^i satisfies the reality condition

$$\Psi^i \;=\; i\gamma^2\, \epsilon^{ij} \Psi_j^*. \tag{A.4}$$

It can be decomposed into 4D Weyl spinors ψ_i as

$$\Psi^i \;=\; \begin{pmatrix} \psi^i \\ -\epsilon^{ij}\,\bar\psi_j \end{pmatrix}, \qquad \overline{\Psi}_i \;=\; \left(\epsilon_{ij}\psi^j, \; \bar\psi_i \right), \tag{A.5}$$

where $\epsilon^{12} = -\epsilon_{12} = 1$. For 4D Weyl spinors, we use the conventions of Wess and Bagger [16]. From (A.5), one finds

$$\overline{\Psi}_i \Psi^i \;=\; \epsilon^{ij} \left(\psi_i \psi_j - \bar\psi_i \bar\psi_j \right), \tag{A.6}$$

$$\overline{\Psi}_i \gamma^M \mathcal{D}_M \Psi^i \;=\; \psi_i \sigma^m \mathcal{D}_m \bar\psi_i + \bar\psi_i \bar\sigma^m \mathcal{D}_m \psi_i - i\epsilon^{ij} \left(\psi_i \mathcal{D}_5 \psi_j + \bar\psi_i \mathcal{D}_5 \bar\psi_j \right).$$

B Component field Lagrangians

The superfield Lagrangians (2.7), (2.13), (2.15) have the following expansions in component fields.

$$\begin{aligned}
\mathcal{L}_{\mathrm{bulk}} \;=\; & \frac{1}{g_{(5)}^2} \, \mathrm{tr} \left(-\frac{1}{4} F^{mn} F_{mn} - \mathcal{D}^m \phi^\dagger \mathcal{D}_m \phi - \frac{i}{2} \lambda_i \sigma^m \mathcal{D}_m \bar\lambda_i - \frac{i}{2} \bar\lambda_i \bar\sigma^m \mathcal{D}_m \lambda_i + F^\dagger F \right. \\
& \left. -\frac{i}{\sqrt{2}} \epsilon^{ij} \left(\lambda_i [\phi^\dagger, \lambda_j] + \bar\lambda_i [\phi, \bar\lambda_j] \right) - \frac{1}{2} \epsilon^{ij} \left(\lambda_i \partial_5 \lambda_j + \bar\lambda_i \partial_5 \bar\lambda_j \right) + [\phi, \phi^\dagger]D + \frac{1}{2} D^2 \right. \\
& \left. -\frac{1}{2} \partial_5 A^m \partial_5 A_m + \frac{1}{\sqrt{2}} \partial_5 A^m \mathcal{D}_m \left(\phi + \phi^\dagger \right) - \frac{i}{\sqrt{2}} \left(\phi - \phi^\dagger \right) \partial_5 D \right) \tag{B.1}
\end{aligned}$$

$$\mathcal{L}_{(1)} = \delta(x^5)\bigg(-\hat{\mathcal{D}}^m\phi^{(1)\dagger}\hat{\mathcal{D}}_m\phi^{(1)} - \frac{i}{2}\bar{\psi}^{(1)}\bar{\sigma}^m\hat{\mathcal{D}}_m\psi^{(1)} - \frac{i}{2}\psi^{(1)}\sigma^m\hat{\mathcal{D}}_m\bar{\psi}^{(1)} + F^{(1)\dagger}F^{(1)}$$

$$+i\sqrt{2}\left(\phi^{(1)\dagger}\hat{\lambda}_1\psi^{(1)} - \bar{\psi}^{(1)}\hat{\bar{\lambda}}_1\phi^{(1)}\right) + \phi^{(1)\dagger}\hat{D}\phi^{(1)}\bigg) \tag{B.2}$$

$$\mathcal{L}_{(2)} = \delta(x^5 - l)\bigg(-\hat{\mathcal{D}}'^m\phi^{(2)\dagger}\hat{\mathcal{D}}'_m\phi^{(2)} - \frac{i}{2}\bar{\psi}^{(2)}\bar{\sigma}^m\hat{\mathcal{D}}'_m\psi^{(2)} - \frac{i}{2}\psi^{(2)}\sigma^m\hat{\mathcal{D}}'_m\bar{\psi}^{(2)} + F^{(2)\dagger}F^{(2)}$$

$$+i\sqrt{2}\left(\phi^{(2)\dagger}\hat{\lambda}_2\psi^{(2)} - \bar{\psi}^{(2)}\hat{\bar{\lambda}}_2\phi^{(2)}\right) - \phi^{(2)\dagger}\hat{D}'\phi^{(2)}\bigg) \tag{B.3}$$

where $\hat{\mathcal{D}}_m = \partial_m + i\hat{A}_m$, $\hat{\mathcal{D}}'_m = \partial_m + i\hat{A}'_m$ and

$$\hat{A}_m = A_m + m_S^{-2}\left(-i\,\lambda_g\sigma_m\bar{\lambda}_2 + i\,\bar{\lambda}_g\bar{\sigma}_m\lambda_2\right) + \mathcal{O}\left(m_S^{-4}\right), \tag{B.4}$$

$$\hat{\lambda}_1 = \lambda_1 + m_S^{-2}\left(\sqrt{2}\,\lambda_g F^\dagger - i\sqrt{2}\,\sigma^m\bar{\lambda}_g\partial_m\phi - \lambda_2 D_g + i\,\sigma^{mn}\lambda_2 F_{g\,mn}\right) + \mathcal{O}\left(m_S^{-4}\right),$$

$$\hat{D} = D + m_S^{-2}\left(-\lambda_g\sigma^m\partial_m\bar{\lambda}_2 - \partial_m\lambda_2\sigma^m\bar{\lambda}_g + \lambda_2\sigma^m\partial_m\bar{\lambda}_g + \partial_m\lambda_g\sigma^m\bar{\lambda}_2 + \frac{i}{\sqrt{2}}D_g(F - F^\dagger)\right)$$

$$+ \mathcal{O}\left(m_S^{-4}\right),$$

$$\hat{A}'_m = A_m + m_S^{-2}\left(-i\,\lambda_g\sigma_m\bar{\lambda}_1 + i\,\bar{\lambda}_g\bar{\sigma}_m\lambda_1\right) + \mathcal{O}\left(m_S^{-4}\right), \tag{B.5}$$

$$\hat{\lambda}_2 = \lambda_2 + m_S^{-2}\left(-\sqrt{2}\,\lambda_g F + i\sqrt{2}\,\sigma^m\bar{\lambda}_g\partial_m\phi - \lambda_1 D_g + i\,\sigma^{mn}\lambda_1 F_{g\,mn}\right) + \mathcal{O}\left(m_S^{-4}\right),$$

$$\hat{D}' = D + m_S^{-2}\left(\lambda_g\sigma^m\partial_m\bar{\lambda}_1 + \partial_m\lambda_1\sigma^m\bar{\lambda}_g - \lambda_1\sigma^m\partial_m\bar{\lambda}_g - \partial_m\lambda_g\sigma^m\bar{\lambda}_1 + \frac{i}{\sqrt{2}}D_g(F - F^\dagger)\right)$$

$$+ \mathcal{O}\left(m_S^{-4}\right).$$

To obtain these results, we assumed that the goldstino resides in an $\mathcal{N}_4 = 1$ Maxwell multiplet

$$\Lambda_{g\,\alpha} = \lambda_g + \frac{1}{2}\theta_\alpha D_g - \frac{i}{2}(\sigma^{mn}\theta)_\alpha F_{g\,mn} + i\,\theta\sigma^m\bar{\theta}\,\partial_m\lambda_{g\,\alpha} + \ldots \tag{B.6}$$

Note that the correctly normalized superpartner of the $U(1)$ gauge field strength $F_{g\,mn}$ is $\lambda'_g = 2i\,\lambda_g$. The normalization in (B.6) is such that $\delta_{\xi_2}\Lambda_g = m_S^2\,\xi_2 + \mathcal{O}(m_S^{-4})$. The factor $\frac{1}{2}$ in $\Lambda_{g\,\alpha} = \frac{1}{2}W_{g\,\alpha}$ is necessary for the non-linear action for $W_{g\,\alpha}$ to reproduce the Born-Infeld action for the $U(1)$ gauge field [13, 9].

Replacing $A_m, \lambda_i, D \to g_{(4)}A_m, g_{(4)}\lambda_i, g_{(4)}D$, integrating out the auxiliary fields D, F, $F^\dagger$, D_g, $F^{(i)}$, $F^{(i)\dagger}$ and using the relation $g_{(5)}^2 = 2\pi R\,g_{(4)}^2$ one finds

$$\mathcal{L} = \frac{1}{2\pi R}\,\text{tr}\left(-\frac{1}{4}F^{mn}F_{mn} - \mathcal{D}^m\phi^\dagger\mathcal{D}_m\phi - \frac{i}{2}\lambda_i\sigma^m\mathcal{D}_m\bar{\lambda}_i - \frac{i}{2}\bar{\lambda}_i\bar{\sigma}^m\mathcal{D}_m\lambda_i\right.$$

$$-\frac{ig_{(4)}}{\sqrt{2}}\,\epsilon^{ij}\left(\lambda_i[\phi^\dagger,\lambda_j]+\bar{\lambda}_i[\phi,\bar{\lambda}_j]\right)-\frac{1}{2}\epsilon^{ij}\left(\lambda_i\partial_5\lambda_j+\bar{\lambda}_i\partial_5\bar{\lambda}_j\right)$$

$$-\frac{1}{2}\partial_5 A^m\partial_5 A_m+\frac{1}{\sqrt{2}}\partial_5 A^m\mathcal{D}_m\left(\phi+\phi^\dagger\right)+\frac{1}{4}\left(\partial_5\left(\phi+\phi^\dagger\right)-i\sqrt{2}\,[\phi,\phi^\dagger]\right)^2\Bigg)$$

$$+\delta(x^5)\left(-\hat{\mathcal{D}}^m\phi^{(1)\dagger}\hat{\mathcal{D}}_m\phi^{(1)}-\frac{i}{2}\bar{\psi}^{(1)}\bar{\sigma}^m\hat{\mathcal{D}}_m\psi^{(1)}-\frac{i}{2}\psi^{(1)}\sigma^m\hat{\mathcal{D}}_m\bar{\psi}^{(1)}\right.\tag{B.7}$$

$$\left.+i\sqrt{2}\,g_{(4)}\left(\phi^{(1)\dagger}\hat{\lambda}_1\psi^{(1)}-\bar{\psi}^{(1)}\hat{\bar{\lambda}}_1\phi^{(1)}\right)+g_{(4)}\,\phi^{(1)\dagger}\bar{D}\phi^{(1)}\right)$$

$$+\delta(x^5-l)\left(-\hat{\mathcal{D}}'^m\phi^{(2)\dagger}\hat{\mathcal{D}}'_m\phi^{(2)}-\frac{i}{2}\bar{\psi}^{(2)}\bar{\sigma}^m\hat{\mathcal{D}}'_m\psi^{(2)}-\frac{i}{2}\psi^{(2)}\sigma^m\hat{\mathcal{D}}'_m\bar{\psi}^{(2)}\right.$$

$$\left.+i\sqrt{2}\,g_{(4)}\left(\phi^{(2)\dagger}\hat{\lambda}_2\psi^{(2)}-\bar{\psi}^{(2)}\hat{\bar{\lambda}}_2\phi^{(2)}\right)-g_{(4)}\,\phi^{(2)\dagger}\bar{D}'\phi^{(2)}\right)$$

$$-\frac{1}{2}g_{(4)}^2\left(\phi^{(1)\dagger}\phi^{(1)}\right)^2\delta(x^5)^2-\frac{1}{2}g_{(4)}^2\left(\phi^{(2)\dagger}\phi^{(2)}\right)^2\delta(x^5-l)^2\ +\ \mathcal{O}\left(m_S^{-4}\right)$$

where

$$\begin{aligned}
\bar{D} &= -\frac{i}{\sqrt{2}}\partial_5\left(\phi+\phi^\dagger\right)-[\phi,\phi^\dagger]\\
&\quad+m_S^{-2}\left(-\lambda_g\sigma^m\partial_m\bar{\lambda}_2-\partial_m\lambda_2\sigma^m\bar{\lambda}_g+\lambda_2\sigma^m\partial_m\bar{\lambda}_g+\partial_m\lambda_g\sigma^m\bar{\lambda}_2\right)+\mathcal{O}\left(m_S^{-4}\right)\\
\bar{D}' &= -\frac{i}{\sqrt{2}}\partial_5\left(\phi+\phi^\dagger\right)-[\phi,\phi^\dagger]\\
&\quad+m_S^{-2}\left(\lambda_g\sigma^m\partial_m\bar{\lambda}_1+\partial_m\lambda_1\sigma^m\bar{\lambda}_g-\lambda_1\sigma^m\partial_m\bar{\lambda}_g-\partial_m\lambda_g\sigma^m\bar{\lambda}_1\right)+\mathcal{O}\left(m_S^{-4}\right)
\end{aligned}\tag{B.8}$$

and $\hat{\lambda}_i$ are as given in (B.4), (B.5) but with D_g and F, $F^\dagger$ set to zero.

C   Feynman rules and loop-integrals

To compute the Feynman diagrams, we convert the Lagrangian (B.7) to the conventions where the Minkowski metric is $\eta_{mn}=\mathrm{diag}(1,-1,-1,-1)$.

The propagator for a 4D Dirac spinor is

$$\overrightarrow{\Psi\qquad\bar{\Psi}}\ \equiv\ \overset{\square}{\Psi\bar{\Psi}}\ =\ \frac{i}{p^2-m^2}\left(\gamma^m p_m+\mathbb{1}_4 m\right).\tag{C.1}$$

This can be decomposed into its Weyl spinor components. Using (A.2) and

$$\Psi=\begin{pmatrix}\psi_1\\\bar{\psi}_2\end{pmatrix},\qquad\bar{\Psi}=\left(\psi_2,\bar{\psi}_1\right),\tag{C.2}$$

one finds

$$\overbrace{\Psi\bar{\Psi}} = \frac{i}{p^2 - m^2}\begin{pmatrix} m & \sigma^m p_m \\ \bar{\sigma}^m p_m & m \end{pmatrix} \equiv \begin{pmatrix} \overbrace{\psi_1\psi_2} & \overbrace{\psi_1\bar{\psi}_1} \\ \underbrace{\psi_2\psi_2} & \underbrace{\psi_2\psi_1} \end{pmatrix}. \tag{C.3}$$

Thus, the Weyl spinor propagators are given by

$$\overleftarrow{\psi_1 \quad \bar{\psi}_1} = \frac{i\,\sigma^m p_m}{p^2 - m^2}, \qquad\qquad \overleftarrow{\bar{\psi}_2 \quad \psi_2} = \frac{i\,\bar{\sigma}^m p_m}{p^2 - m^2},$$

$$\overleftarrow{\psi_1 \quad \psi_2} = \frac{i\,m}{p^2 - m^2}, \qquad\qquad \overleftarrow{\bar{\psi}_2 \quad \bar{\psi}_1} = \frac{i\,m}{p^2 - m^2}. \tag{C.4}$$

The propagators for a scalar field is

$$\overleftarrow{\phi \qquad \phi^\dagger} = \frac{i}{p^2 - m^2}. \tag{C.5}$$

For the brane fields $\phi^{(i)}$, $\psi^{(i)}$, we use the above propagators with $m = 0$. For the bulk fields, we have to sum over all Kaluza-Klein modes with masses $m_n = \frac{n}{R}$, $n = 0, \ldots, \infty$.

The only vertex that we need for our computation is

$$= g_{(4)}\, t_r^a\, \sqrt{2}, \tag{C.6}$$

where we assumed that the brane fields $\phi^{(i)}$, $\psi^{(i)}$ transform according to a representation r of the gauge group G and t_r^a, $a = 1, \ldots, \dim(G)$, are the gauge group generators in this representation.

Our conventions for the invariants that can be formed out of the representation matrices are

$$t_r^a t_r^a = C_2(r)\, \mathbb{1}_{\dim(r)}, \qquad \mathrm{tr}(t_r^a t_r^b) = d^2(r)\, \delta^{ab}. \tag{C.7}$$

The normalization of the Lagrangians in this article is chosen such that $d^2(\mathrm{fund}) = \frac{1}{2}$.

The following identities are useful to do the loop-integrations:

$$\int \frac{d^d q}{(2\pi)^d} \frac{1}{q^2(q-k)^2} = \int_0^1 dx \int \frac{d^d q}{(2\pi)^d} \frac{1}{[q^2 + x(1-x)k^2]^2}, \tag{C.8}$$

$$\int \frac{d^d q}{(2\pi)^d} \frac{q^m}{q^2(q-k)^2} = \int_0^1 dx \int \frac{d^d q}{(2\pi)^d} \frac{(q+xk)^m}{[q^2 + x(1-x)k^2]^2}$$

$$= k^m \int_0^1 dx \int \frac{d^d q}{(2\pi)^d} \frac{x}{[q^2 + x(1-x)k^2]^2}, \tag{C.9}$$

$$\int \frac{d^d q}{(2\pi)^d} \frac{q\cdot k}{q^2(q-k)^2} = \int \frac{d^d q}{(2\pi)^d} \frac{-\frac{1}{2}(q-k)^2 + \frac{1}{2}q^2 + \frac{1}{2}k^2}{q^2(q-k)^2} = \frac{1}{2}k^2 \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^2(q-k)^2}, \tag{C.10}$$

$$\int \frac{d^d q}{(2\pi)^d} \frac{1}{[q^2 + x(1-x)k^2]^2} = \frac{i}{(4\pi)^{d/2}} \frac{\Gamma(2-d/2)}{[x(1-x)k^2]^{2-d/2}}, \tag{C.11}$$

$$\int_0^1 dx \, x[x(1-x)]^\alpha = \frac{1}{2} \int_0^1 dx \, [x(1-x)]^\alpha, \tag{C.12}$$

$$\int_0^1 dx \, \ln\left(x(1-x)\right) = -2. \tag{C.13}$$

To find the $\epsilon \to 0$ limit of the dimensionally regularized loop-integrals, we need the ϵ-expansions

$$A^\epsilon = 1 + \epsilon \ln(A) + \frac{1}{2} \epsilon^2 \ln(A)^2 + \mathcal{O}\left(\epsilon^3\right), \tag{C.14}$$

$$\Gamma(\epsilon/2) = \frac{2}{\epsilon} - \gamma + \epsilon \left(\frac{\pi^2}{24} + \frac{\gamma^2}{4}\right) + \mathcal{O}\left(\epsilon^2\right), \tag{C.15}$$

$$\Gamma(d-\epsilon) = \Gamma(d) \left(1 - \epsilon \, \psi(d) + \frac{1}{2} \epsilon^2 \left(\psi'(d) + \psi(d)^2\right) + \mathcal{O}\left(\epsilon^3\right)\right), \tag{C.16}$$

$$\zeta(d-\epsilon) = \zeta(d) \left(1 - \epsilon \frac{\zeta'(d)}{\zeta(d)} + \frac{1}{2} \epsilon^2 \frac{\zeta''(d)}{\zeta(d)} + \mathcal{O}\left(\epsilon^3\right)\right), \tag{C.17}$$

where

$$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)} = \sum_{n=1}^{x-1} \frac{1}{n} - \gamma, \quad \text{for } x \in \mathbb{N}, \tag{C.18}$$

$$\psi'(x) = \frac{d}{dx}\psi(x), \qquad \zeta'(x) = \frac{d}{dx}\zeta(x). \tag{C.19}$$

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