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Decapitating Tadpoles

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We argue that perturbative quantum field theory and string theory can be consistently modified in the infrared to eliminate, in a radiatively stable manner, tadpole instabilities that arise after supersymmetry breaking. This is achieved by deforming the propagators of classically massless scalar fields and the graviton so as to cancel the contribution of their zero modes. In string theory, this modification of propagators is accomplished by perturbatively deforming the world-sheet action with bi-local operators similar to those that arise in double-trace deformations of AdS/CFT. This results in a perturbatively finite and unitary S-matrix (in the case of string theory, this claim depends on standard assumptions about unitarity in covariant string diagrammatics). The S-matrix is parameterized by arbitrary scalar VEVs, which exacerbates the vacuum degeneracy problem. However, for generic values of these parameters, quantum effects produce masses for the nonzero modes of the scalars, lifting the fluctuating components of the moduli.

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1. Introduction and Summary

Consider a flat space field theory or string theory with one or more classically massless scalars. After supersymmetry breaking, these scalars (and the trace of the graviton) typically develop tadpoles at generic points on the classical moduli space. As a result, perturbation theory around generic points on the classical moduli space does not produce a sensible S-matrix. This is because the zero-momentum tadpole can attach itself to any diagram by the massless propagator,

$$\frac{1}{k^2}|_{k=0} = \infty, \tag{1.1}$$

rendering all amplitudes quantum mechanically divergent.

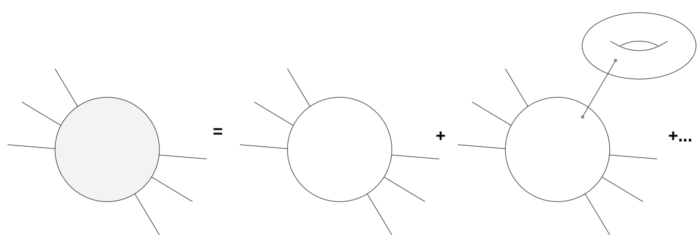

Fig. 1: In the presence of tadpoles, the flat space S matrix does not exist due to divergences.

This IR divergence is usually interpreted as a signal that one must shift the massless field to an extremum of the radiatively generated effective potential. In string theory, this is accomplished by adding the corresponding vertex operator to the worldsheet action [1][2](and *e.g.* [3][4]). The equations of motion satisfied by the shifted field can be deduced cleanly from the condition that BRST trivial modes decouple in the string S-matrix [5][6].[1]

We would like to suggest that there is another way to construct a perturbatively consistent (*i.e.* unitary) theory beginning with this classical background. Instead of shifting

[1] In the case where the scalar being shifted to its extremum is the ubiquitous dilaton, this often leads to either a trivial S-matrix, in the case that the string coupling is driven to zero, or a background which is not well described by perturbation theory, in the case that the dilaton is driven to strong coupling in some region of spacetime. In backgrounds of recent interest that fix the dilaton at a nonzero value via flux stabilization or nongeometrical monodromies, this problem may be avoided (though so far in those cases spacetime techniques have proven more practical than worldsheet analysis).

the massless fields, we will consider changing their propagators. For example, for scalars, we will consider (an IR and UV regulated version of)

$$\frac{-i}{k^2+i\epsilon} \to \frac{-i(1+F(k))}{k^2+i\epsilon} \tag{1.2}$$

where $F(k)$ is chosen to preserve unitarity (and in string theory, worldsheet consistency conditions) while satisfying $F(0) = -1$ in order to cancel the contribution of the zero mode.[2] This effectively changes the equations of motion for the field whose tadpoles we are decapitating, so that any point on the classical moduli space becomes a solution of the deformed equations of motion.

This change is effected in string theory by the perturbative application of the following non-local string theory (NLST) [7][8] deformation of the worldsheet action (again to be regulated in the IR and UV in a manner to be explained in detail in the body of the paper)

$$\delta S_{ws} = \int \frac{d^dk}{(2\pi)^d} \frac{F(k)}{k^2+i\epsilon} \int V^{(k)} \int V^{(-k)} \tag{1.3}$$

where $F(k)$ is chosen to have support only on-shell, on the cone $k^2 = 0$. For simplicity, we will in fact take $F(k)$ to only have support at $k = 0$, though for scalars there may be other options, and will define it as the limit of a smooth function. Here, $\int V$ is an integrated vertex operator; the two factors of the bilocal product can be inserted on the same Riemann surface or on otherwise disconnected surfaces. Diagrammatically, each propagator line is thus replaced by the right hand side of (1.2), so here is the basic mechanism for removal (which we will refer to as "decapitation") of tadpoles:

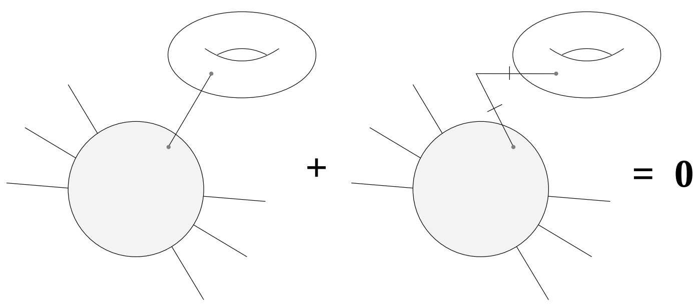

Fig. 2: Cancellation of tadpole divergence via deformation of propagator. The wedge denotes the contribution of the $F(k)$ term from (1.3) in (1.2).

[2] Note that the tadpole only sources the zero mode ($\int d^dx \lambda_1 \phi(x) = \lambda_1 \phi_0$), as is clear diagrammatically from the fact that energy momentum conservation forces the tadpole propagator to $k = 0$.

In addition to decapitating dilaton and moduli scalar tadpoles, we will decapitate the tadpole associated with the trace of the zero momentum graviton in a similar way.

At the same time that we decapitate the tadpole, we remove the zero modes of the massless scalars and graviton from the set of external states we consider in the S matrix. More generally we will focus on the physical S matrix with generic incoming momenta or with external states constructed from smooth wavepackets. In string theory, this is accomplished by rescaling the vertex operators describing external states in our S-matrix by

$$V^{(p)} \to \sqrt{1+F(p)}\; V^{(p)}. \tag{1.4}$$

We will choose $F(k)$ so that (1.3) does not contribute to S-matrix elements except via its cancellation of the massless tadpoles. This will ensure the perturbative unitarity of the resulting theory, provided that the tadpole-free diagrams in the original theory satisfy the cutting rules; this is manifest in simple field theoretic examples and is thought to hold in string perturbation theory.

As we will explain in detail in the bulk of the paper, this effectively removes the *space-time average* of the tadpole for the field in a radiatively stable way, while retaining the quantum-generated self energy for nonzero-momentum modes, including mass renormalization lifting moduli. In simple examples (where the tadpole *is* constant in spacetime) this leads to a nontrivial nonsupersymmetric perturbative S-matrix in flat space. We will study this explicitly for theories for which the tadpole is generated perturbatively.[3] The S matrix so constructed agrees at tree level with the classical S matrix of the undeformed theory, but exists quantum mechanically (at least in perturbation theory). In this S-matrix the fluctuating (nonzero) modes of the moduli are lifted, while the zero mode values (VEVs) of the moduli constitute parameters (couplings) on which the S-matrix amplitudes depend.

In quantum field theory, the perturbative S matrix we construct this way is equivalent (for external states carrying generic momenta or arranged into smooth wavepackets) to that which one would obtain from simply fine tuning away order by order the linear term in the potential expanded about any value for the VEV of the scalar field (or fine tuning away the cosmological constant in the case of gravity). Such a prescription would not be radiatively stable. Our prescription of a nonlocal shift in the propagator is radiatively stable. So, by enlarging the space of possible backgrounds to include nonlocal deformations, one can

[3] We expect that similar results will hold in situations with dynamical supersymmetry breaking.

realize in a radiatively stable manner a system which would otherwise require unnatural fine tuning. In perturbative string theory, one cannot directly fine tune the spacetime effective action in any case, but the decapitation prescription (1.3) can be implemented directly and again provides the same effect in a radiatively stable way. It is also worth noting that it seems likely that the full theory in the presence of $F(k)$, including the possibility of expanding around backgrounds other than flat space, is not equivalent to that which one would obtain from fine tuning away the tadpole.

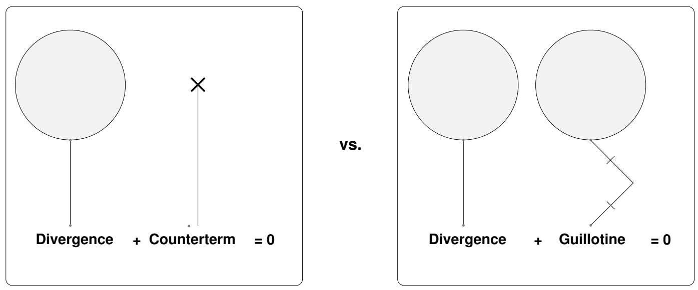

Fig. 3: A counterterm for the tadpole requires delicate order-by-order fine-tuning, and depends critically on the UV cutoff. By contrast, decapitation automatically generates contributions cancelling the tadpoles to all orders once the tree level deformation has been specified, and thus does not involve fine tuning.

Even if we focus on the radiatively stable description in terms of the modified tree-level propagator, we cannot regard this prescription as a solution to the cosmological constant problem per se since in the real world the tadpole is not constant in spacetime. Our prescription removing the zero mode does not address the issue of phase transitions (variation in time) and does not cancel the cosmological term in different localized spatial domains (variation in space) [9]. Indeed, one of the appealing features of our construction is that the metric responds normally to localized sources of stress-energy; it is only the tadpoles due to the cosmological term which are removed by the procedure. It will be interesting to explore more systematically the space of consistent IR modifications, and to try to implement in string theory deformations with a better chance of solving the real-world vacuum energy problem.

Finally, it should be mentioned that, although we will argue for perturbative consistency (unitarity) of our S-matrix, we will have nothing to say here about nonperturbative stability and consistency.

Our argument may appear at odds with standard assumptions about the unity and predictivity of string theory, which are supported by some spectacular results of recent years. Ordinary string/M theory has been unified significantly by string dualities, and formulated nonperturbatively in some backgrounds by matrix theory and AdS/CFT. However, these beautiful results, while conceptually unifying the framework, have not yet rendered the theory highly predictive. Indeed, the space of a priori possible string phenomenologies has grown tremendously with the advent of nonperturbative gauge symmetries, D-branes, and dual descriptions of large N gauge sectors; focusing on elegant possibilities such as [10] may be well motivated from phenomenological considerations and simplicity but has not yet been seen as a prediction of the full theory, which can apparently accommodate arbitrarily large gauge groups and matter content. In addition, the different backgrounds of the theory, while mathematically arising from a unified framework, may not be physically connected due to their very different UV and/or IR behavior [11][12]. In the context of AdS/CFT the equivalence of quantum field theory and string theory shows that string theory need not be more predictive than field theory. In the context of string compactification there is growing evidence that many quantities in the low energy theory can be effectively tuned by choosing the background [13][14][15]. The most urgent issue in evaluating a potential new class of backgrounds of string theory is its physical consistency. The question of vacuum selection in the full quantum theory is an issue that must certainly be addressed but may well fall outside the scope of perturbation theory. In any case, if our backgrounds can ultimately be eliminated by some concrete physical consistency requirement going beyond those we address in this paper, it would serve as further evidence for the unity and predictivity of string theory.

Regardless, our proposal, which will be checked in detail the bulk of this paper, may seem outlandish on first sight. Let us begin therefore by sharing some of the motivations leading to this idea, before embarking on a systematic analysis of our prescription and its physical features.

1.1. Motivation from AdS/CFT double-trace couplings

Bilocal deformations of the general form of (1.3), namely

$$\delta S_{ws} \sim \sum_{I,J} c_{IJ} \int V^{(I)} \int V^{(J)} \tag{1.5}$$

have been derived perturbatively on the string theory side of AdS/CFT dual pairs perturbed by double trace deformations [7][8]. In some AdS/CFT examples [16][17][18][19][20][21], running marginally-relevant double-trace couplings on the field theory side are generated dynamically [22][23][20][21][24] and affect some amplitudes in the theory at large N [7][22].

On the field theory side, the space of couplings includes both single-trace and arbitrary multitrace deformations. These couplings are all on the same footing in field theory (aside from their effect on the structure of the 't Hooft expansion). In specifying a field theory, one chooses a renormalization group trajectory accounting for the behavior of all the couplings. Depending on how one organizes the perturbation expansion, this may involve cancelling divergent amplitudes with counterterms. The coefficients of these counterterms are determined by appropriate renormalization conditions.

Applying the dictionary of [7], this suggests that one should enlarge the space of string backgrounds one considers to include those deformed from ordinary string theory by perturbations of the form (1.5). As in field theory, and in the case of local deformations of string theory, appropriate consistency conditions will restrict this space of backgrounds to a physical subspace.

Moreover, in the context of AdS/CFT, UV divergences requiring counterterms on the field theory side map to IR divergences on the string theory side. These IR divergences may therefore entail a corresponding renormalization prescription, including contributions of the form (1.5) required to cancel divergences, similarly to the way counterterms for double-trace couplings cancel UV divergences on the field theory side [25][26].

This idea is difficult to apply directly in the context of AdS/CFT with dynamically generated double-trace interactions in perturbation theory, because of the usual difficulty involved in describing the string theory side at large curvature. In this paper, we will take this as motivation and apply these ideas directly to flat space string theory, studying the deformation of the form (1.3) and placing on its coefficient $F(k)$ appropriate "renormalization conditons" to ensure the finiteness and consistency of the resulting S matrix.[4]

[4] Another approach to flat space was adopted in [8], by taking a scaling limit of double-trace deformed AdS/CFT to flat space; there one found divergences from insertion of a bilocal product of 0-momentum vertex operators, not smoothed by an integral over k as we have done in (1.3). In [27], NLST deformations naturally arose in describing the squeezed states obtained from particle creation in an asymptotically flat time-dependent background; again this is different because our deformation (1.3) involves both positive and negative frequency modes and does not constitute a squeezed state in the original flat space background.

1.2. Outline of the paper

In section 2 we will present our prescription in detail and show how it cancels tadpole divergences in a radiatively stable manner and lifts the nonzero modes of the moduli. In section 3 we will address the question of other effects of the deformation, and show that the deformation does not contribute for generic external momenta (and therefore smooth wavepackets) to S-matrix elements except via its cancellation of massless tadpoles. This in particular ensures spacetime unitarity and Lorentz invariance of the resulting S-matrix, given plausible assumptions about superstring perturbation theory. In section 4 we will assemble and discuss some basic physical features of the construction, and discuss many future directions.

1.3. Related work

The notion of modifying gravity in the IR and generalizing renormalization to that context is an old idea which has also been explored recently in [28][29][30][9][31]. The work [28][30] has pursued the possibility of a consistent modification of gravity in the IR arising in a brane configuration in a higher dimensional bulk spacetime in the presence of an Einstein term with large coefficient on the brane worldvolume. The work [9] has provided many insights into the requirements an IR modification of gravity must satisfy in order to be able to address the cosmological constant problem including the effects of phase transitions, while maintaining consistency with known physics, and has proposed concrete examples and mechanisms for satisfying these requirements. It would be interesting if an NLST prescription such as the one we employ here to produce a consistent flat-space non-supersymmetric S-matrix could provide a way to formulate a consistent string-theoretic embedding of the effective field theory examples of [9]. The approach of [29] is complementary to ours in a sense we will remark on in the following. Bilocal worldsheet terms appeared in the work [32] on generating effective field theory from string theory, as well as in the more recent context of the AdS/CFT double trace deformations just reviewed.

2. The prescription, and cancellation of tadpole divergences

In this section we will lay out in detail the prescription motivated and summarized in the last section.

2.1. Tadpoles, Divergences, and Regulators

In string theory, similarly to field theory, the contribution of a massless tadpole to an S-matrix element is by a factor of the zero momentum propagator $G_2(k=0)$ times the one-point function of the massless vertex operator at zero momentum. This multiplies the rest of the diagram given by one insertion of the massless vertex operator at zero momentum along with the insertions of vertex operators describing the external states in the amplitude,

$$\mathcal{A}^h|_{Tadpole} \sim \langle \int V_1^{(k_1)} \dots \int V_n^{(k_n)} \int V^{(0)} \rangle_{\Sigma_{\tilde{h}}} \times G_2(k=0) \times \langle \int V^{(0)} \rangle_{\Sigma_{h-\tilde{h}}}. \tag{2.1}$$

This is represented diagramatically as follows:

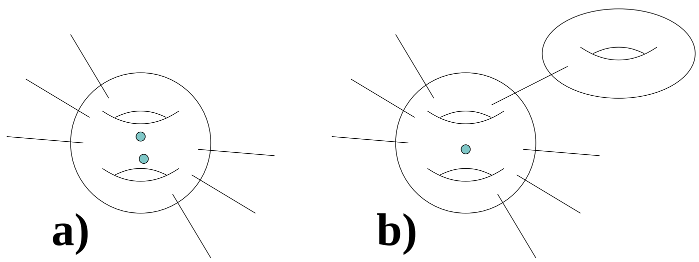

Fig. 4: a) a generic h-loop amplitude; b) the contribution of the one-loop tadpole to this amplitude is a product of the $(h-1)$-loop amplitude, the one-loop tadpole, and a propagator.

At least in the bosonic string, both the massless tadpole diagram and the remaining contributions to the amplitude can be represented as a collection of field theoretic diagrams constructed from (an infinite number of) hermitian irreducible vertices and propagators [33][34]. In this decomposition, all spacetime IR divergences arise from propagator contributions, not from the effective vertices. In the superstring we expect a similar decomposition to hold, and we will assume this, though to our knowledge this has not been proven. This field theoretic decomposition will be important in the following, particularly for our analysis of unitarity in §3 (as in [34]).

While the tadpole is finite in the absence of tachyons (thanks to the soft UV properties of string loops) the on-shell massless propagator is divergent, and requires regularization. We will discuss two natural ways to do this in the case of scalar fields, one of which generalizes to the graviton. We will work in signature $(+,-,\dots,-)$, and denote by d the number of dimensions in which the field whose tadpole we are decapitating propagates.

We begin by discussing classically massless scalar fields. In field theory, a simple method of IR regulation, in situations where it is consistent with gauge invariance, is the by-hand introduction of a small mass μ to be taken to zero at the end of each calculation giving the regulated propagator,

$$\frac{1}{k^2-\mu^2+i\epsilon}. \tag{2.2}$$

In string theory, infrared regularization is most directly expressed in terms of a cut-off on the appropriate Schwinger parameter arising in the propagator of the field theory decomposition summarized above. In particular, the closed string propagator is

$$\lim_{T_c\to\infty, T_0\to 0}\sum_{states}\int_{T_0}^{T_c} dT e^{-T(L_0+\tilde{L}_0)} \tag{2.3}$$

In flat space, for a state corresponding to a spacetime excitation with mass m and momentum k this gives

$$\begin{aligned} G_2(k;T_c,T_0) &\sim \int_{T_0}^{T_c} dT e^{T(k^2-m^2+i\epsilon)} \\ &= \frac{1}{k^2-m^2+i\epsilon}\left(e^{T_c(k^2-m^2+i\epsilon)}-e^{T_0(k^2-m^2+i\epsilon)}\right) \end{aligned} \tag{2.4}$$

Taking $T_c\to\infty$, $T_0\to 0$ reproduces the usual pole $\frac{1}{k^2-m^2+i\epsilon}$. For finite (but large) T_c, as $k^2\to m^2$ we obtain an IR regulated result

$$G_2(k^2\to m^2;T_c)\sim T_c \tag{2.5}$$

One may define these momentum integrals in appropriate circumstances by Euclidean continuation; in that case, T_0 represents a UV cutoff which we may also employ. We can relate the two regulation schemes near the IR limit $k\to 0$ by taking T_c to be a function of k^2 and μ^2 given by the solution to

$$\frac{1}{k^2+i\epsilon}\left(e^{T_c(k^2+i\epsilon)}-e^{T_0(k^2+i\epsilon)}\right)\equiv\frac{1}{k^2-\mu^2+i\epsilon}. \tag{2.6}$$

We will mostly consider the hard (*i.e.* μ-independent) T_c regulator, but will use the μ regulator in sufficiently simple quantum field theory examples.

2.2. The Deformation

We will consider our deformation both in perturbative quantum field theory and string theory. In the μ regularization scheme in quantum field theory, we deform the propagator by

$$\frac{iF(k)}{k^2-\mu^2+i\epsilon}. \tag{2.7}$$

where $F(k)$ will be specified shortly. One can also employ the Schwinger parameterization and regularization in quantum field theory.

In string theory, in terms of the Schwinger cutoff, we implement the following NLST deformation, adding to the worldsheet action

$$\delta S_{ws} \propto \int d^dk \frac{F(k)}{k^2+i\epsilon}(e^{T_c(k^2+i\epsilon)} - e^{T_0(k^2+i\epsilon)}) \int V^{(k)} \int V^{(-k)} \tag{2.8}$$

where $\int V$ are integrated vertex operators corresponding to the massless particles whose tadpoles we wish to decapitate.

As in [7][8], we treat this deformation perturbatively. This introduces an infinite array of new diagrams in which the vertex operators in (2.8) attach to Riemann surfaces in all possible combinations (including diagrams in which the two members of the bi-local pair of vertex operators sit on different, otherwise disconnected, Riemann surfaces).

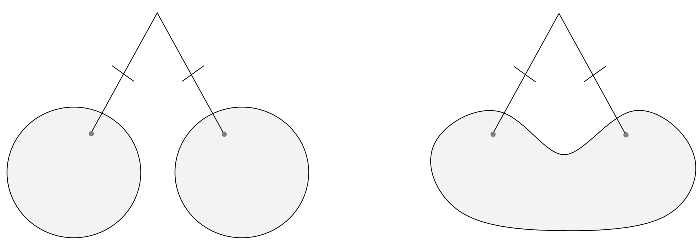

Fig. 5: The bi-local deformation can connect two Riemann surfaces or attach to a single Riemann surface.

Because our two vertex operators in the bi-local term carry momentum k and $-k$ respectively, they occur precisely in the same way as the propagator for the corresponding low-energy field, and thus the effect of the deformation is to shift the propagator:

$$G_2(k;T_c) \to \frac{i(1+F(k))}{k^2+i\epsilon}\left(e^{T_c(k^2+i\epsilon)} - e^{T_0(k^2+i\epsilon)}\right). \tag{2.9}$$

In terms of the μ cutoff, the full momentum-space propagator is parameterized as

$$\frac{i(1+F(k))}{k^2-\mu^2+i\epsilon}. \tag{2.10}$$

In identifying our deformation with a shift in the propagator, we have not implemented any extra subtraction prescription (such as normal ordering) to remove divergences when the $V^{(k)}$ approach the $V^{(-k)}$. As we will see in detail in §3, this divergence integrates to zero once we regulate the theory and does not require any such subtraction procedure. (That is, in the field theoretic organization of the string diagrams which we are using [33][34], all such divergences arise in the propagator, which we have regulated.)

$F(k)$ is constrained as follows.

1. In order to preserve conformal invariance of the worldsheet theory, we demand that $F(k)$ vanish when k is off-shell.
2. In order to cancel the divergences coming from tadpoles, we need
$$F(0) = -1 + \mathcal{O}(\frac{1}{T_c}), \tag{2.11}$$
and in order to precisely cancel the zero mode propagator, we will require
$$F(0) = -1. \tag{2.12}$$
The latter condition ensures that we remove the full zero mode propagator from the tadpole contribution, rather than leaving behind a contribution scaling like an extra massive tadpole as would occur if we kept a nontrivial $\mathcal{O}(\frac{1}{T_c})$ contribution allowed by (2.11).
3. We require $F(k)$ to be consistent with unitarity of the resulting perturbative S matrix. The simplest way to ensure this, which we will employ here, is to choose an $F(k)$ such that the deformation of the propagator does not contribute except in precisely cancelling the tadpole contributions, leaving behind tadpole-free diagrams which satisfy the cutting rules.

One choice of F we have found consistent with the criteria $1-3$ is

$$F(k) = \lim_{\eta\to 0} F_\eta(k) = \lim_{\eta\to 0} \frac{\eta^2}{(k^0+|\vec{k}|-i\eta)(k^0-|\vec{k}|-i\eta)}. \tag{2.13}$$

In a tadpole diagram, (2.7)(2.8) appears integrated with the energy-momentum conserving delta function $\delta^d(k)$ for the propagator in the tadpole part of the diagram. This picks out the integrand evaluated at $k = 0$, for which the factor (2.13) becomes $\frac{\eta^2}{(-i\eta)^2} = -1$. The entire propagator strictly at $k = 0$ is then (in the Schwinger parameterization)

$$G_2(k=0) = \lim_{T_c \to \infty, T_0 \to 0} (1-1)(T_c - T_0) = 0, \tag{2.14}$$

or, in the massive QFT regularization scheme,

$$(1-1)\frac{i}{-\mu^2 + i\epsilon} = 0, \tag{2.15}$$

as depicted in fig. 2. Again, we refer to this mechanism for decoupling the zero mode as decapitating the tadpole.

$F_\eta(k)$ in (2.13) can be written as

$$F_\eta(k) = \pi^2(k^0 + |\vec{k}| + i\eta)(k^0 - |\vec{k}| + i\eta)\delta_\eta(k^0 + |\vec{k}|)\delta_\eta(k^0 - |\vec{k}|) \tag{2.16}$$

where $\delta_\eta(x) = \frac{1}{\pi}\frac{\eta}{x^2+\eta^2}$ is a regulated Dirac delta distribution. As such, when $F(k)$ is integrated against a smooth function, it vanishes. As we have seen, when integrated against $\delta^d(k)$ (which is of course not smooth at $k = 0$) it is -1, so that the deformation cancels the tadpole divergence. We will see that these properties of $F(k)$ imply that its only contribution to physical S-matrix elements (ones at generic external momenta or set up as scattering amplitudes of smooth wavepackets) is precisely its cancellation of the tadpole divergences.

Although we will work with the specific form (2.13) for $F(k)$, any choice satisfying criteria 1–3 is suitable. Any such F which preserves Lorentz symmetry will give an identical perturbative S matrix, so any parameters involved in this choice are not physical, at least perturbatively.

We will perform computations with the following order of limits: we first send $\epsilon \to 0$ and $\eta \to 0$, then remove our IR regulator by taking $T_c \to \infty$ (alternatively, $\mu \to 0$). The $\epsilon \to 0$ and $\eta \to 0$ prescriptions are applied integral by integral, diagram by diagram (*i.e.* these limits are taken before summing over infinite series of diagrams). We refer to this regularization scheme as *the padded room*.

This prescription involves two minor subtleties. Before taking $\eta \to 0$, our deformation (2.7)(2.8) includes off- shell (non-BRST invariant) vertex operators $V^{(\pm k)}$ with $k^2 \neq 0$. Calculating the effects of our deformation perturbatively, as we are doing, thus involves diagrams with insertions of off-shell vertex operators. In tadpole diagrams, energy-momentum conservation projects the deformation onto $k = 0$ so this issue does not arise. In other diagrams, we need to define our prescription and check that the non gauge-invariant contributions vanish as $\eta \to 0$ (the limit we are taking in which $F(k)$ has support only at $k = 0$). Our prescription for the finite η theory before taking the limit $\eta \to 0$ is to work in a specific gauge (fixing the worldsheet metric up to moduli to be integrated over) and calculate correlation functions of the (on-shell and off-shell) vertex operators in the worldsheet CFT on this Riemann surface as in [5]. We will see in §3 that the integration over k in (2.7)(2.8) involves $F(k)$ convolved with a smooth integrand in the regulated theory, so that the deformation makes a vanishing contribution as $\eta \to 0$. This will depend simply on the local behavior of the $V^{(\pm k)}$ near other vertex operators and degenerations of the surface.

In regulating the theory to produce a finite integral over k for general diagrams, note that a UV regulator is also important in intermediate steps of the calculation. For finite η, the wedge propagator scales as η^2/k^4 for large k, (in the UV), which is not soft enough to prevent UV divergences in the diagrams we are adding with wedge propagators in loops. These must be regulated. Once we regulate in the UV, all such diagrams are proportional to (positive powers of) η, and these terms all vanish diagram-by-diagram in the UV regulated theory once we impose our limit ($\eta \to 0$). In the Schwinger parameterization, we can regularize in the UV with our parameter T_0 in computations in which the loop integrals are defined by Euclidean continuation.[5] Alternatively we can simply cut off the k integrals at some scale M_{UV}. We will see in §3 that all such loop contributions will vanish regardless of the details of the choice of UV regulator. (Note that in the tadpole diagrams, the UV behavior is irrelevant since the momentum k is strictly zero.)

This prescription (2.12)(2.14)(2.15) for cancelling divergences caused by radiative tadpoles is reminiscent of the prescription for renormalization of UV divergences via counterterms in quantum field theory. Although our deformation has a large effect in cancelling the divergences from tadpoles, it can be treated perturbatively via (stringy) Feynman diagrams much like counterterms in quantum field theory. In both cases, the (infinite) corrections appear in one to one correspondence with divergences in the uncorrected theory, cancelling them precisely.

[5] In rotating from Lorentzian to Euclidean loop momentum integrals, an extra pole must be included from (2.13); however this pole does not contribute anything in our regulated theory, as will become clear in §3.

2.3. Radiative corrections: stability and moduli masses

It is important to ask whether the specific form of $F(k)$ required by the criteria of the previous subsection is preserved by loop corrections. By construction it is immediate that loop corrections to the "head" of the tadpole do not affect the decapitation, which occurs at the level of the "neck" (*i.e.* at the level of the propagator, regardless of the form of the one-point amplitude to which it attaches).

In fact, loop corrections to the propagator itself also preserve the cancellation of divergences. To see that this is the case, take the 1PI self-energy, Σ, and use it to correct the propagator including the modification (2.15) in the tree-level propagator. One finds (for example in the field theoretic regularization scheme)

$$G_{2,Ren}(k;\mu) = \frac{1+F}{k^2-\mu^2}\left(1+\Sigma\frac{1+F}{k^2-\mu^2}+\left(\Sigma\frac{1+F}{k^2-\mu^2}\right)^2+\cdots\right) = \frac{1+F}{k^2-\mu^2-(1+F)\Sigma}. \tag{2.17}$$

The fact that $1+F(k)$ remains in the numerator of the corrected propagator clearly shows that the cancellation persists at zero momentum and the renormalization of the propagator does not change the fact that the tadpoles (now with renormalized propagator for the neck) are decapitated.

Furthermore, this exhibits the following important physical feature of our construction. Nonzero modes in (2.17) are not affected by $F(k)$, and are subject to generic mass renormalizations included in the quantum self-energy Σ. For models in which this renormalization produces positive mass squared for all the scalars (*i.e.* models in which the second derivative of the effective potential is positive in all directions about the starting value), the fluctuating modes of the moduli are lifted! One example of this is the $O(16)\times O(16)$ heterotic string, whose one-loop potential energy in Einstein frame is proportional to $+e^{(5/2)\Phi}$. Another example would be a pair of D-branes with a repulsive force between them.

On the other hand, there are models in which some of the moduli have negative mass squared at one loop, leading to tachyonic instabilities for nonzero modes. The resulting striped phases may be interesting to study, but for now let us discard these cases since these instabilities will drive us away from the simplest case of Poincaré invariant flat space. Examples of this latter class of 1-loop tachyonic backgrounds include Scherk-Schwarz compactifications and D-brane–anti-D-brane systems.

In this analysis it is important to follow the padded room regularization prescription specifying that the limit $\eta \to 0$ be taken diagram by diagram. In particular, for finite η, the right hand side of (2.17) has poles in the complex k plane corresponding to solutions of the linearized field equations with exponential growth along the spacetime coordinates x^μ.[6] As we take $\eta \to 0$, these solutions revert to oscillating solutions; summing the resulting diagrams then gives the finite result above. If instead we were to sum these diagrams before taking $\eta \to 0$, thereby studying the RHS of (2.17) first at finite η, we would expect divergences arising from these exponentially growing solutions (similar to divergences caused by tachyons in loop diagrams). Importantly, this order of operations is explicitly disallowed in our regularization prescription; the limit $\eta \to 0$ is part of the definition of each diagram and must be taken before doing the sum in (2.17). In fact, as we will see in §3, diagram by diagram our deformation does not contribute in loop propagators; $F(k)$ integrated against the rest of the amplitude vanishes unambiguously, diagram by diagram.

A related issue is the question of whether nonperturbatively the decapitated theory has other background solutions, different from flat space, with consistent (in particular, unitary) physics. (For example, in the presence of our deformation, could one still start with a solution in scalar field theory with the scalar field rolling down the potential hill and expand around this solution to produce a consistent theory?) If there exist other solutions which are in fact connected physically to our flat space solution, it would be interesting to study nonperturbative dynamics that may select which background will arise naturally when this framework is considered in a cosmological context. This very interesting question we leave for future work.

2.4. Decapitating the graviton tadpole

We so far formulated our deformation for massless scalar fields. The tadpole generated for the (trace of the) graviton is the cosmological constant and is of particular interest.[7]

Since the graviton tadpole (cosmological constant) is one of the main motivations for pursuing this direction, we wish to generalize our prescription to a modification of the

[6] We thank the authors of [9] and N. Kaloper and E. Martinec for emphasizing this issue to us.

[7] We could restrict our attention to scalars by considering tadpoles for the scalars arising in the open string sector on D-branes with broken supersymmetry (see the next subsection).

graviton propagator which cancels its zero mode. In particular, for the procedure under discussion to be useful in a simple closed string example (like the $O(16) \times O(16)$ heterotic string) we need to decapitate the graviton also so as to avoid generating large curvature.

It may also be interesting in some circumstances to decapitate the scalars but shift the gravity background in the standard way to obtain dS or AdS space. That said, we content ourselves in the following to the most simple case of asymptotically flat space, leaving generalizations to future work.

In expanding about flat space, Lorentz invariance implies that the only tadpole contribution from the gravitational sector comes from the trace of the graviton. The trace can be gauged away for nonzero momentum, but at zero momentum the gauge transformation required to do so would not vanish at infinity. The worldsheet manifestation of this is the presence of an extra BRST-invariant vertex operator at zero momentum transforming as a spacetime scalar, which we will denote by $V_{trG}^{(k=0)}$. (This mode is degenerate with but independent from the zero-momentum mode of the dilaton.) Defining

$$V_{trG}^{(k)} \equiv : V_{trG}^{(k=0)} e^{ikX} : \tag{2.18}$$

we add to the worldsheet action

$$\delta S_{ws}^{G} = \int \frac{d^d k}{(2\pi)^d} \frac{F(k)}{k^2 + i\epsilon} \left(e^{T_c(k^2+i\epsilon)} - e^{T_0(k^2+i\epsilon)} \right) \int V_{trG}^{(k)} \int V_{trG}^{(-k)} \tag{2.19}$$

As in the case of the scalar fields, before taking $\eta \to 0$ this involves off-shell vertex operators $V_{trG}^{(\pm k)}$ with $k \neq 0$ included in (2.19). Again, we can compute in a fixed gauge and show that these contributions vanish when $\eta \to 0$.

As in the scalar case, this suffices to cancel all tadpole divergences at any loop order. (Note that in contrast to the scalar case, the self-energy of the graviton of course does not include a mass by gauge invariance.) Since we only modified the zero mode of the graviton, we do not expect problems with gauge (diffeomorphism) invariance to be introduced by our prescription; gauge transformations which die at infinity cannot act on the strict zero mode of the graviton. Acting only on the zero mode also ensures that the graviton responds to ordinary local sources of stress-energy in the usual way, as we will exhibit for the S-matrix in §3.

2.5. Open string examples

It is worth emphasizing that we may consider tadpoles for scalars independently of gravitons by considering a non-supersymmetric combination of D-branes in a supersymmetric bulk theory. In such a situation, any closed string tadpoles can be absorbed in radial variation of the fields (if the D-branes are at sufficiently high codimension). In order to produce an S- matrix with positive mass squared for the nonzero modes of the scalars, we can for example choose a pair of branes which repel each other at long distance. (Note that we may not choose an attractive potential $V(r) \sim -\frac{1}{r^n}$ such as arises in a simple D-brane-anti-D-brane system since $V''(r) < 0$ in that case; we can instead choose a repulsive potential $V(r) \sim +\frac{1}{r^n}$ which has $V''(r) > 0$.) In such a system we may decapitate the tadpoles for scalars on one or both of the branes (shifting the nondecapitated fields to the appropriate time-dependent solutions describing motion of the corresponding brane).

2.6. BRST analysis

In [5][6], the loop corrected equations of motion for massless fields was derived by requiring that BRST trivial modes decouple from string S-matrix elements. One considers a diagram

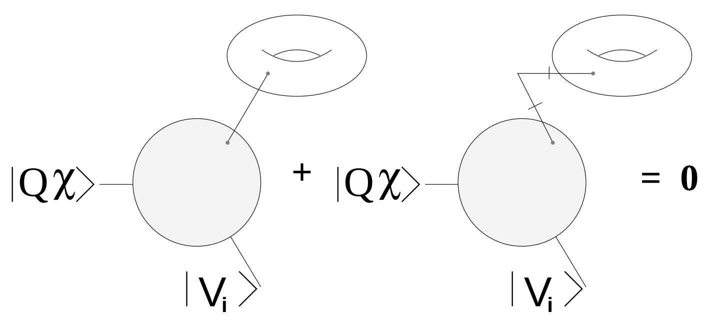

Fig. 6: Before decapitation, the tadpole spoils the decoupling of BRST trivial modes. Decapitation adds a diagram precisely cancelling this anomaly.

with one BRST trivial vertex operator $Q_B\chi \equiv \oint_z j_B\chi(z,\bar{z})$ and any number of physical vertex operators V_i. One can deform the contour of integration away from $\chi(z,\bar{z})$ so that the BRST operator Q_B acts on the other insertions in the diagram. Q_B kills the remaining (physical) vertex operators. It also formally kills the on-shell vertex operators occuring in the subdiagrams arising from factorizing the diagram on a tadpole; $[Q_B, V^{(k)}] \propto k^2 = 0$. However, if we remove the IR regulator, this factorized diagram contains also a factor of

$1/k^2$ from the propagator. So in this order of limits, there remains a finite contribution spoiling the decoupling of the BRST trivial mode. One can deduce from this the equations of motion a shift in the field must obey in order to compensate this anomaly [5][6].

In this calculation, the IR regulator is unnecessary since the BRST charge Q_B pulls down a factor of k^2 cancelling the divergent $1/k^2$ propagator. Let us then consider this anomalous diagram in our case, again removing the IR regulator (sending $T_c \to 0$). Our deformation (2.8)(2.19) precisely cancels the anomaly, since the $1/k^2$ propagator contribution is multiplied by $k^2(1+F(k))|_{k=0} = k^2 \times 0$.

2.7. Effective field theory description

A useful heuristic way to describe our prescription is to consider the momentum-space effective action for a scalar field ϕ whose tadpole we are decapitating. The presence of the discontinuous object $F(k) = \lim_{\eta\to 0} F_\eta(k)$ complicates the analysis of the field theory (the limit $\eta \to 0$ being taken diagram by diagram in the S matrix as we explained in §2.1§2.2). We will ignore all such subtleties in this subsection with the aim of gaining some further intuition for the physics of the deformation. Taking into account the modification we have made to the propagator, this effective action is

$$\int d^dk \left[\phi(k)\Big(\frac{k^2-\mu^2}{1+F(k)} - \lambda_2\Big)\phi(-k)\right] - \lambda_1\phi(0) - \int d^dk \int d^dk' \lambda_3 \phi(k)\phi(k')\phi(-k-k') - \dots \tag{2.20}$$

This leads to the equation of motion

$$\phi(-k) = \lambda_1 \delta^d(k) \frac{1}{\frac{k^2-\mu^2}{1+F(k)} - \lambda_2} \tag{2.21}$$

plus subleading terms involving the higher ($\lambda_{n>2}$) terms in the effective potential. Because of the $F(0) = -1$ contribution, the right hand side here is of the form $f(y)\delta(y)$ with $f(0) = 0$, so this vanishes. That is, $\phi(0)$ is not forced to shift by the tadpole once we include our modification of the kinetic term (corresponding to our original modification of the propagator).

This description involving a nonlocally modified action may be useful but we will mostly stick to the S-matrix formalism (natural in perturbative string theory) we have been developing.

2.8. In contrast

Before returning to the S-matrix description, it is worth noting at this point that our prescription is different from two somewhat similar manipulations that might be confused with it.

Removing the zero mode by boundary conditions

First, in field theory one might consider removing the zero mode of a massless field by putting the system in a box with appropriate boundary conditions. For example, consider a scalar field with a tadpole (say a linear potential) in a box. Imposing Dirichlet boundary conditions removes the zero mode. However, since this does not change the basic equation of motion, half of the remaining modes still respond to the linear term in the potential. Adiabatically decompactifying the box therefore leads to an unstable theory.

Decapitation works not by selecting particular solutions of the original equation of motion, but by changing the equations of motion. In our case (2.20), there is no linear term for nonzero modes, and hence no instability left in the system once we remove the zero mode by our decapitation prescription. Also, our analysis of decapitation involves a regulation prescription compatible with an S matrix description, whereas introducing a box as an IR regulator would not have this feature.

String IR modifications

As discussed in [7], the bilocal deformation $\delta S \sim \int V \int V$ can be obtained by deforming the action locally by

$$\delta S = \int d^2 z \lambda V \tag{2.22}$$

and integrating over λ with a Gaussian weight.

Recently a modification of string theory has been proposed in [29] which involves considering fluctuating couplings λ on the worldsheet. In our case (2.22), $\lambda(k)$ is a constant on the worldsheet, whereas in [29], $\lambda = \lambda(z, \bar{z})$ is a fast varying function of the worldsheet coordinates, and in particular explicitly does not include a worldsheet zero mode.

3. Effects of deformation on general diagrams and unitarity

We have so far established that our modification removes the tadpole divergences associated with massless fields. We must now address the question of what other effects the modification has, and in particular determine whether the S-matrix resulting from our deformation is unitary.

Because of the simplicity of the $F(k)$ we chose for our deformation, we will see in fact that it does not contribute to physical S-matrix amplitudes beyond its cancellation of tadpole divergences, and that unitarity is therefore satisfied.

In particular, as we have seen, $F(k)$ vanishes when integrated against any smooth function (its nonvanishing contribution cancelling the tadpole arises from its integration against a delta function $\delta^d(k)$). The question is then whether the k-dependence of the integrand in amplitudes obtained by bringing down powers of (2.8) and (2.19) is sufficiently smooth, modulo (non-smooth) $\delta^d(k)$ factors coming from tadpole contributions. (Note that we are working with a UV cutoff which ensures no divergence from the UV end of the k integration.) Generic diagrams involving smooth wavepackets integrated over external momenta as well as ordinary loop momentum integrals indeed turn out to have this property in the padded room, *i.e.* in our regularization prescription.

Thus we are interested in the k-dependence of amplitudes with insertions of $V^{(\pm k)}$, near potential singularities in the integrand. The $V^{(\pm k)}$ can be slightly off shell before we take the limit $\eta \to 0$, and we define their amplitudes by working in a gauge-fixed worldsheet path integral. The possible singularities in the integrand arise as the $V^{(\pm k)}$ approach other vertex operators $V^{(p_i)}$ or degenerating internal lines carrying momenum p_i. In both cases, the behavior is determined locally on the Riemann surface and has the structure $\int \frac{d^2z}{|z|^{2+2k\cdot p_i}} \sim \frac{1}{(k+p_i)^2-m_i^2+i\epsilon}$. As in the UV, these potential divergences are cut off in the IR by our regularization prescription.

3.1. Non-1PI contributions

Let us consider first diagrams for which cutting an $F(k)$ contribution to the propagator (which we will refer to as a "wedge propagator" contribution) breaks the diagram in two. For this non-1PI propagator there are two cases. One is what we have already accounted for: the wedge propagator attaches to the head of a tadpole (with no incoming momentum); in this case the wedge contribution cancels the divergence from the tadpole (in fact the whole massless propagator contribution) by construction. The second case is that the the wedge propagator in question connects to a subdiagram with incoming momenta q_i, so that generically there is nonzero momentum $k \equiv \sum_{i=1}^n q_i$ flowing through the wedge propagator.

At generic incoming momentum, since $k \equiv \sum_{i=1}^{n} q_i \neq 0$, $F(k)$ does not contribute (since $F(k \neq 0) = 0$). Similarly, if we consider a smooth wavepacket in the incoming momenta q_i, the relevant part of the amplitude is

$$\int \prod_i d^d q_i f(q_i) F(\sum_i q_i) \frac{i}{(\sum_i q_i)^2 + i\epsilon} \left(e^{[(\sum_i q_i)^2 + i\epsilon]T_0} - e^{[(\sum_i q_i)^2 + i\epsilon]T_c} \right) \tag{3.1}$$

We can change basis in the q_i to obtain an integral over $\sum_i q_i$ (the argument of F in this amplitude); it is then clear that the integrand is sufficiently smooth at $\sum_i q_i = 0$ and because of the convolution with F this amplitude vanishes.[8]

Forces between D-branes

One type of one-particle reducible diagram of particular interest is that describing the force between D-branes, so let us study this explicitly. Here we have at leading order

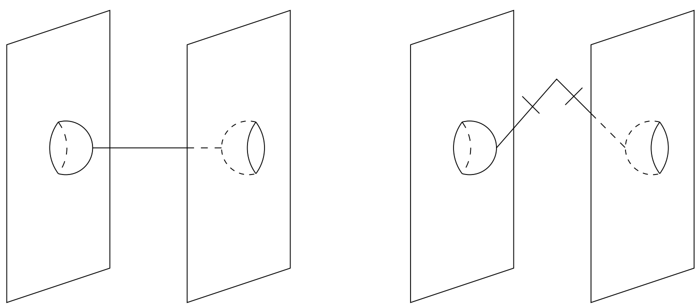

Fig. 7: Diagrams contributing to the force between parallel branes.

The correction term is proportional to

$$\int d^n \vec{k}_\perp \frac{F(\vec{k}_\perp)}{\vec{k}_\perp^2 + i\epsilon} \left(e^{-T_c(\vec{k}_\perp^2 + i\epsilon)} - e^{-T_0(\vec{k}_\perp^2 + i\epsilon)} \right) \tag{3.2}$$

where $\vec{k}_\perp$ denotes the momenta in the n transverse directions to the D-brane. This contribution vanishes, as can be seen by plugging in the above expression for F in terms of delta functions (2.16). So as expected from the general arguments above, we see explicitly here that the force between gravitational sources such as D-branes is not changed by our decapitation of the tadpoles of the closed strings exchanged.

[8] In fact for normalizable wavepackets $f(q_i)$ is not only smooth at $\sum_i q_i = 0$ but vanishes there.

3.2. 1PI contributions

Consider a general contribution involving wedges which carry loop momentum (*i.e.* a diagram which is 1PI with respect to cutting at least some of the wedges). We would like to know if this diagram is nonzero (and if it is nonzero, we would like to know if it preserves unitarity of the S-matrix).

Let us focus on one wedge at a time, with momentum k. If the Riemann surfaces are smooth and the vertex operators are separated from each other and from the $V^{(k)}$'s, then the integrand will be nonsingular. The potential divergences as k varies come from the degenerations of the Riemann surface approaching the $V^{(\pm k)}$'s and/or the approach of vertex operators to each other. These can always be viewed as IR divergences or poles in the S-matrix. So we can focus on the region of the moduli space of the Riemann surface near IR limits and poles. (Again, note that any UV divergences are cut off.)

Using this, the structure of the potentially singular part of the k-dependent integrand in the amplitude is

$$\int d^dkF(k)\frac{i}{k^2+i\epsilon}(e^{T_c(k^2+i\epsilon)}-e^{T_0(k^2+i\epsilon)}) \\ \prod_i\int d^dp_if(p_i)\frac{1}{(k+p_i)^2-m_i^2+i\epsilon}(e^{T_c((k+p_i)^2-m_i^2+i\epsilon)}-e^{T_0((k+p_i)^2-m_i^2+i\epsilon)}), \tag{3.3}$$

times a factor of T_c if the $V^{(\pm k)}$ approach each other (*c.f.* (2.5)). Here the p_i are linear combinations of some subset of the momenta (including in general both loop and external momenta). That is, the propagators in (3.3) come from pieces of the diagram in which a $V^{(\pm k)}$ line hits a line carrying momentum p_i. In the case that p_i is a linear combination of external momenta, then we take the function $f(p_i)$ to be a nontrivial smooth wavepacket.[9] When p_i involves a loop momentum, then $f(p_i)$ encodes any further momentum dependence in the amplitude beyond the pole contribution, and again is a smooth function.

As before, whether this contribution survives is determined by whether the integrand as a function of k can become singular as k varies. This is clearly averted here since the only singularities of the integrand are the poles from the propagators, and for finite T_c, the expansion of the exponentials for small $(k+p_i)^2-m_i^2$ kills the factor of $(k+p_i)^2-m_i^2$ in the denominator. So we see that the F terms do not contribute in loop (1PI) propagators, just as we found for non-1PI propagators in physical S-matrix amplitudes.

[9] This wavepacket should die off fast enough for large momentum so as not to introduce new UV divergences; we may in any case include a UV cutoff M_{UV} on the external momentum integrals as well as on the internal ones.

3.3. New tadpole diagrams which vanish

It is worth mentioning that the tadpole contributions formally include the following diagrams introduced by our modification:

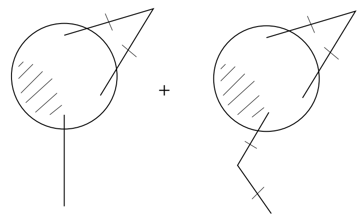

Fig. 8: 0+(-0)=0.

However, these diagrams cancel. Not only do they cancel each other exactly via decapitation, but they are separately zero because as we have just derived, the F's do not contribute in loops. This is related to the comment in §2 about the absence of a need for a normal-ordering prescription for the product $V^{(k)}V^{(-k)}$.

3.4. Explicit evaluation at one loop

The above general arguments suffice to establish that our deformation proportional to F does not contribute except in decapitating the tadpoles. It is nonetheless instructive to work out explicitly a simple 1-loop example in quantum field theory to illustrate the effect.

Let us consider a one-loop graph involving two virtual massless scalar particles (whose tadpoles we are decapitating) with total momentum p running through it and loop momentum k. This is given by (up to an overall real constant)

$$\lim_{\mu\to 0}\lim_{\eta\to 0}\int \frac{d^dk}{(2\pi)^d}\frac{1+F_\eta(k)}{k^2-\mu^2+i\epsilon}\frac{1+F_\eta(k-p)}{(k-p)^2-\mu^2+i\epsilon} \tag{3.4}$$

Let us perform the k^0 integral by treating it as a contour integral, closing the contour at infinity in the lower half plane. This is possible because the integrand falls off for large $|k^0|$. This picks up the residues of poles at $k^0 = \sqrt{\vec{k}^2+\mu^2-i\epsilon}$ and $k^0 = p^0 + \sqrt{(\vec{k}-\vec{p})^2+\mu^2-i\epsilon}$. (Note that this follows even in the presence of the F terms because we constructed $F_\eta(k)$ to have no poles in the lower half k_0 plane.) Letting $E_k \equiv \sqrt{|\vec{k}|^2+\mu^2}$, this gives

$$i\int \frac{d^{d-1}\vec{k}}{(2\pi)^{d-1}}\Bigg(\frac{1+F_\eta(E_k-i\epsilon,\vec{k})}{2\sqrt{\vec{k}^2+\mu^2-i\epsilon}}\frac{1+F_\eta(E_k-i\epsilon-p^0,\vec{k}-\vec{p})}{(E_k-i\epsilon-p^0)^2-(\vec{k}-\vec{p})^2-\mu^2+i\epsilon}+ \\ +\frac{1+F_\eta(E_{k-p}-i\epsilon,\vec{k}-\vec{p})}{2E_{k-p}-i\epsilon}\frac{1+F(p^0+E_{k-p}-i\epsilon,\vec{k})}{(p^0+E_{k-p}-i\epsilon)^2-\vec{k}^2-\mu^2+i\epsilon}\Bigg) \tag{3.5}$$

For generic external momentum p, the denominators in this expression never vanish (for finite μ, which is taken to zero at the very end of the computation) where either of the F factors have support. Hence as argued for general diagrams in the above subsections, here we see explicitly that the deformation does not contribute in loop propagators.

3.5. Unitarity

Because the F terms do not contribute to amplitudes except in cancelling massless tadpole contributions, we expect that perturbative unitarity is satisfied. This is manifest in simple quantum field theories such as ϕ^3 theory expanded about $\phi_0 = 0$: once the diagrams including tadpoles are removed the remaining diagrams satisfy the cutting rules for perturbative unitarity (see figure 9). This result is clear also from the equivalence of the S matrix resulting from decapitation and that obtained by simply fine tuning away the tadpole contribution order by order; the latter also removes the tadpole diagrams leaving behind finite ones satisfying the cutting rules.

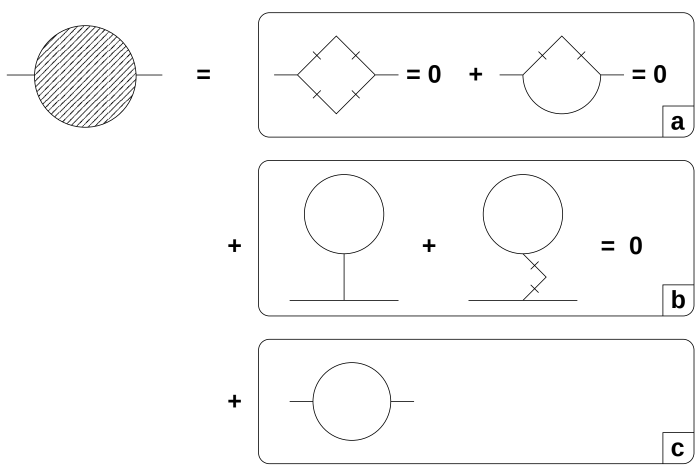

Fig. 9: The one loop two-point function in decapitated ϕ^3 theory. (a) All diagrams with wedges in loops vanish identically. (b) Decapitation ensures that all tadpole diagrams cancel. (c) The remaining diagram respects the cutting rules by construction.

In ordinary bosonic string theory, one can formally argue for perturbative unitarity by decomposing string diagrams into quantum field theory diagrams made from propagators and hermitian vertices (the latter containing no boundaries of moduli space and therefore no poles) (see *e.g.* [33] and the discussion in [34], chapter 9), and then appealing to the field theory argument based on Cutkosky rules. As we have shown in this section, the effect of our deformation is precisely to cancel massless tadpole contributions in this field theory language; the remaining diagrams satisfy the cutting rules as usual if they do in the undeformed theory. So if the superstring perturbation theory works similarly to the bosonic case in this regard, *i.e.* if it is decomposable into diagrams, formed from propagators and vertices, which satisfy the cutting rules for unitarity (which seems plausible though it has not been proved), then we can conclude that our deformation produces a unitary theory.

Since the remaining diagrams describe forces that fall off with distance, we expect cluster decomposition to hold in our theories. (This is again manifest in the perturbative quantum field theory examples where the result is equivalent to that one would obtain tuning away the tadpoles.)

Note that since we have shown that tadpole-free diagrams are unaffected by our modification, the analogous modifications of perturbative supersymmetric theories would have no effect on the physical S matrix. (An interesting future direction is to apply our construction, perhaps field theoretically, to models of low-energy supersymmetry with dynamical (nonperturbative) supersymmetry breaking.)

4. Discussion

Having argued for the unitarity of our S-matrix, let us now recap and assemble the salient physical features of our system. Our prescription leads to a class of unitary non-supersymmetric perturbative S-matrices in flat space, parameterized by the VEVs of the classical moduli, whose fluctuating modes are generically lifted. We accomplished this by rendering non-dynamical the zero modes of fields (moduli and the graviton) which would otherwise be destabilized by tadpoles, via a modification of the propagator for these fields in the deep infrared. On the worldsheet this modification arises as a perturbative NLST deformation. The tree-level S-matrix is the same as in the unmodified theory; in particular the response of gravity to localized sources of stress-energy is as in ordinary general relativity and has not been removed by our mechanism.

The tadpoles in our examples are uniform over spacetime, and have been effectively removed. It is worth emphasizing that this is not true of the cosmological term in the real world, which is subject to phase transitions (variation in time) as well as possible variation among different spatial domains. Further, we have not so far identified a dynamical mechanism for selecting our theory. In this regard, it will be very interesting to study more systematically the space of consistent IR deformations along these lines.

One result of our analysis which is in some sense disappointing is the presence of parameters descending from the VEVs of the moduli fields. Again, these arise because we can implement our decapitation construction expanding about any point in the classical moduli space having positive 1-loop quadratic terms in the potential for all the moduli. The point in the moduli space from which we start controls the couplings in the S matrix, while the decapitation construction removes the tadpoles which would otherwise generically drive the moduli away from the starting point. Our construction (for any choice of $F(k)$ satisfying our criteria in §2) does not entail any parameters coming from $F(k)$, though it is possible that more general choices of $F(k)$ that do affect non-tadpole diagrams could also lead to consistent perturbative S matrices in flat space or otherwise.

Continuous parameters are of course also seen in flat space SUSY models with moduli spaces and in SUSY and non-SUSY versions of (deformations of) the AdS/CFT correspondence (where the values of the field theory couplings in the UV form a continuum of parameters). The novelty here is the persistence of such a continuum after supersymmetry breaking, in a background preserving maximal (Poincaré) symmetry. (This also has something of an analogue in known backgrounds–in flux compactifications even after supersymmetry breaking, one has a finely spaced set of discrete parameters which can allow one to effectively tune contributions to the low-energy effective action, including the cosmological term [13][14][15][35][36].)

This work leaves open the possibility that our perturbative string theories may not complete to nonperturbatively consistent theories. It was only relatively recently that ordinary perturbative string theories have been (in many cases) understood to fit into a nonperturbative framework via string/M theory duality, matrix theory, and AdS/CFT. We do not have any concrete results on this question; perhaps something could be learned by considering nonperturbative features of decapitation in spontaneously broken gauge theories.[10] Also, it is possible that the assumption we make about the undeformed superstring

[10] Work on a related question of whether or not similar modifications might be consistent in the Higgs sector of the Standard Model is in progress [37].

diagrams satisfying perturbative unitarity relations as in quantum field theory along the lines of the bosonic case [33][34] is wrong because of subtleties associated with superstring perturbation theory. This loophole we find less plausible but in the absence of a proof it certainly remains a possibility.

Although (as in the previously known cases listed above) the parameters add to the lack of predictivity in perturbative string theory, there is a very appealing robust prediction in this class of models. Namely, our construction provides a mechanism for solving the moduli problem, in that for generic values of the parameters in our S-matrix, the fluctuating modes of the moduli are lifted.

While in this paper we considered perturbative diagrams producing tadpoles, our construction may also apply to situations in which SUSY is broken dynamically at low energies. As an IR effect, we can describe our modification in field theory terms, and low-energy field theoretic SUSY breaking models may be amenable also to such a deformation. (Also, in some circumstances classical SUSY breaking superpotentials may be dual to dynamical ones.)

Similarly we may ask about non-flat backgrounds. It will be interesting to consider whether we can decapitate scalar tadpoles but not the graviton tadpole, leading to a de Sitter or anti de Sitter solution. It is also important to understand much better the space of consistent string backgrounds, in particular to understand how much fine tuning of initial conditions is required to land on the flat space backgrounds we have exhibited in this paper.

Along similar lines, one may consider IR deformations of this sort which involve different forms for $F(k)$. In particular one can imagine introducing a length scale L above which the decapitation acts nontrivially, rather than simply acting at zero momentum. As in [9], this may bring the approach closer to applying to the real world cosmological term.

An important theme of this subject is the application of renormalization ideas to infrared divergences. Our prescription here is analogous to renormalization via counterterms in that the finite result we obtain arises from cancellation of quantities that diverge as the cutoff is removed. It would be very interesting to pursue the possibility of IR renormalization using instead an analogue of Wilsonian renormalization involving coarse-graining in momentum space.

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