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Combined Results on *b*-Hadron Production Rates and Decay Properties

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ALEPH, CDF, DELPHI, L3, OPAL, SLD

Prepared [1] from Contributions to the 2000 Summer conferences.

Abstract

Combined results on b-hadron lifetimes, b-hadron production rates, $\mathrm{B_d^0}-\overline{\mathrm{B_d^0}}$ and $\mathrm{B_s^0}-\overline{\mathrm{B_s^0}}$ oscillations, the decay width difference between the mass eigenstates of the $\mathrm{B_s^0}-\overline{\mathrm{B_s^0}}$ system, the average number of c and $\bar{c}$ quarks in b-hadron decays, and searches for CP violation in the $\mathrm{B_d^0}-\overline{\mathrm{B_d^0}}$ system are presented. They have been obtained from published and preliminary measurements available in Summer 2000 from the ALEPH, CDF, DELPHI, L3, OPAL and SLD Collaborations. These results have been used to determine the parameters of the CKM unitarity triangle.

[1]The members of the working groups involved in this activity are: D. Abbaneo, J. Alcaraz, V. Andreev, E. Barberio, M. Battaglia, S. Blyth, G. Boix, M. Calvi, P. Checchia, P. Coyle, L. Di Ciaccio, P. Gagnon, R. Hawkings, O. Hayes, P. Henrard, T. Hessing, I.J. Kroll, O. Leroy, D. Lucchesi, M. Margoni, S. Mele, H.G. Moser, F. Muheim, F. Palla, D. Pallin, F. Parodi, M. Paulini, E. Piotto, P. Privitera, Ph. Rosnet, P. Roudeau, D. Rousseau, O. Schneider, C. Shepherd-Themistocleous, F. Simonetto, P. Spagnolo, A. Stocchi, D. Su, T. Usher, C. Weiser, B. Wicklund and S. Willocq.

Contents

1 Introduction

This paper contains an update of similar results [1] which were based on measurements made available in Summer 1999. Experimental results, made available in Summer 2000, have been included and new sections have been prepared on charm counting in b-hadron decays, on direct $\sin(2\beta)$ measurements and on other searches for CP violation in the $\mathrm{B^0_d}-\overline{\mathrm{B^0_d}}$ system. A large fraction of these measurements have been used, in addition, to determine the CKM unitarity triangle parameters. These studies have been limited to combining measurements obtained by experiments which were running before the start of the asymmetric B factories. As compared with the previous report [1], apart from the new sections, the main improvements have been obtained on b-lifetime and oscillation measurements.

Accurate determinations of b-hadron decay properties provide constraints on the values of the elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [2]. The $|\mathrm{V}_{cb}|$ and $|\mathrm{V}_{ub}|$ elements can be obtained from semileptonic decay rates into charmed and non-charmed hadrons, and measurements of the oscillation frequencies in $\mathrm{B^0_d}-\overline{\mathrm{B^0_d}}$ and $\mathrm{B^0_s}-\overline{\mathrm{B^0_s}}$ systems give access to $|\mathrm{V}_{td}|$ and $|\mathrm{V}_{ts}|$.

Elements of the CKM matrix govern weak transitions between quarks. Experimental results are obtained from processes involving b-hadrons. Effects from strong interactions have thus to be controlled and b-hadrons are also a good laboratory in this respect. Lifetime differences between the different weakly decaying hadrons can be related to interactions between the heavy quark and the light quark system inside the hadron. The polarization of Λ^0_b baryons produced by b-quarks of known polarization, emitted from Z decays at LEP or SLC, indicates how polarization is transmitted from the heavy quark to the baryon(s) in the hadronization process. Rates and decay properties of D^{**} states[1] produced in b-hadron semileptonic decays are needed to obtain accurate determinations of $|\mathrm{V}_{cb}|$ as they provide constraints on experimental and theoretical related uncertainties. Decays of D^{**} states are also an important source of background in other channels, and their properties have to be measured. Finally, it is necessary to measure the production rates of the different weakly decaying b-hadrons emitted during the hadronization of b-quarks created in high energy collisions, because all these states have different properties so the study of any one of them requires control of the background from the others.

Results obtained on b-hadron lifetimes, b-hadron production rates, $\mathrm{B^0_d}-\overline{\mathrm{B^0_d}}$ and $\mathrm{B^0_s}-\overline{\mathrm{B^0_s}}$ oscillations, b-hadron semileptonic decays, on the mean number of c and $\overline{c}$ quarks in b-hadron decays, on the decay width difference between the $\mathrm{B^0_s}-\overline{\mathrm{B^0_s}}$ system mass eigenstates and on CP violation in the $\mathrm{B^0_d}-\overline{\mathrm{B^0_d}}$ system made available during Summer 2000, are presented here. These quantities have been obtained by averaging published and preliminary measurements released publicly by the ALEPH, CDF, DELPHI, L3, OPAL and SLD experiments. In addition, these results have been used to provide averaged values for $|\mathrm{V}_{cb}|$, $|\mathrm{V}_{ub}|$ and to obtain the parameters defining the unitarity triangle, within the Standard Model framework.

Whenever possible, the input parameters used in the various analyses have been adjusted to common values, and all known correlations have been taken into account. Close contacts have been established between representatives from the experiments and members of the different working groups in charge of the averages, to ensure that the data are

[1] The notation D^{**} includes all charm mesons and non-resonant charmed final states which are not simply D or D^* mesons.

prepared in a form suitable for combinations. Working group activities are coordinated by a steering group[2].

Section 2 presents the values of the common input parameters that contribute to the systematic uncertainties presented in this note. Studies on the production rates of weakly decaying b-hadrons, on some characteristics of D^{**} mesons in semileptonic b-decays, and on the Λ_b^0 polarization are also presented. As some measured quantities are needed for the evaluation of others, an iterative procedure has been adopted to obtain stable results.

Section 3 describes the averaging of the b-hadron lifetime measurements. The combined values are compared with expectations from theory. The inclusive b-hadron and $\overline{\mathrm{B_d^0}}$ meson[3] lifetimes are needed in the determination of $|\mathrm{V}_{ub}|$ and $|\mathrm{V}_{cb}|$ in order to convert measured branching fractions into partial widths that can then be compared with theory.

In Section 4, oscillations of neutral B mesons are studied. Also the production rates of the different b-hadrons in jets induced by a b-quark are determined using direct measurements together with the constraints provided by B mixing. The $\overline{\mathrm{B_d^0}}$ production rate is an important input for the $|\mathrm{V}_{cb}|$ measurement using $\overline{\mathrm{B_d^0}} \to \mathrm{D}^{*+}\ell^-\overline{\nu_\ell}$ decays, and the sensitivity to $\mathrm{B_s^0}$-$\overline{\mathrm{B_s^0}}$ oscillations depends on the $\overline{\mathrm{B_s^0}}$ production rate.

In Section 5, a limit on the decay width difference between mass eigenstates of the $\mathrm{B_s^0}$-$\overline{\mathrm{B_s^0}}$ system is given.

The average number of c and $\overline{c}$ quarks in b-hadron decays is studied in Section 6. Combined measurements of the inclusive semileptonic branching fraction and of charm production in b-hadron decays have been compared with theoretical expectations.

The determination of $|\mathrm{V}_{cb}|$ and $|\mathrm{V}_{ub}|$ presented in Sections 7 and 8 includes only LEP results. The determination of $|\mathrm{V}_{ub}|$ is based on a technique which uses the lepton momentum and the mass of the hadronic system. The accuracy of these results (especially that of $|\mathrm{V}_{cb}|$) depends mostly on theoretical uncertainties. Appendix F gives details on the theoretical inputs used.

Measurements of the time dependence of the $J/\psi \mathrm{K}_s^0$ decay channel for initially produced $\mathrm{B_d^0}$ and $\overline{\mathrm{B_d^0}}$ mesons have been used, in Section 9, to evaluate the phase angle β of CP violation. Other searches for CP violation involving b-quarks have been summarized also in this Section.

In Section 10, the values of the CKM matrix parameters, $\overline{\rho}$ and $\overline{\eta}$, obtained from the measurements of b-decay and oscillation properties are given. These values are compared with those deduced from measurements of CP violation in the K system. As the two approaches give compatible results, a global average is obtained and parameters of the CKM unitarity triangle are determined.

A summary of all results obtained by the different working groups is given in Section 11. In addition, Appendices A-E contain, respectively, the individual measurements of the production rates of narrow D^{**} states, Λ_b^0 polarization, b-hadron lifetimes, direct measurements of b-meson and b-baryon production rates, and measurements on c-hadron production in b decays, that have been used in the present averages.

[2]The present members of the Heavy Flavour Steering Group are: D. Abbaneo, J. Alcaraz, E. Barberio, M. Battaglia, S. Blyth, D. Su, P. Gagnon, R. Hawkings, S. Mele, F. Palla, M. Paulini, P. Roudeau, O. Schneider, A. Stocchi, Ch. Weiser, B. Wicklund and S. Willocq.

[3]Throughout the paper charge conjugate states are implicitly included unless stated otherwise.

2 Common input parameters

The b-hadron properties used as common input parameters in these averages are given in Table 1. Most of the quantities have been taken from results obtained by the LEPEWWG [3, 4] or quoted by the PDG [5]. The others, which concern the production rates and decay properties of D^{**} mesons in b-hadron semileptonic decays and the value of the Λ_b^0 polarization, are explained later in this section.

Quantity	Symbol	Value	Reference
Fraction of b events	R_b	0.21652 ± 0.00069	[3]
Fraction of c events	R_c	0.1702 ± 0.0034	[3]
Beam energy fraction	$< x_E >$	0.702 ± 0.008	[4]
b-hadron sl. BR	$BR(b \to \ell^- \overline{\nu_\ell} X)$	0.1056 ± 0.0021	[3]
Cascade b sl. decay (r.s.)	$BR(b \to \overline{c} \to \ell^- \overline{\nu_\ell} X)$	$0.0162 \pm^{0.0044}_{0.0036}$	[4]
Cascade b sl. decay (w.s.)	$BR(b \to c \to \ell^+ \nu_\ell X)$	0.0801 ± 0.0026	[3]
c-hadron sl. BR	$BR(c \to \ell^+ \nu_\ell X)$	0.0984 ± 0.0032	[3]
b quarks from gluons	$P(g \to b\overline{b})$	0.00254 ± 0.00050	[3]
c quarks from gluons	$P(g \to c\overline{c})$	0.0299 ± 0.0039	[3]
b decay charged mult.	n^b_{ch}	4.955 ± 0.062	[4]
b-hadron mixing	$\overline{\chi}$	0.1194 ± 0.0043	[3]
χ_d at the $\Upsilon(4S)$	$\chi_d(\Upsilon(4S))$	0.182 ± 0.015	[6, 7]
D^{**} in sl. b decays	$BR(\overline{B^0_d} \to D^{**+} \ell^- \overline{\nu_\ell})$	0.0304 ± 0.0038	Sect. 2.1
D^{*+} in D^{**} sl. b decays	$BR(B^- \to D^{*+} \pi^- \ell^- \overline{\nu_\ell})$	0.0129 ± 0.0016	Sect. 2.2
D^{*+} in τ sl. b decays	$BR(\overline{B^0_d} \to D^{*+} \tau^- \overline{\nu_\tau} X)$	0.0127 ± 0.0021	Sect. 2.3
D^{*+} in double charm	$BR(b \to D^{*+} X_{\overline{c}} (\to \ell^- X))$	0.008 ± 0.003	Sect. 2.3
Λ_b^0 polarization	$\mathcal{P}(\Lambda_b^0)$	$-0.45 \pm^{0.19}_{0.17}$	Sect. 2.5
τ in sl. b decays	$BR(b \to \tau^- \overline{\nu_\tau} X)$	0.026 ± 0.004	[5]
J/ψ in b decays	$BR(b \to J/\psi X$	0.0116 ± 0.0010	[5]
D^0 branching fraction	$BR(D^0 \to K^- \pi^+)$	0.0383 ± 0.0009	[5]
D^+ branching fraction	$BR(D^+ \to K^- \pi^+ \pi^+)$	0.090 ± 0.006	[5]
D_s^+ branching fraction	$BR(D_s^+ \to \phi \pi^+)$	0.036 ± 0.009	[5]
Λ_c^+ branching fraction	$BR(\Lambda_c^+ \to p K^- \pi^+)$	0.050 ± 0.013	[5]
D^{*+} branching fraction	$BR(D^{*+} \to D^0 \pi^+)$	0.677 ± 0.005	[5]

Table 1: *Common set of input parameters used for the derivation of the various measurements presented in this paper. The first set of results has been taken from those obtained by the LEPEWWG [3,4] or quoted by the PDG [5], the second set corresponds to averages obtained in the present report and, for the sake of completeness, in the last set, branching fractions in charm decays needed for the production rates of these particles are listed as well. The abbreviated notations sl., r.s. and w.s. correspond, respectively, to semileptonic, right sign and wrong sign.*

Note the following:

- R_b, R_c *and* $< x_E >$

  These values apply only to b-hadrons produced in Z decays. R_b and R_c are the respective branching fractions of the Z boson into $b\overline{b}$ and $c\overline{c}$ pairs in hadronic events,

$< x_E >$ is the mean fraction of the beam energy taken by a weakly decaying b-hadron.

- *shape of the b-quark fragmentation function*

  The value of $< x_E >$ is given in Table 1. To evaluate the corresponding systematic uncertainty, parameter(s) governing b-quark fragmentation functions have been varied in accordance with the uncertainty quoted for $< x_E >$. Fragmentation functions taken from two models [8,9] have been chosen to estimate the systematic uncertainties coming from the shape of the function. These models typically yield results on either side of those obtained using the Peterson function [10], which is commonly used by the experiments. For analyses which are rather insensitive to the fragmentation uncertainty, it is considered adequate to use only the Peterson model and to inflate the uncertainty on $< x_E >$ to ± 0.02.

- *the inclusive semileptonic branching fraction of b-hadrons*

  The average LEP value for $\mathrm{BR}(b \to \ell^- \overline{\nu_\ell} X) = (10.56 \pm 0.11(\mathrm{stat.}) \pm 0.18(\mathrm{syst.}))\%$ is taken from a LEPEWWG fit which combines the heavy flavour measurements performed at the Z without including forward-backward asymmetry measurements.

  The largest contribution to the systematic error comes from the uncertainty on the semileptonic decay model. Including asymmetry measurements gives an inclusive b-hadron semileptonic branching fraction of $(10.56 \pm 0.19)\%$ [3]. In the following the former value has been used.

  In the absence of direct measurements of the semileptonic branching fractions for the different B meson states, it has been assumed, when needed in the following analyses, that all b-hadron semileptonic widths are equal. This hypothesis is strictly valid for B^- and $\overline{\mathrm{B}^0_\mathrm{d}} \to c\ell^- \overline{\nu_\ell}$ decays because of isospin invariance originating from the $b \to c$ transition, which is $\Delta \mathrm{I} = 0$ (similar considerations apply also to $\mathrm{D}^{0,+} \to s\ell^+\nu_\ell$ decays). It is not valid in B^- and $\overline{\mathrm{B}^0_\mathrm{d}} \to \mathrm{u}\ell^- \overline{\nu_\ell}$ decays, but the induced difference between B^- and $\overline{\mathrm{B}^0_\mathrm{d}}$ total semileptonic decay rates can be neglected. It has been assumed valid also for $\overline{\mathrm{B}^0_\mathrm{s}}$ mesons and b-baryons, but for b-baryons an uncertainty of 15% has been added, estimated by comparing the lifetime ratios and semileptonic branching fraction ratios for b-mesons and b-baryons [11]. Exclusive semileptonic branching fraction averages, given in the following for the $\overline{\mathrm{B}^0_\mathrm{d}}$ meson, have been obtained using this hypothesis. Results for other b-hadron flavours can be obtained using, in addition, the corresponding lifetime ratios. The latter are obtained from b-hadron lifetimes given in Section 3.

- *gluon splitting to heavy quarks*

  The quantities $\mathrm{P}(g \to c\overline{c})$ and $\mathrm{P}(g \to b\overline{b})$ are defined as the ratios $\frac{\mathrm{BR}(\mathrm{Z} \to q\overline{q}g,\ g \to Q\overline{Q})}{\mathrm{BR}(\mathrm{Z} \to \mathrm{hadrons})}$ in which, respectively, Q is a c or a b quark.

- *b-hadron decay multiplicity*

  The value given in Table 1 is an average of DELPHI [12] and OPAL [13] measurements which does not include charged decay products from the long lived particles $\mathrm{K^0_S}$ and Λ^0.

2.1 Inclusive D^{**} production rate in *b*-hadron semileptonic decays

All results are quoted for the $\overline{B^0_d}$ taken as reference but include all available information. Corresponding values of the branching fractions for the other b-hadron states, B_i, can be obtained by multiplying $\overline{B^0_d}$ results by the lifetime ratio $\tau(B_i)/\tau(\overline{B^0_d})$.

The inclusive b-hadron semileptonic branching fraction into D^{**} mesons has been measured in three different ways:

- by subtracting the contributions of $\overline{B^0_d} \rightarrow (D^+ + D^{*+})\ell^-\overline{\nu_\ell}$ from the total semileptonic branching fraction of $\overline{B^0_d}$ mesons, yielding:

$$BR(\overline{B^0_d} \rightarrow D^{**+}\ell^-\overline{\nu_\ell}) \;=\; (3.66 \pm 0.37)\% \tag{1}$$

  using the inclusive semileptonic branching fraction given in Table 1 and published values for the exclusive rates [5].
- from a measurement at the $\Upsilon(4S)$ of the rate of final states with a D^{*+} and using models to account for the other channels [14]:

$$BR(\overline{B^0_d} \rightarrow D^{**+}\ell^-\overline{\nu_\ell}) \;=\; (2.7 \pm 0.7)\% \tag{2}$$

- from an inclusive measurement of final states in which a D or a D^{*+} meson is accompanied by a charged hadron and assuming that non-strange D^{**} decay channels involve only $D\pi$ and $D^*\pi$ final states:

$$\begin{aligned} BR(\overline{B^0_d} \rightarrow D^{**+}\ell^-\overline{\nu_\ell}) &= (2.16 \pm 0.30 \pm 0.30)\%\ [15] \\ &= (3.40 \pm 0.52 \pm 0.32)\%\ [16] \end{aligned} \tag{3}$$

As the χ^2 of these four measurements is equal to 2.56 per degree of freedom, and considering that systematic uncertainties may have been underestimated, the uncertainty on the weighted average has been multiplied by 1.6, giving:

$$BR(\overline{B^0_d} \rightarrow D^{**+}\ell^-\overline{\nu_\ell}) \;=\; (3.04 \pm 0.38)\% \tag{4}$$

2.2 D^{**} decays to D^* mesons in semileptonic *b*-decays

Semileptonic decays to excited charm states which subsequently decay to a D^{*+} are a source of correlated (physics) background in studies of $\overline{B^0_d}$ meson properties. It is appropriate to express the different measurements in terms of the parameter b^{**}, defined as the branching fraction of $\overline{B^0_d}$ semileptonic decays involving D^{**} final states in which a D^*, charged or neutral, is produced:

$$b^{**} = BR(\overline{B^0_d} \rightarrow D^{**+}\ell^-\overline{\nu_\ell}) \times BR(D^{**+} \rightarrow D^*X) \tag{5}$$

This is to differentiate such D^{**} decays from those into a D meson directly. Throughout this section it is assumed that decays of D^{**} mesons involve at most one pion (or one kaon for strange states).

The following measurements have been interpreted in terms of the quantity b^{**} and the production fractions (f_{B_i}) and lifetimes ($\tau(B_i)$) of the different types of weakly decaying b-hadrons:

- semi-inclusive measurements of semileptonic decays in which a D^{*+} and a charged pion have been isolated:

$$\begin{aligned}
\mathrm{BR}(b \to \mathrm{D}^{*+}\pi^- \mathrm{X}\ell^-\overline{\nu_\ell}) &= (4.73 \pm 0.77 \pm 0.55) \times 10^{-3} \text{ [15]} \\
&= (4.8 \pm 0.9 \pm 0.5) \times 10^{-3} \text{ [16]} \\
&= f_{\mathrm{B}^+} \frac{2}{3} \mathrm{b}^{**} \frac{\tau(\mathrm{B}^-)}{\tau(\overline{\mathrm{B}^0_\mathrm{d}})} \qquad (6)
\end{aligned}$$

- the ARGUS measurement [14]:

$$\begin{aligned}
\mathrm{BR}(\overline{\mathrm{B}^0_\mathrm{d}} \to \mathrm{D}^{**+}\ell^-\overline{\nu_\ell}) &= (2.7 \pm 0.7)\% \\
&= \frac{\mathrm{b}^{**}}{0.77} \qquad (7)
\end{aligned}$$

  where the value of 0.77 corresponds to the modelling used for D^{**} decays in that analysis;

- the inclusive production rate of charged D^* mesons in semileptonic b decays:

$$\begin{aligned}
\mathrm{BR}(b \to \mathrm{D}^{*+}\mathrm{X}\ell^-\overline{\nu_\ell}) &= (2.75 \pm 0.17 \pm 0.16)\% \text{ [16]} \\
&= (2.86 \pm 0.18 \pm 0.23)\% \text{ [17]} \\
&= f_{\mathrm{B_d}} \mathrm{b}^* + f_{\mathrm{B_d}} \frac{1}{3}\mathrm{b}^{**} + f_{\mathrm{B}^+}\frac{2}{3}\mathrm{b}^{**}\frac{\tau(\mathrm{B}^-)}{\tau(\overline{\mathrm{B}^0_\mathrm{d}})} \qquad (8) \\
&\quad + f_{\mathrm{B_s}} \frac{\alpha}{2}\mathrm{b}^{**}\frac{\tau(\overline{\mathrm{B}^0_\mathrm{s}})}{\tau(\overline{\mathrm{B}^0_\mathrm{d}})}
\end{aligned}$$

  where the quantity b^* is the exclusive semileptonic branching fraction:

$$\mathrm{b}^* = \mathrm{BR}(\overline{\mathrm{B}^0_\mathrm{d}} \to \mathrm{D}^{*+}\ell^-\overline{\nu_\ell}) = (4.67 \pm 0.26)\% \qquad (9)$$

  which has been obtained by averaging results quoted by the PDG [5] for corresponding decays of $\overline{\mathrm{B}^0_\mathrm{d}}$ and B^- mesons, and the scaling factor $\alpha = 0.75 \pm 0.25$ has been introduced in (8) to account for a possible SU(3) flavour violation when comparing $\mathrm{D}_\mathrm{s}^{**+} \to \mathrm{D}^{*+}\mathrm{K}^0$ and $\mathrm{D}^{**0} \to \mathrm{D}^{*+}\pi^-$ decays.

In the above, the lifetimes ($\tau(\mathrm{B}_i)$) and production fractions (f_{B_i}) are taken from Sections 3 and 4 respectively.

These measurements form a coherent set of results yielding:

$$\mathrm{b}^{**} = \mathrm{BR}(\overline{\mathrm{B}^0_\mathrm{d}} \to \mathrm{D}^{**+}\ell^-\overline{\nu_\ell}) \times \mathrm{BR}(\mathrm{D}^{**+} \to \mathrm{D}^*\mathrm{X}) = (1.82 \pm 0.21 \pm 0.08)\% \qquad (10)$$

From this result, and using the same hypotheses, it is possible to derive other quantities of interest in several analyses presented in this paper such as:

$$\mathrm{BR}(\mathrm{B}^- \to \mathrm{D}^{**0}(\to \mathrm{D}^{*+}\pi^-)\ell^-\overline{\nu_\ell}) = \frac{2}{3}\mathrm{b}^{**}\frac{\tau(\mathrm{B}^-)}{\tau(\overline{\mathrm{B}^0_\mathrm{d}})} = (1.29 \pm 0.16)\% \qquad (11)$$

$$\mathrm{BR}(\overline{\mathrm{B}^0_\mathrm{d}} \to \mathrm{D}^{*+}\pi^0\ell^-\overline{\nu_\ell}) = \frac{1}{3}b^{**} = (0.61 \pm 0.08)\% \qquad (12)$$

$$\mathrm{BR}(\overline{\mathrm{B}^0_\mathrm{s}} \to \mathrm{D}^{*+}\mathrm{K}^0\ell^-\overline{\nu_\ell}) = \frac{1}{2}b^{**}\frac{\tau(\overline{\mathrm{B}^0_\mathrm{s}})}{\tau(\overline{\mathrm{B}^0_\mathrm{d}})}\alpha = (0.65 \pm 0.23)\%. \qquad (13)$$

It is also possible to determine the fraction of $\overline{\mathrm{B}^0_\mathrm{d}}$ semileptonic decays which contain a D^{*+}:

$$\frac{\mathrm{BR}(\overline{\mathrm{B}^0_\mathrm{d}} \to \mathrm{D}^{*+}\ell^-\overline{\nu_\ell}) + \mathrm{BR}(\overline{\mathrm{B}^0_\mathrm{d}} \to \mathrm{D}^{*+}\pi\ell^-\overline{\nu_\ell})}{\mathrm{BR}(\overline{\mathrm{B}^0_\mathrm{d}} \to \ell^-\overline{\nu_\ell}\mathrm{X})} = 0.50 \pm 0.03 \tag{14}$$

and the fraction of b-quark jets in which the D^{*+} comes from a D^{**} decay:

$$\frac{\mathrm{BR}(b \to \mathrm{D}^{*+}\pi\ell^-\overline{\nu_\ell})}{\mathrm{BR}(b \to \mathrm{D}^{*+}\ell^-\overline{\nu_\ell}) + \mathrm{BR}(b \to \mathrm{D}^{*+}\pi\ell^-\overline{\nu_\ell})} = 0.31 \pm 0.04 \tag{15}$$

As these results (10) to (15) are highly correlated they are represented in Table 1 by a single entry, chosen to be result (11).

2.3 Other semileptonic decays to D^{*+} mesons

Other mechanisms giving a D^{*+} accompanied by a lepton are important because they are further sources of background for the exclusive channel $\overline{\mathrm{B}^0_\mathrm{d}} \to \mathrm{D}^{*+}\ell^-\overline{\nu_\ell}$.

2.3.1 Charged D^* production in semileptonic decays of b-hadrons involving τ leptons

The value of $\mathrm{BR}(\overline{\mathrm{B}^0_\mathrm{d}} \to \mathrm{D}^{*+}\tau^-\overline{\nu_\tau}\mathrm{X})$ quoted in Table 1 is obtained from the average measured value of $\mathrm{BR}(b \to \tau\overline{\nu_\tau}\mathrm{X})$ taken from the PDG [5] and assuming that a D^{*+} is produced in $(50 \pm 10)\%$ of the cases, as measured in semileptonic decays involving light leptons. The uncertainty on this last number has been increased, as compared to the value obtained in Equation (14), to account for a possible different behaviour of decays when a τ is produced instead of a light lepton because of differences in the masses involved in the final state [18].

2.3.2 Charged D^* production in double charm semileptonic ($b \to \overline{c} \to \ell$) decays

The double charm production rate in b-decays was measured by the ALEPH [19] and CLEO [20] Collaborations, isolating the contributions of different D meson species. From these measurements the inclusive semileptonic branching fraction, involving a wrong sign charmed meson, has been obtained [4]:

$$\mathrm{BR}(b \to \overline{c} \to \ell) \;=\; 0.0162^{+0.0044}_{-0.0036}. \tag{16}$$

The product $\mathrm{BR}(\mathrm{b} \to \mathrm{D}^{*+}\mathrm{X}_{\overline{c}}\mathrm{Y}) \times \mathrm{BR}(\mathrm{X}_{\overline{c}} \to \ell^-\mathrm{X})$ quoted in Table 1 is then determined assuming again that a D^{*+} is produced in $(50 \pm 10)\%$ of the cases.

2.4 Production of narrow D** states in *b*-hadron semileptonic decays

A modelling of b-hadron semileptonic decays requires detailed rate measurements of the different produced states which can be resonant or non-resonant $\mathrm{D}^{(*)}n\pi$ systems[4], each having characteristic decay properties. As an example, there are four orbitally excited states with L=1. They can be grouped in two pairs according to the value of the spin of the light system, $j = \mathrm{L} \pm 1/2$ (L=1).

States with $j = 3/2$ can have $\mathrm{J^P} = 1^+$ and 2^+. The 1^+ state decays only through $\mathrm{D}^*\pi$, and the 2^+ through $\mathrm{D}\pi$ or $\mathrm{D}^*\pi$. Parity and angular momentum conservation imply that in the 2^+ the D^* and π are in a D wave but allow both S and D waves in the 1^+. However, if the heavy quark spin is assumed to decouple, conservation of $j(=3/2)$ forbids S waves even in the 1^+. An important D wave component, and the fact that the masses of these states are not far from threshold, imply that the $j = 3/2$ states are narrow. These states have been observed with a typical width of 20 MeV/c^2, in accordance with the expectation.

On the contrary, $j = 1/2$ states can have $\mathrm{J^P} = 0^+$ and 1^+, so they are expected to decay mainly through an S wave and to be broad resonances with typical widths of several hundred MeV/c^2. The first experimental evidence for broad 1^+ states has been obtained by CLEO [21].

At present, information on the composition of b-hadron semileptonic decays involving D** is rather scarce. ALEPH [15], CLEO [22] and DELPHI [23] have reported evidence for the production of narrow resonant states (D_1 and D_2^*), which can be summarised as follows:

$$\mathrm{BR}(\overline{\mathrm{B}} \to \mathrm{D}_1 \ell^- \overline{\nu_\ell}) = (0.63 \pm 0.10)\% \tag{17}$$

$$\mathrm{BR}(\overline{\mathrm{B}} \to \mathrm{D}_2^* \ell^- \overline{\nu_\ell}) = (0.23 \pm 0.08)\% \text{ or } < 0.4\% \text{ at the 95\% CL} \tag{18}$$

$$\mathrm{R}^{**} = \frac{\mathrm{BR}(\overline{\mathrm{B}} \to \mathrm{D}_2^* \ell^- \overline{\nu_\ell})}{\mathrm{BR}(\overline{\mathrm{B}} \to \mathrm{D}_1 \ell^- \overline{\nu_\ell})} = 0.37 \pm 0.14 \text{ or } < 0.6 \text{ at the 95\% CL} \tag{19}$$

where R** is the ratio between the production rates of D_2^* and D_1 in b-meson semileptonic decays. The original measurements are listed in Appendix A. Absolute rates for the D_1 and D_2^* mesons have been obtained assuming that the former decays always into $\mathrm{D}^*\pi$ only and the latter decays into $\mathrm{D}\pi$ in $(71 \pm 7)\%$ of the cases, as measured by ARGUS [24] and CLEO [25].

It is of interest to note that these results are very different from naive expectations from HQET in which $\mathrm{R}^{**} \simeq 1.6$ [26]. This implies large $1/m_c$ corrections in these models, as was noted for instance in [27]. These corrections not only reduce the expected value for R**, they also enhance the expected production rate of $j = 1/2$ states which can become of similar importance as the $j = 3/2$ rate.

Comparing Equations (17-18) and (4), it can be deduced that narrow D** states account for less than one third of the total D** rate. If, in addition, model expectations for the production rates of $j = 1/2$ states are used, it can be concluded that non resonant production of $\mathrm{D}^{(*)}\pi$ states is as important as, or even larger than,

[4] In the following the study has been limited to n=1.

the production of resonant states. As a consequence, models like ISGW [28, 29], which contain only resonant charmed states, do not provide a complete description of b-hadron semileptonic decays.

2.5 Λ_b^0 polarization

Even for unpolarized $e^{\pm}$ beams, the b-quarks produced in Z decays are highly polarized: $\mathcal{P}_b = -0.94$. Hard gluon emission and quark mass effects are expected to change $\mathcal{P}_b$ by only 3% [30]. However, during the hadronization process of the heavy quark, part or all of the initial b-quark polarization may be lost by the final weakly decaying b-hadron state. The b-mesons always decay finally to spin zero pseudoscalar states, which do not retain any polarization information. In contrast, Λ_b^0 baryons are expected to carry the initial b-quark polarization since the light quarks are arranged, according to the constituent quark model, in a spin-0 and isospin-0 singlet. However, b-quark fragmentation into intermediate $\Sigma_b^{(*)}$ states can lead to a depolarization of the heavy quark [31].

The $\Lambda\ell$ final state is used to select event samples enriched with Λ_b^0 semileptonic decays, and the polarization is measured using the average values of the lepton and neutrino energies. Samples of events enriched in b-mesons, which carry no polarization information, are used to calibrate the measurements.

Measurements from ALEPH [32], DELPHI [33] and OPAL [34], collected in Appendix B, have been averaged, assuming systematic uncertainties to be correlated, yielding:

$$\mathcal{P}(\Lambda_b^0) \quad = \quad -0.45^{+0.17}_{-0.15} \pm 0.08 \tag{20}$$

where the uncertainty is still dominated by the statistics. The following sources of systematic uncertainties, common to all measurements, have been considered:

- b-quark fragmentation,
- Λ production in semileptonic and inclusive decays of the Λ_c^+,
- Λ_c^+ polarization,
- theoretical uncertainties in the modelling of polarized Λ_b^0 semileptonic decays.

3 Averages of b-hadron lifetimes

Best estimates for the various b-hadron lifetimes, for the ratio of the B^+ and B_d^0 lifetimes, and for the average lifetime of a sample of b-hadrons produced in b-jets have been obtained by the B lifetime working group[5]. Details on the procedure used to combine the different measurements can be found in [35].

The measurements used in the averages presented here are listed in Appendix C. Possible biases can originate from the averaging procedure. They depend on the statistics and time resolution of each measurement and in the way systematic uncertainties have been included. These effects have been studied in detail using simulations and have been measured to be at a level which can be neglected as compared to the statistical accuracy.

[5]The present members of the B lifetime working group are: J. Alcaraz, L. Di Ciaccio, T. Hessing, I.J. Kroll, H.G. Moser and C. Shepherd-Themistocleous.

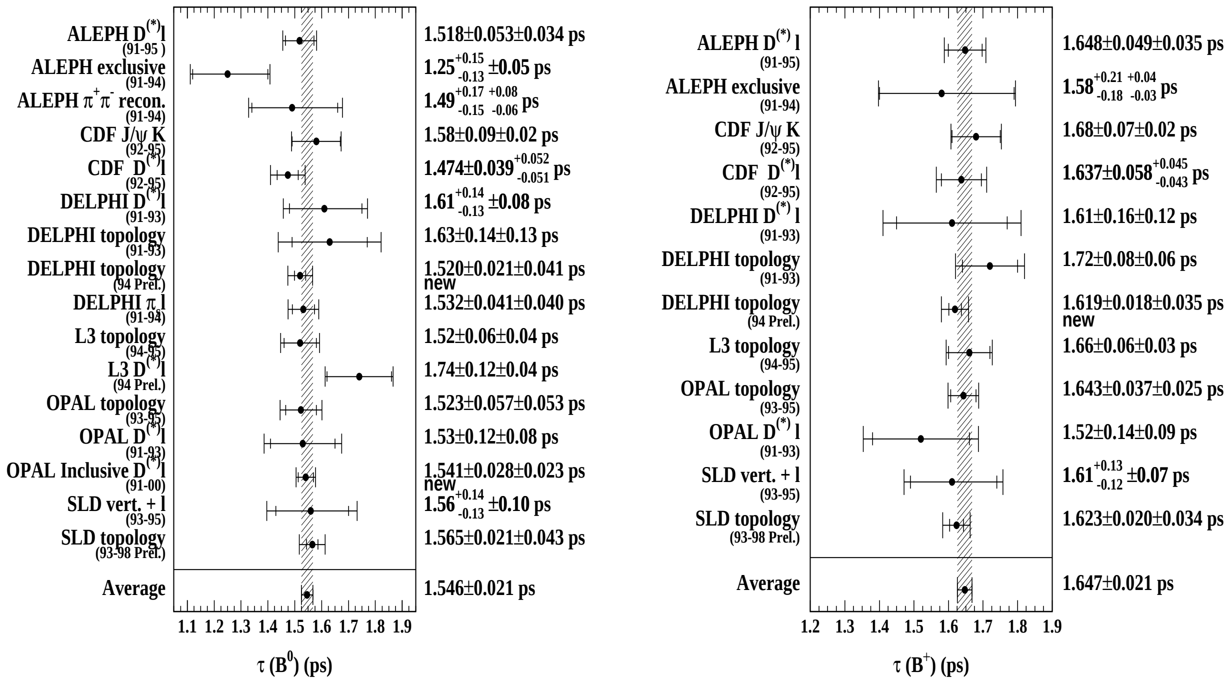

Figure 1: *Left:* $\mathrm{B_d^0}$ *lifetime measurements. Right:* $\mathrm{B^+}$ *lifetime measurements. Hatched areas correspond to present averages of the measurements given in the corresponding figures. Internal error bars correspond to statistical uncertainties and full error bars include systematics.*

3.1 Dominant sources of systematic uncertainties

The dominant sources of systematic uncertainties leading to correlations between measurements are briefly reviewed below (more details can be found in [35]). These are the background estimation, the evaluation of the b-hadron momentum, and the decay length reconstruction. Depending on the level of correlation of the various components, the obtained systematic uncertainty on the average can be smaller than the total systematic uncertainty affecting individual measurements.

The background can be due either to physics processes leading to a final state similar to that used to tag the signal, or to accidental combinations of tracks which simulate the decay of interest. When "physics" background is present, the experimental uncertainties on the branching fractions of the background processes and on the lifetimes of the background particles lead to a systematic error which is correlated between different experiments. When the background is combinatorial, the amount and/or the lifetime of the background "particles" is normally extracted from the data using the sidebands of mass distributions or wrong sign combinations. In this case the related systematic uncertainty is usually not correlated between experiments. But, in the measurement of the b-baryon lifetime using $\Lambda\ell$ correlations, the amount of accidental background is obtained from the wrong sign combinations and a correction, common to all analyses, has to be applied to this number to take into account the production asymmetry of accidental $\Lambda\ell$ pairs.

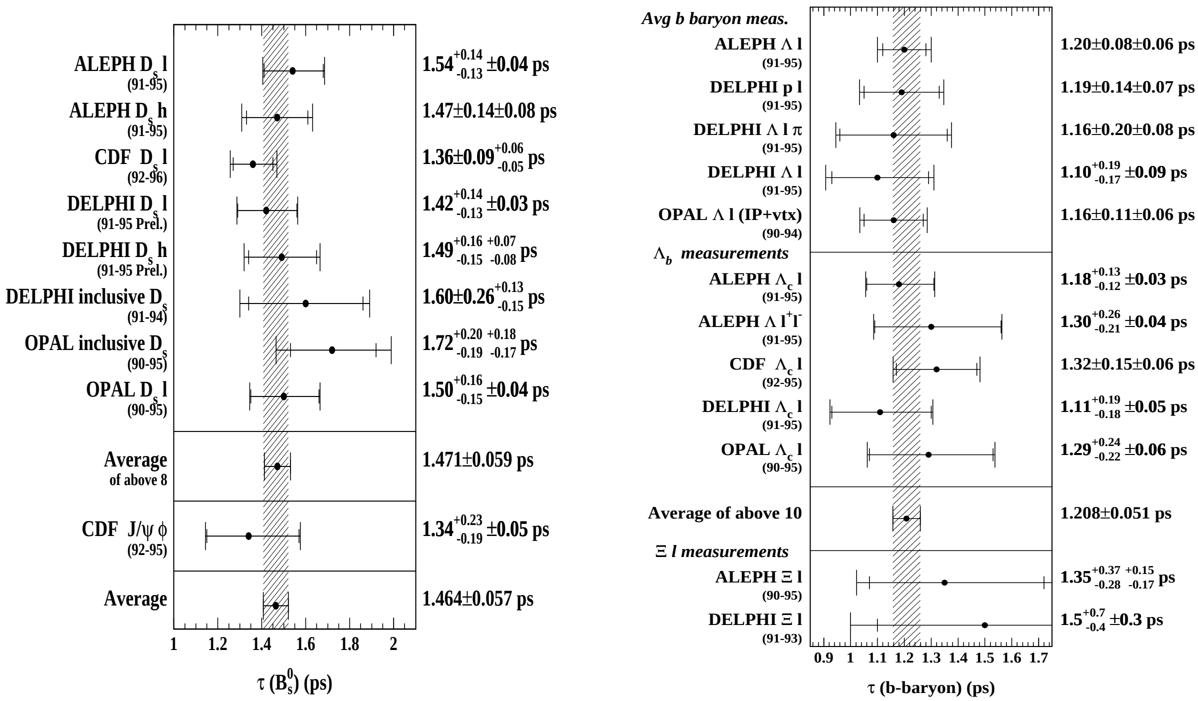

Figure 2: *Left:* $\mathrm{B_s^0}$ *lifetime measurements, Right: b-baryon lifetime measurements. Hatched areas correspond to present averages of the measurements given in the corresponding figures. Internal error bars correspond to statistical uncertainties and full error bars include systematics.*

Most of the exclusive b-lifetime measurements are based on the reconstruction of b-decay length and momentum. In most analyses the b-particles are only partially reconstructed, and their energies are estimated from the energies of the detected decay products. In all cases, systematic uncertainties have been evaluated for the following effects:

- determination of the b-quark fragmentation function,
- branching fractions of b- and c-hadrons,
- b-hadron masses,
- b-baryon polarization and
- modelling of neutral hadronic energy.

Finally there are uncertainties correlated within an experiment. They are due to primary and secondary vertex reconstruction procedures, detector resolution, tracking errors, B flight direction reconstruction and detector alignment.

3.2 Measurements of $\mathrm{B_d^0}$ and $\mathrm{B^+}$ lifetimes

Apart from the measurements by CDF using large samples of exclusive $\mathrm{B_d^0} \to \mathrm{J}/\psi \mathrm{K}^{(*)0}$ and $\mathrm{B^+} \to \mathrm{J}/\psi \mathrm{K^+}$ decays, the most precise measurements of $\mathrm{B_d^0}$ and $\mathrm{B^+}$ lifetimes originate from two classes of partially reconstructed decays. In the first class the decay $\mathrm{B} \rightarrow \overline{\mathrm{D}^{(*)}}\ell^{+}\nu_{\ell}\mathrm{X}$ is used in which the charge of the charmed meson distinguishes between neutral and charged b-mesons. In the second class the charge attached to the b-decay vertex is used to achieve this separation.

The following sources of correlated systematic uncertainties have been considered: background composition (includes D^{**} branching fraction uncertainties as obtained in Section 2), momentum estimation, lifetimes of $\mathrm{B}_{\mathrm{s}}^{0}$ and b-baryons (as obtained in Sections 3.3 and 3.4), and fractions of $\mathrm{B}_{\mathrm{s}}^{0}$ and b-baryons produced in Z decays (as measured in Section 4.1).

The world average lifetimes of $\mathrm{B}_{\mathrm{d}}^{0}$ and B^{+} mesons are:

$$\tau(\mathrm{B}_{\mathrm{d}}^{0}) = (1.546 \pm 0.021)\ \mathrm{ps} \tag{21}$$

$$\tau(\mathrm{B}^{+}) = (1.647 \pm 0.021)\ \mathrm{ps}. \tag{22}$$

The various measurements [36–50] and the average values are shown in Figure 1. The respective values for the χ^2/NDF ratio of the fits are 9.1/16 and 2.2/12.

3.3 $\mathrm{B}_{\mathrm{s}}^{0}$ lifetime measurements

The most precise measurements of the $\mathrm{B}_{\mathrm{s}}^{0}$ lifetime originate from partially reconstructed decays in which a D_{s}^{-} meson has been completely reconstructed (see Figure 2-left).

The following sources of correlated systematic uncertainties have been considered: average b-hadron lifetime used for backgrounds, $\mathrm{B}_{\mathrm{s}}^{0}$ decay multiplicity, and branching fractions of beauty and charmed hadrons.

The world average lifetime of $\mathrm{B}_{\mathrm{s}}^{0}$ mesons is equal to:

$$\tau(\mathrm{B}_{\mathrm{s}}^{0}) = (1.464 \pm 0.057)\ \mathrm{ps} \tag{23}$$

This result has been obtained neglecting a possible difference, $\Delta\Gamma_{\mathrm{B}_{\mathrm{s}}^{0}}$, between the decay widths of the two mass eigenstates of the $\mathrm{B}_{\mathrm{s}}^{0} - \overline{\mathrm{B}_{\mathrm{s}}^{0}}$ system. The measurement of $\Delta\Gamma_{\mathrm{B}_{\mathrm{s}}^{0}}$ is explained in Section 5. The various measurements [39, 51–58] and the average value are shown in Figure 2-left.

3.4 *b*-baryon lifetime measurements

The most precise measurements of the b-baryon lifetime originate from two classes of partially reconstructed decays (see Figure 2-right). In the first class, decays with an exclusively reconstructed Λ_{c}^{+} baryon and a lepton of opposite charge are used. In the second class, more inclusive final states with a baryon (p, $\overline{\mathrm{p}}$, Λ or $\overline{\Lambda}$) and a lepton have been used.

The following sources of correlated systematic uncertainties have been considered: experimental time resolution within a given experiment, b-quark fragmentation distribution into weakly decaying b-baryons, Λ_{b}^{0} polarization, decay model, and evaluation of the b-baryon purity in the selected event samples. As the measured b-hadron lifetime is proportional to the assumed b-hadron mass, the central values of the masses are scaled to $\mathrm{m}(\Lambda_b) = (5624 \pm 9)\ \mathrm{MeV/c^2}$ and $\mathrm{m}(b\text{-baryon}) = (5670 \pm 100)\ \mathrm{MeV/c^2}$, before computing the averages. Uncertainties related to the decay model are dominated by assumptions on the fraction of n-body decays. To be conservative it is assumed that they are correlated

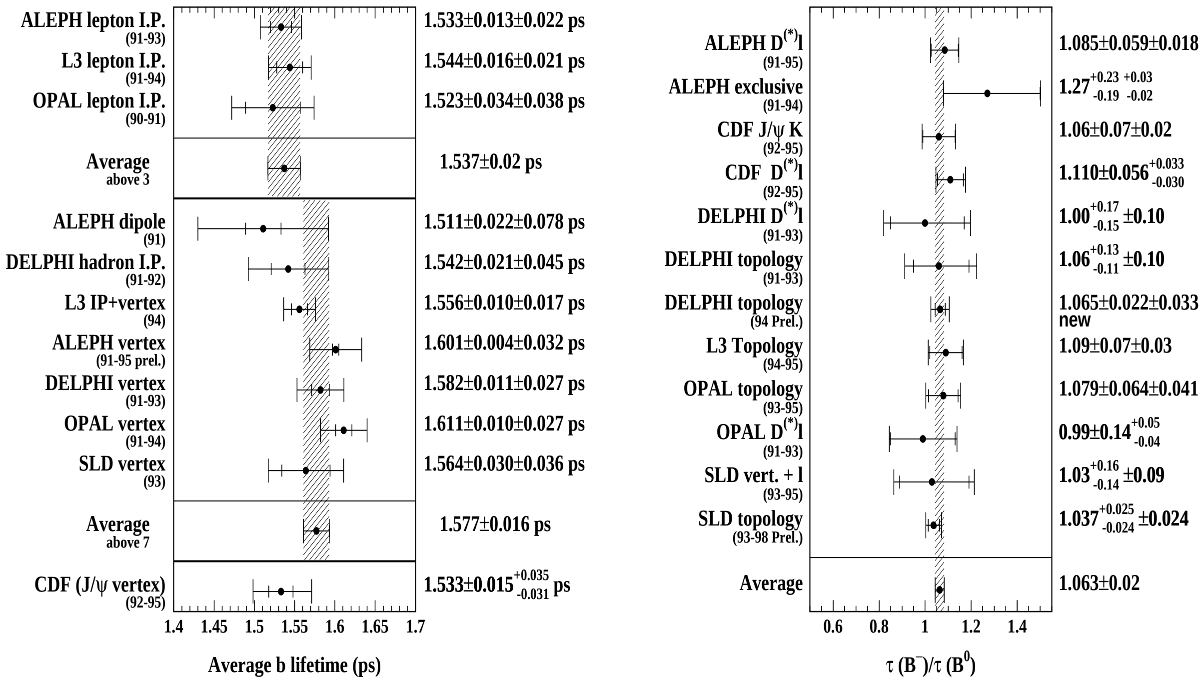

Figure 3: *Left: Measurements of the average b-hadron lifetime. Right: Ratio between the* B^- *and* $\overline{\mathrm{B}^0_\mathrm{d}}$ *lifetimes. Hatched areas correspond to present averages of the measurements given in the corresponding figures. Internal error bars correspond to statistical uncertainties and full error bars include systematics.*

whenever given as an error. Furthermore, in computing the average, results have been corrected for the effect of the measured value of the Λ_b^0 polarization and it has been assumed that b-baryons have the same fragmentation distribution as all b-hadrons (Section 2).

The world average lifetime of b-baryons is then:

$$\tau(b - \text{baryon}) \quad = \quad (1.208^{+0.051}_{-0.050}) \text{ ps} \tag{24}$$

This value and the single measurements [59–64] are given in Figure 2-right. Keeping only $\Lambda_c^\pm \ell^\mp$ and $\Lambda \ell^- \ell^+$ final states, as representative of the Λ_b^0 baryon, the following lifetime is obtained:

$$\tau(\Lambda_b^0) \quad = \quad (1.229^{+0.081}_{-0.079}) \text{ ps} \tag{25}$$

Averaging the measurements based on the $\Xi^\mp \ell^\mp$ final states gives [65, 66] a lifetime value for a sample of events containing Ξ_b^0 and Ξ_b^- baryons:

$$\tau(\Xi_b) \quad = \quad (1.39^{+0.34}_{-0.28}) \text{ ps} \tag{26}$$

3.5 Average b-hadron lifetime

Available results have been divided into three different sets and a separate average has been computed for each set (see Figure 3-left):

(1) measurements at LEP based on the identification of a lepton from a b-hadron decay:

$$\tau_b^{sl} = (1.537 \pm 0.020) \text{ ps} \tag{27}$$

(2) measurements at LEP and SLD which accept any b-hadron decay:

$$\tau_b^{incl.} = (1.577 \pm 0.016) \text{ ps} \tag{28}$$

(3) measurement at CDF based on the identification of a J/ψ from a b-hadron decay:

$$\tau_b^{\mathrm{J}/\psi} = (1.533^{+0.038}_{-0.034}) \text{ ps} \tag{29}$$

The reason for this division is that, since the lifetimes of the individual b-hadron species are different, a meaningful average lifetime can only be computed for samples in which the composition is the same.

The following sources of correlated systematic uncertainties have been considered when evaluating the averages: b- and c-quark fragmentation and decay models, $\mathrm{BR}(b \to \ell)$, $\mathrm{BR}(b \to c \to \overline{\ell})$, τ_c, $\mathrm{BR}(c \to \overline{\ell})$ and the charged track multiplicity in b-hadron decays (see Table 1).

The measurement in Set 2 is the average b-hadron lifetime for a sample of weakly decaying b-hadrons produced in Z decays.

$$\tau_b = f_{\mathrm{B_d}}\tau(\mathrm{B^0_d}) + \mathrm{f}_{\mathrm{B^+}}\tau(\mathrm{B^+}) + \mathrm{f}_{\mathrm{B_s}}\tau(\mathrm{B^0_s}) + \mathrm{f}_{b\text{-baryon}}\tau(b - \text{baryon}) \tag{30}$$

The measurements of Set 1 are based on samples containing more B^+ and less b-baryons than in Set 2, and thus are expected to correspond to a higher average lifetime. In practice this difference should be small. Assuming that all b-hadrons have the same semileptonic partial width, the average lifetimes in the inclusive and semileptonic samples can be related using the measured values of the production rates (Section 4) and lifetimes of the different types of b-hadrons, giving:

$$\frac{\tau_b^{sl}}{\tau_b} = \frac{\sum_i f_{\mathrm{B}_i}\tau(\mathrm{B}_i)^2}{\sum_i f_{\mathrm{B}_i}\tau(\mathrm{B}_i)} = 1.006 \pm 0.003 \tag{31}$$

where the dominant uncertainty comes from the possible difference of 15% assumed between the semileptonic width of b-baryons as compared to other b-hadrons.

Combining the ten measurements corresponding to $\tau_b^{incl.}$ and τ_b^{sl}, with the correction of Equation (31) included, the average b-hadron lifetime is:

$$\tau_b = (1.561 \pm 0.014) \text{ ps}. \tag{32}$$

The global χ^2/NDF is equal to 10.1/10. It can be noted that there is a 2.1σ difference between the partial averages obtained using semileptonic decays or more inclusive final states which may indicate an underestimate of systematic uncertainties. Measurements using $\mathrm{J}/\psi\mathrm{X}$ final states, which may correspond to a different composition in b-hadron species, as compared to any of the previous sets of measurements, have not been included in the average.

Lifetime ratio	Measured value
$\tau(\mathrm{B}^+)/\tau(\mathrm{B}^0_\mathrm{d})$	1.063 ± 0.020
$\tau(\mathrm{B}^0_\mathrm{s})/\tau(\mathrm{B}^0_\mathrm{d})$	0.946 ± 0.039
$\tau(b-\mathrm{baryon})/\tau(\mathrm{B}^0_\mathrm{d})$	0.780 ± 0.035

Table 2: *Ratios of b-hadron lifetimes relative to the* B^0_d *lifetime.*

3.6 *b*-hadron lifetime ratios and expectations from theory

Ratios of other b-hadron lifetimes to the B^0_d lifetime have also been obtained, using the averages of the individual lifetimes and the direct measurements of the ratio between B^+ and B^0_d lifetimes shown in Figure 3-right. The results are given in Table 2.

Inclusive properties of heavy hadron decays have been analyzed using the Wilson formalism of Operator Product Expansion. Total or partial decay widths are expressed as a series of local operators with increasing dimension, where coefficients contain inverse powers of the b-quark mass starting as $1/m_b^2$ [76]. Average values of the local operators are also expanded in inverse powers of the b-quark mass using the Heavy Quark Effective Theory. Terms entering into this expansion receive contributions from QCD operating in the non-perturbative regime and have been determined using approximations or models. Within the factorization approximation, they obtain [77]:

$$\frac{\tau(\mathrm{B}^+)}{\tau(\mathrm{B}^0_\mathrm{d})} \;=\; 1 + 0.05 \times \frac{f_B^2}{(200\ \mathrm{MeV})^2},\; \frac{\tau(\mathrm{B}^0_\mathrm{s})}{\tau(\mathrm{B}^0_\mathrm{d})} \;=\; 1 \pm \mathcal{O}(1\%),\; \frac{\tau(\Lambda^0_b)}{\tau(\mathrm{B}^0_\mathrm{d})} \;=\; 0.90 - 0.95. \quad (33)$$

For b-meson lifetimes, these predictions had encountered theoretical criticism [78] but the improvement in the experimental accuracy on B^0_d and B^+ lifetime determinations shows that they were in agreement with the measurements. In addition, a recent lattice study [79] finds a result quite consistent with [77].

However a significant discrepancy is observed for baryons. It remains to be clarified if this problem is related only to the validity of the quark models used to determine the parameters in the case of baryons, as was suggested in reference [77], or to more basic defects in the whole theoretical picture. Lattice evaluations for b-baryons are starting to be available [80] and corrections appear to be larger for baryons than for mesons. Present results indicate that contributions involving spectator partons may account for half of the discrepancy. The remaining difference can come from a violation of quark-hadron duality or, owing to present uncertainties, to contributions of higher order terms. It should be noted that a possible violation of duality in inclusive b-decays does not impair the use of OPE to extract $|\mathrm{V}_{cb}|$ from measurements of the inclusive b-hadron semileptonic decay width (Section 7.1) because the latter corresponds to a completely different situation in which such violations are expected to be smaller [81]. Possible evidence for duality violation in b-semileptonic decays has to be searched for directly using the corresponding final states.

4 $\mathrm{B^0_d}$ and $\mathrm{B^0_s}$ oscillations and *b*-hadron production fractions

The four LEP Collaborations and CDF and SLD have published or otherwise released measurements of Δm_d [82–87] and lower limits for Δm_s [88–92]. Combined results, as well as estimates of b-hadron fractions, have been prepared by the B oscillation working group[6].

The estimates of the b-fractions, described in Section 4.1, are important inputs needed for the combination of the $\mathrm{B^0_d}$ and $\mathrm{B^0_s}$ oscillation results. The procedure used to combine Δm_d results is explained in Section 4.2, followed by a description of the common systematic uncertainties in the analyses, and the result for the mean value of Δm_d. Then combined results on the $\mathrm{B^0_s}$ oscillation amplitude, as well as an overall lower limit on Δm_s are presented in Section 4.3.

4.1 Measurements of *b*-hadron production rates in *b*-jets

In this analysis, the relative production rates of the different types of weakly-decaying b-hadrons are assumed to be similar in b-jets originating from Z decays and in high transverse momentum b-jets in $\mathrm{p\overline{p}}$ collisions at 1.8 TeV. This hypothesis can be justified by considering the last steps of jet hadronization to be a non-perturbative QCD process occurring at a scale of order Λ_{QCD}.

Direct information on production rates is available from measurements of branching fraction products using channels with characteristic signatures. Dedicated analyses have also been developed which are sensitive to the production of charged and neutral b-hadrons [93] and of b-baryons [94] without requiring the knowledge of specific branching fractions. All the measurements used in the present analysis are listed in Appendix D.

At CDF and LEP, the $\mathrm{\overline{B^0_s}}$ production rate has been evaluated using events with a D_s^+ accompanied by a lepton of opposite sign in the final state (Table 25). Since the rate of these events is given by $f_{\mathrm{B_s}} \times \mathrm{BR}(\mathrm{B^0_s} \to \mathrm{D^-_s}\ell^+\nu_\ell \mathrm{X})$, it is necessary to evaluate $\mathrm{BR}(\mathrm{B^0_s} \to \mathrm{D^-_s}\ell^+\nu_\ell \mathrm{X})$. This has been done by assuming, from SU(3) symmetry, that the partial semileptonic decay widths into D, D^* and D^{**} final states are the same for all b-mesons. A lower value for the $\mathrm{B^0_s}$ semileptonic branching fraction with a D_s^- in the final state is obtained by assuming that all $\overline{\mathrm{D}_s^{**}}$ states decay into a non-strange D meson. The branching fraction corresponding to $\mathrm{B^0_s} \to \mathrm{D^{**-}_s}\ell^+\nu_\ell$ is obtained using Equation (1) multiplied by the lifetime ratio $\tau(\mathrm{B^0_s})/\tau(\mathrm{B^0_d})$. This corresponds to a lower limit on D_s^- production because the possibility that D_s^{**-} states decay into $\mathrm{D}_s^-\overline{\mathrm{K}}\mathrm{K}$ or $\mathrm{D}_s^-\eta^{(\prime)}$ has been neglected. In addition, assuming that only the measured $\mathrm{D}^{(*)}\pi$ final state in $\mathrm{B^0_d}$ or B^+ semileptonic decays corresponds to $\mathrm{D}^{(*)}\mathrm{K}$ transitions for the $\mathrm{B^0_s}$, an upper value can be obtained for the $\mathrm{B^0_s}$ decay of interest using the relation:

$$\Gamma(\mathrm{B^0_d} \to \overline{\mathrm{D}^{(*)0}}\pi^-\ell^+\nu_\ell) = \frac{4}{3}\Gamma(\mathrm{B^0_s} \to \overline{\mathrm{D}^{(*)0}}\mathrm{K}^-\ell^+\nu_\ell) \qquad (34)$$

based on isospin symmetry. In practice, the lower and upper values are compatible within uncertainties. This allows $f_{\mathrm{B_s}} = (12.2^{+4.5}_{-3.1})\%$ to be extracted from the LEP measure-

[6] The present members of the B oscillation working group are: V. Andreev, E. Barberio, G. Boix, C. Bourdarios, P. Checchia, O. Hayes, R. Hawkings, O. Leroy, S. Mele, H-G. Moser, F. Parodi, M. Paulini, P. Privitera, P. Roudeau, O. Schneider, A. Stocchi, T. Usher, C. Weiser and S. Willocq.

b-hadron fractions	correlation coefficients
$f_{\mathrm{B_s}} = (9.2 \pm 2.4)\%$	
$f_{b\text{-baryon}} = (10.5 \pm 2.0)\%$	$\rho(f_{\mathrm{B_s}}, f_{b\text{-baryon}}) = -0.36$
$f_{\mathrm{B_d}} = f_{\mathrm{B^+}} = (40.1 \pm 1.3)\%$	$\rho(f_{\mathrm{B_d}}, f_{\mathrm{B_s}}) = -0.68$, $\rho(f_{\mathrm{B_d}}, f_{b\text{-baryon}}) = -0.45$

Table 3: *Average values of b-hadron production rates and their correlation coefficients obtained from direct measurements.*

ments. Using the same assumptions, the CDF Collaboration has measured the ratio $f_{\mathrm{B_s}}/(f_{\mathrm{B^+}} + f_{\mathrm{B_d}}) = 0.213 \pm 0.068$ from final states with an electron and a charm meson ($\mathrm{D^0}$, $\mathrm{D^+}$, $\mathrm{D^{*+}}$, D_s^+) [98]. CDF has published a second measurement of the same ratio, using double semileptonic B decays with $\phi\ell$ and $\mathrm{K}^*\ell$ final states as characteristic signatures for $\mathrm{B_s^0}$ and non-strange B mesons [100], giving $f_{\mathrm{B_s}}/(f_{\mathrm{B^+}} + f_{\mathrm{B_d}}) = 0.210 \pm 0.073$ [7].

In a similar way, the fraction of b-baryons is estimated from the measured production rates of $\Lambda_c^+\ell^-$ [101,102] and $\Xi^-\ell^-$ [103,104] final states (Table 26) yielding, respectively, $f_{\Lambda_b^0} = (11.6^{+4.6}_{-3.1})\%$ and $f_{\Xi_b^-} = (1.1^{+0.6}_{-0.4})\%$. The value for $\mathrm{BR}(\Lambda_b^0 \to \Lambda_c^+ \mathrm{X}\ell^-\overline{\nu_\ell})$ has been obtained considering that there could be one Λ_c^+ produced in every decay or, at the other extreme, that all excited charmed baryons, assumed to be produced with a rate similar to D^{**} final states in B mesons, decay into a D meson and a non-charmed baryon. Similar considerations have been applied to Ξ_b semileptonic decays. The semileptonic decay width $\Gamma(\Xi_b \to \Xi_c \mathrm{X}\ell^-\overline{\nu_\ell})$ has been taken to be equal to $\Gamma(\Lambda_b^0 \to \Lambda_c^+ \mathrm{X}\ell^-\overline{\nu_\ell})$ and it has been assumed that, within present uncertainties, $\mathrm{BR}(\Xi_c^0 \to \Xi^-\mathrm{X}) = \mathrm{BR}(\Lambda_c^+ \to \Lambda^0\mathrm{X})$. The total b-baryon production rate is then: $f_{b\text{-baryon}} = (13.7^{+4.8}_{-3.2})\%$, assuming the same production rates for Ξ_b^0 and Ξ_b^- baryons. This is then averaged with a direct measurement of $f_{b\text{-baryon}} = (10.2 \pm 2.8)\%$ from the number of protons in b-events [94] to give a result of $f_{b\text{-baryon}} = (11.7 \pm 2.1)\%$ from LEP measurements alone. Finally, the CDF Collaboration has measured the production rate of b-baryons relative to non-strange B mesons, $f_{b\text{-baryon}}/(f_{\mathrm{B^+}} + f_{\mathrm{B_d}}) = 0.118 \pm 0.042$, using $\Lambda_c^+ e^-$ final states [98].

The fraction of $\mathrm{B^+}$ mesons has been estimated by subtracting $f_{\Xi_b^-}$ from the fraction of all charged b-hadrons measured using the charge of a large sample of inclusively reconstructed secondary vertices [93]: $f_{\mathrm{B^+}} = (41.4 \pm 1.6)\%$ (Table 27).

All these measurements have been combined to obtain average production rates for $\mathrm{B_s^0}$, b-baryons, $\mathrm{B^+}$ and $\mathrm{B_d^0}$ [8], imposing that:

$$f_{\mathrm{B^+}} + f_{\mathrm{B_d}} + f_{\mathrm{B_s}} + f_{b\text{-baryon}} = 1 \quad (35)$$

and

$$f_{\mathrm{B^+}} = f_{\mathrm{B_d}}. \quad (36)$$

The results obtained are given in Table 3. Correlated systematics between the different measurements, coming mainly from the poorly measured branching fractions of D_s^+ and Λ_c^+ charmed hadrons, have been taken into account. It may be noted that constraint (36)

[7] The uncertainties quoted for these two results from CDF include the uncertainty on the evaluation of the branching fraction for the D_s^+ meson into $\phi\pi$; original measurements are given in Table 25.

[8] The production of $\mathrm{B_c}$ mesons and of other weakly decaying states made of several heavy quarks has been neglected.

is expected to be true for b-mesons even though it is known to be wrong for c-mesons. In both b- and c-jets, isospin invariance of strong interactions predicts similar production rates of mesons in which the heavy quark is associated to a $\overline{u}$ or $\overline{d}$ antiquark. Strong and electromagnetic decays of these states result in different rates for weakly decaying mesons with a u or a d flavour because the $\mathrm{D}^* \to \mathrm{D}\pi$ decays occur very close to threshold, and the threshold prevents the transition $\mathrm{D}^{*+} \to \mathrm{D}^+\pi^-$. However for B^0_d and B^+ mesons, no asymmetry is expected: B^* mesons decay electromagnetically, leaving the flavour of the light spectator quark in the b-meson unchanged; non-strange B^{**} mesons decaying through strong interactions and having masses away from threshold will not induce any asymmetry either; B^{**0}_s mesons decay also by strong interactions into $\mathrm{B}^{0(*)}_d\overline{\mathrm{K}^0}$ or $\mathrm{B}^{+(*)}\mathrm{K}^-$ with equal probabilities, although a possible tiny difference between these two rates can be expected for decays of narrow states occurring very close to threshold, because of the mass difference between K^0 and K^+ mesons[9].

Additional information on the production rates can be obtained from measurements of the time-integrated mixing probability of b-hadrons. For an unbiased sample of semileptonic b-hadron decays in b-jets, with fractions $g_{\mathrm{B_d}}$ and $g_{\mathrm{B_s}}$ of B^0_d and B^0_s mesons, this mixing probability is equal to:

$$\overline{\chi} = g_{\mathrm{B_s}}\ \chi_s + g_{\mathrm{B_d}}\ \chi_d \tag{37}$$

where χ_d is the time-integrated mixing probability for B^0_d mesons (see Section 4.2) and χ_s=1/2 is the corresponding quantity for B^0_s mesons[10]. As already mentioned in Section 2, the semileptonic width is assumed to be the same for all b-hadron species, implying $g_i = f_i\ \mathrm{R}_i$, where $\mathrm{R}_i = \frac{\tau_i}{\tau_b}$ are the lifetime ratios. This leads to the relation:

$$f_{\mathrm{B_s}} \ = \ \frac{1}{\mathrm{R}_s}\ \frac{(1+r)\ \overline{\chi} - (1 - f_{b\text{-baryon}}\ \mathrm{R}_{b\text{-baryon}})\ \chi_d}{(1+r)\ \chi_s - \chi_d} \tag{38}$$

where $r \ = \ \mathrm{R}_u/\mathrm{R}_d \ = \ \tau(\mathrm{B}^+)/\tau(\mathrm{B}^0_\mathrm{d})$. This is used to extract another determination of $f_{\mathrm{B_s}}$ from the b-baryon fraction of Table 3, the lifetime ratio averages of Table 2, the world average value of χ_d from Equation (44) of Section 4.2, and the $\overline{\chi}$ average from the LEPEWWG (see Table 1). This new estimate of $f_{\mathrm{B_s}}$, $f^{\mathrm{mixing}}_{\mathrm{B_s}} \ = \ (10.1 \pm 1.4)\%$, is then combined with the b-hadron rates from direct measurements, taking into account correlations[11] and imposing the conditions (35) and (36). The final b-hadron fractions are displayed in Table 4.

The production rate for $b\overline{s}$ states is expected to be rather similar to the probability ($\gamma_s \sim 12\%$) for creating an $s\overline{s}$ pair during the hadronization process. But, because of their masses, non strange B^{**} mesons are not expected to decay into $\mathrm{B}^{0(*)}_s\overline{\mathrm{K}}$, whereas strange B^{**} mesons are expected to decay mainly into non-strange weakly decaying b-mesons. As a result, the fraction of $\overline{\mathrm{B}^0_\mathrm{s}}$ in b-jets is expected to be given by $f_{\mathrm{B_s}} \simeq \gamma_s\ [1 - \mathrm{P}(b \to \overline{\mathrm{B}}^{**})] \simeq$ 10%, as observed.

[9] The mass difference between B^0_d and B^+ mesons is compatible with zero within ± 0.3 MeV/c^2.

[10] The assumption $\chi_s = \frac{1}{2}$ can be justified by the existence of limits on Δm_s obtained from $\mathrm{D_s}$-lepton analyses, which have negligible dependence on $f_{\mathrm{B_s}}$.

[11] There is a small statistical correlation between χ_d and $\overline{\chi}$, arising from the fact that a few Δm_d analyses at LEP are based on the same samples of dilepton events as the ones used to extract $\overline{\chi}$. This correlation is ignored, with a negligible effect on the final result.

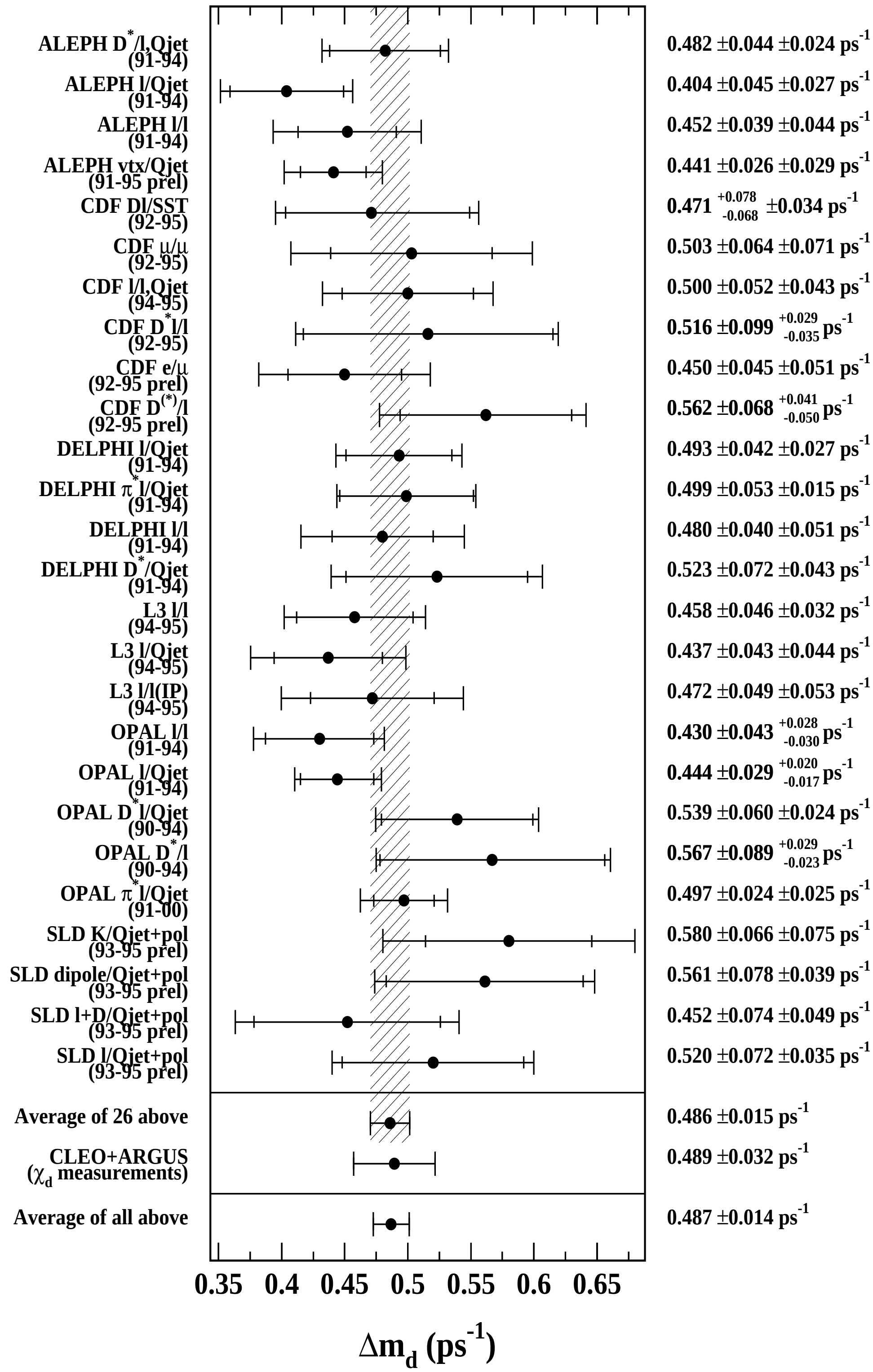

Figure 4: *Individual and combined measurements of Δm_d at LEP, CDF and SLD. Note that the individual measurements are as quoted in the original publications, but their average includes the effects of adjustments to a common set of input parameters. The last average also includes χ_d measurements [5] performed by CLEO and ARGUS experiments*

b-hadron fractions	correlation coefficients
$f_{\mathrm{B_s}} = (9.8 \pm 1.2)\%$	
$f_{b\text{-baryon}} = (10.3 \pm 1.8)\%$	$\rho(f_{\mathrm{B_s}}, f_{b\text{-baryon}}) = 0.01$
$f_{\mathrm{B_d}} = f_{\mathrm{B^+}} = (39.9 \pm 1.1)\%$	$\rho(f_{\mathrm{B_d}}, f_{\mathrm{B_s}}) = -0.55,\ \rho(f_{\mathrm{B_d}}, f_{b\text{-baryon}}) = -0.84$

Table 4: *Average values of b-hadron production rates and their correlation coefficients obtained from direct measurements and including constraints from results on B mixing.*

4.2 Combination method for Δm_d

The measurements of Δm_d are now quite precise and it is important that the correlated systematic uncertainties are correctly handled. Furthermore, many results depend on physics parameters for which different values were used in the original analyses. Before being combined, the measurements of Δm_d are therefore adjusted on the basis of a common consistent set of input values.

For each input parameter the Δm_d measurement and the corresponding systematic uncertainty are linearly rescaled in accordance with the difference between the originally used parameter value and error, and the new common values. This is done for the systematic uncertainties originating from the b-hadron lifetimes, the b-hadron fractions and the mixing parameters $\overline{\chi}$ and χ_d. The statistical and systematic uncertainties of each individual measurement are symmetrized. An alternative approach using asymmetric uncertainties (when quoted as such) would produce negligible differences in the combined result.

The combination procedure makes a common fit of Δm_d and the common input parameters. It is assumed that Δm_d may be expressed as a function $X(Y_1 \ldots Y_{N_{sys}})$ with a weak dependence on the systematic sources Y_α. Expanding X then gives:

$$X(Y_1 \ldots Y_{N_{sys}}) \approx X^0 + \sum_{\alpha=1}^{N_{sys}} Y_\alpha \sigma_\alpha^{syst} \tag{39}$$

where the quantities σ_α^{syst} are the correlated errors on Δm_d from systematic sources α, and $Y_\alpha = (z_\alpha - z_\alpha^{fit})/\delta_\alpha$. In this last expression, z_α and z_α^{fit} are the input and fitted values of the systematic parameter α, and δ_α is the variation used to calculate σ_α^{syst}.

The following χ^2 is then constructed:

$$\chi^2 = \sum_{i=1}^{N} \left(\frac{X_i - \sum_{\alpha=1}^{N_{sys}} Y_\alpha \sigma_{i\alpha}^{syst} - X^0}{\sigma_i^{uncor}} \right)^2 + \sum_{\alpha=1}^{N_{sys}} Y_\alpha^2, \tag{40}$$

where X_i are the measurements of Δm_d and σ_i^{uncor} the quadratic sum of the statistical and uncorrelated systematic uncertainties on X_i. This χ^2 is minimized with respect to the parameters X^0 and Y_α; the result for Δm_d is taken as the value of X^0 at the minimum (and the values of Y_α at the minimum are ignored). This method gives the same results as a χ^2 minimization with inversion of a global correlation matrix. As several measurements also have a statistical correlation, the first sum in Equation (40) is then generalized to handle an $N \times N$ error matrix describing the statistical and uncorrelated systematic uncertainties.

The following sources of systematic uncertainties, common to analyses from different experiments, have been considered:

- b-lifetime measurements (Section 3). Different measurements use the b-hadron lifetimes in different ways as input: some use the actual lifetimes, and others use ratios of lifetimes. As this leads to complicated correlations between the analyses, the following procedure is adopted. For each measurement a single combined value of all b-lifetime-related systematic uncertainties is computed. This systematic is then treated as fully correlated with all other such numbers from the other measurements. Tests show that this procedure gives a negligible bias and results in a conservative evaluation of the uncertainty.

- b-quark fragmentation (Section 2). In some analyses the b-hadron momentum dependence is treated through the variation of the ϵ parameter in the Peterson fragmentation function, and in others through the mean scaled b-hadron energy $< x_E >$. In such a case, the corresponding systematic uncertainties are treated as fully correlated, without attempting to adjust the individual results to a common value of ϵ or $< x_E >$.

- fraction of cascade decays (Section 2), related mainly to the value of BR($b \to c \to \ell^+ \nu_\ell X$).

- fraction of B^+ mesons remaining in B^0_d enriched samples for analyses using D^* mesons. This depends on decay branching fractions and on the mistag probability.

- production rate of D^{**} mesons in b-hadron semileptonic decays (Section 2).

- b-hadron fractions (Section 4.1).

- D^0 lifetime ([5]).

Common systematics due to purely experiment-dependent factors (i.e. common to different results in a particular experiment) have not been included in the above list, but nonetheless are treated as correlated in the fit. Following this procedure, a combined value:

$$\Delta m_d^{\mathrm{LEP+CDF+SLD}} = (0.486 \pm 0.015)\ \mathrm{ps}^{-1} \tag{41}$$

is obtained. Using the relation[12]:

$$\chi_d = \frac{1}{2}\,\frac{(\Delta m_d\ \tau_{B_d})^2}{(\Delta m_d\ \tau_{B_d})^2 + 1} \tag{42}$$

and the B^0_d lifetime of Section 3, the above Δm_d result can be converted to:

$$\chi_d^{\mathrm{LEP+CDF+SLD}} = 0.181 \pm 0.008. \tag{43}$$

Averaging with the time-integrated mixing results obtained by ARGUS and CLEO at the $\Upsilon(4S)$ (see Table 1), yields finally [13]:

$$\chi_d^{\mathrm{ALL}} = 0.181 \pm 0.007, \tag{44}$$

[12]Equation (42) assumes that there is no decay width difference in the $B^0_d - \overline{B^0_d}$ system (current expectations are at the 10^{-3} level for $\Delta\Gamma_{B^0_d}/\Gamma_{B^0_d}$), and that effects from CP violation in the considered final states can be neglected.

[13]In the following expressions, ALL refers to ARGUS, CDF, CLEO, LEP and SLD measurements.

or equivalently

$$\Delta m_d^{\mathrm{ALL}} = (0.487 \pm 0.014)\ \mathrm{ps}^{-1} = (3.21 \pm 0.09)\ 10^{-4}\ \mathrm{eV}/c^2. \tag{45}$$

The individual measurements of Δm_d and their combined value are shown in Figure 4. The determination of Δm_d as described above and of the b-hadron fractions described in Section 4.1 cannot be performed sequentially: the values of the fractions are needed to perform the Δm_d fit, and the best estimates of the fractions can be obtained only once the final Δm_d average is known. This circular dependence has been handled by including the calculation of the fractions in the Δm_d fitting procedure, in such a way that the final results quoted for Δm_d and the b-hadron fractions form a consistent set.

4.3 Combination method for B_s^0 oscillation amplitudes and derived limits on Δm_s

No experiment has yet directly observed B_s^0 oscillations, so the task of a combination procedure is to calculate an overall limit or to quantify the evidence for a signal from the information provided by each measurement. This is done using the amplitude method of reference [105]. At each value of Δm_s, in the range of interest, an amplitude is measured in each analysis, where the expected value of the amplitude is unity at the true frequency. An overall limit on Δm_s is then inferred from the combined amplitude spectrum by excluding regions of Δm_s where the amplitude is incompatible with unity.

Studies with Monte Carlo simulation show that the measured amplitude has a Gaussian distribution around its expected value, which is zero for frequencies much lower than the true one, and unity at the true frequency. In addition, the error on the amplitude depends on the statistical power of the analysis but not on the true value of the oscillation frequency. Therefore, if at a given frequency (Δm_s) a measured amplitude (μ) with error σ is found, this value of the frequency can be excluded at the 95% C.L. if the probability of measuring an amplitude equal or smaller than μ, for a true amplitude of unity, is smaller than 5%, that is $\int_{-\infty}^{\mu} G(A-1,\sigma)dA < 0.05$, or equivalently $\mu + 1.645\sigma < 1$ where G is the Gaussian function.

Before combining the measured amplitudes, the individual central values and systematic uncertainties are adjusted, using the same procedure as for Δm_d measurements, to common values of the b-hadron fractions (Table 4), b-hadron lifetimes (Section 3), and Δm_d (Equation (45)). The adjustment to a common value of $f_{\mathrm{B_s}}$ is performed first, and needs a special treatment for certain analyses, as explained below. Although the final value of $f_{\mathrm{B_s}}$ is calculated under the assumption that B_s^0 mixing is maximal, studies indicate that the effect on the combined Δm_s result is negligible.

The statistical uncertainty on the measured amplitude is expected to be inversely proportional to the B_s^0 purity of the analysed sample [105]. This purity is of the order of $f_{\mathrm{B_s}}$ for inclusive analyses. For analyses where a full or partial B_s^0 reconstruction is performed, the purity is much less dependent on the assumed value of $f_{\mathrm{B_s}}$. Therefore, the statistical uncertainties on the amplitudes measured in inclusive analyses have been multiplied by $f_{\mathrm{B_s}}^{\mathrm{used}}/f_{\mathrm{B_s}}^{\mathrm{new}}$, where $f_{\mathrm{B_s}}^{\mathrm{used}}$ is the B_s^0 fraction assumed in the corresponding analysis, and $f_{\mathrm{B_s}}^{\mathrm{new}}$ is the B_s^0 fraction from Table 4. This correction is also applied on the central values of the amplitude measured in inclusive analyses, after having checked that the relative uncertainty on the amplitude is essentially independent of $f_{\mathrm{B_s}}$ for large enough values of Δm_s.

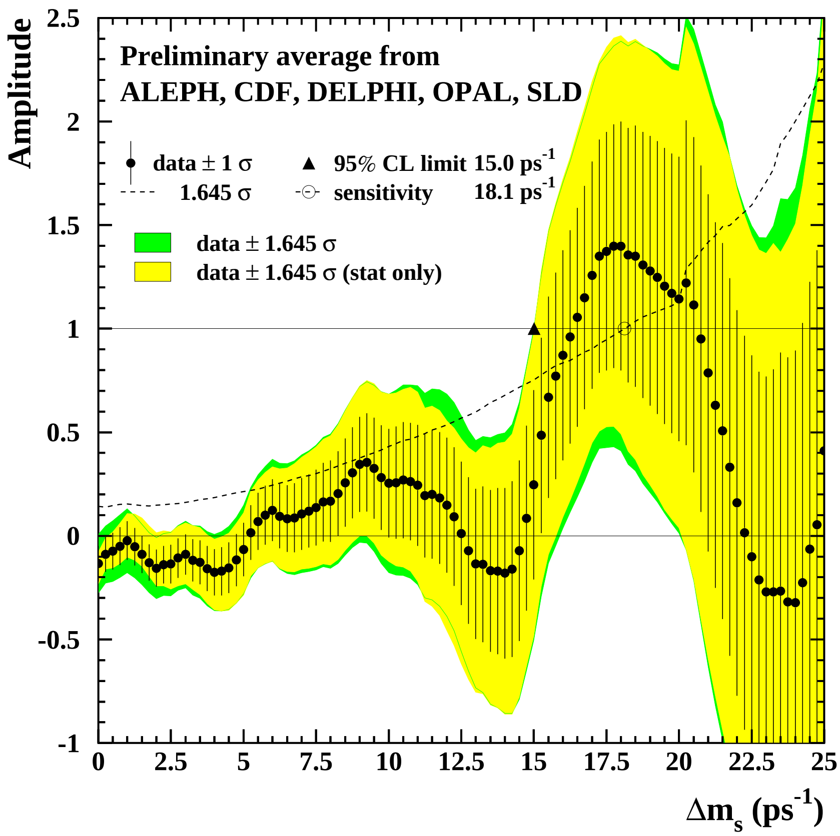

Figure 5: *Combined* $\mathrm{B_s^0}$ *oscillation amplitude as a function of* Δm_s*. A 95% CL lower limit on* Δm_s *of 15.0* ps^{-1} *is derived from this spectrum. Points with error bars are the fitted amplitude values and corresponding uncertainties, including systematics. The dark shaded area is obtained by multiplying these uncertainties by 1.645 such that the integral of the probability distribution, assumed to be Gaussian, is equal to 5% above this range. The light shaded area is obtained in a similar way but including only statistical uncertainties in the fit; as a consequence, central values of the two regions do not necessarily coincide.*

The amplitudes (measured at each value of Δm_s) are averaged using the same procedure as for the combination of Δm_d results. In addition to systematics which are correlated within the individual experiments, the following sources of correlated systematic uncertainties have been taken into account:

- b-hadron lifetime measurements,
- b-quark fragmentation,
- direct and cascade semileptonic branching fractions of b-hadrons,
- b-hadron fractions,
- Δm_d measurements,
- $\Delta\Gamma_{\mathrm{B_s^0}}$ measurements.

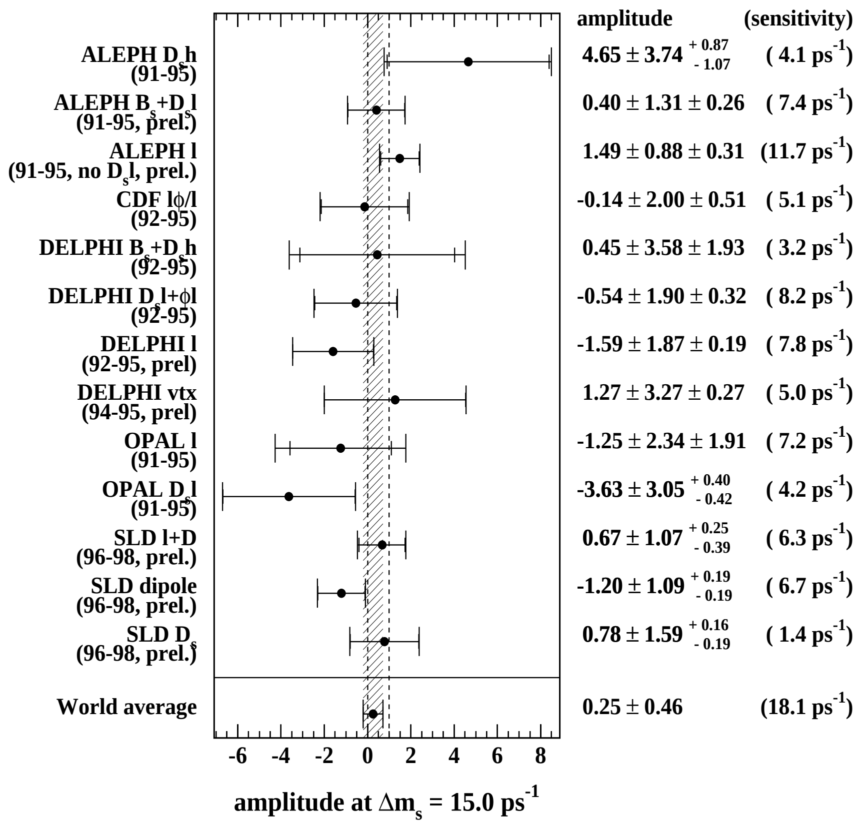

Figure 6: *Measurements of the* $\mathrm{B}_{\mathrm{s}}^{0}$ *oscillation amplitude. The amplitudes are given at* $\Delta m_s = 15\ \mathrm{ps}^{-1}$*, along with the relevant statistical and systematic errors. The exclusion sensitivities are given within parentheses in the column on the right. Note that the individual measurements are as quoted in the original publications, but the average, corresponding to the hatched area, includes the effects of adjustments to a common set of input parameters. The dashed lines correspond to amplitude values of 0 and 1.*

Data using the amplitude method have been provided by ALEPH, DELPHI, OPAL [14], CDF and SLD. The combined amplitude spectrum is shown in Figure 5. All values of Δm_s below 15.0 ps^{-1} are excluded at the 95% CL. The exclusion sensitivity to Δm_s, defined as the largest value of Δm_s that would have been excluded if the combined amplitudes were zero at all values of Δm_s, is 18.1 ps^{-1}. In Figure 6 the values of the amplitude at $\Delta m_s = 15\ \mathrm{ps}^{-1}$ from the various analyses, are shown. In addition, the individual exclusion sensitivities are given.

Positive measured amplitudes are found for frequency values near and above the sensitivity limit. Some values deviate from zero by more than twice the total estimated error. The likelihood profile obtained from the amplitude spectrum, using the prescription of [105], shows a minimum at 17.2 ps^{-1}. The likelihood value at this minimum is 2.3 units below the asymptotic likelihood value for $\Delta m_s \to \infty$. Because the measurements

[14] The result released by the OPAL Collaboration in September 2000, obtained from an analysis of the D_s^+-lepton channel, has been included in the combination.

at different frequencies are correlated, it is not possible to calculate analytically the probability that, in the explored frequency range, a fluctuation as or more unlikely occurs in a sample where the true frequency is far beyond the sensitivity. Therefore, an estimation procedure based on fast Monte Carlo experiments has been developed [106]: the above probability is found to be around 3%.

Note that the combined sensitivity for a "five-sigma discovery" of $\mathrm{B_s^0}-\overline{\mathrm{B_s^0}}$ oscillations, defined as the value of Δm_s for which the uncertainty on the combined amplitude is equal to 0.2, is 8 ps^{-1}.

5 Limit on the decay width difference for mass eigenstates in the $\mathrm{B_s^0}$-$\overline{\mathrm{B_s^0}}$ system

The CKM picture of weak charge-changing transitions predicts that the $\mathrm{B_s^0}$ and its charge conjugate mix. This results in new states $\mathrm{B_s^{heavy}}$ and $\mathrm{B_s^{light}}$, with masses $\mathrm{m_{B_s^0}^{heavy}}$ and $\mathrm{m_{B_s^0}^{light}}$ and (probably) different widths $\Gamma_{\mathrm{B_s^0}}^{\mathrm{heavy}}$ and $\Gamma_{\mathrm{B_s^0}}^{\mathrm{light}}$.

Neglecting CP violation, the mass eigenstates are also CP eigenstates, the $\mathrm{B_s^{light}}$ being CP even and $\mathrm{B_s^{heavy}}$ being CP odd. The decay of a $\overline{\mathrm{B_s^0}}$ meson via the quark subprocess $b(\overline{s}) \to c\overline{c}s(\overline{s})$ gives rise to predominantly CP-even eigenstates, thus the CP-even eigenstate should have the greater decay rate and hence the shorter lifetime. For convenience of notation, in the following we therefore substitute $\Gamma_{\mathrm{B_s^0}}^{\mathrm{light}} \equiv \Gamma_{\mathrm{B_s^0}}^{\mathrm{short}}$ and $\Gamma_{\mathrm{B_s^0}}^{\mathrm{heavy}} \equiv \Gamma_{\mathrm{B_s^0}}^{\mathrm{long}}$, and define $\Gamma_{\mathrm{B_s^0}} = 1/\tau_{\mathrm{B_s^0}} = (\Gamma_{\mathrm{B_s^0}}^{\mathrm{long}} + \Gamma_{\mathrm{B_s^0}}^{\mathrm{short}})/2$ and $\Delta\Gamma_{\mathrm{B_s^0}} = \Gamma_{\mathrm{B_s^0}}^{\mathrm{short}} - \Gamma_{\mathrm{B_s^0}}^{\mathrm{long}}$.

Theoretical calculations [107] of the ratio $\Delta\Gamma_{\mathrm{B_s^0}}/\Gamma_{\mathrm{B_s^0}}$ at next-to-leading order give:

$$\frac{\Delta\Gamma_{\mathrm{B_s^0}}}{\Gamma_{\mathrm{B_s^0}}} = \left(\frac{\mathcal{F}_{\mathrm{B_s^0}}}{230\ \mathrm{MeV}}\right)^2 [0.007 \times \mathcal{B}_b(m_b) + (0.132^{+0.011}_{-0.027})\overline{\mathcal{B}_s}(m_b) - (0.078 \pm 0.018)] \quad (46)$$

where $\mathcal{F}_{\mathrm{B_s^0}}$ is the $\mathrm{B_s^0}$ decay constant, $\mathcal{B}_b(m_b)$ and $\overline{\mathcal{B}_s}(m_b) = \frac{m^2_{\mathrm{B_s^0}}\mathcal{B}_s(m_b)}{(\overline{m_b}+\overline{m_s})^2}$ are bag parameters and where $\overline{m_q}$ are quark masses evaluated in the $\overline{\mathrm{MS}}$ scheme at the scale $m_b = 4.8\ \mathrm{GeV/c^2}$. The third term in square brackets in Equation (46) is an estimate of the $1/m_b$ correction in the factorization approximation [108]. Using the values from recent lattice calculations [107, 112] of $\mathcal{B}_b(m_b) = 0.9 \pm 0.1$, $\overline{\mathcal{B}_s}(m_b) = 1.25 \pm 0.10$ and $\mathcal{F}_{\mathrm{B_s^0}} = (220 \pm 30)$ MeV (see Section 10) yields $\Delta\Gamma_{\mathrm{B_s^0}}/\Gamma_{\mathrm{B_s^0}} = 0.085^{+0.035}_{-0.046}$, where the dominant uncertainties are from $\mathcal{F}_{\mathrm{B_s^0}}$, from the scale dependence of perturbative QCD corrections and from the uncertainty on $1/m_b$ corrections. Due to progress in lattice QCD evaluations, uncertainties on the $\mathcal{B}$ parameters are no longer dominant.

Assuming that the CKM matrix is unitary, the width difference of the $\mathrm{B_s^0}-\overline{\mathrm{B_s^0}}$ mass eigenstates can be related to the mass difference of $\mathrm{B_d^0}-\overline{\mathrm{B_d^0}}$ mass eigenstates in an expression in which non-perturbative QCD parameters enter through ratios, which are better determined than individual absolute values of contributing quantities [111, 112]. The dependence in the $\mathrm{B_s^0}$ decay constant also disappears. Assuming that there is no new physics contributions in present determinations of the CKM parameters (see Section 10) they obtain:

$$\Delta\Gamma_{\mathrm{B_s^0}}/\Gamma_{\mathrm{B_s^0}} = (4.7 \pm 2.2)\%. \quad (47)$$

The width difference and the mass difference are correlated ($\Delta\Gamma_{\mathrm{B_s^0}}/\Delta m_s = \frac{3}{2}\pi\frac{m_b^2}{m_t^2}$ to first approximation [113]), thus offering the possibility of measuring Δm_s via the lifetime difference rather than the oscillation frequency. This could be particularly important if the oscillation frequency is too fast to be measured with the present experimental proper time resolution. In addition, if $\Delta\Gamma_{\mathrm{B_s^0}}$ does turn out to be sizable, the observation of CP violation and the measurement of CKM phases from untagged $\mathrm{B_s^0}$ samples can be imagined [114]. Within the present level of uncertainties the two evaluations of $\Delta\Gamma_{\mathrm{B_s^0}}/\Gamma_{\mathrm{B_s^0}}$, given in Equations (46) and (47), are compatible and progress is needed to reduce uncertainties related to the scale dependence of perturbative QCD and to $1/m_b$ corrections before these theoretical evaluations can be used to set a limit on Δm_s.

The existing experimental constraints on the width difference and the combination of these constraints are reported here[15].

5.1 Experimental constraints on $\Delta\Gamma_{\mathrm{B_s^0}}/\Gamma_{\mathrm{B_s^0}}$

Experimental information on $\Delta\Gamma_{\mathrm{B_s^0}}$ can be extracted by studying the proper time distribution of data samples enriched in $\mathrm{B_s^0}$ mesons. An alternative method based on measuring the branching fraction $\mathrm{B}_s \to \mathrm{D}_s^{(*)+}\mathrm{D}_s^{(*)-}$ has also been proposed [116]. The available results are summarised in Table 5. The values of the limit on $\Delta\Gamma_{\mathrm{B_s^0}}/\Gamma_{\mathrm{B_s^0}}$ quoted in the last column of this table have been obtained by the working group.

Methods based on double exponential lifetime fits to samples containing a mixture of CP eigenstates have a quadratic sensitivity to $\Delta\Gamma_{\mathrm{B_s^0}}$ (inclusive, semileptonic, D_s^{+}-hadron), whereas methods based on isolating a single CP eigenstate have a linear dependence on $\Delta\Gamma_{\mathrm{B_s^0}}$ ($\phi\phi$, $\mathrm{J}/\psi\phi$). The latter are therefore, in principle, more sensitive to $\Delta\Gamma_{\mathrm{B_s^0}}$; but they tend to suffer from reduced statistics.

In order to obtain an improved limit on $\Delta\Gamma_{\mathrm{B_s^0}}$, the results based on fits to the proper time distributions are used to apply a constraint on the allowed range of $1/\Gamma_{\mathrm{B_s^0}}$. The world average $\mathrm{B_s^0}$ lifetime is not used, as its meaning is not clear if $\Delta\Gamma_{\mathrm{B_s^0}}$ is non-zero. Instead, it is chosen to constrain $1/\Gamma_{\mathrm{B_s^0}}$ to the world average $\tau_{\mathrm{B_d^0}}$ lifetime ($1/\Gamma_{\mathrm{B_s^0}} \equiv 1/\Gamma_{\mathrm{B_d^0}} = \tau_{\mathrm{B_d^0}} = (1.546 \pm 0.021)$ ps). This is well motivated theoretically, as the total widths of the $\mathrm{B_s^0}$ and $\mathrm{B_d^0}$ mesons are expected to be equal within less than one percent [77], [117] and $\Delta\Gamma_{\mathrm{B_d^0}}$ is expected to be small.

Further information on the various individual measurements is now given.

- *L3 inclusive b-sample:* in an unbiased inclusive B sample, all $\mathrm{B_s^0}$ decay modes are measured, including decays into CP eigenstates. An equal number of $\mathrm{B_s^{short}}$ and $\mathrm{B_s^{long}}$ mesons are therefore selected, and the proper time dependence of the $\mathrm{B_s^0}$ signal is given by:

$$P_{incl}(t) = \frac{1}{2}(\Gamma^{\mathrm{long}}_{\mathrm{B_s^0}} \exp(-\Gamma^{\mathrm{long}}_{\mathrm{B_s^0}} t) + \Gamma^{\mathrm{short}}_{\mathrm{B_s^0}} \exp(-\Gamma^{\mathrm{short}}_{\mathrm{B_s^0}} t)). \quad (48)$$

If the proper time dependence of this sample is fitted assuming only a single exponential lifetime then, using the definitions of $\Gamma_{\mathrm{B_s^0}}$ and $\Delta\Gamma_{\mathrm{B_s^0}}$ and assuming that $\Delta\Gamma_{\mathrm{B_s^0}}/\Gamma_{\mathrm{B_s^0}}$ is small, the measured lifetime is given by:

$$\tau_{\mathrm{B_s^{incl.}}} = \frac{1}{\Gamma_{\mathrm{B_s^0}}} \frac{1}{1 - \left(\frac{\Delta\Gamma_{\mathrm{B_s^0}}}{2\Gamma_{\mathrm{B_s^0}}}\right)^2}. \quad (49)$$

[15] The present members of the $\Delta\Gamma_{\mathrm{B_s^0}}$ working group are: P. Coyle, D. Lucchesi, S. Mele, F. Parodi and P. Spagnolo.

Experiment	Selection	Measurement	$\Delta\Gamma_{B_s^0}/\Gamma_{B_s^0}$
L3 [44]	inclusive b-sample		< 0.67
DELPHI [54]	$\overline{B_s^0} \to D_s^+\ell^-\overline{\nu_\ell}X$	$\tau_{B_s^{semi.}} = (1.42^{+0.14}_{-0.13} \pm 0.03)$ ps	< 0.46
OTHERS [115]	$\overline{B_s^0} \to D_s^+\ell^-\overline{\nu_\ell}X$	$\tau_{B_s^{semi.}} = (1.46 \pm 0.07)$ ps	< 0.30
ALEPH [116]	$B_s^0 \to \phi\phi X$	$BR(B_s^{short} \to D_s^{(*)+}D_s^{(*)-}) = (23 \pm 10^{+19}_{-9})\%$	$0.26^{+0.30}_{-0.15}$
ALEPH [116]	$B_s^0 \to \phi\phi X$	$\tau_{B_s^{short}} = (1.27 \pm 0.33 \pm 0.07)$ ps	$0.45^{+0.80}_{-0.49}$
DELPHI [54]†	$\overline{B_s^0} \to D_s^+$ hadron	$\tau_{B_s^{D_s-had.}} = (1.53^{+0.16}_{-0.15} \pm 0.07)$ ps	< 0.69
CDF [39]	$B_s^0 \to J/\psi\phi$	$\tau_{B_s^{J/\psi\phi}} = (1.34^{+0.23}_{-0.19} \pm 0.05)$ ps	$0.33^{+0.45}_{-0.42}$

† The value quoted for the measured lifetime differs slightly from the one quoted in Table 21 because it corresponds to the present status of the analysis in which the information on $\Delta\Gamma_{B_s^0}$ has been obtained.

Table 5: *Experimental constraints on $\Delta\Gamma_{B_s^0}/\Gamma_{B_s^0}$. The upper limits, which have been obtained by the working group, are quoted at the 95 %C.L.*

L3 effectively incorporates $P_{incl}(t)$ into the proper time fit of an inclusive b-sample and applies the constraint $1/\Gamma_{B_s^0} = (1.49 \pm 0.06)$ ps.

- *DELPHI $\overline{B_s^0} \to D_s^+\ell^-\overline{\nu_\ell}X$:* in a semileptonic selection, the ratio of short and long B_s^0's selected is proportional to the ratio of the decay widths and the proper time dependence of the signal is:

$$P_{semi}(t) = \frac{\Gamma_{B_s^0}^{short}\,\Gamma_{B_s^0}^{long}}{(\Gamma_{B_s^0}^{short} + \Gamma_{B_s^0}^{long})}(\exp(-\Gamma_{B_s^0}^{short}t) + \exp(-\Gamma_{B_s^0}^{long}t)). \tag{50}$$

If this sample is fitted assuming only a single exponential lifetime for the B_s^0, then the measured lifetime is, always in the limit that $\Delta\Gamma_{B_s^0}/\Gamma_{B_s^0}$ is small, given by:

$$\tau_{B_s^{semi.}} = \frac{1}{\Gamma_{B_s^0}}\frac{1 + \left(\frac{\Delta\Gamma_{B_s^0}}{2\Gamma_{B_s^0}}\right)^2}{1 - \left(\frac{\Delta\Gamma_{B_s^0}}{2\Gamma_{B_s^0}}\right)^2}. \tag{51}$$

The single lifetime fit is thus more sensitive to the effects of $\Delta\Gamma_{B_s^0}$ in the semileptonic than in the fully inclusive case. Information on $\Delta\Gamma_{B_s^0}$ is obtained by scanning the likelihood as a function the two parameters $1/\Gamma_{B_s^0}$ and $\Delta\Gamma_{B_s^0}/\Gamma_{B_s^0}$ and applying the $\tau_{B_d^0}$ constraint.

- *OTHERS:* other analyses of the B_s^0 semileptonic lifetime [115] have not explicitly considered the possibility of a non-zero $\Delta\Gamma_{B_s^0}$ value. Nevertheless, the fact that the single exponential lifetime for this case (Equation (51)) is sensitive to $\Delta\Gamma_{B_s^0}$ allows information on $\Delta\Gamma_{B_s^0}$ to be extracted. The average B_s^0 semileptonic lifetime has been recalculated excluding the DELPHI result just discussed, and information on $\Delta\Gamma_{B_s^0}$ has been obtained by using Equation (51) and applying the $\tau_{B_d^0}$ constraint. The validity of Equation (51) in the presence of background contributions has been verified using a toy Monte Carlo [119].

- *ALEPH* $\mathrm{B^0_s} \to \phi\phi\mathrm{X}$ *(counting method):* only those $\mathrm{B^0_s}$ decays which are CP eigenstates can contribute to a width difference between the CP even and CP odd states. An analysis [120] of such decays shows that $\mathrm{B^0_s} \to \mathrm{D_s^{(*)+}D_s^{(*)-}}$ is by far the dominant contribution and is almost 100% CP even. Under this assumption, $\Delta\Gamma_{\mathrm{B^0_s}} = \Gamma^{\mathrm{short}}_{\mathrm{B^0_s}}(\mathrm{B_s^{short}} \to \mathrm{D_s^{(*)+}D_s^{(*)-}})$ where:

$$\mathrm{BR}(\mathrm{B_s^{short}} \to \mathrm{D_s^{(*)+}D_s^{(*)-}}) = \frac{\Gamma^{\mathrm{short}}_{\mathrm{B^0_s}}(\mathrm{B_s^{short}} \to \mathrm{D_s^{(*)+}D_s^{(*)-}})}{\Gamma^{\mathrm{heavy}}_{\mathrm{B^0_s}}} = \frac{\Delta\Gamma_{\mathrm{B^0_s}}}{\Gamma_{\mathrm{B^0_s}}(1+\frac{\Delta\Gamma_{\mathrm{B^0_s}}}{2\Gamma_{\mathrm{B^0_s}}})}. \quad (52)$$

  ALEPH [116] has measured this branching fraction, and this is the only constraint on $\Delta\Gamma_{\mathrm{B^0_s}}/\Gamma_{\mathrm{B^0_s}}$ which does not rely on a measurement of the average $\mathrm{B^0_s}(\mathrm{B^0_d})$ lifetime.

- *ALEPH* $\mathrm{B^0_s} \to \phi\phi\mathrm{X}$ *(lifetime method):* as the decay $\mathrm{B^0_s} \to \mathrm{D_s^{(*)+}D_s^{(*)-}} \to \phi\phi\mathrm{X}$, is predominantly CP even, the proper time dependence of the $\mathrm{B^0_s}$ component of the $\phi\phi$ sample is therefore just a simple exponential with the appropriate lifetime:

$$P_{single}(t) = \Gamma^{\mathrm{short}}_{\mathrm{B^0_s}} \exp(-\Gamma^{\mathrm{short}}_{\mathrm{B^0_s}} t). \quad (53)$$

  This lifetime is related to $\Delta\Gamma_{\mathrm{B^0_s}}$ and $\Gamma_{\mathrm{B^0_s}}$ via the expression:

$$\frac{\Delta\Gamma_{\mathrm{B^0_s}}}{\Gamma_{\mathrm{B^0_s}}} = 2(\frac{1}{\Gamma_{\mathrm{B^0_s}}\ \tau_{\mathrm{B_s^{short}}}} - 1). \quad (54)$$

  ALEPH [116] has measured the lifetime of $\phi\phi X$ events and extracted information on $\Delta\Gamma_s/\Gamma_s$ with the help of the world average $\mathrm{B^0_s}$ lifetime obtained from semileptonic $\mathrm{B^0_s}$ decays. The result listed in Table 5 has been obtained by the working group with the assumption $1/\Gamma_{\mathrm{B^0_s}} \equiv \tau_{\mathrm{B^0_d}}$.

- *DELPHI inclusive* D_s^+*:* a fully inclusive D_s^+ selection is expected to have an increased CP-even content, as the $\mathrm{B^0_s} \to \mathrm{D_s^{(*)+}D_s^{(*)-}}$ contribution is enhanced by the selection criteria. If f_{DsDs} is the fraction of $\mathrm{D}_s^{(*)+}\mathrm{D}_s^{(*)-}$ in the sample, then the proper time dependence is expected to be:

$$P(t)_{D_s-had.} = f_{DsDs} P_{single}(t) + (1 - f_{DsDs}) P_{semi}(t). \quad (55)$$

  For the DELPHI analysis a value $f_{DsDs} = (22 \pm 7)\%$ is estimated from simulation. Scanning the likelihood as a function $1/\Gamma_{\mathrm{B^0_s}}$ and $\Delta\Gamma_{\mathrm{B^0_s}}/\Gamma_{\mathrm{B^0_s}}$ and applying the $\tau_{\mathrm{B^0_d}}$ constraint yields an upper limit on $\Delta\Gamma_{\mathrm{B^0_s}}/\Gamma_{\mathrm{B^0_s}}$.

- *CDF* $\mathrm{B^0_s} \to \mathrm{J}/\psi\phi$*:* the final state $\mathrm{B^0_s} \to \mathrm{J}/\psi\phi$ is thought to be predominantly CP even (i.e. measures mainly $\tau_{\mathrm{B_s^{short}}}$) [120]. An update [118] of the CDF measurement of the polarization in $\mathrm{B^0_s} \to \mathrm{J}/\psi\phi$ decays measures the fraction of CP even in the final state to be $f_{short} = (79 \pm 19)\%$ and supports this expectation. For this case, the proper time dependence of the $\mathrm{B^0_s}$ component of the sample is:

$$P_{\mathrm{J}/\psi\phi}(t) = f_{short} P_{single}(\Gamma^{\mathrm{short}}_{\mathrm{B^0_s}}, t) + (1 - f_{short}) P_{single}(\Gamma^{\mathrm{long}}_{\mathrm{B^0_s}}, t). \quad (56)$$

  CDF measures the lifetime of $\mathrm{J}/\psi\phi$ events and information on $\Delta\Gamma_{\mathrm{B^0_s}}/\Gamma_{\mathrm{B^0_s}}$ is obtained after applying the $1/\Gamma_{\mathrm{B^0_s}} \equiv \tau_{\mathrm{B^0_d}}$ constraint and including the experimental uncertainty on f_{short}.

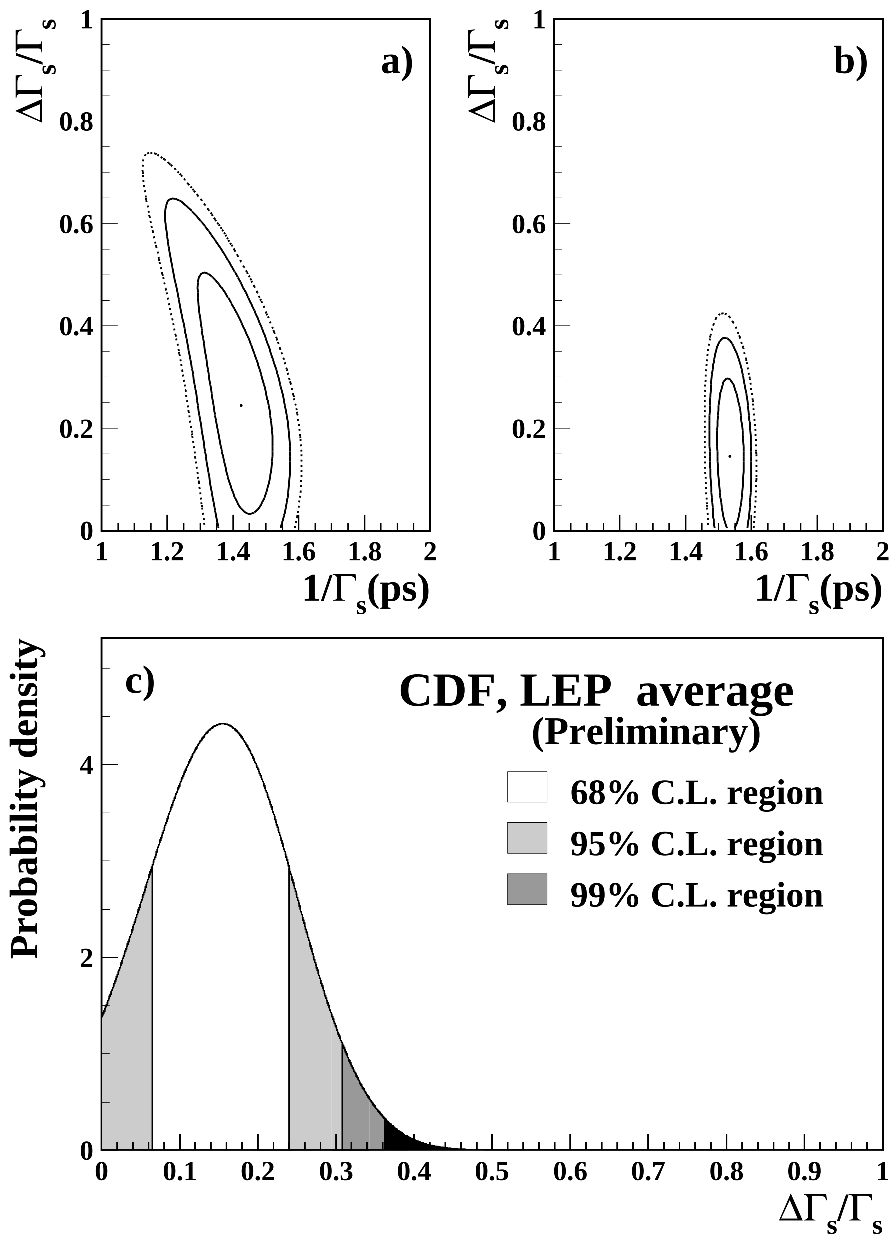

Figure 7: *a) 68%, 95% and 99% C.L. contours of the negative log-likelihood distribution in the plane* $1/\Gamma_{\mathrm{B_s^0}}$-$\Delta\Gamma_{\mathrm{B_s^0}}/\Gamma_{\mathrm{B_s^0}}$*. b) Same as a) but with the constraint* $1/\Gamma_{\mathrm{B_s^0}} \equiv \tau_{\mathrm{B_d^0}}$ *supposed to be exact. c) Probability density distribution for* $\Delta\Gamma_{\mathrm{B_s^0}}/\Gamma_{\mathrm{B_s^0}}$ *after applying the constraint; the three shaded regions show the limits at the 68%, 95% and 99% C.L. respectively.*

5.2 Combined limit on $\Delta\Gamma_{\mathrm{B_s^0}}$

In order to combine the analyses summarised in Table 5, the result of each analysis has been converted to a two-dimensional log-likelihood in the $(1/\Gamma_{\mathrm{B_s^0}}, \Delta\Gamma_{\mathrm{B_s^0}}/\Gamma_{\mathrm{B_s^0}})$ plane. This log-likelihood has either been provided by each experiment or reconstructed from the measured lifetimes using the expected dependence of this quantity on $1/\Gamma_{\mathrm{B_s^0}}$ and $\Delta\Gamma_{\mathrm{B_s^0}}/\Gamma_{\mathrm{B_s^0}}$. The latter procedure was necessary for the OTHERS and CDF entries of Table 5. The L3 analysis is not included in the average as the two-dimensional likelihood was not provided and could not be reconstructed from the available information. The $\tau_{\mathrm{B_d^0}}$ constraint is not applied on $1/\Gamma_{\mathrm{B_s^0}}$ at this stage. Systematic uncertainties are included in the individual log-likelihood distributions.

The log-likelihood distributions have been summed and the variation of the global negative log-likelihood function has been measured with respect to its minimum $(\Delta\mathcal{L})$. The 68%, 95% and 99% C.L. contours of the combined negative log-likelihood are shown in Figure 7a. The corresponding limit on $\Delta\Gamma_{\mathrm{B_s^0}}/\Gamma_{\mathrm{B_s^0}}$ is:

$$\begin{aligned}\Delta\Gamma_{\mathrm{B_s^0}}/\Gamma_{\mathrm{B_s^0}} &= 0.24^{+0.16}_{-0.12}\\ \Delta\Gamma_{\mathrm{B_s^0}}/\Gamma_{\mathrm{B_s^0}} &< 0.53 \text{ at the 95\% C.L.}\end{aligned} \tag{57}$$

An improved limit on $\Delta\Gamma_{\mathrm{B_s^0}}/\Gamma_{\mathrm{B_s^0}}$ can be obtained by applying the $\tau_{\mathrm{B_d^0}} = (1.546 \pm 0.021)$ ps constraint. When expressed as a probability density, this constraint is:

$$f_C(1/\Gamma_{\mathrm{B_s^0}}) = \frac{1}{\sqrt{2\pi}\sigma_{\tau_{\mathrm{B_d^0}}}} \exp\left(-\frac{(1/\Gamma_{\mathrm{B_s^0}} - \tau_{\mathrm{B_d^0}})^2}{2\sigma^2_{\tau_{\mathrm{B_d^0}}}}\right). \tag{58}$$

Using Bayes theorem, the probability density for $\Delta\Gamma_{\mathrm{B_s^0}}/\Gamma_{\mathrm{B_s^0}}$, with the constraint [16] applied, is obtained by convoluting $f_C(1/\Gamma_{\mathrm{B_s^0}})$ with the 2-D probability density for $1/\Gamma_{\mathrm{B_s^0}}$ and $\Delta\Gamma_{\mathrm{B_s^0}}/\Gamma_{\mathrm{B_s^0}}$ $(\mathcal{P}(\tau_{\mathrm{B_s^0}}, \Delta\Gamma_{\mathrm{B_s^0}}/\Gamma_{\mathrm{B_s^0}}))$, and normalizing the result to unity:

$$\mathcal{P}(\Delta\Gamma_{\mathrm{B_s^0}}/\Gamma_{\mathrm{B_s^0}}) = \frac{\int \mathcal{P}(1/\Gamma_{\mathrm{B_s^0}}, \Delta\Gamma_{\mathrm{B_s^0}}/\Gamma_{\mathrm{B_s^0}}) f_C(1/\Gamma_{\mathrm{B_s^0}}) d(1/\Gamma_{\mathrm{B_s^0}})}{\int \mathcal{P}(1/\Gamma_{\mathrm{B_s^0}}, \Delta\Gamma_{\mathrm{B_s^0}}/\Gamma_{\mathrm{B_s^0}}) f_C(1/\Gamma_{\mathrm{B_s^0}}) d(1/\Gamma_{\mathrm{B_s^0}}) d\Delta\Gamma_{\mathrm{B_s^0}}/\Gamma_{\mathrm{B_s^0}}} \tag{59}$$

where $\mathcal{P}(1/\Gamma_{\mathrm{B_s^0}}, \Delta\Gamma_{\mathrm{B_s^0}}/\Gamma_{\mathrm{B_s^0}})$ is proportional to $\exp(-\Delta\mathcal{L})$. This probability has been normalized by taking only positive values of $\Delta\Gamma_{\mathrm{B_s^0}}/\Gamma_{\mathrm{B_s^0}}$. The two-dimensional log-likelihood obtained, after including the constraint, supposed to be exact, is shown in Figure 7b. The resulting probability density distribution for $\Delta\Gamma_{\mathrm{B_s^0}}/\Gamma_{\mathrm{B_s^0}}$ is shown in Figure 7c. The corresponding limit on $\Delta\Gamma_{\mathrm{B_s^0}}/\Gamma_{\mathrm{B_s^0}}$ is:

$$\begin{aligned}\Delta\Gamma_{\mathrm{B_s^0}}/\Gamma_{\mathrm{B_s^0}} &= 0.16^{+0.08}_{-0.09}\\ \Delta\Gamma_{\mathrm{B_s^0}}/\Gamma_{\mathrm{B_s^0}} &< 0.31 \text{ at the 95\% C.L.}\end{aligned} \tag{60}$$

If an additional 2% uncertainty, assumed to be Gaussian distributed, is incorporated to account for the theory assumption $\tau_{\mathrm{B_d^0}} = 1/\Gamma_{\mathrm{B_s^0}}$, the effect on the result is small:

$$\begin{aligned}\Delta\Gamma_{\mathrm{B_s^0}}/\Gamma_{\mathrm{B_s^0}} &= 0.17^{+0.09}_{-0.10}\\ \Delta\Gamma_{\mathrm{B_s^0}}/\Gamma_{\mathrm{B_s^0}} &< 0.32 \text{ at the 95\% C.L.}\end{aligned} \tag{61}$$

[16] A flat, a priori probability density distribution has been assumed for $\Delta\Gamma_{\mathrm{B_s^0}}/\Gamma_{\mathrm{B_s^0}}$.

6 Average number of c and $\overline{c}$ quarks produced in b-hadron decays

The measured value of the b-hadron semileptonic decay branching fraction is on the low side of theoretical expectations [121]. One way to reconcile theory with experiment consists in assuming that the c-quark effective mass is lower than used in these evaluations. Such considerations imply also that decays of the type $b \to c\overline{c}s(d)$ correspond to a larger decay rate. The average number of c and $\overline{c}$ quarks contributing in b-hadron decays is thus negatively correlated with the expected value for $\mathrm{BR}(b \to \ell \mathrm{X})$ and a simultaneous measurement of these two quantities may help to clarify the theoretical picture. Another way to lower the expected semileptonic branching fraction of b-hadrons consists in adding new physics contributions to the $b \to sg$ transition [122]. This possibility has been constrained by the measurement of the decay rate of b-hadrons with no-open charm in the final state.

Experimentally, the number of c and $\overline{c}$ quarks contributing in b-hadron decays can be obtained by measuring the production fractions of charmed hadrons and charmonium states. Measurements originate from four sources:

- open-charm counting using exclusive signals of reconstructed charmed hadrons,
- charmonium production,
- inclusive measurements of the distribution of charged track impact parameters and of secondary vertices allowing the determination of $b \to \mathrm{D}\overline{\mathrm{D}}\mathrm{X}$ and $b \to 0\mathrm{D}$ charm decay rates,
- $b \to \mathrm{D}_i\overline{\mathrm{D}}_j\mathrm{X}$ branching fraction measurements in which $\overline{\mathrm{D}}_j$, and sometimes D_i, are completely reconstructed.

Measurements done at LEP and at the $\Upsilon(4\mathrm{S})$ have been analyzed using the same values for external parameters and the same hypotheses to account for missing measurements. Consequently, present results may differ from previous determinations [123, 124] (see Section 6.5).

6.1 Open-charm counting in b-hadron decays

Exclusive signals of D^0, D^+, D_s^+, Λ_c^+ and $\Xi_c^{0,+}$ charmed hadrons and of their anti-particles, produced in b-hadron decays, have been obtained by the ALEPH [125], CLEO [126, 128], DELPHI [129] and OPAL [130] Collaborations.

Values for the products of the production rates for a given charmed or anti-charmed hadron by its decay branching fraction into the reconstructed final state: $\mathrm{P}(b \to \mathrm{X}_c \text{ or } \overline{\mathrm{X}_c}) \times \mathrm{BR}(\mathrm{X}_c \to \mathrm{X})$ are given in Appendix E. LEP results obtained by DELPHI and OPAL are quoted using the value of R_b given in Table 1 whereas ALEPH measurements are independent of R_b.

To account for correlations between the different measurements when evaluating averaged values, systematic uncertainties have been split into three categories: those which are typical of the considered measurement, those which are common to several channels within a given experiment and those which are common to several experiments.

LEP and $\Upsilon(4\mathrm{S})$ measurements have been averaged separately. Using the values of the decay branching fractions for charmed particles listed in Table 1, production rates of the

Decay channel	LEP	CLEO
$\mathrm{BR}(b \to \mathrm{D}^0, \overline{\mathrm{D}}^0\mathrm{X})$	$(59.3 \pm 2.3 \pm 1.4)\%$	$(64.9 \pm 2.4 \pm 1.5)\%$
$\mathrm{BR}(b \to \mathrm{D}^+, \mathrm{D}^-\mathrm{X})$	$(22.5 \pm 1.0 \pm 1.5)\%$	$(24.0 \pm 1.3 \pm 1.6)\%$
$\mathrm{BR}(b \to \mathrm{D}_s^+, \mathrm{D}_s^-\mathrm{X})$	$(17.3 \pm 1.1 \pm 4.3)\%$	$(11.8 \pm 0.9 \pm 2.9)\%$
$\mathrm{BR}(b \to \Lambda_c^+, \overline{\Lambda}_c^-\mathrm{X})$	$(10.2 \pm 1.0 \pm 2.7)\%$	$(5.5 \pm 1.3 \pm 1.4)\%$

Table 6: *Measured production rates of charmed hadrons in b-hadron decays. The second quoted uncertainty is due to the error on charmed hadrons branching fractions.*

different charmed hadrons are then obtained and given in Table 6. Because of $\overline{\mathrm{B}_s^0}$ and b-baryon production in b-quark jets at high energy, it is expected that branching fractions into D_s^+ and Λ_c^+ are larger at LEP than at the $\Upsilon(4\mathrm{S})$, as observed.

To obtain the total number of c and $\overline{c}$ quarks produced in b-hadron decays these values have to be corrected for:

- unmeasured open-charm states such as strange-charm baryons and
- charmonium production.

6.1.1 Strange-charm baryon production in b-hadron decays

There are currently no absolute measurement of any strange-charm baryon branching fractions. As a result, the evaluation of the production rates of these baryons has to rely on models. Production of these states have been observed by CLEO [131] in b-meson decays. They obtain:

$$\overline{\mathrm{BR}}(\overline{\mathrm{B}} \to \Xi_c^0, \overline{\Xi}_c^0\mathrm{X}) = (2.4 \pm 1.3)/f^0\ \%,\ \mathrm{BR}(\mathrm{B} \to \Xi_c^+, \overline{\Xi}_c^-\mathrm{X}) = (1.5 \pm 0.7)/f^+\ \%. \quad (62)$$

The quoted uncertainties correspond only to statistical errors and the quantities f^q are the fractions of semileptonic decays of Ξ_c^q baryons containing a Ξ, which are unknown at present and usually taken to be equal to unity. Comparing the central value of the total $\Xi_c + \overline{\Xi}_c$ rate (3.9%) with the corresponding measurement of $\Lambda_c^+ + \overline{\Lambda}_c^-$ production (5.5%), using results quoted in Table 6, it appears that the production rate of strange-charm baryon can be much larger than naively expected. This would imply that there is a mechanism that enhances the production of $\Xi_c^{0,+}$ states but searches for this process have not yet been successful as explained in the following.

Two production mechanisms can be considered for strange-charm baryons in b-decays. In the first one, a strange diquark-antidiquark pair is created from the vacuum, with the probability f_s, giving rise to a strange-charm baryon in place of a Λ_c^+. As there are two possible weakly decaying strange-charm baryon states, the production rate can be evaluated to be of the order of:

$$\mathrm{BR}_1(b \to \Xi_c^{0,+}, \overline{\Xi}_c^{0,-} X) = \frac{2f_s}{1-2f_s} \times \mathrm{BR}(b \to \Lambda_c^+, \overline{\Lambda}_c^- X). \quad (63)$$

Using $f_s = 0.10$ with an (arbitrary) uncertainty of 50%, this gives:

$$\mathrm{BR}_1(b \to \Xi_c^{0,+}, \overline{\Xi}_c^{0,-}\mathrm{X}) = (0.25 \pm 0.12)\mathrm{BR}(b \to \Lambda_c^+, \overline{\Lambda}_c^- X). \quad (64)$$

The second mechanism has been invoked for decays in which the virtual W^- couples to a $(\bar{c}s)$ pair. The s quark can form a strange-charm baryon with the c-quark from the $b \to c W^-$ decay. A possible, but indirect, signature for this mechanism consists of the presence of $\overline{\Lambda_c^-}$ in $\overline{B}$ meson decays as observed by CLEO [131]. They measure [17] the ratio:

$$R_{\Lambda_c^+} = \frac{BR(\overline{B} \to \overline{\Lambda_c}^- X)}{BR(\overline{B} \to \Lambda_c^+ X)} = 0.17 \pm 0.13 \tag{65}$$

It is then possible to evaluate the fraction of $\overline{\Lambda_c^-}$ baryons produced in this mechanism, relative to the inclusive Λ_c^+ and $\overline{\Lambda_c^-}$ production rate:

$$BR(\overline{B} \to \overline{\Lambda_c^-} X) = \frac{R_{\Lambda_c^+} \times BR(\overline{B} \to \Lambda_c^+, \overline{\Lambda_c^-} X)}{1 + R_{\Lambda_c^+}}. \tag{66}$$

To evaluate, from this result, the fraction of events with a $\Xi_c^{0,+}$ baryon is not direct because a $\overline{\Lambda_c^-}$ can be produced without an accompanying strange-charm baryon and also a strange-charm baryon, formed with the cs quark pair, may not be accompanied by a $\overline{\Lambda_c^-}$. It has been assumed, in the following, that Ξ_c production from the second mechanism is similar to $\overline{\Lambda_c^-}$ production giving:

$$BR_2(\overline{B} \to \Xi_c^{0,+}, \overline{\Xi_c^{0,-}} X) = (0.15 \pm 0.10) BR(\overline{B} \to \Lambda_c^+, \overline{\Lambda_c^-} X). \tag{67}$$

The total production rate of strange-charm baryons, in b-meson decays, is then evaluated, summing the two contributions and inflating the errors, to be:

$$BR(\overline{B} \to \Xi_c^{0,+}, \overline{\Xi_c^{0,-}} X) = (0.4 \pm 0.3) BR(\overline{B} \to \Lambda_c^+, \overline{\Lambda_c^-} X) \tag{68}$$

In the absence of accurate results on $\Xi_c^{0,+}$ production in b-meson decays owing to the small statistical evidence for the signals and the indetermination of the decay branching fractions of these states, the quoted central value and uncertainty are supposed to accomodate the various possible scenarios for $\Xi_c^{0,+}$ production.

For b-hadrons produced in the hadronization of b-quark jets, an additional source of strange-charm baryons can originate from Ξ_b decays. It will be assumed that:

$$BR(\Xi_b \to \Xi_c^{0,+}) = BR(\Lambda_b^0 \to \Lambda_c^+). \tag{69}$$

and the value of $BR(\Lambda_b^0 \to \Lambda_c^+)$ is obtained by comparing the inclusive production rates of Λ_c^+ measured at CLEO and LEP given in Table 6:

$$BR(\Lambda_b^0 \to \Lambda_c^+) = \frac{1}{f_{\Lambda_b^0}} [BR(b \to \Lambda_c^+, \overline{\Lambda_c^-})_{LEP} - (1 - f_{\Lambda_b^0}) BR(b \to \Lambda_c^+, \overline{\Lambda_c^-})_{CLEO}] \tag{70}$$

$f_{\Lambda_b^0}$ is the fraction of b-baryons produced in a b-jet, as measured in Section 4.1.

[17] The published value for this ratio $(0.19 \pm 0.14 \pm 0.04)$ has been corrected using the present value for the oscillation parameter of B_d^0 mesons.

Decay channel	measured rate (%)	direct production rate (%)	reference
$\mathrm{BR}(b \to \mathrm{J}/\psi \mathrm{X})$	1.153 ± 0.051	0.812 ± 0.064	CLEO/LEP average
$\mathrm{BR}(b \to \psi' \mathrm{X})$	0.355 ± 0.049	0.355 ± 0.049	CLEO/LEP average
$\mathrm{BR}(b \to \chi_c^1 \mathrm{X})$	0.414 ± 0.051	0.383 ± 0.051	[133]
$\mathrm{BR}(b \to \chi_c^2 \mathrm{X})$	0.10 ± 0.05	0.07 ± 0.05	[133]

Table 7: *Charmonium production in b-hadron decays.*

6.1.2 Average result for open charm production

Adding $\Xi_c^{0,+}$ particles, open charm production measured at LEP and CLEO are respectively:

$$n(\mathrm{open-charm})_{LEP} = 1.130 \pm 0.039 \pm 0.063. \quad (71)$$

$$n(\mathrm{open-charm})_{CLEO} = 1.084 \pm 0.041 \pm 0.045. \quad (72)$$

where the last uncertainty comes from c-hadron branching fraction measurements. Including correlated systematics between the two results which mainly originate from c-hadron branching fractions, the average production of open-charm in b-hadron decays is:

$$n(\mathrm{open-charm}) = 1.095 \pm 0.059. \quad (73)$$

The dominant uncertainties originate from the poorly known decay branching fractions of the D_s^+, Λ_c^+ and $\Xi_c^{0,+}$ charmed hadrons.

6.2 Charmonium production

Bound $(c\bar{c})$ states (charmonium) contribute also to charm quark production in b-hadron decays. J/ψ production is measured to be similar at LEP [5] and at the $\Upsilon(4S)$ [132]:

$$\mathrm{BR}(\mathrm{B} \to \mathrm{J}/\psi \mathrm{X}) = (1.16 \pm 0.10)\%(LEP), \ = (1.15 \pm 0.06)\%(CLEO) \quad (74)$$

and in the following it has been assumed that charmonium production is the same for all b-hadrons.

Combining LEP and CLEO results for J/ψ and ψ' production and using only CLEO [133] measurements for the χ_c^1 and χ_c^2 rates, measured branching fractions have been given in Table 7. These values have to be corrected for cascade decays to determine the direct production rates of charmonium states. This has been done in the third column of Table 7 using corresponding branching fractions given in the PDG [5].

The model of [134] has been used to account for unmeasured states as χ_c^0, η_c and h_c.

$$\mathrm{BR}(b \to \chi_c^0,\ \eta_c,\ h_c\mathrm{X}) = (0.04 \pm 0.04)\%,\ (0.4 \pm 0.2)\%,\ (0.2 \pm 0.2)\% \quad (75)$$

Theoretical expectations for contributions of other $(c\bar{c})$ states have been considered and found to be negligible [135]. The total production rate for charmonium in b-hadron decays is then evaluated to be:

$$\mathrm{BR}(b \to (c\bar{c})\mathrm{X}) = (2.3 \pm 0.3)\%. \quad (76)$$

The quoted uncertainty depends mainly on the evaluation of η_c and h_c production rates. It may be noted that no model can account for the measured χ_c^1 and χ_c^2 rates. Colour singlet models predict no χ_c^2 production, as observed essentially, but they predict also a χ_c^1 rate which is too low by an order of magnitude. Colour octet models, in the contrary, can explain the measured χ_c^1 rate but predict that the χ_c^2 rate is larger by a factor 5/3 at variance with the observations.

Using the average value for open-charm production given in Equation (73), the average number of charm quarks in b-hadron decays is then evaluated to be:

$$n_c + n_{\overline{c}} = n(\text{open} - \text{charm}) + 2 \cdot \text{BR}(b \to (c\overline{c})\text{X}) = 1.141 \pm 0.059 \tag{77}$$

6.3 Measurements of *b*-hadron decays into no and two open-charm particles

DELPHI [136] and SLD [137] have measured, using respectively the distributions of charged track impact parameters relative to the beam interaction point position and of secondary vertices, the fractions of b-hadron decays in which zero or two charmed hadrons are produced (see Table 32 in Appendix E). These values have been averaged, giving:

$$\text{BR}(b \to 0\text{DX}) = (4.3 \pm 1.8)\% \tag{78}$$

$$\text{BR}(b \to \text{D}\overline{\text{D}}\text{X}) = (22.3 \pm 5.6)\%. \tag{79}$$

Quoted uncertainties for the measured $\text{D}\overline{\text{D}}$ rates by DELPHI [136] and SLD [137] have been increased because the corresponding χ^2 for the average of the two results is equal to 10.7. The difference between these two measurements may indicate an underestimate of systematic uncertainties in the present analyses. Without this correction, the average $\text{BR}(b \to \text{D}\overline{\text{D}}\text{X})$ was equal to $(19.1 \pm 3.8)\%$.

The value obtained for no-open charm corresponds to the sum of $b \to u\overline{u}d$, $b \to sg$ and $b \to (c\overline{c})\text{X}$ decay processes.

From these values the average number of charm quarks produced in b-hadron decays amounts to:

$$n_c + n_{\overline{c}} = 1 - \text{BR}(b \to 0\text{DX}) + \text{BR}(b \to \text{D}\overline{\text{D}}\text{X}) + 2\text{BR}(b \to (c\overline{c})\text{X}) = 1.226 \pm 0.060. \tag{80}$$

6.4 Measurements of *b*-hadron decays with a $\overline{c}$ quark in the final state

Branching fractions of b-hadrons into two charm hadrons can be obtained by reconstructing completely these two hadrons or only the wrong-sign one ($\overline{\text{B}} \to \overline{\text{D}}\ \text{X}$). These measurements can be expressed in terms of the wrong-sign charm branching fraction corrected for the contribution of the tiny $b \to u$ transition:

$$\text{BR}'(b \to \overline{\text{D}}_i X) = \text{BR}(b \to \overline{\text{D}}_i X)(1 - \left|\frac{V_{ub}}{V_{cb}}\right|^2 \alpha_u). \tag{81}$$

The quantity $\alpha_u = 2 \pm 1$ accounts for the larger phase space available in a $b \to u$ as compared to a $b \to c$ transition. The total double-charm rate can be obtained by summing over all types of produced wrong-sign charm hadrons:

$$\text{BR}(b \to \text{D}\overline{\text{D}}\text{X}) = \sum_i \text{BR}'(b \to \overline{\text{D}}_i X). \tag{82}$$

6.4.1 Results at the $\Upsilon(4S)$

CLEO [138] has measured the ratio of wrong sign relative to right sign D^0 and D^+ mesons, called D in the following expressions, in b-meson decays [18].

$$r_D = \frac{BR(\overline{B} \to \overline{D}X)}{BR(\overline{B} \to DX)} = 0.088 \pm 0.026 \pm 0.005 \tag{83}$$

The probability of having a $\overline{D}^0$ or a D^- from the virtual W^- is then:

$$BR(\overline{B} \to \overline{D}X) = \frac{r_D \times BR(\overline{B} \to D,\ \overline{D}X)}{1 + r_D} \tag{84}$$

In non-strange b-meson decays the largest fraction of produced D_s^- mesons is expected to originate from $W^- \to \overline{c}s$ final states. The measured value by CLEO [127] includes D_s^+ and D_s^- mesons and has to be corrected for D_s^+ production at the lower vertex when, after the $b \to c$ transition, the c quark hadronises into a D_s^+ meson by combining with a $\overline{s}$ quark.

$$BR(\overline{B} \to D_s^- X) = BR(\overline{B} \to D_s^+,\ D_s^- X) - \mathcal{P}^{\text{low. V}}_{\overline{B} \to D_s^+} \tag{85}$$

CLEO [139] has searched also for the production of right sign D_s^+ mesons, in $\overline{B}$ decays and obtains a relative fraction equal to $0.16 \pm 0.08 \pm 0.03$. The lower vertex production rate of D_s^+ mesons can then be obtained corresponding to $(2 \pm 1)\%$. The correction which has to be applied in Equation (85) concerns the production of strange D_s^+ mesons at the lower vertex, in double-charm decays, and its value may differ from this measured number because kinematical and dynamical situations are different. In the following it has been assumed that:

$$\mathcal{P}^{\text{low. V}}_{\overline{B} \to D_s^+} = (2 \pm 2)\% \tag{86}$$

and the same value has been used for charm baryon production, at the lower vertex. We introduce the notation:

$$p_{sdq} = \mathcal{P}^{\text{low. V}}_{\overline{B} \to D_s^+} = \mathcal{P}^{\text{low. V}}_{\overline{B} \to \Lambda_c^+}. \tag{87}$$

The branching fraction into wrong-sign $\overline{\Lambda}_c^-$ baryons $BR(\overline{B} \to \overline{\Lambda}_c^- X)$ has been given already in Equation (66).

6.4.2 ALEPH measurements

The ALEPH Collaboration [140] has measured several branching fractions of b-hadron decays into two charmed hadrons by reconstructing completely their decay final states (see Appendix E, Table 33)

From decays with no strange c-meson, emitted in the final state, it is possible to obtain:

$$BR'(\overline{B} \to \overline{D}X) = \frac{BR(b \to D\overline{D}X)}{(f_{B^+} + f_{B_d})(1 - 2p_{sdq}) + (f_{B_s} + f_{b\text{-baryon}})p_{nsdq}}. \tag{88}$$

The quantity p_{nsdq} corresponds to the probability, supposed to be the same for $\overline{B}_s^0$ and b-baryons, that there is, respectively, no D_s^+ and no charm baryon in their decay final

[18] The measured value $0.100 \pm 0.026 \pm 0.016$ has been corrected using the new value of the B_d^0 oscillation parameter given in Table 1.

	$BR'(\overline{B} \to \overline{D}X)$	$BR'(\overline{B} \to D_s^- X)$	$BR'(\overline{B} \to \overline{\Lambda}_c^- X)$
ALEPH	$0.094 \pm 0.032 \pm 0.006$	$0.148 \pm 0.035 \pm 0.043$	–
DELPHI	$0.090 \pm 0.022 \pm 0.002$	–	–
CLEO	$0.070 \pm 0.020 \pm 0.002$	$0.096 \pm 0.022 \pm 0.029$	0.008 ± 0.006
Average	0.082 ± 0.013	0.098 ± 0.037	0.008 ± 0.006

Table 8: *Probability of producing a charm hadron from the upper vertex in b-decays estimated from ALEPH, DELPHI and CLEO data, together with the average.*

state, emitted at the lower vertex. The central value, measured in b-hadron semileptonic decays into D^{**} states has been used as an estimate but its attached uncertainty has been increased to account for the fact that this number is used in case of double-charm decays:

$$p_{nsdq} = \mathcal{P}^{\text{low. V}}_{\mathrm{B_s^0} \to D} = \mathcal{P}^{\text{low. V}}_{b-baryon \to D} = (30 \pm 10)\%. \tag{89}$$

In a similar way, it is possible to relate the measured branching fraction corresponding to DD_s^-X and $\overline{D}D_s^+X$ final states to the branching fraction into wrong sign D or D_s^- mesons:

$$\begin{aligned} BR(b \to DD_s^-X \ + \ \overline{D}D_s^+X) = & [(f_{B^+} + f_{B_d} + f_{b\text{-baryon}})p_{sdq} + f_{B_s}p_{nsdq}]BR'(\overline{B} \to \overline{D}X) \\ & + [(f_{B^+} + f_{B_d})(1 - 2p_{sdq}) + (f_{B_s} + f_{b\text{-baryon}})p_{nsdq}]BR'(\overline{B} \to D_s^-X) \end{aligned} \tag{90}$$

6.4.3 DELPHI measurement

DELPHI [141] has measured the fractions, r_{D^0} and r_{D^+}, of wrong-sign $\overline{D^0}$ and D^- in b-hadron decays:

$$r_{D^0} = \frac{BR(b \to \overline{D}^0\ X)}{BR(b \to D^0\ X)} = (12.9 \pm 2.8)\%, \ \ r_{D^+} = \frac{BR(b \to D^-\ X)}{BR(b \to D^+\ X)} = (12.3 \pm 6.7)\% \tag{91}$$

Using the values of the inclusive rates measured at LEP, given in Table 6, and correcting for the $b \to u$ contribution, a value for $BR'(\overline{B} \to \overline{D}X)$ has been obtained (Table 8).

6.4.4 Average of exclusive measurements

Production rates for wrong sign c-hadrons measured by CLEO, DELPHI and ALEPH have been combined and their averages are given in Table 8. From these values, and using Equation (80), the number of c and $\overline{c}$ quarks produced in b-meson decays has been obtained:

$$n_c + n_{\overline{c}} = 1.191 \pm 0.040. \tag{92}$$

6.5 Average number of c and $\overline{c}$ quarks from all measurements

Measurements done at the $\Upsilon(4S)$ and at the Z have been combined separately [19]; they are found to be compatible and an overall average has been obtained.

[19] The value corresponding to $\Upsilon(4S)$ measurements differs from the one given in [123] (1.10 ± 0.05) mainly because of different rates taken for the production of charmed baryons in b-meson decays and

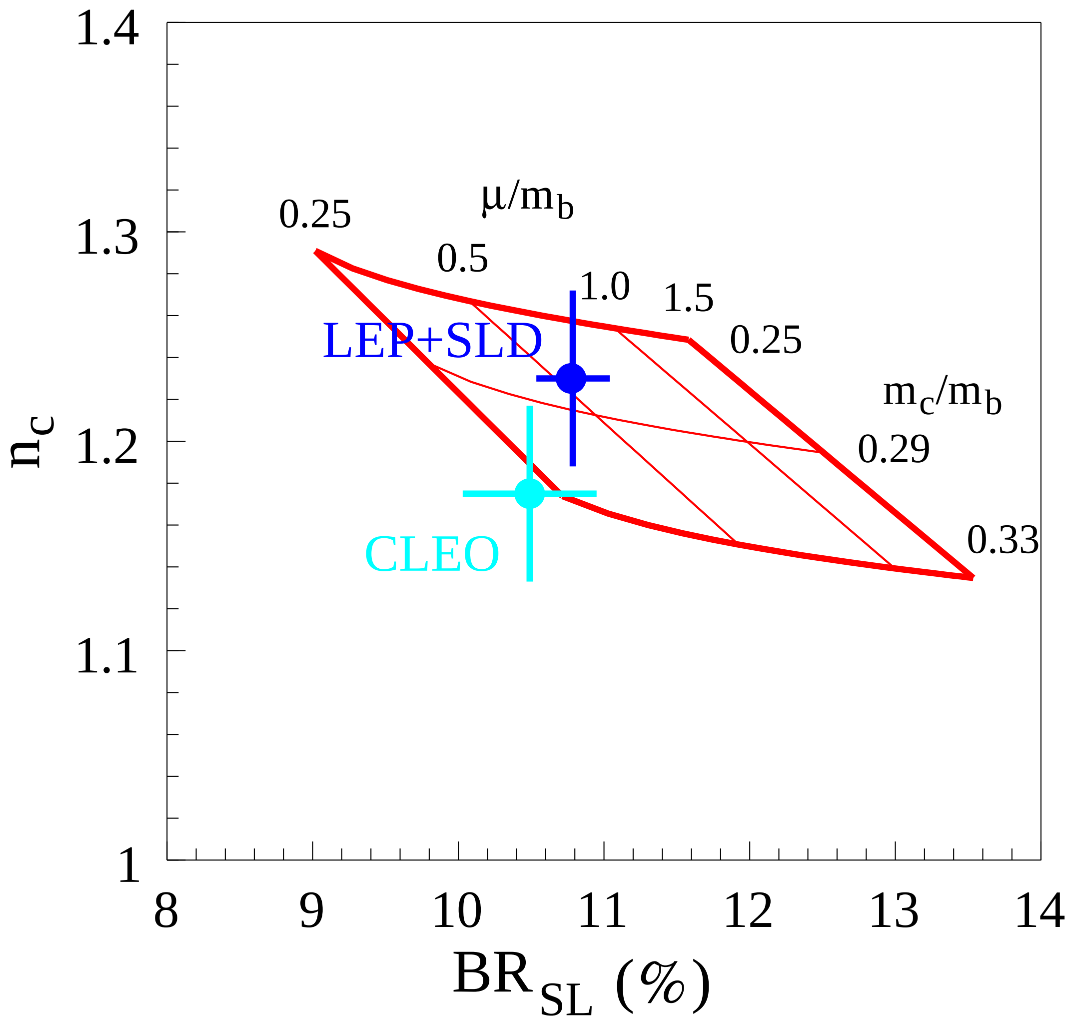

Figure 8: *Comparison between the measured number of c and $\overline{c}$ quarks in b-hadron decays and of the inclusive semileptonic branching fraction, with theoretical predictions. The value of the semileptonic branching fraction reported for CLEO is taken from [142].*

$$n_c + n_{\overline{c}}(\Upsilon(4S)) = 1.175 \pm 0.042 \tag{93}$$

$$n_c + n_{\overline{c}}(LEP + SLD) = 1.230 \pm 0.042 \tag{94}$$

$$n_c + n_{\overline{c}}(\Upsilon(4S) + LEP + SLD) = 1.206 \pm 0.033 \tag{95}$$

These values and the corresponding measured semileptonic branching fractions of b-hadrons have been indicated in Figure 8 and compared with theoretical estimates [78]. The value of the semileptonic branching fraction measured at LEP has been corrected so that it corresponds to a sample containing the same amount of $\overline{\mathrm{B}^0_{\mathrm{d}}}$ and B^+ mesons to be compared with the corresponding measurement at the $\Upsilon(4S)$:

$$\mathrm{BR}(b \to \ell^- \overline{\nu_\ell} X)(\mathrm{corrected}) = \frac{\frac{1}{2}[\tau(\mathrm{B}^+) + \tau(\mathrm{B}^0_{\mathrm{d}})]}{\tau_b} \mathrm{BR}(b \to \ell^- \overline{\nu_\ell} X). \tag{96}$$

by the inclusion, in the present average, of double charm rates measured by CLEO. The value obtained for $LEP + SLD$ measurements differs from the one given in [124] (1.171 ± 0.040) because the inclusive double charm rate measured by SLD, made available later in year 2000, has now been included.

Using the lifetime values given in Section 3, this correction amounts to 1.02 ± 0.02.

Present results favour a rather standard value for the charm quark mass and a low scale, μ, at which QCD corrections have to be evaluated:

$$\frac{m_c}{m_b} = 0.30 \pm 0.02, \ \frac{\mu}{m_b} \sim 0.35 \tag{97}$$

It should be noted that heavy quark masses entering into this evaluation [78] have been defined using a different renormalization scheme as compared to the one used in Sections 7 and 8 for the determination of $|V_{cb}|$ and $|V_{ub}|$. As a consequence, their values cannot be directly compared. In Equation (97), m_Q are pole masses defined at one-loop in perturbation theory. Using recent determinations of $\overline{\rm MS}$ heavy quark masses [143, 144]:

$$\overline{m_b}(\overline{m_b}) = (4.25 \pm 0.08)\ \mathrm{GeV/c^2} \text{ and } \overline{\mathrm{m_c}}(\overline{\mathrm{m_c}}) = (1.23 \pm 0.09)\ \mathrm{GeV/c^2} \tag{98}$$

and the expression, at one loop, relating these two definitions of quark masses:

$$m_Q^{pole,1} = \overline{m_Q}(\overline{m_Q}) \left(1 + \frac{4}{3}\frac{\alpha_S(\overline{m_Q})}{\pi}\right), \tag{99}$$

one obtains:

$$m_b^{pole,1} = (4.7 \pm 0.1)\ \mathrm{GeV/c^2} \text{ and } \mathrm{m_c^{pole,1}} = (1.4 \pm 0.1)\ \mathrm{GeV/c^2}. \tag{100}$$

The ratio of these two quantities agrees with Equation (97).

The $0c$ rate can be obtained from the measured 0D rate given in Equation (78) after having subtracted charmonium production:

$$\mathrm{BR}(\overline{\mathrm{B}} \rightarrow 0c) = (2.0 \pm 1.8)\%. \tag{101}$$

This value is in agreement with the theoretical estimate of $(2.1\pm0.8)\%$ [145] which includes $b \rightarrow u,\ b \rightarrow sg,\ sgg$ transitions evaluated at NLO. There is no room for a large additional $0c$ component as advocated for instance in [146].

The $c\overline{c}$ rate can be obtained by combining results given in Equation (79), Table 8 and adding the charmonium component:

$$\mathrm{BR}(\overline{\mathrm{B}} \rightarrow c\overline{c}) = (22.3 \pm 3.2)\%. \tag{102}$$

This value is in reasonable agreement with theoretical expectations: $(21 \pm 6)\%$ as already illustrated in Figure 8. These predictions depend mainly on the assumed value for the c-quark mass.

The main conclusions from this study are that inclusive semileptonic b-hadron decays can be explained by theory once QCD corrections are evaluated at a rather low scale, of the order of 0.35 m_b, and that the measured number of charmed quarks is in agreement with the value expected using independent determinations of b and c-quark masses. The large variation of the estimates for the b-hadron semileptonic branching fraction versus μ and the low value favoured for this parameter indicate that computations of higher order QCD corrections are needed. As already explained, it would be of interest to have a similar analysis done using values for quark masses evaluated in the same renormalization scheme as in Sections 7 and 8.

7 Average of LEP $|V_{cb}|$ measurements

Within the framework of the Standard Model of electroweak interactions, the elements of the Cabibbo-Kobayashi-Maskawa mixing matrix are free parameters, constrained only by the requirement that the matrix be unitary. The Operator Product Expansion (OPE) and Heavy Quark Effective Theory (HQET) provide means to determine $|V_{cb}|$ with relatively small theoretical uncertainties, by studying the decay rates of inclusive and exclusive semileptonic b-decays respectively. Relevant branching fractions have to be determined experimentally. Inputs from theory are needed to obtain the values of the matrix elements.

There are two methods to measure $|V_{cb}|$: the inclusive method, which uses the semileptonic decay width of b-decays and the OPE; and the exclusive method, where $|V_{cb}|$ is extracted by studying the exclusive $\overline{B^0_d} \rightarrow D^{*+}\ell^-\overline{\nu}_\ell$ decay process using HQET. The $B \rightarrow D\ell^-\overline{\nu_\ell}$ channel has not been averaged to date.

In this note, both methods are used to determine values for $|V_{cb}|$, which are then combined to produce a single average[20]. The semileptonic b-decay width, determined by the LEP heavy flavour electroweak fit to ALEPH, DELPHI, L3 and OPAL data, is used to determine $BR(b \rightarrow \ell^-\overline{\nu_\ell}X)$. Results from ALEPH [147], DELPHI [148] and OPAL [149] are used to perform a LEP $|V_{cb}|$ average in the $\overline{B^0_d} \rightarrow D^{*+}\ell^-\overline{\nu}_\ell$ decay channel. These measurements are combined using a method similar to that used by the B oscillations working group. OPAL has published two measurements, quoted in the following $\mathrm{OPAL_{inc}}$ and $\mathrm{OPAL_{exc}}$, using respectively a partial or a complete reconstruction of the D^{*+} decay products.

Theoretical input parameters needed to extract $|V_{cb}|$ from actual measurements are detailed in Appendix F. In both methods, the extraction of $|V_{cb}|$ is systematics limited and the dominant error is from theory. Thus it is important that a consistent approach to theoretical uncertainties is adopted by the authors, even though some arbitrariness in uncertainty definitions remain (see Appendix F). Further work from theorists is required to provide uncertainties in a consistent framework and to identify methods to experimentally control these uncertainties. In both analyses, the validity of the quark/hadron duality hypothesis has been assumed. This assumption implies that the cross sections and the decay rates defined in the region of the time-like momenta are calculable in QCD after a "smearing" or "averaging" procedure is applied. In semileptonic decays, it is considered to be valid [81] as the integration over the lepton and neutrino phase space provides the "smearing" over the invariant hadronic mass (global duality). Global duality cannot be derived from first principles and it is an assumption in many QCD applications. Ultimately experimental data, for example the moment-type analyses of the lepton momentum distribution in b-hadron semileptonic decays, will decide the real size of any deviations from global duality. As explained in [81], violations of duality will affect more the exclusive than the inclusive determination of $|V_{cb}|$.

As compared with previous averages [1], a new analysis from OPAL has been included and the same form factors parametrization, as used in CLEO, has been adopted.

[20]The present members of the $|V_{cb}|$ working group are: D. Abbaneo, E. Barberio, S. Blyth, M. Calvi, P. Gagnon, R. Hawkings, M. Margoni, S. Mele, D. Rousseau and F. Simonetto.

7.1 Inclusive $|\mathrm{V}_{cb}|$ determination

In the inclusive method, the partial width for semileptonic B meson decays to charmed mesons is related to $|\mathrm{V}_{cb}|$ using the following expression (Appendix F):

$$
\begin{aligned}
|\mathrm{V}_{cb}| &= 0.0411\sqrt{\frac{1.55}{0.105}\Gamma(b\to\ell^-\overline{\nu_\ell}\mathrm{X}_c)}\left(1-0.024\left(\frac{\mu_\pi^2-0.5}{0.2}\right)\right)\times \\
& (1\pm 0.030(pert.)\pm 0.020(m_b)\pm 0.024(1/m_b^3)). \qquad (103)
\end{aligned}
$$

where $\mathrm{X_c}$ represents all final states containing a charmed quark. The definition adopted to fix the values of the running b- and c-quark masses and the scale at which they are evaluated are given in Appendix F. The rest of the expression, within parentheses, represents the correction to the muon decay formalism, depending on the b- and c-quark masses and on the strong coupling constant; μ_π^2 is the average of the square of the b-quark momentum inside the b-hadron.

In Z decays, a mixture of $\overline{\mathrm{B^0_d}}$, $\mathrm{B^-}$, $\overline{\mathrm{B^0_s}}$ and b-baryons is produced, such that the inclusive semileptonic branching fraction measured at LEP is an average over the different hadrons produced:

$$
\begin{aligned}
\mathrm{BR}(b\to\ell^-\overline{\nu_\ell}\mathrm{X}_c) &= f_{\mathrm{B_d}}\frac{\Gamma(\overline{\mathrm{B^0_d}}\to\ell^-\overline{\nu_\ell}\mathrm{X}_c)}{\Gamma(\overline{\mathrm{B^0_d}})}+f_{\mathrm{B^+}}\frac{\Gamma(\mathrm{B^-}\to\ell^-\overline{\nu_\ell}\mathrm{X_c})}{\Gamma(\mathrm{B^-})} \\
&+f_{\mathrm{B_s}}\frac{\Gamma(\overline{\mathrm{B^0_s}}\to\ell^-\overline{\nu_\ell}\mathrm{X}_c)}{\Gamma(\overline{\mathrm{B^0_s}})}+f_{b\text{-baryon}}\frac{\Gamma(b-\mathrm{baryon}\to\ell^-\overline{\nu_\ell}\mathrm{X}_c)}{\Gamma(b-\mathrm{baryon})} \\
&\simeq \Gamma(b\to\ell^-\overline{\nu_\ell}\mathrm{X}_c)(f_{\mathrm{B_d}}\tau_{B^0}+f_{\mathrm{B^+}}\tau_{B^-}+f_{\mathrm{B_s}}\tau_{B_s}+f_{b\text{-baryon}}\tau_{b-\mathrm{baryon}}) \\
&= \Gamma(b\to\ell^-\overline{\nu_\ell}\mathrm{X}_c)\tau_b \qquad (104)
\end{aligned}
$$

where τ_b is the average b-hadron lifetime. Therefore the semileptonic width of b-hadrons can be obtained using the inclusive semileptonic branching fraction and the average b-hadron lifetime. The two last equalities in Equation (104) assume that all b-hadrons have the same semileptonic width. This hypothesis may be incorrect for b-baryons. Taking into account the present precision of LEP measurements of b-baryon semileptonic branching fractions and lifetimes, an estimate of the correction to Equation (104) is about 1.5% (see Section 2).

The average LEP value for $\mathrm{BR}(b\to\ell^-\overline{\nu_\ell}\mathrm{X}) = (10.56\pm 0.11(\mathrm{stat.})\pm 0.18(\mathrm{syst.}))\%$ is taken from the global fit which combines the heavy flavour measurements performed at the Z (see Section 2). The $\mathrm{BR}(b\to\ell^-\overline{\nu_\ell}\mathrm{X}_u)$ contribution is subtracted from $\mathrm{BR}(b\to\ell^-\overline{\nu_\ell}\mathrm{X})$, using the LEP average value given in Section 8. For the average b-hadron lifetime, the world average value of τ_b is used, as obtained in Equation (32).

7.1.1 Sources of systematic errors

The systematic errors assumed at present in the determination of the inclusive semileptonic decay width can be grouped into the following categories:

- *errors related to the efficiency and purity of the b-tagging algorithm*

  As a lifetime b-tag is involved, effects due to the uncertainties in the sample composition in terms of different heavy hadrons, and uncertainties in the heavy hadron lifetimes, are considered.

- *input parameters influencing the signal and background normalisation*

  The values of the production fractions: $f_{\mathrm{B_d}}$, $f_{\mathrm{B^+}}$, $f_{\mathrm{B_s}}$, $f_{b\text{-baryon}}$ were taken from Section 4.1. The branching fractions $\mathrm{BR}(b \to \overline{c} \to \ell)$, $\mathrm{BR}(b \to \tau \to \ell)$, $\mathrm{BR}(b \to \mathrm{J}/\psi \to \ell^+\ell^-)$, and the rates of gluon splitting $\mathrm{P}(\mathrm{g} \to \mathrm{c\overline{c}})$ and $\mathrm{P}(\mathrm{g} \to \mathrm{b\overline{b}})$ have been fixed to the values of [3]. R_b, $\mathrm{R_c}$, $\mathrm{BR}(b \to c \to \overline{\ell})$ and $\mathrm{BR}(c \to \overline{\ell})$ are parameters of the LEPEWWG fit. Values for all these quantities are listed in Table 1.

- *the average fraction of the beam energy carried by the weakly decaying b-hadron*

  Different models have been considered for the shape of the fragmentation function and the free parameters of the models have been determined from the data.

- *Λ_b polarization*

  These effects on the lepton spectra have been included.

- *semileptonic decay models*

  The average LEP value for $\mathrm{BR}(b \to \ell^- \overline{\nu_\ell} \mathrm{X})$ is taken from the global LEPEWWG fit which combines the heavy flavour measurements performed at the Z [3], but removing the forward-backward asymmetry measurements (see Section 2).

- *detector specific items*

  These include: lepton efficiencies, misidentification probabilities, detector resolution effects, jet reconstruction, etc.

The errors listed in the last item are uncorrelated among the different experiments. The others have been split into their uncorrelated and correlated parts. The error on the average b-hadron lifetime is assumed to be uncorrelated with the error on the semileptonic branching fraction. The propagation of these errors to the error on $|\mathrm{V}_{cb}|$ is done assuming that they are Gaussian in the branching fraction and the lifetime, respectively.

7.1.2 Inclusive $|\mathrm{V}_{cb}|$ average

Using the expression given in Equation (103), the following value is obtained:

$$|\mathrm{V}_{cb}|^{incl.} = (40.7 \pm 0.5(\mathrm{exp.}) \pm 2.4(\mathrm{theo.})) \times 10^{-3} \tag{105}$$

where the first error is experimental and the second is from theory. The experimental contributions due to the semileptonic branching fraction and the lifetime are $\pm 0.37 \times 10^{-3}$ and $\pm 0.18 \times 10^{-3}$, respectively. The dominant systematic uncertainty, of theoretical origin, comes from the determination of the kinetic energy of the b-quark inside the b-hadron as explained in Appendix F.

7.2 Exclusive $|\mathrm{V}_{cb}|$ determination

In the exclusive method, the value of $|\mathrm{V}_{cb}|$ is extracted by studying the decay rate for the process $\overline{\mathrm{B^0_d}} \to \mathrm{D}^{*+} \ell^- \overline{\nu}_\ell$ as a function of the recoil kinematics of the D^{*+} meson. The decay rate is parameterized as a function of the variable w, defined as the product of the four-velocities of the D^{*+} and the $\overline{B^0_d}$ mesons. This variable is related to the square of the four-momentum transfer from the $\overline{B^0_d}$ to the $\ell^- \overline{\nu}_\ell$ system, q^2, by:

$$w = \frac{m^2_{D^{*+}} + m^2_{B^0_d} - q^2}{2 m_{B^0_d} m_{D^{*+}}}, \tag{106}$$

and its values range from 1.0, when the D^{*+} is produced at rest in the $\overline{B^0_d}$ rest frame, to about 1.50. Using HQET, the differential partial width for this decay is given by:

$$\frac{d\Gamma}{dw} = \mathcal{K}(w) \mathcal{F}^2_{D^*}(w) \left|V_{cb}\right|^2 \tag{107}$$

where $\mathcal{K}(w)$ contains kinematic factors and $\mathcal{F}_{D^*}(w)$ is the hadronic form factor for the decay.

$$\mathcal{K}(w) = m^3_{D^*}(m_B - m_{D^*})^2 \sqrt{w^2-1}(w+1)^2 \left(1 + \frac{4w}{w+1} \frac{1-2wr+r^2}{(1-r)^2}\right) \tag{108}$$

with $r = m_{D^*}/m_B$.

Although the shape of the form factor, $\mathcal{F}_{D^*}(w)$, is not known, its magnitude at zero recoil, corresponding to $w = 1$, can be estimated using HQET. It is found to be convenient to express $\mathcal{F}_{D^*}(w)$ in terms of the axial form factor $h_{A_1}(w)$ and of the reduced helicity form factors $\tilde{H}_0$ and $\tilde{H}_\pm$:

$$\mathcal{F}_{D^*}(w) = h_{A_1}(w) \sqrt{\frac{\tilde{H}^2_0 + \tilde{H}^2_+ + \tilde{H}^2_-}{1 + \frac{4w}{w+1}\frac{1-2wr+r^2}{(1-r)^2}}}. \tag{109}$$

The reduced helicity form factors are themselves expressed in terms of the ratios between the other HQET form factors $(h_V(w),\ h_{A_2}(w),\ h_{A_3}(w))$ and $h_{A_1}(w)$:

$$\tilde{H}_0(w) = 1 + \frac{w-1}{1-r}\left[1 - R_2(w)\right] \tag{110}$$

$$\tilde{H}_\pm(w) = \frac{\sqrt{1-2wr+r^2}}{1-r}\left[1 \mp \sqrt{\frac{w-1}{w+1}} R_1(w)\right] \tag{111}$$

with

$$R_1(w) = \frac{h_V(w)}{h_{A_1}(w)} \text{ and } R_2(w) = \frac{h_{A_3}(w) + r h_{A_2}(w)}{h_{A_1}(w)}. \tag{112}$$

Values for $R_1(w)$ and $R_2(w)$ have been obtained by CLEO [150] using different models.

The unknown function $h_{A_1}(w)$ is approximated with an expansion around $w = 1$ due to Caprini, Lellouch and Neubert (CLN) [151]:

$$h_{A_1}(w) = h_{A_1}(1) \times \left[1 - 8\rho^2_{A_1} z + (53\rho^2_{A_1} - 15)z^2 - (231\rho^2_{A_1} - 91)z^3\right], \tag{113}$$

where $\rho^2_{A_1}$ is the slope parameter at zero recoil and $z = \frac{\sqrt{w+1}-\sqrt{2}}{\sqrt{w+1}+\sqrt{2}}$. An alternative parametrization, obtained earlier, can be found in [152].

In the heavy quark limit ($m_b \to \infty$), $\mathcal{F}_{D^*}(1) = h_{A_1}(1)$ coincides with the Isgur-Wise function [153, 154] which is normalised to unity at the point of zero recoil. Corrections to

$\mathcal{F}_{D^*}(1)$ have been calculated to take into account the effects of finite quark masses and QCD corrections [155]. They yield $\mathcal{F}_{D^*}(1) = 0.88 \pm 0.05$ (Appendix F).

Experiments determine the product $\mathcal{F}^2_{D^*}(1)\,|\mathrm{V}_{cb}|^2$ by fitting this quantity and the slope $\rho^2_{A_1}$, using the expression (107), convoluted with the experimental resolution on the w variable. Since the phase space factor $\mathcal{K}(w)$ tends to zero as $w \to 1$, the decay rate vanishes at $w = 1$ and the accuracy of the extrapolation relies on achieving a reasonably constant reconstruction efficiency in the region close to $w = 1$.

7.2.1 Sources of systematic uncertainties

The systematic uncertainties in the determination of $|\mathrm{V}_{cb}|$ using the semileptonic decay $\overline{\mathrm{B}^0_\mathrm{d}} \to \mathrm{D}^{*+}\ell^-\overline{\nu}_\ell$ can be grouped into the following categories:

- *normalisation*: $\overline{\mathrm{B}^0_\mathrm{d}}$ meson production rate, $\mathrm{D}^{(*)}$ branching fraction to the tagged final states (including topological BR), $\overline{\mathrm{B}^0_\mathrm{d}}$ lifetime (this is needed to obtain the $\overline{\mathrm{B}^0_\mathrm{d}} \to \mathrm{D}^{*+}\ell^-\overline{\nu}_\ell$ decay partial width), and the $b \to \mathrm{B}$ fragmentation function (which influences the reconstruction efficiency).

- *background from physical processes:* comprising $\overline{\mathrm{B}^0_\mathrm{d}} \to \mathrm{D}^{*+}\tau^-\overline{\nu}_\tau$, $\overline{\mathrm{B}^0_\mathrm{d}} \to \mathrm{D}^{*+}\mathrm{X}_{\overline{c}}$ (followed by the semileptonic decay $\tau/\mathrm{X}_{\overline{c}} \to \ell^-\overline{\nu}_\ell\mathrm{X}$) and, particularly, the intermediate production of excited charm mesons D^{**} which subsequently decay to a D^{*+};

- *detector specific items*: selection efficiency (lepton identification, tracking, vertexing), non physics background (combinatorial, hadron mis-identification), resolution, fitting, etc. This last set is treated as uncorrelated among experiments, and therefore will be ignored in the following discussion.

7.2.2 Normalisation

The $\overline{\mathrm{B}^0_\mathrm{d}}$ production rate at LEP is given by the product:

$$\frac{\Gamma(\mathrm{Z} \to b\bar{b})}{\Gamma(\mathrm{Z} \to \mathrm{hadrons})}\ \mathrm{BR}(b \to \overline{\mathrm{B}^0_\mathrm{d}}) \;=\; \mathrm{R}_b\ f_{\mathrm{B_d}} \tag{114}$$

where R_b is taken from [3] (Table 1) and $f_{\mathrm{B_d}}$ from Section 4.1. The values for the charm meson decay branching fractions are taken from the PDG [5]. Correlations among some of them (e.g. $\mathrm{D}^0 \to \mathrm{K}^-\pi^+$ with $\mathrm{D}^0 \to \mathrm{K}n\pi$, etc.) are included. Analyses based on the inclusive reconstruction of pions from D^{*+} cascade decays may be affected by the knowledge of the topological D^0 branching fractions; they are taken from MARKIII measurements [156]. The $\overline{\mathrm{B}^0_\mathrm{d}}$ lifetime determined in Section 3.2 is used. Knowledge of the $\overline{\mathrm{B}^0_\mathrm{d}}$ fragmentation function is necessary in order to compute the fraction of $\overline{\mathrm{B}^0_\mathrm{d}}$ which were not reconstructed because they did not have enough energy to be detected. $\overline{\mathrm{B}^0_\mathrm{d}}$ hadrons produced in e^+e^- annihilations carry on average a large fraction, $< x_E >$, of the beam energy (Table 1); consequently only a small fraction of them are outside the selection acceptance.

7.2.3 Physics background

Semileptonic decays of b-hadrons to charm excited states (D^{**}) which then decay to a D^{*+} are the major source of correlated (physics) background. The D^{**} can be either a narrow resonant state, D_1(narrow) or D_2^*, or a broad and/or a non-resonant state, D_1(broad) or D_0^*. The production rates of the different D^{**} states (see Sections 2.2 and 2.4) and the variation of their corresponding form factors as a function of w, have also to be considered. The Isgur-Wise model [28] predicts a sizeable D^{**} background rate

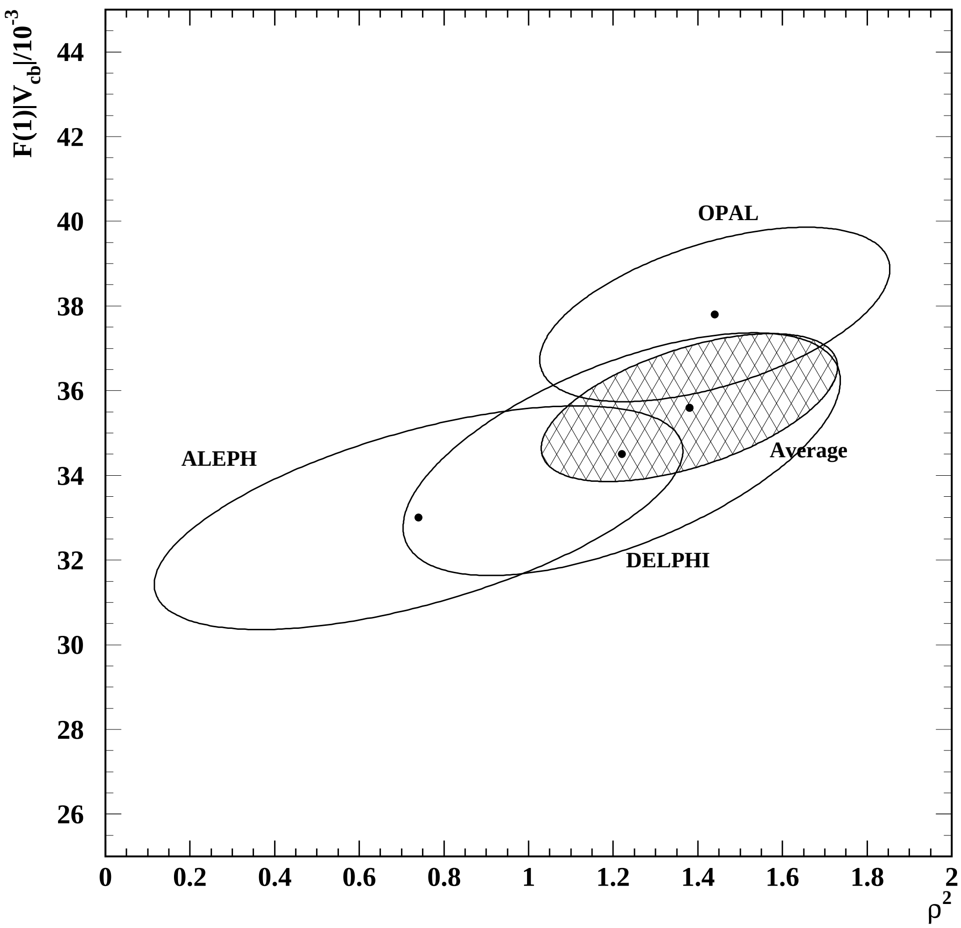

Figure 9: *Corrected measurements and LEP average of the quantities $\mathcal{F}_{D^*}(1)|V_{cb}|$ and ρ^2 using the exclusive method. The error ellipses, centred on the different measurements, correspond to contours at the 68% C.L. level and include systematics. The hatched ellipse corresponds to the average of the three experiments.*

near the end-point spectrum. As a consequence, in this model the error on the overall amount of D^{**} has a large effect on $|V_{cb}|$ while having a negligible contribution on the slope parameter $\rho^2_{A_1}$. However, HQET predicts that, in the infinite charm mass limit, the rate near $w = 1$ is suppressed by a further factor $(w^2 - 1)$ when compared with the signal [26], [157]. In this case, the D^{**} rate uncertainty would have a large effect on the slope with only a small influence on $|V_{cb}|$ [148]. However, models in this extreme case fail to predict the ratio R^{**} (Equation (19)) between the production rates of the two narrow states.

A treatment which accounts for $\mathcal{O}(1/m_c)$ corrections is proposed in [27]. Several possi-

Experiment	$\mathcal{F}_{D^*}(1)\|V_{\rm cb}\|$ ($\times 10^3$)	$\rho^2_{A_1}$
ALEPH [147]	$33.0 \pm 2.1 \pm 1.6$	$0.74 \pm 0.25 \pm 0.41$
DELPHI [148]	$34.5 \pm 1.4 \pm 2.5$	$1.22 \pm 0.14 \pm 0.36$
$\rm OPAL_{inc}$ [149]	$37.9 \pm 1.3 \pm 2.4$	$1.21 \pm 0.15 \pm 0.35$
$\rm OPAL_{exc}$ [149]	$37.5 \pm 1.7 \pm 1.8$	$1.42 \pm 0.21 \pm 0.24$

Table 9: *Experimental results on $\mathcal{F}_{D^*}(1)|V_{cb}|$ and $\rho^2_{A_1}$ corrected for common inputs.*

ble approximations of the form factors are provided, depending on five different expansion schemes and on three different input parameters. In the calculation of the systematic errors each scheme proposed was tested and for each scheme, the input parameters were varied over the range consistent with the measured value of R**. The systematic error was then computed as half the difference between the two most extreme results. The branching ratios of the processes which affect the $|\mathrm{V}_{cb}|$ value are taken from Section 2. For the average, the ALEPH analysis which was based on the Isgur-Wise model [28] , has been updated to use the same model as the others.

To account for the lack of knowledge of the non-resonant part of the D** background, the fraction of narrow resonances is varied from 0 to 100%. In the average, this error is added in quadrature with the one derived from the D** shape. Since HQET models predict the variation of the D** form factors as a function of w only for resonant states the present approach does not account for the shape variation from non-resonant $\mathrm{D}^{(*)}\pi$ background states if they have marked differences as compared to D** resonant states.

7.2.4 Corrections applied to the measurements

Since the four LEP measurements have been performed using different methods and inputs, they must be put on the same footing before being averaged. Corrections for changing to the standard input parameters, listed in Table 1 or evaluated in previous Sections, have been calculated as for the Δm_d measurement (see Section 4.2). The central value of each analysis is adjusted according to the difference between the used and desired parameter values and the associated systematic error. The systematic error itself is then scaled to reflect the desired uncertainty on the input parameter. Table 9 lists the corrected results. The uncertainty on the variation with w of the $\overline{\mathrm{B}^0_{\rm d}} \to \mathrm{D}^{**+}\ell^-\overline{\nu}_\ell$ form factors is taken to be fully correlated between experiments. Published results are using theoretically predicted values for the form factor ratios $R_1(w)$ and $R_2(w)$ defined in Equation (112). In the present analysis, experimental results on these quantities, obtained by CLEO [150] have been used and a corresponding systematic uncertainty has been derived. The dominant systematic uncertainties on $\mathcal{F}_{D^*}(1)|V_{cb}|$ are listed in Table 10. The largest comes from the $\overline{\mathrm{B}^0_{\rm d}} \to \mathrm{D}^{**+}\ell^-\overline{\nu}_\ell$ contribution.

7.2.5 Exclusive $|\mathbf{V}_{cb}|$ average

The combination method for $\mathcal{F}_{D^*}(1)|V_{cb}|$ and $\rho^2_{A_1}$ is the same as the method used for Δm_d (see Section 4.2) generalized to the combination of two or more correlated parameters. The LEP average (see Figure 10-right) gives:

$$\mathcal{F}_{D^*}(1)|V_{cb}| \quad = \quad (35.6 \pm 0.8(\mathrm{stat.}) \pm 1.5(\mathrm{syst.})) \times 10^{-3} \tag{115}$$

	$\frac{\Delta\mathcal{F}(1)\lvert V_{cb}\rvert}{\mathcal{F}(1)\lvert V_{cb}\rvert}$				$\Delta\rho^2_{A_1}$			
Source	A	D	O_{inc}	O_{exc}	A	D	O_{inc}	O_{exc}
Correlated errors								
$\Gamma_{b\bar{b}}/\Gamma_{had}$	0.2	0.3	0.2	0.2	-	-	-	-
$BR(b \to \overline{B^0_d})$	1.1	1.8	1.3	1.3	-	0.02	-	-
$BR(D^{*+} \to D^0\pi^+)$	0.4	0.4	0.4	0.4	-	0.01	-	-
$BR(D^0 \to K^+\pi^-)$	0.6	-	-	0.3	-	-	-	0.01
$BR(D^0 \to K3\pi)$	1.3	-	-	-	-	-	-	-
$BR(D^0 \to K^0 2\pi)$	0.6	-	-	-	-	-	-	-
$BR(D^0 \to K^+\pi^-\pi^0)$	-	-	-	2.6	-	-	-	0.02
$BR(D \to Kn\pi)$	-	0.2	-	-	-	-	-	-
D^{**} rate	0.9	1.5	0.4	0.7	0.02	0.07	0.03	0.05
D^{**} shape	1.0	5.3	4.1	1.0	0.06	0.19	0.15	0.13
$B^- \to D^*X_c$	0.3	0.2	0.3	0.2	-	0.01	0.01	-
$B^- \to D^*\tau\bar{\nu}$	0.1	0.2	0.1	0.1	-	-	-	-
Fragmentation	0.9	1.0	1.0	0.5	0.01	-	0.11	-
τ_b lifetime	0.9	0.9	0.7	0.6	-	-	-	-
R_1 and R_2	2.4	1.1	1.0	1.0	0.4	0.3	0.2	0.2
Uncorrelated errors								
Combinatorial and fake D^0	1.1	0.5	-	1.2	-	0.07	-	0.01
Fake lepton	0.7	-	1.2	0.2	-	-	-	-
ℓ efficiency/modelling	0.7	1.1	-	1.2	-	0.03	-	-
Selection efficiency/modelling	1.2	2.6	2.9	3.6	0.02	0.08	0.12	0.01
MC statistics	1.6	0.2	-	-	0.05	-	-	-
w resolution	1.5	2.1	2.2	1.4	0.07	0.07	0.12	0.035
Total Systematic	4.0	7.2	6.0	4.8	0.41	0.36	0.35	0.24
Statistical	6.5	4.0	3.5	4.5	0.25	0.14	0.15	0.21

Table 10: *Dominant systematic uncertainties on $\mathcal{F}_{D^*}(1)|V_{cb}|$ and $\rho^2_{A_1}$ expressed, respectively, as relative and absolute values.*

$$\rho^2_{A_1} = 1.38 \pm 0.08 \pm 0.26 \tag{116}$$

The confidence level of the fit is 12%. The error ellipses of the corrected measurements and of the LEP average are shown on Figure 9.

The theoretical estimate, $\mathcal{F}_{D^*}(1)$=0.88 ± 0.05 (see Appendix F), is used to determine:

$$|V_{cb}|^{excl.} = (40.5 \pm 1.9(\text{exp.}) \pm 2.3(\text{theo.})) \times 10^{-3}. \tag{117}$$

7.3 Overall $|V_{cb}|$ average

The combined $|V_{cb}|$ average (see Figure 10-right) can be extracted taking into account correlations between the inclusive and exclusive methods. The most important source of correlations comes from theoretical uncertainties in the evaluation of μ_π, the average momentum of the b-quark inside the b-hadron (Appendix F). In the determination of experimental systematic uncertainties, theoretical uncertainties in the modelling of $b \to \ell$

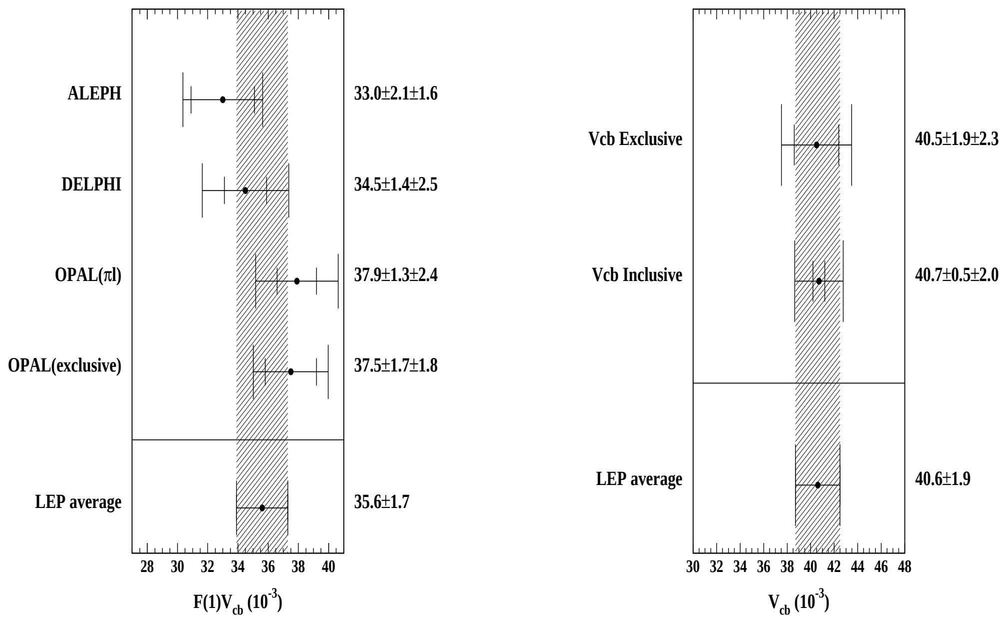

Figure 10: *Left: Corrected* $\mathcal{F}_{D^*}(1)|V_{cb}|$ *values and LEP average using the exclusive method. The values shown in the plot have been adjusted by the working group and are those used to perform the average. The original values can be found in the experimental papers. Right:* $|\mathrm{V}_{cb}|$ *LEP average.*

decays are taken as fully correlated. Uncertainties from lepton identification and background also contribute, but to a much lesser extent. All other sources provide negligible contributions to the correlated error. The combined value is:

$$|\mathrm{V}_{cb}| = (40.6 \pm 1.9) \times 10^{-3} \tag{118}$$

where, within the total error of 1.9×10^{-3}, 1.5×10^{-3} comes from uncorrelated sources and 1.0×10^{-3} from correlated sources.

8 Average of LEP $|\mathrm{V}_{ub}|$ measurements

The LEP combined determinations of $\mathrm{BR}(b \to \ell^- \overline{\nu_\ell} \mathrm{X}_u)(\ell^- = e^-$ or $\mu^-)$ and the derivation of $|\mathrm{V}_{ub}|$ have been obtained by combining the results reported by the ALEPH [158], DELPHI [159] and L3 [160] Collaborations[21]. The three analyses rely on different techniques to measure the inclusive yield of $b \to u$ transitions in semileptonic b-hadron decays. The experimental details can be found in the original publications. All three experiments reported evidence for the $b \to \ell^- \overline{\nu_\ell} \mathrm{X}_u$ transition and measured its rate $\mathrm{BR}(b \to \ell^- \overline{\nu_\ell} \mathrm{X}_u)$. DELPHI fitted $|\mathrm{V}_{ub}| / |\mathrm{V}_{cb}|$ directly to the fraction of candidate $b \to \ell^- \overline{\nu_\ell} \mathrm{X}_u$ decays in the selected data sample. For this averaging, the corresponding value of $\mathrm{BR}(b \to \ell^- \overline{\nu_\ell} \mathrm{X}_u)$

[21] The present members of the $|\mathrm{V}_{ub}|$ working group are: D. Abbaneo, M. Battaglia, P. Henrard, S. Mele, E. Piotto and Ph. Rosnet.

has been derived by using the value of $|V_{cb}|$ obtained in Section 7.3 and the relationship between $|V_{ub}|$ and $BR(b \to \ell^- \overline{\nu_\ell} X_u)$ given below.

In order to average these results, the sources of systematic uncertainties have been divided into two categories. The first contains (a) uncorrelated systematics due to experimental systematics, such as lepton identification, b-tagging, vertexing efficiency and energy resolution, and (b) uncorrelated systematics from signal modelling and background description. The second contains correlated systematic uncertainties deriving from the simulation of signal $b \to u$ and background $b \to c$ transitions. The contributions from the statistical, experimental, and uncorrelated and correlated modelling uncertainties are summarised in Table 11.

Experiment	BR	(stat.)	(exp.)	(uncorrelated)	(correlated)
ALEPH [158]	1.73	$\pm$ 0.48	$\pm$ 0.29	$\pm$ 0.29 $\left(\begin{smallmatrix}\pm 0.29 & b\to c\\ \pm 0.04 & b\to u\end{smallmatrix}\right)$	$\pm$ 0.47 $\left(\begin{smallmatrix}\pm 0.43 & b\to c\\ \pm 0.19 & b\to u\end{smallmatrix}\right)$
DELPHI [159]	1.69	$\pm$ 0.37	$\pm$ 0.39	$\pm$ 0.18 $\left(\begin{smallmatrix}\pm 0.13 & b\to c\\ \pm 0.13 & b\to u\end{smallmatrix}\right)$	$\pm$ 0.42 $\left(\begin{smallmatrix}\pm 0.34 & b\to c\\ \pm 0.20 & b\to u\end{smallmatrix}\right)$
L3 [160]	3.30	$\pm$ 1.00	$\pm$ 0.80	$\pm$ 0.68 $\left(\begin{smallmatrix}\pm 0.68 & b\to c\\ - & b\to u\end{smallmatrix}\right)$	$\pm$ 1.40 $\left(\begin{smallmatrix}\pm 1.29 & b\to c\\ \pm 0.54 & b\to u\end{smallmatrix}\right)$

Table 11: *The results for $10^3 \times BR(b \to \ell^- \overline{\nu_\ell} X_u)$ from the LEP experiments with the statistical, experimental, model uncorrelated, and model correlated uncertainties.*

The correlated systematics are summarised in Table 12. Differences in the analysis techniques adopted by the three experiments are reflected by the different sizes of the systematics uncertainties estimated from each common source. Important common systematics arise from the D topological branching fractions and the rate of $D \to K^0$ decays. D decays represent a potential source of background for $b \to u$ decays because both are characterized by a small hadronic mass and a low charged multiplicity. The sensitivity to the topological branching fractions is reduced in the DELPHI analysis by applying a rescaling of the mass M_X of the hadronic system, based on the reconstructed $\ell^- \overline{\nu_\ell} X$ mass, and by the use of identified kaons for separating signal from background events. This explains the different sensitivity of the ALEPH and DELPHI analyses to these two important sources of systematic uncertainty. ALEPH and L3 are sensitive to the uncertainties in the b fragmentation function due to the kinematical variables used for discriminating $b \to \ell^- \overline{\nu_\ell} X_u$ from $b \to \ell^- \overline{\nu_\ell} X_c$ decays. The DELPHI result is sensitive to the assumed production rates of b hadron species due to the use of kaon anti-tagging to reject $b \to c$, thus rejecting also B^0_s and Λ^0_b decays; and it is sensitive to the contribution of $D^{(*)}\pi$ and D^{**} states in semileptonic decays because the resulting difference in the vertex topology is also used for discriminating $b \to u$ from $b \to c$ decays.

The systematic uncertainties on the $b \to u$ signal have been grouped into *inclusive model* and *exclusive model* classes, which are assumed to be fully correlated. The first corresponds to the uncertainty in the modelling of the kinematics of the b-quark in the heavy hadron. It has been estimated from the spreads of the results obtained with the ACCMM [161], the Dikeman-Shifman-Uraltsev [162], and the parton [163] models in the ALEPH and DELPHI analyses. In the case of the L3 analysis, the uncertainties in the single pion and in the lepton energy spectra were evaluated from the discrepancies between the model of Ref [164] and the ISGW [28] model respectively to the JETSET 7.4 [165] prediction. The *exclusive model* uncertainty corresponds to the modelling of the hadronic final state in the $b \rightarrow \ell^- \overline{\nu_\ell} X_u$ decay. These uncertainties have been estimated by replacing the parton shower fragmentation model in JETSET [165] with the fully exclusive ISGW2 [29] model by ALEPH and DELPHI, and by propagating a 100% uncertainty on the $B \rightarrow \pi \ell^- \overline{\nu_\ell}$ rate by L3. ALEPH and L3 have also taken into account the uncertainty from the modelling of the charmless semi-leptonic decay of b-baryons. This has not been considered by DELPHI as they remove decays containing identified protons and kaons, thus suppressing the contribution of b-baryons as mentioned above. In addition to these sources, ALEPH has estimated a $b \rightarrow u$ uncertainty from the energy cut-off value Λ for the hybrid model adopted [166]; DELPHI allowed for the b-quark pole mass m_b and the expectation value of the kinetic energy operator $< p_b^2 >$ uncertainties which have been assumed to be uncorrelated.

The three measurements of $\mathrm{BR}(b \rightarrow \ell^- \overline{\nu_\ell} X_u)$ have been averaged using the Best Linear Unbiased Estimate (B.L.U.E.) technique [167]. Using the inputs from Table 11 and Table 12, the LEP average value for $\mathrm{BR}(b \rightarrow \ell^- \nu X_u)$ was found to be:

$$\begin{aligned} \mathrm{BR}(b \rightarrow \ell^- \overline{\nu_\ell} X_u) &= (1.74 \pm 0.37(\mathrm{stat.+exp.}) \pm 0.38(b \rightarrow c) \pm 0.21(b \rightarrow u)) \times 10^{-3} \\ &= (1.74 \pm 0.57) \times 10^{-3} \end{aligned} \tag{119}$$

with a confidence level for the combination of 0.70 (see Figure 11).

The magnitude of the matrix element V_{ub} has been extracted using the following relationship derived in the context of (OPE) [199], [200]:

$$\begin{aligned} |V_{ub}| &= 0.00445 \sqrt{\frac{\mathrm{BR}(b \rightarrow X_u \ell^- \overline{\nu_\ell})}{0.002} \frac{1.55\mathrm{ps}}{\tau_b}} \times \\ & \quad (1 \pm 0.010(\mathrm{pert}) \pm 0.030(1/m_b^3) \pm 0.035(m_b)) \end{aligned} \tag{120}$$

by assuming $m_b = (4.58 \pm 0.06)\ \mathrm{GeV/c^2}$ (Appendix F).

From the LEP average of $\mathrm{BR}(b \rightarrow \ell^- \overline{\nu_\ell} X_u)$ obtained above, the b-lifetime value, τ_b, obtained in Section 3, and the quoted $|V_{ub}|$ uncertainty coming from OPE (± 4.7 % relative), a probability density function for $|V_{ub}|$ has been calculated. The resulting distribution is shown in Figure 11-right, where all errors are convoluted together assuming that they

Source	ALEPH	DELPHI	L3
b species	0.01	0.12	-
b fragmentation	0.22	0.03	0.32
$b \rightarrow \ell$ model	0.11	0.08	1.24
$c \rightarrow \ell$ model	0.14	0.13	0.12
D topological BR's	0.31	0.06	-
$\mathrm{BR}(D \rightarrow K^0)$	0.08	0.19	-
D^{**}, $D^{(*)}\pi$ production	0.04	0.19	-
$b \rightarrow u$ inclusive model	0.18	0.08	0.25
$b \rightarrow u$ exclusive model	0.05	0.18	0.20
Λ_b decay model	0.04	-	0.44

Table 12: *Correlated sources of systematic uncertainties (in units of 10^{-3}) entering in the measurement of* $\mathrm{BR}(b \rightarrow \ell^- \overline{\nu_\ell} X_u)$.

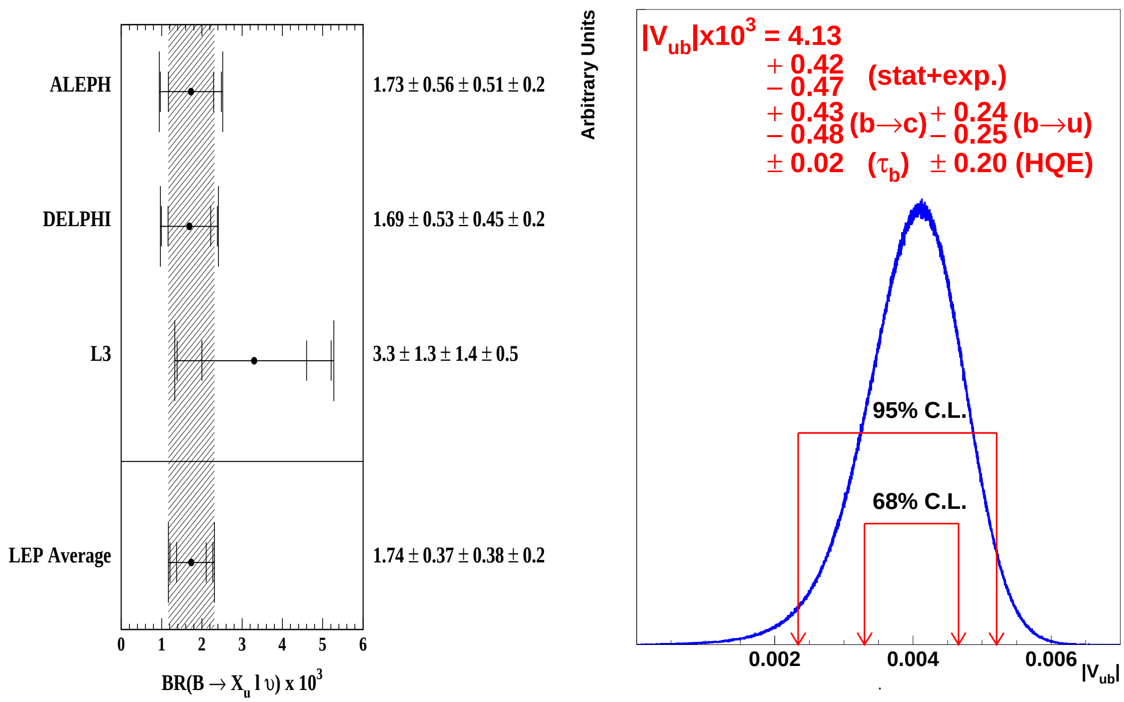

Figure 11: *Left: the determinations of* $\mathrm{BR}(b \to \ell^- \overline{\nu_\ell} \mathrm{X}_u)$ *by ALEPH, DELPHI and L3 and the resulting LEP average. Right: the probability density function for* $|\mathrm{V}_{ub}|$ *corresponding to the LEP average value of* $\mathrm{BR}(b \to \ell^- \overline{\nu_\ell} \mathrm{X}_u)$ *with the value of the median and two confidence intervals indicated. Unphysical negative entries have been discarded and the probability density function renormalised accordingly.*

are Gaussian in $\mathrm{BR}(b \to \ell^- \overline{\nu_\ell} \mathrm{X}_u)$, with the exception of the OPE error assumed to be Gaussian in $|\mathrm{V}_{ub}|$. The small part of this function in the negative, unphysical region, corresponding to only 0.14%, has been discarded and the probability density function renormalised accordingly. The median of this function has been chosen as the best estimate of $|\mathrm{V}_{ub}|$, and $\pm 34.135\%$ and $\pm 47.725\%$ of the integral of the probability density function around this value have been used to define the 1σ and 2σ confidence regions, denoted hereafter as the 68% and 95% confidence levels, obtaining:

$$|\mathrm{V}_{ub}| = (4.13^{+0.63}_{-0.75}) \times 10^{-3} \text{ at the 68\% C.L.} \tag{121}$$

and

$$|\mathrm{V}_{ub}| = (4.13^{+1.18}_{-1.71}) \times 10^{-3} \text{ at the 95\% C.L.} \tag{122}$$

All the uncertainties have been included in these estimates. The application of the above procedure for each error source separately yields the following detailed result for the 68% confidence level:

$$\begin{aligned} |\mathrm{V}_{ub}| \;&=\; (4.13^{+0.42}_{-0.47}\ (\text{stat.} + \text{det.})^{+0.43}_{-0.48}\ (b \to c \text{ syst.})^{+0.24}_{-0.25}\ (b \to u \text{ syst.}) \\ &\quad \pm 0.02\ (\tau_b) \pm 0.20\ (\text{OPE})) \times 10^{-3}. \end{aligned} \tag{123}$$

9 Results on CP violation in the $\mathrm{B_d^0} - \overline{\mathrm{B_d^0}}$ system

The decay $\mathrm{B_d^0} \to \mathrm{J}/\psi \mathrm{K_S^0}$ is known as the gold-plated mode to establish the existence of CP violation in b-hadron decays, due to its clean experimental signature and low theoretical uncertainty. The final state is a CP eigenstate, to which both $\mathrm{B_d^0}$ and $\overline{\mathrm{B_d^0}}$ can decay. The interference between their direct and indirect decays via $\mathrm{B_d^0} - \overline{\mathrm{B_d^0}}$ mixing leads to a time-dependent CP asymmetry given by:

$$A(t) = \frac{\Gamma(\mathrm{B_d^0} \to \mathrm{J}/\psi \mathrm{K_S^0}) - \Gamma(\overline{\mathrm{B_d^0}} \to \mathrm{J}/\psi \mathrm{K_S^0})}{\Gamma(\mathrm{B_d^0} \to \mathrm{J}/\psi \mathrm{K_S^0}) + \Gamma(\overline{\mathrm{B_d^0}} \to \mathrm{J}/\psi \mathrm{K_S^0})} = -\sin(2\beta)\sin(\Delta m_d t). \tag{124}$$

Here $\Gamma(\mathrm{B_d^0} \to \mathrm{J}/\psi \mathrm{K_S^0})$ represents the rate of particles that were produced as $\mathrm{B_d^0}$ decaying to $\mathrm{J}/\psi \mathrm{K_S^0}$ at proper time t.

The first published attempt at a direct measurement was made by OPAL [168]. They selected 24 candidates with an estimated purity of 60% and reported a value of $\sin 2\beta = 3.2^{+1.8}_{-2.0} \pm 0.5$. CDF published an analysis based on 395 candidates with a purity of about 40% [169]. They measured $\sin 2\beta = 0.79^{+0.41}_{-0.44}$, statistical and systematic errors combined. Finally ALEPH [170] selected 23 candidates with an estimated purity of 71% and measured $\sin 2\beta = 0.84^{+0.84}_{-1.05}$, where the uncertainty is dominated by the statistics. The three log-likelihoods of these measurements have been summed to give a combined result, i.e. neglecting any correlation between the systematic errors of the different experiments (expected to be small). It corresponds to [170]:

$$\sin 2\beta = 0.88^{+0.36}_{-0.39}. \tag{125}$$

A second way to search for CP-violating effects uses inclusive semileptonic or fully inclusive decays. In the $\mathrm{B_d^0}$-$\mathrm{\bar{B}_d^0}$ system, the mass eigenstates $|\mathrm{B_1}\rangle$ and $|\mathrm{B_2}\rangle$ can be described in terms of the weak eigenstates $|\mathrm{B_d^0}\rangle$ and $|\mathrm{\bar{B}_d^0}\rangle$ by:

$$\begin{aligned} |\mathrm{B_1}\rangle &= \frac{(1+\epsilon_B+\delta_B)|\mathrm{B_d^0}\rangle + (1-\epsilon_B-\delta_B)|\mathrm{\bar{B}_d^0}\rangle}{\sqrt{2(1+|\epsilon_B+\delta_B|^2)}} \\ |\mathrm{B_2}\rangle &= \frac{(1+\epsilon_B-\delta_B)|\mathrm{B_d^0}\rangle - (1-\epsilon_B+\delta_B)|\mathrm{\bar{B}_d^0}\rangle}{\sqrt{2(1+|\epsilon_B-\delta_B|^2)}} \end{aligned} \tag{126}$$

where the complex parameters ϵ_B and δ_B parameterise indirect CP and CPT violation [171]. Predictions for $\mathrm{Re}(\epsilon_\mathrm{B})$ are of the order of 10^{-3} in the Standard Model and up to an order of magnitude larger in superweak models [172]. For $\mathrm{B_s^0}$ mesons, the effects are expected to be at least an order of magnitude smaller, because of the relative sizes of $\mathrm{Im}(\mathrm{V_{td}})$ and $\mathrm{Im}(\mathrm{V_{ts}})$.

The value of $\mathrm{Re}(\epsilon_\mathrm{B})$ has been measured by ALEPH, DELPHI and OPAL by studying time dependent charge asymmetries in semileptonic and fully inclusive b-hadron decays [173]. The methods are similar to those used for measuring $\mathrm{B_d^0}$ oscillations. CDF has also obtained a measurement using dimuon events [174]. Assuming $\mathrm{Re}(\epsilon_\mathrm{B}) = 0$ for $\mathrm{B_s^0}$ mesons, and that CPT violation is negligible (*i.e.* $\mathrm{Im}(\delta_\mathrm{B}) = 0$), the LEP and CDF results have been averaged to obtain:

$$\mathrm{Re}(\epsilon_\mathrm{B})/(1+|\epsilon_\mathrm{B}|^2) = -0.0024 \pm 0.0039. \tag{127}$$

This value compares well with a recent CLEO result using inclusive lepton and dilepton events, $\mathrm{Re}(\epsilon_{\mathrm{B}})/(1+|\epsilon_{\mathrm{B}}|^2) = 0.0035 \pm 0.0103 \pm 0.0015$ [175].

The measurements of the CPT-violation parameter $\mathrm{Im}(\delta_{\mathrm{B}})$ obtained by DELPHI and OPAL [173] have also been averaged to give

$$\mathrm{Im}(\delta_{\mathrm{B}}) = -0.016 \pm 0.012. \tag{128}$$

10 Determination of the parameters of the unitarity triangle

As, in the present report, measurements of $|\mathrm{V}_{cb}|$, $|\mathrm{V}_{ub}|$, Δm_d, and limits on Δm_s and on CP violation in the $\mathrm{B^0_d}-\overline{\mathrm{B^0_d}}$ system have been collected from results made available by CDF, LEP and SLD Collaborations, it has been considered appropriate to give also an updated overview of the constraints provided by these measurements, and by additional results already published by other collaborations, on the CKM parameters within the Standard Model framework. In the Standard Model, weak interactions among quarks are codified in a 3×3 unitary matrix: the CKM matrix.

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \tag{129}$$

This matrix can be parametrized using three real quantities and one phase which cannot be removed by redefining the quark field phases. This phase leads to the violation of the CP symmetry. In the Wolfenstein parametrization [176], these four parameters are labelled as λ, A, $\overline{\rho}$ and $\overline{\eta}$ [22]. Experimentally there is a hierarchy in the magnitude of the elements of the CKM matrix and the following expression corresponds to an expansion valid up to $\mathcal{O}(\lambda^6)$ without any prejudice on the possible values of the other parameters [177].

$$V_{CKM} = \begin{pmatrix} 1-\frac{\lambda^2}{2}-\frac{\lambda^4}{8} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda + \frac{A^2\lambda^5}{2}(1-2\rho) - iA^2\lambda^5\eta & 1-\frac{\lambda^2}{2}-\lambda^4(\frac{1}{8}+\frac{A^2}{2}) & A\lambda^2 \\ A\lambda^3[1-(1-\frac{\lambda^2}{2})(\rho+i\eta)] & -A\lambda^2(1-\frac{\lambda^2}{2})[1+\lambda^2(\rho+i\eta)] & 1-\frac{A^2\lambda^4}{2} \end{pmatrix} \tag{130}$$

It has been obtained using the following definitions for the parameters:

$$V_{us} = \lambda,\ V_{cb} = A\lambda^2,\ V_{ub} = A\lambda^3(\rho - i\eta). \tag{131}$$

The Standard Model allows the amplitudes for all processes involving weak transitions to be evaluated in terms of the values of these parameters. Measurements of semileptonic decays of strange and b-hadrons are the main sources of informations on λ and A, respectively. Measurements of $|\epsilon_K|$, $\left|\frac{V_{ub}}{V_{cb}}\right|$, Δm_d and Δm_s are expected to define compatible values for $\overline{\rho}$ and $\overline{\eta}$. The latter provide a set of four constraints which are obtained by comparing measured and expected values of the corresponding quantities, in the framework of the Standard Model or of any other given model. These constraints depend, in addition, on parameters which have to be taken from theory or from other measurements.

[22] The parameters $\overline{\rho}$ and $\overline{\eta}$ are related to the original ρ and η, introduced by Wolfenstein, using the expressions $\overline{\rho} = \rho \times (1-\frac{\lambda^2}{2})$ and $\overline{\eta} = \eta \times (1-\frac{\lambda^2}{2})$.

10.1 The unitarity triangle

The unitarity of the CKM matrix implies that six equations of the type:

$$\sum_{i=1,3} V^{*}_{ij} V_{ik} \; = \; 0 \text{ and } \sum_{i=1,3} V_{ji} V^{*}_{ki} \; = \; 0, \tag{132}$$

corresponding to the products of two columns or two lines, have to be satisfied. They define six triangles of the same area. The triangle, obtained from the product between the third column and the complex conjugate of the first one, is expected to have sides of similar sizes. Using the Wolfenstein parametrization one obtains:

$$V^{*}_{ud} V_{ub} = A\lambda^3(\overline{\rho} - i\overline{\eta}), \;\; V^{*}_{cd} V_{cb} = -A\lambda^3, \;\; V^{*}_{td} V_{tb} = A\lambda^3(1 - \overline{\rho} + i\overline{\eta}) \tag{133}$$

The three expressions are proportional to $A\lambda^3$, which can be factored out, and the geometrical representation of Equation (133), in the $(\overline{\rho}, \overline{\eta})$ plane, is a triangle with vertices at C(0,0), B(1,0) and A$(\overline{\rho}, \overline{\eta})$.

The lengths of the sides of the triangle can be related to the values of the modulus of CKM matrix elements:

$$\overline{AC} = \frac{1 - \frac{\lambda^2}{2}}{\lambda} \frac{\mid V_{ub} \mid}{\mid V_{cb} \mid} = \sqrt{\overline{\rho}^2 + \overline{\eta}^2} \quad (|\mathrm{V_{ub}}| \text{ and } |\mathrm{V_{cb}}| \text{ measts.})$$

$$\overline{AB} = \frac{\mid V_{td} \mid}{\lambda \mid V_{cb} \mid} = \sqrt{(1 - \overline{\rho})^2 + \overline{\eta}^2} \quad (\Delta \mathrm{m_d} \text{ and } |\mathrm{V_{cb}}| \text{ measts.})$$

$$\overline{AB} = \frac{1 - \frac{\lambda^2}{2}}{\lambda} \frac{\mid V_{td} \mid}{\mid V_{ts} \mid} = \sqrt{(1 - \overline{\rho})^2 + \overline{\eta}^2} \quad (\Delta \mathrm{m_d} \text{ and } \Delta \mathrm{m_s} \text{ measts.}) \tag{134}$$

these expressions are valid up to $\mathcal{O}(\lambda^4)$ [23]. Angles of the triangle, opposite to the sides BC, AC and AB are designed respectively as α, β and γ.

10.2 The constraints which depend on $\overline{\rho}$ and $\overline{\eta}$

Four measurements which provide constraints on the possible range of variation of the $\overline{\rho}$ and $\overline{\eta}$ parameters have been considered.

10.2.1 Charmless b-hadron semileptonic decays

The relative rate between charmed and charmless b-hadron semileptonic decays, proportional to $\left|\frac{V_{ub}}{V_{cb}}\right|^2$, allows to measure the length of the side AC of the triangle. Measurements of $|\mathrm{V}_{ub}|$ obtained at LEP from inclusive $b \to u \ell^- \overline{\nu_\ell} \mathrm{X}$ transitions (Section 8) and by the CLEO Collaboration of the exclusive $\overline{\mathrm{B}} \to \rho \ell^- \overline{\nu_\ell}$ decay channel [178] have been used. The CLEO result is:

$$|\mathrm{V}_{ub}| = (32.5 \pm 2.9 \pm 5.5) \times 10^{-4} \tag{135}$$

where the second uncertainty is theoretical and has been obtained using several models to describe the decay form factors. The p.d.f. for the CLEO measurement is thus a convolution of Gaussian and flat distributions. The p.d.f. for the LEP result is given in Section 8. Combining the two distributions, in practice an almost Gaussian p.d.f. is obtained corresponding to:

$$|\mathrm{V}_{ub}| = (35.5 \pm 3.6) \times 10^{-4} \tag{136}$$

[23] By definition, the length of the side BC is equal to unity.

10.2.2 The $B^0_d - \overline{B^0_d}$ oscillation parameter Δm_d

In the Standard Model, the mass difference between the mass eigenstates of the $B^0_d - \overline{B^0_d}$ system can be written:

$$\Delta m_d = \frac{G_F^2}{6\pi^2} m_W^2 m_{B^0_d} \eta_c S(\frac{m_t^2}{m_W^2}) A^2 \lambda^6 [(1-\overline{\rho})^2 + \overline{\eta}^2] f_{B_d}^2 \hat{B}_{B_d} \tag{137}$$

The most uncertain parameter in Equation (137) is $f_{B_d}\sqrt{\hat{B}_{B_d}}$. Its value has been taken from recent lattice QCD results. It depends on the values of the B^0_d decay constant, f_{B_d}, and of the renormalization scale invariant parameter $\hat{B}_{B_d}$. It should be noted that the final accuracy on $\overline{\rho}$ and $\overline{\eta}$ is weakly dependent on the assumed uncertainty on this parameter [179]. The value of $\eta_c = 0.55 \pm 0.01$ corresponds to a perturbative QCD short-distance NLO correction [180]. S is the Inami-Lim function whose expression can be found also in [180]. The value of m_t if the $\overline{\mathrm{MS}}$ top mass, $m_t^{\overline{\mathrm{MS}}}(m_t^{\overline{\mathrm{MS}}}) = (167 \pm 5)\mathrm{GeV/c^2}$, deduced from measurements of the physical mass performed by CDF and D0 [181]. Measurements of Δm_d constrain the length of the side AB of the triangle.

10.2.3 The ratio between $B^0_d - \overline{B^0_d}$ and $B^0_s - \overline{B^0_s}$ oscillation periods

In the Standard Model, the mass difference between the mass eigenstates of the $B^0_s - \overline{B^0_s}$ system is independent of the values of the parameters $\overline{\rho}$ and $\overline{\eta}$. As a consequence, a measurement of this quantity can be used as a constraint on the effects of strong interactions appearing in the parameters $f_{B_q}\sqrt{B_{B_q}}$. The ratio between the values of the mass difference between the mass-eigenstates, measured for the two systems of neutral b-mesons, can be expressed as:

$$\frac{\Delta m_d}{\Delta m_s} = \frac{m_{B_d} f_{B_d}^2 \hat{B}_{B_d}}{m_{B_s} f_{B_s}^2 \hat{B}_{B_s}} \frac{\lambda^2}{(1-\frac{\lambda^2}{2})^2} [(1-\overline{\rho})^2 + \overline{\eta}^2]. \tag{138}$$

This expression depends on the ratio of b-decay constants and bag parameters for B^0_s and B^0_d mesons respectively:

$$\xi = \frac{f_{B_s}\sqrt{\hat{B}_{B_s}}}{f_{B_d}\sqrt{\hat{B}_{B_d}}} \tag{139}$$

and is expected to have a smaller relative uncertainty than the two contributing quantities from B^0_d and B^0_s mesons.

10.2.4 CP violation in the kaon system

CP violation in the kaon system can be expressed in terms of the quantity $|\epsilon_K|$:

$$|\epsilon_K| = C_\epsilon A^2 \lambda^6 \hat{B}_K \overline{\eta}\, [-\eta_1 S(\frac{m_c^2}{m_W^2}) + \eta_2 S(\frac{m_t^2}{m_W^2}) A^2 \lambda^4 (1-\overline{\rho}) + \eta_3 S(\frac{m_c^2}{m_W^2}, \frac{m_t^2}{m_W^2})] \tag{140}$$

where

$$C_\epsilon = \frac{G_F^2 f_K^2 m_K m_W^2}{6\sqrt{2}\pi^2 \Delta m_K}. \tag{141}$$

Parameter	Value	Gaussian σ	Uniform half-width	Ref.
λ	0.2237	0.0033		[182]
$\lvert V_{cb}\rvert$	40.6×10^{-3}	1.9×10^{-3} , see text		Sect. 7
$\lvert V_{ub}\rvert$	35.5×10^{-4}	3.6×10^{-4}	–	Sect. 10.2.1
$\lvert\epsilon_K\rvert$	2.280×10^{-3}	0.019×10^{-3}	–	[5]
Δm_d	0.487 ps^{-1}	0.014 ps^{-1}	–	Sect. 4
Δm_s	$>$ 15.0 ps^{-1} at 95% C.L.	see text		Sect. 4
m_t	167 GeV	5 GeV	–	[181]
m_b	4.23 GeV	0.07 GeV	–	[183]
m_c	1.3 GeV	0.1 GeV	–	[5]
$\hat{B}_K$	0.87	0.06	0.13	[182]
$f_{B_d}\sqrt{\hat{B}_{B_d}}$	230 MeV	25 MeV	20 MeV	[182]
$\xi = \frac{f_{B_s}\sqrt{\hat{B}_{B_s}}}{f_{B_d}\sqrt{\hat{B}_{B_d}}}$	1.14	0.03	0.05	[182]
α_s	0.119	0.03	–	[182]
η_1	1.38	0.53	–	[182]
η_2	0.574	0.004	–	[182]
η_3	0.47	0.04	–	[182]
η_c	0.55	0.01	–	[182]

Table 13: *Values of the quantities entering into the expressions of $|\epsilon_K|$, $\left|\frac{V_{ub}}{V_{cb}}\right|$, Δm_d and Δm_s. In the third column the Gaussian and the flat part of the error are given explicitely. Values for the other parameters have been taken from [5].*

Functions S are again appropriate Inami-Lim functions, including the next to leading order corrections which enter also in the parameters η_i. The most uncertain parameter, in Equation (140) is $\hat{B}_K$. Its central value and corresponding uncertainty have been taken from lattice QCD calculations.

10.3 Probability distributions for $\overline{\rho}$, $\overline{\eta}$ and other quantities

Expressions for $\left|\frac{V_{ub}}{V_{cb}}\right|$, Δm_d, $\Delta m_d/\Delta m_s$ and $|\epsilon_K|$ relate the corresponding measurements of these quantities ($c_j,\ j = 1, 4$) to the CKM matrix parameters $\overline{\rho}$ and $\overline{\eta}$, via the set of ancillary parameters $(x_1, x_2, ..., x_N)$ corresponding to all experimentally determined or theoretically calculated quantities. The bi-dimensional probability distribution for $\overline{\rho}$ and $\overline{\eta}$ is obtained using the Bayes theorem.

$$\mathcal{P}(\overline{\rho}, \overline{\eta}, x_1, ..., x_\mathrm{N}|c_1, ..., c_\mathrm{M}) \propto \prod_{j=1,\mathrm{M}} \mathcal{P}_j(c_j|\overline{\rho}, \overline{\eta}, x_1, ..., x_\mathrm{N}) \times \prod_{i=1,\mathrm{N}} \mathcal{P}_i(x_i) \times \mathcal{P}(\overline{\rho}, \overline{\eta}). \quad (142)$$

Values used for all relevant parameters and their corresponding probability distributions are given in Table 13. Central values and uncertainties taken for the parameters obtained from lattice QCD evaluations have been discussed in [182] where more details can be found on the present determination of the CKM unitarity triangle parameters.. Rather similar studies have been done to obtain the favoured values for $\overline{\rho}$ and $\overline{\eta}$ and for their corresponding uncertainties [184]. A different approach, named scanning, has been used in [185]. It has been verified that, in practice, similar 95% C.L. regions for $\overline{\rho}$ and $\overline{\eta}$ are obtained if similar central values and corresponding uncertainties are used for the different parameters [182].

10.4 The unitarity triangle from all constraints

The region in the ($\overline{\rho}$, $\overline{\eta}$) plane selected by the measurements of the four constraints is given in Figure 12. The resulting values of the unitarity triangle parameters are:

$$\begin{gathered}\overline{\rho} = 0.224 \pm 0.038,\ \overline{\eta} = 0.317 \pm 0.040 \\ \sin(2\beta) = 0.698 \pm 0.066,\ \sin(2\alpha) = -0.42 \pm 0.23 \text{ and } \gamma = (54.8 \pm 6.2)^\circ. \end{gathered} \tag{143}$$

Several comments can be made:

- $\sin(2\beta)$ is determined precisely. This value can be compared with measurements of this quantity using $\mathrm{J}/\psi\mathrm{K}^0_\mathrm{S}$ events which are analyzed in Section 9 and with new results obtained by BaBar [186] and BELLE [187] collaborations. Combining all direct measurements one obtains:

$$\sin(2\beta) = 0.49 \pm 0.16. \tag{144}$$

  At present there is no discrepancy with the indirect approach.

- The angle γ is known with a 10% accuracy. It may be noted that the probability that γ is greater than 90° is only 0.03%. The central value for the angle γ is much smaller than that obtained in recent fits of rare b-meson two-body decays, based on the factorization hypothesis [188], which found values centered on $\gamma \sim 110^\circ$. Before invoking new physics, the validity of the hypotheses used to analyze two-body decays of b-mesons have to be scrutinized.

10.5 The unitarity triangle from b-physics measurements of its sides

Without including the constraint given by the measurement of the $|\epsilon_K|$ parameter of CP violation in kaon physics [24], the region of the ($\overline{\rho}$, $\overline{\eta}$) plane favoured by semileptonic decays and oscillations of b-hadrons is shown in Figure 13.

The corresponding values of the $\overline{\eta}$ and $\sin(2\beta)$ parameters defining the unitarity triangle are:

$$\overline{\eta} = 0.302^{+0.052}_{-0.061},\ \sin(2\beta) = 0.678^{+0.078}_{-0.101} \tag{145}$$

The selected region is well compatible with the one corresponding to CP violation in kaon physics, displayed as a band. This comparison is already a test of the compatibility between the measurements of the sides of the unitarity triangle and the contraints coming from the CP violation phase, obtained before the "B factories era".

[24] The numerical constraint provided by the measurement of $|\epsilon_K|$ has not been applied but the variation of $\overline{\eta}$ has been restricted to positive values only.

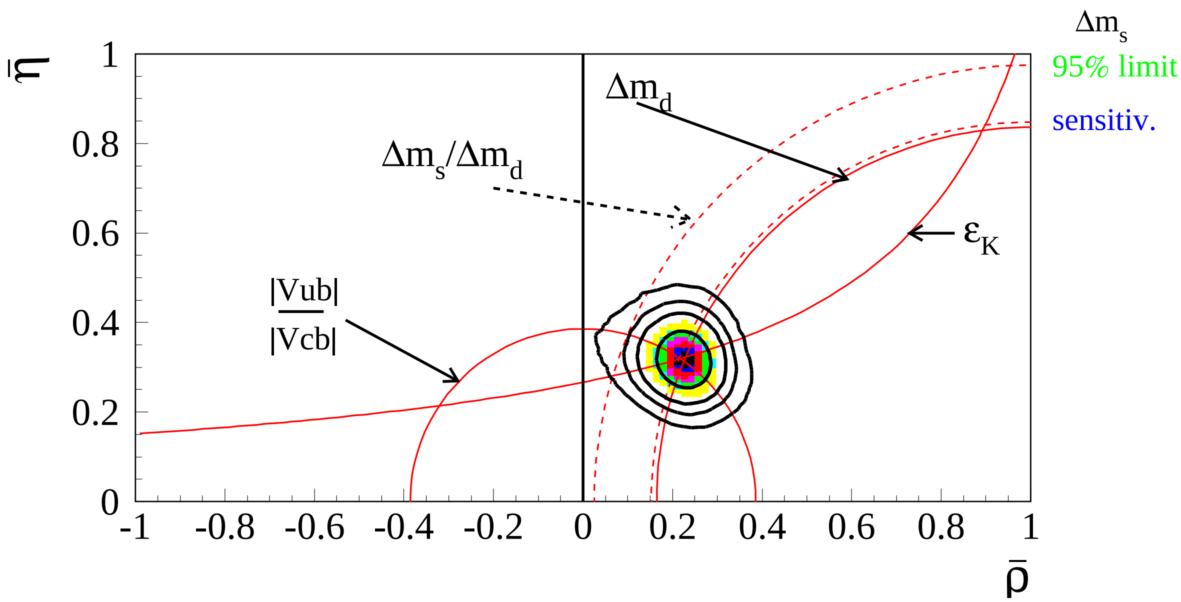

Figure 12: *Allowed regions for $\overline{\rho}$ and $\overline{\eta}$ using the parameters listed in Table 13. The contours at 68 %, 95 %, 99% and 99.9% probability are shown. The full lines correspond to the central values of the constraints given by the measurements of $\frac{|V_{ub}|}{|V_{cb}|}$, $|\epsilon_K|$ and Δm_d. The two dotted curves correspond, from left to right respectively to the 95 % upper limit and to the exclusion sensitivity reached in the experimental study of $\mathrm{B_s^0} - \overline{\mathrm{B_s^0}}$ oscillations.*

The value of the $\hat{B}_K$ parameter given by the constraints from b-physics on $\overline{\rho}$ and $\overline{\eta}$ and using the Standard Model expression and the measured value for $|\epsilon_K|$ is equal to:

$$\hat{B}_K = 0.90^{+0.30}_{-0.14}. \tag{146}$$

It can be compared with the result mentioned before, from lattice QCD: $\hat{B}_K = 0.87 \pm 0.06 \pm 0.13$ (Table 13). Rather low values of $\hat{B}_K$, as obtained from the chiral perturbation theory [189] are not favoured by the present analysis.

10.6 Other consistency checks

In a similar way as the measurement of $|\epsilon_K|$ which has been removed from the set of constraints to determine $\overline{\rho}$, $\overline{\eta}$ and $\hat{B}_K$, other constraints can be removed in turn.

10.6.1 Results without including the Δm_d measurement

If the Δm_d measurements are removed from the fit, the following result is obtained:

$$f_{B_d}\sqrt{\hat{B}_{B_d}} = (230 \pm 12)\mathrm{MeV}. \tag{147}$$

As the accuracy on this value is better than present theoretical uncertainties, from lattice QCD, reported in Table 13, this explains why the characteristics of the unitarity triangle are weakly dependent on the assumed value for this quantity.

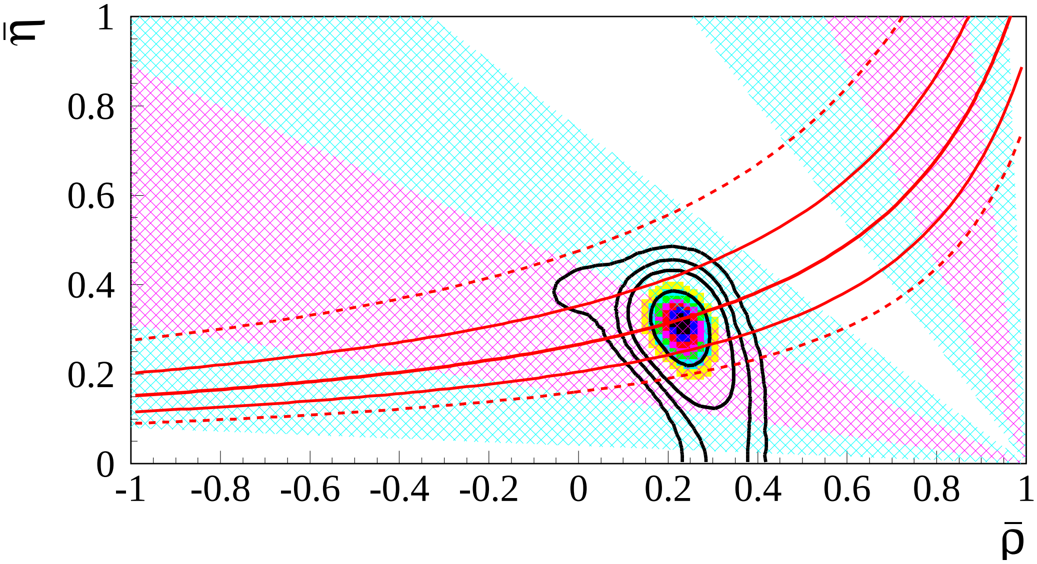

Figure 13: *The allowed regions (at 68%, 95%, 99% and 99.9% probability) for $\overline{\rho}$ and $\overline{\eta}$ using the constraints given by the measurements of $\frac{|V_{ub}|}{|V_{cb}|}$, Δm_d and Δm_s. The constraint due to $|\epsilon_K|$ is not included. The regions (at 68% and 95% probability) selected by the measurements of $|\epsilon_K|$ [continuous (1σ) and dotted (2σ) curves] and* $\sin(2\beta)$ *[darker (1σ) and clearer (2σ) zones] are shown. For* $\sin(2\beta)$ *the two solutions are displayed.*

10.6.2 Results without including the limit on Δm_s

Without including measurements on the Δm_s oscillation amplitude, obtained in Section 4.3, the probability distribution for the variable Δm_s has been obtained, giving the result:

$$\Delta m_s = (16.3 \pm 3.4)\ \mathrm{ps}^{-1} \text{ and } 9.7 \leq \Delta m_s \leq 23.2\ \mathrm{ps}^{-1} \text{ at 95\% probability.} \tag{148}$$

These values can be compared with recent lattice QCD determinations of Δm_s giving $(15.8 \pm 2.3 \pm 3.3)$ ps^{-1} [190] or (18.2 ± 4.7) ps^{-1} [191]. It should be noted that these predictions are independent of the values of the parameters $\overline{\rho}$ and $\overline{\eta}$.

Including results of Section 4.3, on searches for $\mathrm{B_s^0}-\overline{\mathrm{B_s^0}}$ oscillations, the expected range for Δm_s gets narrower:

$$\Delta m_s = (17.3^{+1.5}_{-0.7})\ \mathrm{ps}^{-1} \text{ and } 15.6 \leq \Delta m_s \leq 20.5\ \mathrm{ps}^{-1} \text{ at 95\% probability.} \tag{149}$$

A signal for $\mathrm{B_s^0}-\overline{\mathrm{B_s^0}}$ oscillations is thus expected at the TeVatron.

11 Summary of all results

This paper provides precise combined results (see Tables 14 and 15), from measurements submitted to the 2000 Summer Conferences, on parameters which govern production and decay properties of b-hadrons emitted in high energy b-quark jets.

At the Z pole, the polarization of b-baryons is found to be significantly different from zero but it is reduced as compared to the initial b-quark polarization of -0.94. Production of $\Sigma_b^{(*)}$ baryons has been invoked to explain this difference.

***b*-hadron lifetimes**	CDF-LEP-SLD, Section 3
$\tau(\mathrm{B_d^0})$	(1.546 ± 0.021) ps
$\tau(\mathrm{B^+})$	(1.647 ± 0.021) ps
$\tau(\mathrm{B_s^0})$	(1.464 ± 0.057) ps
$\tau(\Lambda_b^0)$	$(1.229^{+0.081}_{-0.079})$ ps
$\tau(\Xi_b)$	$(1.39^{+0.34}_{-0.28})$ ps
$\tau(b-\mathrm{baryon})$	$(1.208^{+0.051}_{-0.050})$ ps
τ_b	(1.561 ± 0.014) ps
$\frac{\tau(\mathrm{B^-})}{\tau(\overline{\mathrm{B_d^0}})}$	1.063 ± 0.020
***b*-hadron production rates**	CDF-LEP, Section 4
$f_{\mathrm{B_s}}$	$(9.8 \pm 1.2)\%$
$f_{b\text{-baryon}}$	$(10.3 \pm 1.8)\%$
$f_{\mathrm{B_d}} = f_{\mathrm{B^+}}$	$(39.9 \pm 1.1)\%$
$\rho(f_{\mathrm{B_s}}, f_{b\text{-baryon}})$	0.01
$\rho(f_{\mathrm{B_d}}, f_{\mathrm{B_s}})$	-0.55
$\rho(f_{\mathrm{B_s}}, f_{b\text{-baryon}})$	-0.84
$\mathbf{B^0 - \overline{B^0}}$ **oscillations**	ARGUS-CDF-CLEO-LEP-SLD, Section 4
Δm_d	(0.487 ± 0.014) ps^{-1}
χ_d	0.181 ± 0.007
Δm_s	> 15.0 ps^{-1}at the 95% C.L.
Limit on $\mathbf{\Delta\Gamma_{B_s^0}}$	CDF-LEP-SLD, Section 5
$\Delta\Gamma_{\mathrm{B_s^0}}/\Gamma_{\mathrm{B_s^0}}$	< 0.31 at the 95% C.L.
$\mathbf{\Lambda_b^0}$ **polarization in Z decays**	LEP, Section 2
$\mathcal{P}(\Lambda_b^0)$	$-0.45^{+0.17}_{-0.15} \pm 0.08$
$\mathbf{D^{**}}$ **properties in semileptonic B decays**	ARGUS-CLEO-LEP, Section 2
$\mathrm{BR}(\overline{\mathrm{B_d^0}} \to \mathrm{D^{**+}}\ell^-\overline{\nu_\ell})$	$(3.04 \pm 0.38)\%$
$\mathrm{BR}(\overline{\mathrm{B_d^0}} \to \mathrm{D^{**+}}\ell^-\overline{\nu_\ell}) \times \mathrm{BR}(\mathrm{D^{**+}} \to \mathrm{D^*X})$	$(1.82 \pm 0.21 \pm 0.08)\%$
$\frac{\mathrm{BR}(\overline{\mathrm{B_d^0}} \to \mathrm{D^{*+}}\ell^-\overline{\nu_\ell}) + \mathrm{BR}(\overline{\mathrm{B_d^0}} \to \mathrm{D^{*+}}\pi\ell^-\overline{\nu_\ell})}{\mathrm{BR}(\overline{\mathrm{B_d^0}} \to \ell^-\overline{\nu_\ell}\mathrm{X})}$	0.50 ± 0.03
$\mathrm{BR}(\overline{\mathrm{B}} \to \mathrm{D_1}\ell^-\overline{\nu_\ell})$	$(0.63 \pm 0.10)\%$
$\mathrm{BR}(\overline{\mathrm{B}} \to \mathrm{D_2^*}\ell^-\overline{\nu_\ell})$	$< 0.4\%$ at the 95% CL.

Table 14: *Summary of the results obtained on b-hadron production rates and decay properties using data available by mid 2000. More details on systematic uncertainties can be found in the corresponding sections.*

The accuracy on $\overline{\mathrm{B_d^0}}$ and $\mathrm{B^-}$ lifetimes is better than 2%. The $\mathrm{B^-}$ lifetime is significantly larger than the $\overline{\mathrm{B_d^0}}$ lifetime, in agreement with original expectations based on OPE and parton-hadron duality [77]. But the clear difference between measured and expected lifetimes of b-baryons remains to be explained. It may point to a failure of parton-hadron duality in inclusive B decays or to a failure of quark models used to evaluate the mean value of operators contributing in b-baryon decays.

The accuracy on b-hadron production rates in b-quark jets has improved by 30% as compared to published values [5].

Charm counting	LEP-SLD, Section 6
$n_c + n_{\overline{c}}$	1.230 ± 0.042
BR($b \rightarrow 0c$)	$(2.0 \pm 1.8)\%$
BR($b \rightarrow c\overline{c}$)	$(22.3 \pm 3.2)\%$
Measurement of $\|\mathbf{V}_{cb}\|$	LEP, Section 7
BR($b \rightarrow \ell X$)	$(10.58 \pm 0.07 \pm 0.18)\%$
$\mathcal{F}_{D^*}(1)\|V_{cb}\|$	$(35.6 \pm 0.8(\text{stat.}) \pm 1.5(\text{syst.})) \times 10^{-3}$
ρ^2	$1.38 \pm 0.08 \pm 0.26$
$\|\mathrm{V}_{cb}\|^{incl.}$	$(40.7 \pm 0.5(\text{exp.}) \pm 2.4(\text{theo.})) \times 10^{-3}$
$\|\mathrm{V}_{cb}\|^{excl.}$	$(40.5 \pm 1.9(\text{exp.}) \pm 2.3(\text{theo.})) \times 10^{-3}$
$\|\mathrm{V}_{cb}\|$	$(40.6 \pm 1.9) \times 10^{-3}$
Measurement of $\|\mathbf{V}_{ub}\|$	LEP, Section 8
$\|\mathrm{V}_{ub}\|$	$(4.04^{+0.62}_{-0.74}) \times 10^{-3}$
CP violation in B^0_d	CDF-LEP, Section 9
$\sin(2\beta)$	$0.88^{+0.36}_{-0.39}$
$\mathrm{Re}(\epsilon_\mathrm{B})/(1+\|\epsilon_\mathrm{B}\|^2)$	-0.0024 ± 0.0039
$\mathrm{Im}(\delta_\mathrm{B})$	-0.016 ± 0.012
Fitted S.M. observables	All , Section 10
$\overline{\rho}$	0.224 ± 0.038
$\overline{\eta}$	0.317 ± 0.040
$\sin(2\beta)$	0.698 ± 0.066
$\sin(2\alpha)$	-0.42 ± 0.23
γ	$(54.8 \pm 6.2)^\circ$

Table 15: *Summary of the results obtained on charm counting and semileptonic* b*-hadron decays. Constraints on the parameters of the CKM unitarity triangle are also given. The last free results correspond of the expected values of the corresponding parameters when they are not included as constraints in the fit. More details on systematic uncertainties can be found in the corresponding sections.*

In conjunction with the better determination of the $\overline{\mathrm{B}^0_\mathrm{d}}$ lifetime, the improvement in the B^0_d production rate has given a more precise determination of $|\mathrm{V}_{cb}|$ from $\overline{\mathrm{B}^0_\mathrm{d}} \rightarrow \mathrm{D}^{*+}\ell^-\overline{\nu_\ell}$ decays. The $\overline{\mathrm{B}^0_\mathrm{s}}$ fraction in jets is close to 10% with an uncertainty of 12%. Its determination is an important input in searches for B^0_s-$\overline{\mathrm{B}^0_\mathrm{s}}$ oscillations.

The mass difference between mass eigenstates of the B^0_d-$\overline{\mathrm{B}^0_\mathrm{d}}$ system is measured with a relative precision close to 3%. The corresponding quantity for the B^0_s-$\overline{\mathrm{B}^0_\mathrm{s}}$ system is still unmeasured, in spite of impressive progress in the sensitivity reached by the experiments.

A combined result on the decay width difference between mass eigenstates of the B^0_s-$\overline{\mathrm{B}^0_\mathrm{s}}$ system has been obtained which remains far from theoretical expectations.

Studies of inclusive charm production in b-hadron decays have shown that there is no room left for a large $0c$ component originating from new physics. Theory can reproduce the values for the semileptonic b-hadron branching fraction and $n_c + n_{\overline{c}}$ using a standard value for the charm quark mass and a low value for the scale at which QCD corrections have to be evaluated.

Because of the very accurate measurements obtained on the inclusive b-lifetime and semileptonic branching fraction, the accuracy of $|\mathrm{V}_{cb}|$ determined from inclusive semileptonic decays is entirely limited by theoretical uncertainties. An experimental control of these uncertainties is thus needed. The measurement of $|V_{cb}|$ from exclusive $\overline{B^0_d} \rightarrow D^{*+}X\ell^-\overline{\nu_\ell}$ decays is limited by uncertainties related to D^{**} production and decay mechanisms. Present experimental results on production characteristics of these states in b-hadron semileptonic decays have been detailed in Section 2 of the present report. The fact that the theoretical uncertainties on the determinations of $|V_{cb}|$ from inclusive and exclusive measurements are largely uncorrelated has been used in evaluating a global average.

The measurement of $|V_{ub}|$ from LEP experiments has reached an accuracy similar to previous determinations at the $\Upsilon(4S)$ but with a better sensitivity over a larger fraction of the phase space than in measurements using the lepton energy end-point region.

Because of progress in lattice QCD during the last years, of improvements in the determination of production and decay properties of b-hadrons given in the previous Sections and, in particular, of improvements on the sensitivity of analyses on $B^0_s - \overline{B^0_s}$ oscillations it is possible, within the framework of the Standard Model, to constrain the ranges of possible values for the parameters of the CKM unitarity triangle ($\overline{\rho},\ \overline{\eta},\ \sin 2\alpha,\ \sin 2\beta,\ \gamma$). They have been obtained using all available information from b-physics, CP violation in K decays and lattice QCD. Removing, in turn, the corresponding constraints, expected values for $\hat{B}_K,\ \Delta m_s$ and $f_{B_d}\sqrt{\hat{B}_{B_d}}$ have been also determined which can be compared with future accurate direct determinations of these quantities.

New measurements from LEP and SLD are still expected in the year 2001 and will improve present determinations of b-hadron production and decay properties, which already provide stringent constraints on the shape of the CKM unitarity triangle at the start of the era of asymmetric B factories.

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A Production rates of the D_1 and D_2^* mesons in semileptonic b-decays

Experiment	Channel	Value $\times 10^3$	Ref.
ALEPH	$BR(b \to \overline{B})BR(\overline{B} \to D_1^+ \ell^- \overline{\nu_\ell})BR(D_1^+ \to D^{*0}\pi^+)$	$2.06^{+0.55}_{-0.51}{}^{+0.29}_{-0.40}$	[15]
ALEPH	$BR(b \to \overline{B})BR(\overline{B} \to D_1^0 \ell^- \overline{\nu_\ell})BR(D_1^0 \to D^{*+}\pi^-)$	$1.68^{+0.40}_{-0.36}{}^{+0.28}_{-0.29}$	[15]
ALEPH	$BR(b \to \overline{B})BR(\overline{B} \to D_1^0 \ell^- \overline{\nu_\ell})BR(D_1^0 \to D^{*+}\pi^-)$	$3.62^{+1.78}_{-1.48} \pm 0.77$	[15]
CLEO	$BR(B^- \to D_1^0 \ell^- \overline{\nu_\ell})BR(D_1^0 \to D^{*+}\pi^-)$	$3.73 \pm 0.85 \pm 0.52 \pm 0.24$	[22]
DELPHI	$BR(b \to \overline{B})BR(\overline{B} \to D_1^0 \ell^- \overline{\nu_\ell})BR(D_1^0 \to D^{*+}\pi^-)$	$1.8 \pm 0.5 \pm 0.3$	[23]

Table 16: *Measured values of the* D_1 *production rate in b-semileptonic decays. The third systematic uncertainty, quoted in the CLEO analysis, comes from the variation of the detection efficiency when changing the parameters of the signal form factors in the ISGW2 model [29].*

Experiment	Channel	Value $\times 10^4$	Ref.
ALEPH	$BR(b \to \overline{B})BR(\overline{B} \to D_2^{*+} \ell^- \overline{\nu_\ell})BR(D_2^{*+} \to D^0\pi^+)$	$3.1^{+2.4}_{-2.2}{}^{+0.4}_{-0.6}$	[15]
ALEPH	$BR(b \to \overline{B})BR(\overline{B} \to D_2^{*0} \ell^- \overline{\nu_\ell})BR(D_2^{*0} \to D^{*+}\pi^-)$	$5.1^{+3.0}_{-2.6}{}^{+0.8}_{-0.9}$	[15]
ALEPH	$BR(b \to \overline{B})BR(\overline{B} \to D_2^{*0} \ell^- \overline{\nu_\ell})BR(D_2^{*0} \to D^{+}\pi^-)$	$3.8^{+2.4}_{-1.9}{}^{+0.8}_{-0.8}$	[15]
CLEO	$BR(B^- \to D_2^{*0} \ell^- \overline{\nu_\ell})BR(D_2^{*0} \to D^{*+}\pi^-)$	$5.9 \pm 6.6 \pm 1.0 \pm 0.4$	[22]
DELPHI	$BR(b \to \overline{B})BR(\overline{B} \to D_2^{*0} \ell^- \overline{\nu_\ell})BR(D_2^{*0} \to D^{+}\pi^-,\ D^{*+}\pi^-)$	$11.6 \pm 5.6 \pm 3.2$	[23]

Table 17: *Measured values of the* D_2^* *production rate in b-semileptonic decays. In the original publication from ALEPH, as these values differ from zero only by one or two standard deviations, only upper limits were quoted. The third systematic uncertainty, quoted in the CLEO analysis, comes from the variation of the detection efficiency when changing the parameters of the signal form factors in the ISGW2 model [29].*

B Λ_b^0 polarization measurements

Experiment	Value	Ref.
ALEPH	$-0.23^{+0.24}_{-0.20}{}^{+0.08}_{-0.07}$	[32]
DELPHI	$-0.49^{+0.32}_{-0.30}{}^{+0.17}_{-0.17}$	[33]
OPAL	$-0.56^{+0.20}_{-0.13}{}^{+0.09}_{-0.09}$	[34]

Table 18: *Measured values of the* Λ_b^0 *polarization in* Z *decays.*

C Measurements used in the evaluation of b-hadron lifetimes

In the following Tables, preliminary numbers are labelled with "(p)".

Experiment	Method	Data set	τ_{B^0} (ps)	Reference
ALEPH	$D^{(*)}\ell$	91–95	$1.518 \pm 0.053 \pm 0.034$	[36]
ALEPH	Excl. rec.	91–94	$1.25^{+0.15}_{-0.13} \pm 0.05$	[37]
ALEPH	Partial rec. $\pi^+\pi^-$	91–94	$1.49^{+0.17+0.08}_{-0.15-0.06}$	[37]
CDF	$D^{(*)}\ell$	92–95	$1.474 \pm 0.039^{+0.52}_{-0.51}$	[38]
CDF	Excl. $(J/\psi K)$	92–95	$1.58 \pm 0.09 \pm 0.02$	[39]
DELPHI	$D^{(*)}\ell$	91–93	$1.61^{+0.14}_{-0.13} \pm 0.08$	[40]
DELPHI	Charge sec. vtx.	91–93	$1.63 \pm 0.14 \pm 0.13$	[41]
DELPHI	Inclusive $D^*\ell$	91–93	$1.532 \pm 0.041 \pm 0.040$	[42]
DELPHI (p)	Charge sec. vtx.	94	$1.520 \pm 0.021 \pm 0.041$	[43]
L3	Charge sec. vtx.	94–95	$1.52 \pm 0.06 \pm 0.04$	[44]
L3 (p)	Inclusive $D^*\ell$	94	$1.74 \pm 0.12 \pm 0.04$	[45]
OPAL	$D^{(*)}\ell$	91–93	$1.53 \pm 0.12 \pm 0.08$	[46]
OPAL	Charge sec. vtx.	93–95	$1.523 \pm 0.057 \pm 0.053$	[47]
OPAL	Inclusive $D^*\ell$	91–00	$1.541 \pm 0.028 \pm 0.023$	[48]
SLD	Charge sec. vtx.ℓ	93–95	$1.56^{+0.14}_{-0.13} \pm 0.10$	[49]
SLD (p)	Charge sec. vtx.	93–98	$1.565 \pm 0.021 \pm 0.043$	[50]
Average			1.546 ± 0.021	

Table 19: *Measurements of the* B_d^0 *lifetime.*

Experiment	Method	Data set	τ_{B^+}(ps)	Reference
ALEPH	$D^{(*)}\ell$	91-95	$1.648 \pm 0.049 \pm 0.035$	[36]
ALEPH	Excl. rec.	91-94	$1.58^{+0.21+0.04}_{-0.18-0.03}$	[37]
CDF	$D^{(*)}\ell$	92-95	$1.637 \pm 0.058^{+0.45}_{-0.43}$	[38]
CDF	Excl. $(J/\psi K)$	92-95	$1.68 \pm 0.07 \pm 0.02$	[39]
DELPHI	$D^{(*)}\ell$	91-93	$1.61 \pm 0.16 \pm 0.12$	[40][a]
DELPHI	Charge sec. vtx.	91-93	$1.72 \pm 0.08 \pm 0.06$	[41][a]
DELPHI (p)	Charge sec. vtx.	94	$1.619 \pm 0.018 \pm 0.035$	[43][a]
L3	Charge sec. vtx.	94-95	$1.66 \pm 0.06 \pm 0.03$	[44]
OPAL	$D^{(*)}\ell$	91-93	$1.52 \pm 0.14 \pm 0.09$	[46]
OPAL	Charge sec. vtx.	93-95	$1.643 \pm 0.037 \pm 0.025$	[47]
SLD	Charge sec. vtx. ℓ	93-95	$1.61^{+0.13}_{-0.12} \pm 0.07$	[49]
SLD (p)	Charge sec. vtx.	93-98	$1.623 \pm 0.020 \pm 0.034$	[50]
Average			1.647 ± 0.021	

Table 20: *Measurements of the* B^+ *lifetime.*
a) The combined DELPHI result quoted in [41] is (1.70 ± 0.09) ps.

Experiment	Method	Data set	τ_{B_s} (ps)	Reference
ALEPH	$\mathrm{D}_s\ell$	91-95	$1.54^{+0.14}_{-0.13}\pm 0.04$	[51]
ALEPH	$\mathrm{D}_s h$	91-95	$1.47\pm 0.14 \pm 0.08$	[52]
CDF	$\mathrm{D}_s\ell$	92-96	$1.36\pm0.09^{+0.06}_{-0.05}$	[53]
CDF	Excl. $\mathrm{J}/\psi\phi$	92-95	$1.34^{+0.23}_{-0.19}\pm 0.05$	[39]
DELPHI	$\mathrm{D}_s\ell$	91-95	$1.42^{+0.14}_{-0.13}\pm 0.03$	[54]
DELPHI	$\mathrm{D}_s h$	91-95	$1.53^{+0.16}_{-0.15}\pm0.07$	[55]
DELPHI	D_s inclus.	91-94	$1.60\pm0.26^{+0.13}_{-0.15}$	[56]
OPAL	$\mathrm{D}_s\ell$	90-95	$1.50^{+0.16}_{-0.15}\pm 0.04$	[57]
OPAL	D_s inclus.	90-95	$1.72^{+0.20+0.18}_{-0.19-0.17}$	[58]
Average			1.464 ± 0.057	

Table 21: *Measurements of the* $\mathrm{B^0_s}$ *lifetime.*

Experiment	Method	Data set	τ_{Λ_b} (ps)	Reference
ALEPH	$\Lambda\ell$	91-95	$1.20^{+0.08}_{-0.08}\pm 0.06$	[59]
DELPHI	$\Lambda\ell\pi$ vtx	91-94	$1.16\pm0.20 \pm 0.08$	[61][a]
DELPHI	$\Lambda\mu$ i.p.	91-94	$1.10^{+0.19}_{-0.17}\pm 0.09$	[62][a]
DELPHI	$\mathrm{p}\ell$	91-94	$1.19\pm0.14\pm 0.07$	[61][a]
OPAL	$\Lambda\ell$ i.p.	90-94	$1.21^{+0.15}_{-0.13}\pm 0.10$	[63][b]
OPAL	$\Lambda\ell$ vtx.	90-94	$1.15\pm0.12\pm 0.06$	[63][b]
Avg. above 6			$1.170^{+0.066}_{-0.054}$	
ALEPH	$\Lambda_c\ell$	91-95	$1.18^{+0.13}_{-0.12}\pm 0.03$	[59]
ALEPH	$\Lambda\ell^-\ell^+$	91-95	$1.30^{+0.26}_{-0.21}\pm 0.04$	[59]
CDF	$\Lambda_c\ell$	91-95	$1.32\pm0.15\pm 0.06$	[60]
DELPHI	$\Lambda_c\ell$	91-94	$1.11^{+0.19}_{-0.18}\pm 0.05$	[61][a]
OPAL	$\Lambda_c\ell\&\Lambda\ell^-\ell^+$	90-95	$1.29^{+0.24}_{-0.22}\pm 0.06$	[64]
Avg. above 5	τ_{Λ_b}		$1.229^{+0.081}_{-0.079}$	
Avg. above 11			1.208 ± 0.051	
ALEPH	$\Xi\ell$	90-95	$1.35^{+0.37+0.15}_{-0.28-0.17}$	[65]
DELPHI	$\Xi\ell$	91-93	$1.5\ ^{+0.7}_{-0.4}\pm 0.3$	[66]
Avg. above 2	τ_{Ξ_b}		$1.39^{+0.34}_{-0.28}$	

Table 22: *Measurements of the b-baryon lifetime.*
a) The combined DELPHI result quoted in [61] is (1.14$\pm$*0.08* $\pm$ *0.04) ps.*
b) The combined OPAL result quoted in [63] is (1.16 $\pm$ *0.11* $\pm$ *0.06) ps.*

Experiment	Method	Data set	τ_B (ps)	Reference
ALEPH	Lepton i.p. (3D)	91-93	$1.533 \pm 0.013 \pm 0.022$	[67]
L3	Lepton i.p. (2D)	91-94	$1.544 \pm 0.016 \pm 0.021$	[68][b]
OPAL	Lepton i.p. (2D)	90-91	$1.523 \pm 0.034 \pm 0.038$	[69]
Average set 1			1.537 ± 0.020	
ALEPH	Dipole	91	$1.511 \pm 0.022 \pm 0.078$	[70]
ALEPH (p)	Sec. vert.	91-95	$1.601 \pm 0.004 \pm 0.032$	[71]
DELPHI	All track i.p.(2D)	91-92	$1.542 \pm 0.021 \pm 0.045$	[72][a]
DELPHI	Sec. vert.	91-93	$1.582 \pm 0.011 \pm 0.027$	[73][a]
L3	Sec. vert. + i.p.	91-94	$1.556 \pm 0.010 \pm 0.017$	[68][b]
OPAL	Sec. vert.	91-94	$1.611 \pm 0.010 \pm 0.027$	[74]
SLD	Sec. vert.	93	$1.564 \pm 0.030 \pm 0.036$	[75]
Average set 2			1.577 ± 0.016	
Average sets 1-2			1.564 ± 0.014	
CDF	J/ψ vert.	92-95	$1.533 \pm 0.015 \ ^{+0.035}_{-0.031}$	[39]

Table 23: *Measurements of the average b-hadron lifetime.*
a) The combined DELPHI result quoted in [73] is (1.575 $\pm$ 0.010 $\pm$ 0.026) ps.
b) The combined L3 result quoted in [68] is (1.549 $\pm$ 0.009 $\pm$ 0.015) ps.

Experiment	Method	Data set	Ratio τ_+/τ_0	Reference
ALEPH	$D^{(*)}\ell$	91-95	$1.085 \pm 0.059 \pm 0.018$	[36]
ALEPH	Excl. rec.	91-94	$1.27^{+0.23+0.03}_{-0.19-0.02}$	[37]
CDF	$D^{(*)}\ell$	92-95	$1.110 \pm 0.056^{+0.033}_{-0.030}$	[38]
CDF	Excl. (J/ψK)	92-95	$1.06 \pm 0.07 \pm 0.02$	[39]
DELPHI	$D^{(*)}\ell$	91-93	$1.00^{+0.17}_{-0.15} \pm 0.10$	[40]
DELPHI	Charge sec. vtx.	91-93	$1.06^{+0.13}_{-0.11} \pm 0.10$	[41]
DELPHI	Charge sec. vtx.	94	$1.065 \pm 0.022 \pm 0.033$	[43]
L3	Charge sec. vtx.	94-95	$1.09 \pm 0.07 \pm 0.03$	[44]
OPAL	$D^{(*)}\ell$	91-93	$0.99 \pm 0.14^{+0.05}_{-0.04}$	[46]
OPAL	Charge sec. vtx.	93-95	$1.079 \pm 0.064 \pm 0.041$	[47]
SLD	Charge sec. vtx. ℓ	93-95	$1.03^{+0.16}_{-0.14} \pm 0.09$	[49]
SLD (p)	Charge sec. vtx.	93-98	$1.037^{+0.025}_{-0.024} \pm 0.024$	[50]
Average			1.063 ± 0.020	

Table 24: *Measurements of the ratio* $\tau_{B+}/\tau_{B^0_d}$.

D Measurements of b-hadron production rates

D.1 B_s^0 production rate

Quantity	Value	Ref.
f_{B_s} BR($B_s^0 \to D_s^- \ell^+ \nu_\ell X$) BR($D_s^- \to \phi\pi^-$)	$(2.87 \pm 0.32 \pm ^{0.25}_{0.41}) \times 10^{-4}$	[95]
f_{B_s} BR($B_s^0 \to D_s^- \ell^+ \nu_\ell X$) BR($D_s^- \to \phi\pi^-$)	$(4.2 \pm 1.9) \times 10^{-4}$	[96]
f_{B_s} BR($B_s^0 \to D_s^- \ell^+ \nu_\ell X$) BR($D_s^- \to \phi\pi^-$)	$(3.9 \pm 1.1 \pm 0.8) \times 10^{-4}$	[97]
Average of the three measurements	$(3.00 \pm ^{0.38}_{0.41}) \times 10^{-4}$	
$f_{B_s}/(f_{B^+} + f_{B_d})$ BR($D_s^- \to \phi\pi^-$)	$(7.7 \pm 1.5) \times 10^{-3}$	[98]
$f_{B_s}/(f_{B^+} + f_{B_d})$ BR($D_s^- \to \phi\pi^-$)	$(7.6 \pm 1.8) \times 10^{-3}$	[100]

Table 25: *Inputs used in the calculation of the* B_s^0 *production rate.*

All published results have been multiplied by the branching fraction for the decay $D_s^- \to \phi\pi^-$ to be independent of the assumed central value and uncertainty of this quantity; quoted uncertainties have been reevaluated accordingly.

The ALEPH measurement [99] of f_{B_s} BR($B_s^0 \to D_s^- X$) BR($D_s^- \to \phi\pi^-$) $= (3.1 \pm 0.7 \pm 0.6) \times 10^{-3}$ has not been used to obtain an additional measurement on f_{B_s} because of the model dependence attached to the evaluation of BR($B_s^0 \to D_s^- X$). Instead, the value of f_{B_s}, quoted in Table 4, can be used to extract, from this measurement, the inclusive branching fraction for D_s^- production in B_s^0 decays:

$$\mathrm{BR}(B_s^0 \to D_s^- X)\ \mathrm{BR}(D_s^- \to \phi\pi^-) = (3.1 \pm 0.7 \pm 0.7) \times 10^{-2} \tag{150}$$

and, using the value for BR($D_s^- \to \phi\pi^-$) given in Table 1:

$$\mathrm{BR}(B_s^0 \to D_s^- X) = 0.86 \pm 0.19 \pm 0.29. \tag{151}$$

D.2 b-baryon production rate

Quantity	Value	Ref.
BR($b \to \Lambda_b^0$) BR($\Lambda_b^0 \to \Lambda_c^+ \ell^- \overline{\nu_\ell} X$) BR($\Lambda_c^+ \to pK^-\pi^+$)	$(3.78 \pm 0.31 \pm 0.23) \times 10^{-4}$	[101]
BR($b \to \Lambda_b^0$) BR($\Lambda_b^0 \to \Lambda_c^+ \ell^- \overline{\nu_\ell} X$) BR($\Lambda_c^+ \to pK^-\pi^+$)	$(5.19 \pm 1.14 \pm ^{1.19}_{0.66}) \times 10^{-4}$	[102]
Average of the two measurements	$(3.90 \pm 0.42) \times 10^{-4}$	
$f_{b\text{-baryon}}/(f_{B^+} + f_{B_d})$ BR($\Lambda_c^+ \to pK^-\pi^+$)	$(5.9 \pm 1.4) \times 10^{-3}$	[98]
BR($b \to \Xi_b$) BR($\Xi_b^- \to \Xi^- \ell^- \overline{\nu_\ell} X$)	$(5.4 \pm 1.1 \pm 0.8) \times 10^{-4}$	[103]
BR($b \to \Xi_b$) BR($\Xi_b^- \to \Xi^- \ell^- \overline{\nu_\ell} X$)	$(5.9 \pm 2.1 \pm 1.0) \times 10^{-4}$	[104]
Average of the two measurements	$(5.5 \pm 1.2) \times 10^{-4}$	
$f_{b\text{-baryon}}$	$0.102 \pm 0.007 \pm 0.027$	[94]

Table 26: *Inputs used in the calculation of the b-baryon production rate.*

All published results which are using the Λ_c^+ baryon have been multiplied by the branching fraction for the decay $\Lambda_c^+ \to pK^-\pi^+$ to be independent of the assumed central value and uncertainty of this quantity; quoted uncertainties have been reevaluated accordingly.

D.3 B^+ production rate

Quantity	Value	Ref.
f_{B^+}	0.414 ± 0.016	[93]

Table 27: *Direct measurement of the* B^+ *production rate.*

This value is obtained from the measurement of the production rate of charged weakly decaying b-hadrons. A small correction has been applied to account for Ξ_b^- production, as given in Table 26.

E Measurements of c and $\bar{c}$ production rates in b-hadron decays

Experiment	$P(b \to D^0 \text{ or } \overline{D}^0) \times BR(D^0 \to K^-\pi^+)(\%)$
ALEPH [125]	$2.32 \pm 0.090 \pm 0.048 \pm 0.040(0.035)$
DELPHI [129]	$2.308 \pm 0.075 \pm 0.045 \pm 0.134(0.121)$
OPAL [130]	$2.099 \pm 0.106 \pm 0.076 \pm 0.094(0.053)$
CLEO [126]	$2.487 \pm 0.055 \pm 0.074$

Table 28: *Experimental results on* D^0 *production rates in b-hadron decays.*

Experiment	$P(b \to D^+ + D^-) \times BR(D^+ \to K^-\pi^+\pi^+)(\%)$
ALEPH [125]	$2.13 \pm 0.120 \pm 0.052 \pm 0.073(0.029)$
DELPHI [129]	$2.092 \pm 0.094 \pm 0.057 \pm 0.088(0.060)$
OPAL [130]	$1.752 \pm 0.143 \pm 0.074 \pm 0.098 \pm 0.065$
CLEO [126]	$2.160 \pm 0.083 \pm 0.083$

Table 29: *Experimental results on* D^+ *production rates in b-hadron decays.*

Experiment	$P(b \to D_s^+ + D_s^-) \times BR(D_s^+ \to \phi\pi^+)(\%)$
ALEPH [125]	$0.652 \pm 0.061 \pm 0.022 \pm 0.026(0.018)$
DELPHI [129]	$0.574 \pm 0.078 \pm 0.070 \pm 0.036(0.021)$
OPAL [130]	$0.768 \pm 0.083 \pm 0.048 \pm 0.057(0.027)$
CLEO [127]	$0.424 \pm 0.014 \pm 0.031$

Table 30: *Experimental results on* D_s^+ *production rates in b-hadron decays.*

Experiment	$P(b \to \Lambda_c^+ + \overline{\Lambda}_c^-) \times BR(\Lambda_c^+ \to pK^-\pi^+)(\%)$
ALEPH [125]	$0.48 \pm 0.06 \pm 0.02 \pm 0.019(0.011)$
DELPHI [129]	$0.445 \pm 0.086 \pm 0.026 \pm 0.028(0.022)$
OPAL [130]	$0.564 \pm 0.106 \pm 0.032 \pm 0.034 \pm 0.019$
CLEO [128]	$0.273 \pm 0.051 \pm 0.039$

Table 31: *Experimental results on* Λ_c^+ *production rates in b-hadron decays.*

Experiment	BR($b \to$ 0D X)(%)	BR($b \to$ D $\overline{\mathrm{D}}$ X)(%)
DELPHI [136]	$3.3 \pm 1.8 \pm 1.0$	$13.6 \pm 3.0 \pm 3.0$
SLD [137]	$5.6 \pm 1.1 \pm 2.0$	$24.6 \pm 1.4 \pm 4.0$

Table 32: *Experimental results on no-open charm and two charm hadron branching fractions. The first uncertainty corresponds to statistical errors and un-correlated systematics whereas the second number is for correlated systematic uncertainties*

Decay channel	BR (%)
$b \to \mathrm{D}^0\mathrm{D}_s^-\mathrm{X}$	$9.1^{+2.0}_{-1.8}{}^{+1.3}_{-1.2}{}^{+3.1}_{-1.9}$
$b \to \mathrm{D}^+\mathrm{D}_s^-\mathrm{X}$	$4.0^{+1.7}_{-1.4} \pm 0.7^{+1.4}_{-0.9}$
$b \to \mathrm{D}^0\overline{\mathrm{D}}^0\mathrm{X}$	$5.1^{+1.6}_{-1.4}{}^{+1.2}_{-1.1} \pm 0.3$
$b \to \mathrm{D}^0\mathrm{D}^-,\ \mathrm{D}^+\overline{\mathrm{D}}^0\mathrm{X}$	$2.7^{+1.5}_{-1.3}{}^{+1.0}_{-0.9} \pm 0.2$
$b \to \mathrm{D}^+\mathrm{D}^-\mathrm{X}$	< 0.9 at 90%C.L.
$b \to \mathrm{D}^{*+}\mathrm{D}_s^-\mathrm{X}$	$3.3^{+1.0}_{-0.9} \pm 0.6^{+1.1}_{-0.7}$
$b \to \mathrm{D}^{*+}\overline{\mathrm{D}}^0,\ \mathrm{D}^0\mathrm{D}^{*-}\mathrm{X}$	$3.0^{+0.9}_{-0.8}{}^{+0.7}_{-0.5} \pm 0.2$
$b \to \mathrm{D}^{*+}\mathrm{D}^-,\ \mathrm{D}^+\mathrm{D}^{*-}\mathrm{X}$	$2.5^{+1.0}_{-0.9}{}^{+0.6}_{-0.5} \pm 0.2$
$b \to \mathrm{D}^{*+}\mathrm{D}^{*-}\mathrm{X}$	$1.2^{+0.4}_{-0.3} \pm 0.2 \pm 0.1$

Table 33: *Measurements of different double charm final states by the ALEPH Collaboration [140]. The first error is statistical, the second is the sum of all systematic errors except those from D branching fractions which determine the third error.*

Only measured branching fractions into a D or a $\overline{\mathrm{D}}$ meson have been used in the determination of the double charm rates given in Section 6.4.2. Decay channels with one or two D* can be used to evaluate the branching channel for $b \to \mathrm{D}^+\mathrm{D}^-\mathrm{X}$ which is unmeasured. Two models have been considered. In the first model [25], two parameters are introduced: the total double charm rate with no D_s^+ in the final state and the relative rate, r, for producing a D* instead of a D meson, in the decay of the b-hadron. The value $r = 3$ allows to reproduce the different measured decay rates and gives BR($b \to \mathrm{D}^+\mathrm{D}^-\mathrm{X}$) = 0.5%. In another approach, only non-strange final states have been considered and three parameters have been used corresponding to the respective production rates for $\mathrm{D}\overline{\mathrm{D}}$, $\mathrm{D}\overline{\mathrm{D}}^*$ and $\mathrm{D}^*\overline{\mathrm{D}}^*$ assumed to be equal in all possible charge combinations. These parameters have been fitted using measured decay rates for the different decay channels. The fit prefers no prompt production of $b \to \mathrm{D}\overline{\mathrm{D}}\mathrm{X}$ and, fitting only the two remaining parameters, it gives: BR($b \to \mathrm{D}^+\mathrm{D}^-\mathrm{X}$) = 0.35%.

These two results are compatible with the quoted limit for this channel and the importance of D* production in double-charm b-hadron decays can explain the smallness of the $\mathrm{D}^+\mathrm{D}^-$ contribution.

[25] R. Barate, private communication.

F Theoretical uncertainties relevant to the measurements of $|V_{ub}|$ and $|V_{cb}|$

The determination of the values of $|V_{cb}|$ and $|V_{ub}|$ from present measurements depends on the knowledge of the values of other parameters and on the use of theoretical expressions of limited accuracy.

Some of these parameters, such as α_S (the strong coupling constant), are measured and it can be assumed that the corresponding error has a Gaussian distribution. Other quantities contributing to the error (parameters, series truncated at finite orders, ..) are taken from theory. For a parameter, the quoted uncertainty is expected to correspond to a range which contains its exact value. If the uncertainty is related to a limitation of the theoretical approach in evaluating higher order contributions, the quoted uncertainty is expected to have been deduced from the evaluation of an upper limit on the contribution from neglected terms. To ensure that the combined uncertainty corresponds to intervals that contain the exact values of $|V_{cb}|$ or $|V_{ub}|$, uncertainties of purely theoretical origin have to be added linearly. Other type of uncertainties, assumed to be Gaussian distributed, can be convoluted with the previous one to get the final error distribution.

In practice, there are no complete analytical expressions allowing to update the values of the different parameters and to evaluate correlations. In addition, uncertainties on some parameters have already been obtained by summing in quadrature individual errors. As a result, in the present analysis, systematic uncertainties have been added in quadrature and an interval of $\pm 1\sigma$ centered on the quoted value for $|V_{cb}|$ or $|V_{ub}|$ is supposed to contain the exact value of these parameters with 68% probability (only).

F.1 The b and c-quark masses

The b-quark mass used to determine $|V_{ub}|$ and $|V_{cb}|$ must be running and evaluated at a low scale to improve the convergence of the perturbative QCD series used to compute semileptonic widths of b-hadrons [192]. In this way, concerns [193] originating from the importance of QCD corrections affecting the b-quark mass, getting amplified by the power (about five) at which it is raised in the theoretical expression of $\Gamma_{sl}(b)$, are controlled. Typical scales have to be around 1 GeV.

Values for the b-quark mass, using the definition given in [192], have been obtained at NNLO from analyses [194, 195] of sum rules in the Υ system:

$$m_b^{kin.}(1\ \mathrm{GeV}) = (4.56 \pm 0.06)\ \mathrm{GeV/c^2},\ \mathrm{m_b^{kin.}}(1\ \mathrm{GeV}) = (4.57 \pm 0.04)\ \mathrm{GeV/c^2}. \quad (152)$$

Whereas the pole or the $\overline{\mathrm{MS}}$ b-quark masses are not adapted to extract $|V_{ub}|$ or $|V_{cb}|$, there exist other mass definitions which have similar properties as the kinetic mass mentioned above. These are the 1S (m_b^{1S}) and the potential subtracted (m_b^{PS}) masses [195, 203]. All these values of the b-quark mass have been obtained using sum rules which relate the masses and the electronic decay widths of Υ mesons to moments of the vacuum polarization function, as proposed initially in [196].

In the following the value:

$$m_b^{kin.}(1\ \mathrm{GeV}) = (4.58 \pm 0.06)\ \mathrm{GeV/c^2} \quad (153)$$

has been used where the quoted uncertainty is the quadratic sum of several contributions.

The inclusive and exclusive determinations of $|V_{cb}|$ need also a value for the c-quark mass. At present, it is considered [192] that the mass difference between b and c quarks is under better control than the absolute determination of the c-quark mass itself. Using the definition of the kinetic mass:

$$\overline{M_{H_Q}} = m_Q^{kin.}(\mu) + \overline{\Lambda}(\mu) + \frac{\mu_\pi^2(\mu)}{2m_Q(\mu)} + \frac{\rho_D^3(\mu) - \overline{\rho}^3(\mu)}{4m_Q^2(\mu)} + \mathcal{O}\left(\frac{1}{m_Q^3}\right) \tag{154}$$

in which, $\overline{M_{H_Q}}$ is the spin-averaged heavy hadron mass, $\overline{\Lambda}(\mu)$ is a function which is independent of the heavy quark, $\mu_\pi^2(\mu)$ is the average of the square of the heavy quark momentum inside the hadron, $\rho_D^3(\mu)$ is the expectation value of the Darwin term and $\overline{\rho}^3(\mu)$ is the sum of positive non-local correlators [197].

Taking the difference of the previous expression for b and c quarks one obtains:

$$\begin{aligned} m_b^{kin.}(\mu) - m_c^{kin.}(\mu) = \quad & \overline{M_B} - \overline{M_D} - \tfrac{\mu_\pi^2(\mu)}{2}\left(\tfrac{1}{m_b(\mu)} - \tfrac{1}{m_c(\mu)}\right) \\ & - \tfrac{\rho_D^3(\mu) - \overline{\rho}^3(\mu)}{4}\left(\tfrac{1}{m_b^2(\mu)} - \tfrac{1}{m_c^2(\mu)}\right) + \mathcal{O}\left(\tfrac{1}{m_{b,c}^3}\right). \end{aligned} \tag{155}$$

As the ρ^3 terms are expected to be of order 0.1 GeV^3 for $\mu \sim 1$ GeV, the $\mathcal{O}\left(\frac{1}{m_{b,c}^2}\right)$ correction has already a small value. Numerically this gives:

$$m_b^{kin.}(1\ \mathrm{GeV}) - \mathrm{m_c^{kin.}}(1\ \mathrm{GeV}) = (\ 3.50\ +\ 0.040\ \frac{\mu_\pi^2 - 0.5}{0.1}\ +\ \Delta\mathrm{M}_2\ )\ (\mathrm{GeV/c^2}) \tag{156}$$

where $|\Delta M_2| \leq 0.015$ GeV/c^2.

It can be noted that $m_c^{kin.}(1\ \mathrm{GeV}) \sim 1.08$ GeV/c^2 is obtained in this way. This may seem to be a rather low value but it is in agreement with the independent determination of this quantity obtained using the value of the $\overline{\mathrm{MS}}$ charm quark mass, $\overline{m_c}(\overline{m_c}) = (1.23 \pm 0.09)$ GeV/c^2 [144] and the theoretical expression given in [198], which relates the kinetic and the $\overline{\mathrm{MS}}$ heavy quark mass definitions.

Even if, numerically there is no evidence (by chance?) for such problems, it may be argued that the $1/m_c$ is less under control than the $1/m_b$ expansion as $m_c \sim 1$ GeV. As a result it has been decided to double the theoretical error attached to the inclusive determination of $|V_{cb}|$.

F.2 Measurement of $|V_{ub}|$ using the decay $b \rightarrow \ell^- \overline{\nu_\ell} X_u$

Based on studies developed independently by two groups [199], [200], a relative theoretical uncertainty of 5% has been evaluated for the extraction of $|V_{ub}|$ from the measured inclusive charmless semileptonic rate BR($b \rightarrow \ell^- \overline{\nu_\ell} X_u$). The central values of the two analyses also agree[26], and the following relationship has been adopted in the extraction of $|V_{ub}|$:

$$\begin{aligned} |V_{ub}| = \quad & 0.00445\left(\tfrac{BR(b\rightarrow \ell^- \overline{\nu_\ell} X_u)}{0.002}\right)^{1/2}\left(\tfrac{1.55\ \mathrm{ps}}{\tau_B}\right)^{1/2} \\ & \times\left(1 \pm 0.010_{pert.} \pm 0.030_{1/m_b^3} \pm 0.035_{m_b}\right) \end{aligned} \tag{157}$$

[26] In practice the initial central value quoted in [199] has been corrected and the value obtained in [200] is $|V_{ub}| = 0.00443\left(\frac{BR(b\rightarrow \ell \overline{\nu_\ell} X_u)}{0.002}\right)^{1/2}\left(\frac{1.55\ \mathrm{ps}}{\tau_b}\right)^{1/2} \times (1 \pm 0.020_{pert.} \pm 0.030_{m_b})$.

The measurements performed at LEP reported in Section 8, based on the inclusive selection of $b \to \ell^- \overline{\nu_\ell} \mathrm{X}_u$ decays, are sensitive to a large fraction of the mass distribution of the hadronic system and to the full range of the lepton energy spectrum. This mass distribution M_X has been studied by theorists [204] for the $b \to \ell^- \overline{\nu_\ell} \mathrm{X}_u$ transition. The conclusion was[27] that the additional theoretical uncertainty on $|\mathrm{V}_{ub}|$ from m_b and μ_π^2 is less than 10% if the experimental inclusive technique is sensitive up to at least $M_X \simeq 1.5$ GeV/c^2.

F.3 Measurement of $|\mathrm{V}_{cb}|$ using the decay $\overline{\mathrm{B_d^0}} \to \mathrm{D}^{*+}\ell^-\overline{\nu_\ell}$

It has been proposed to use the parametrization given in [205] to account for the dependence in w and to extract the value at $w=1$ of the differential decay rate. The quantity $\mathcal{F}_{D^*}(1)\,|\mathrm{V}_{cb}|$ is then obtained.

In order to measure $|\mathrm{V}_{cb}|$, the value at $w=1$ of the form factor is taken from theory by evaluating different corrections [206] which have to be applied to the naive expectation of $\mathcal{F}_{D^*}(1) = 1$:

$$|\mathcal{F}_{D^*}(1)|^2 \;=\; \xi_A(\mu) - \Delta^A_{1/m^2} - \Delta^A_{1/m^3} - \sum_{0<\epsilon_i<\mu} |\mathcal{F}_i|^2 \tag{158}$$

The central value of the b quark mass and its uncertainty are the same as given in Section F.1. The main difference between the theoretical analyses described in [206] and [207] comes from the evaluation of the uncertainties on non-perturbative corrections. A different approach can be found in [208].

The detailed balance of uncertainties given in [206], revised for consistency with the values adopted for the different parameters, is the following:

$$\mathcal{F}_{D^*}(1) \;=\; 0.880 - 0.024\frac{\mu_\pi^2 - 0.5\ \mathrm{GeV}^2}{0.1\ \mathrm{GeV}^2} \pm 0.035_{excit.} \pm 0.010_{pert.} \pm 0.025_{1/m^3} \tag{159}$$

The hypotheses or results on which the contributions of the different terms were based are reviewed in the following.

F.3.1 μ_π^2

The value $\mu_\pi^2 = (0.5\pm0.1)$ GeV^2 has been used and it gives ±0.024 uncertainty on $\mathcal{F}_{D^*}(1)$.

From QCD sum rules, the following value has been obtained [209]:

$$\mu_\pi^2 = (0.5 \pm 0.1)\ \mathrm{GeV}^2 \tag{160}$$

and a model-independent lower bound has been established [210]:

$$\mu_\pi^2 > \mu_G^2 \simeq \frac{4}{3}(M^2_{B^*} - M^2_B) \approx 0.4\ \mathrm{GeV}^2 \tag{161}$$

[27] In practice experimental analyses take into account the expected mass distribution of the hadronic system, and the variation of the detection efficiencies as a function of this mass, to evaluate the real importance of this systematic uncertainty.

F.3.2 High mass excitations

In QCD sum rules, used in [206], [207], the effect expected from high mass hadronic states has been introduced in a rather arbitrary way. In [206], it has been assumed that their effect, on the correction terms which behave as m_Q^{-2}, can vary between 0 and 100% of Δ^A_{1/m^2}. Such a variation corresponds to ± 0.035 on $\mathcal{F}_{D^*}(1)$.

F.3.3 Higher order non-perturbative corrections

The value of ± 0.025 uncertainty related to the contribution from terms of order at least m_Q^{-3} is considered as being optimistic from the authors of [206].

F.3.4 Adopted value

Combining in quadrature the uncertainties quoted in Equation (159) gives:

$$\mathcal{F}_{D^*}(1) \ = \ 0.88 \ \pm 0.05. \tag{162}$$

It exists several other determinations of $\mathcal{F}_{D^*}(1)$, collected in Table 34. The first value is

$\mathcal{F}_{D^*}(1)$	ref
0.89 ± 0.08	[202]
0.913 ± 0.042	[157]
0.935 ± 0.035	(lattice) [201]

Table 34: Different values for $\mathcal{F}_{D^*}(1)$.

obtained from the same expressions as used in the present analysis but uncertainties have been combined linearly. The ± 0.08 interval corresponds to a range inside which the exact value of $\mathcal{F}_{D^*}(1)$ is expected. This uncertainty is thus rather similar to the ± 0.1 interval which corresponds to the 95% probability region when using Equation (162). The quoted uncertainty for the second value in Table 34 corresponds also to a range and is two times smaller as compared with the first value. The third result has been obtained using lattice QCD and uncertainties have been combined in quadrature, the uncertainty originating from the quenching approximation remains to be evaluated. From these comparisons it results that the uncertainty adopted on $\mathcal{F}_{D^*}(1)$ agrees with the most conservative estimate.

F.4 Measurement of $|V_{cb}|$ using the inclusive semileptonic decay $b \to \ell^- X$ rate

The expression relating $|V_{cb}|$ to the inclusive semileptonic branching fraction can be found in [206]:

$$\begin{aligned} |V_{cb}| = \quad & 0.0411 \left(\frac{\mathrm{BR}(b \to \ell^- \overline{\nu_\ell} X_c)}{0.105}\right)^{1/2} \left(\frac{1.55 \text{ ps}}{\tau_b}\right)^{1/2} \\ & \times \left(1 - 0.012 \frac{\mu_\pi^2 - 0.5\mathrm{GeV}^2}{0.1\mathrm{GeV}^2}\right) \\ & \times \left(1 \pm 0.015_{pert.} \pm 0.010_{m_b} \pm 0.012_{1/m_Q^3}\right) \end{aligned} \tag{163}$$

The central value of reference [206] has been lowered by 2% to account for a different choice of the b-quark mass. A very similar result for the central value and the uncertainties can be found in [200] [28].

F.4.1 Uncertainties related to quark masses

It has been assumed that $m_b(\mu)$ can be determined with an uncertainty of ± 60 MeV/c^2, as discussed in Section F.1, and that the difference between the b- and c-quark masses is known with an uncertainty of ± 40 MeV/c^2, related to the error on μ_π^2. These variations induce ± 0.010 and ± 0.012 uncertainties on $|\mathrm{V}_{cb}|$, respectively.

F.4.2 Adopted value

Adding in quadrature the quoted uncertainties in Equation (163) leads to a $\pm 2.5\%$ relative uncertainty on $|\mathrm{V}_{cb}|$. Since these errors have not been cross-checked by other theoretical approaches or experimental measurements, as mentioned in Section F.1, it has been decided to inflate the total error by an arbitrary factor of two, leading to a theoretical uncertainty of $\pm 5\%$ (see reference [202] for a more detailed discussion). This implies, in practice, that uncertainties of ± 120 MeV/c^2, ± 80 MeV/c^2, and ± 0.2 GeV^2 have been attached to the values of m_b, $m_b - m_c$, and μ_π^2 respectively.

F.5 Common sources of theoretical errors for the two determinations of $|\mathbf{V}_{cb}|$

Theoretical uncertainties attached to the exclusive and inclusive measurements of $|\mathrm{V}_{cb}|$ are largely uncorrelated. When evaluating the average, only the uncertainties related to μ_π^2 and m_b have been considered fully correlated.

F.6 Sources of theoretical errors entering into the measurement of the ratio $\frac{|\mathbf{V}_{ub}|}{|\mathbf{V}_{cb}|}$

In the evaluation of the ratio $\frac{|\mathrm{V}_{ub}|}{|\mathrm{V}_{cb}|}$, several common experimental and model systematics have a reduced effect.

For the ratio $\frac{|\mathrm{V}_{ub}|_{incl}}{|\mathrm{V}_{cb}|_{incl}}$, uncertainties originating from the determination of m_b have to be considered as fully correlated. Leading effects of order $\mathcal{O}(1/m^3)$ are expected to be uncorrelated between $|\mathrm{V}_{ub}|$ and $|\mathrm{V}_{cb}|$ determinations [202]. This is also expected for perturbative uncertainties.

With the conventions used in the previous sections, the relative errors on the ratio $\frac{|\mathrm{V}_{ub}|_{incl}}{|\mathrm{V}_{cb}|_{incl}}$ are:

$$\pm 0.015(m_b) \;\pm 0.032(pert.) \;\pm 0.024(\mu_\pi^2) \pm 0.038(1/m^3) \;=\; \pm 0.06 \qquad (164)$$

For the ratio $\frac{|\mathrm{V}_{ub}|_{incl}}{|\mathrm{V}_{cb}|_{excl}}$, all uncertainties can be considered as uncorrelated giving:

$$\pm 0.035(m_b) \;\pm 0.014(pert.) \;\pm 0.024(\mu_\pi^2) \pm 0.039(1/m^3) \pm 0.035(excit.) \;=\; \pm 0.07 \quad (165)$$

[28] Using the same values for the lifetime and the semileptonic branching fraction of b-hadrons it gives: $|\mathrm{V}_{cb}| = 0.0426 \times (1 \pm 0.019 \pm 0.017 \pm 0.012)$.

In practice, the ratio $\frac{|\mathrm{V}_{ub}|}{|\mathrm{V}_{cb}|}$ will be obtained using the average of the two measurements of $|\mathrm{V}_{cb}|$ and taking into account common systematics with $|\mathrm{V}_{ub}|$.

F.7 Conclusions and summary

Two groups (at least) have obtained consistent results on central values and uncertainties for $|\mathrm{V}_{ub}|$ (*incl.*) and $|\mathrm{V}_{cb}|$ (*incl.*).

Uncertainties on $|\mathrm{V}_{cb}|$ (*incl.*) have been enlarged, in a rather arbitrary way, by a factor two to have some margin because of possible additional contributions (reliability of $1/m_c$ expansion, need for other techniques to evaluate m_b,...) to the error.

Dedicated experimental studies on D^{**} production and on the distributions of moments of the lepton momentum in the B rest frame are needed to evaluate with better accuracy and greater confidence the most important sources of systematic errors contributing in the theoretical expressions.

As said in the introduction, this proposal is a first conservative attempt, and it is hoped that in future quoted values for the uncertainties can be reduced and/or corrected for possible mistakes.

The following central values and theoretical uncertainties have been used in the measurements of $|\mathrm{V}_{ub}|$ and $|\mathrm{V}_{cb}|$ obtained from combined LEP analyses available for Summer 2000:

- inclusive measurement of $|\mathrm{V}_{ub}|$:

$$\begin{aligned} |\mathrm{V}_{ub}| = \quad & 0.00445 \left(\frac{\mathrm{BR}(b\to\ell^-\overline{\nu_\ell}\mathrm{X}_u)}{0.002}\right)^{1/2} \left(\frac{1.55\ \mathrm{ps}}{\tau_b}\right)^{1/2} \\ & \times \left(1 \pm 0.010_{pert.} \pm 0.030_{1/m_b^3} \pm 0.035_{m_b}\right) \end{aligned} \tag{166}$$

- inclusive measurement of $|\mathrm{V}_{cb}|$:

$$\begin{aligned} |\mathrm{V}_{cb}| = \quad & 0.0411 \left(\frac{\mathrm{BR}(b\to\ell^-\overline{\nu_\ell}\mathrm{X}_c)}{0.105}\right)^{1/2} \left(\frac{1.55\ \mathrm{ps}}{\tau_b}\right)^{1/2} \\ & \times \left(1 - 0.024\frac{\mu_\pi^2 - 0.5\mathrm{GeV}^2}{0.2\mathrm{GeV}^2}\right) \\ & \times \left(1 \pm 0.030_{pert.} \pm 0.020_{m_b} \pm 0.024_{1/m_Q^3}\right) \end{aligned} \tag{167}$$

- exclusive measurement of $|\mathrm{V}_{cb}|$:

$$\mathcal{F}_{D^*}(1) \;=\; 0.880 - 0.024\frac{\mu_\pi^2 - 0.5\ \mathrm{GeV}^2}{0.1\ \mathrm{GeV}^2} \pm 0.035_{excit.} \pm 0.010_{pert.} \pm 0.025_{1/m^3} \tag{168}$$

All uncertainties have been added in quadrature.

The correlated uncertainty between $|\mathrm{V}_{cb}|$ (*incl.*) and $|\mathrm{V}_{cb}|$ (*excl.*) measurements stem from the effect of μ_π^2.

The correlated uncertainty between $|\mathrm{V}_{ub}|$ (*incl.*) and $|\mathrm{V}_{cb}|$ (*incl.*) measurements is limited to the effect of m_b. Hence no correlated uncertainty between $|\mathrm{V}_{ub}|$ (*incl.*) and $|\mathrm{V}_{cb}|$ (*excl.*) measurements is assumed.