

## UNIQUE GRAVITON EXCHANGE SIGNATURES AT LINEAR COLLIDERS

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Many types of new physics can lead to contact interaction-like modifications in  $e^+e^-$  processes below direct production threshold. We examine the possibility of uniquely identifying the effects of graviton exchange from amongst this large set of models by using the moments of the angular distribution of the final state particles. In the case of  $e^+e^- \rightarrow f\bar{f}(W^+W^-)$  we demonstrate that this technique allows for the unique identification of the graviton exchange signature at the  $5\sigma$  level for mass scales as high as  $6(2.5)\sqrt{s}$ .

It is generally expected that new physics beyond the Standard Model(SM) will manifest itself at future colliders that probe the TeV scale. This new physics(NP) may appear either directly, as in the case of new particle production, or indirectly through deviations from the predictions of the SM. Perhaps the most well known example of this indirect scenario would be the observation of deviations in, *e.g.*, various  $e^+e^-$  cross sections due to apparent contact interactions<sup>1</sup>. There are many very different NP scenarios that predict new particle exchanges which lead to contact interactions below direct production threshold; a partial list of known candidates is: a  $Z'$ , leptoquarks,  $R$ -parity violating sneutrinos( $\tilde{\nu}$ ), bileptons, graviton Kaluza-Klein(KK) towers, gauge boson KK towers, and even string excitations. If contact interaction effects are observed one can always try to fit the shifts in the observables to each one of the set of known theories and see which gives the best fit. Alternatively, one can devise a technique which will rather quickly divide the full set of all possible models into distinct subclasses. In this paper we propose such a technique that makes use of the specific modifications in angular distributions induced by new exchanges. This method offers a way to uniquely identify graviton KK tower exchange (or any possible spin-0 exchange).

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Let us consider the normalized cross section for  $e^+e^- \rightarrow f\bar{f}$  in the SM assuming  $m_f = 0$  and  $f \neq e$  for simplicity. This can be written as  $\sigma^{-1}d\sigma/dz = \frac{3}{8}(1+z^2) + A_{FB}(s)z$  where  $z = \cos\theta$  and  $A_{FB}(s)$  is the Forward-Backward Asymmetry which depends upon the electroweak quantum numbers of the fermion,  $f$ , as well as the center of mass energy of the collision,  $\sqrt{s}$ . This structure is particularly interesting in that it is equally valid for a wide variety of New Physics models: composite-like contact interactions,  $Z'$  models, TeV-scale KK gauge bosons, *etc.* In fact, when this expression holds the *only* deviation from the SM for any of these models will be through the variations in the value of  $A_{FB}(s)$  since we have chosen to normalize the cross section.

Now let us consider taking moments of the normalized cross section above with respect to the Legendre Polynomials,  $P_n(z)$ . This can be done easily by re-writing the above as  $\sigma^{-1}d\sigma/dz = 1/2P_0 + 1/4P_2 + A_{FB}(s)P_1$  and recalling that the  $P_n(z)$  are orthonormal. Denoting such moments as  $\langle P_n \rangle$ , one finds that  $\langle P_1 \rangle = 2A_{FB}/3$ ,  $\langle P_2 \rangle = 1/10$  and  $\langle P_{n>2} \rangle = 0$ . In addition we also trivially obtain that  $\langle P_0 \rangle = 1$  since we have normalized the distribution so that this moment carries no new information. Thus, very naively, if we find that the  $\langle P_{n>1} \rangle$  are given by their SM values while  $\langle P_1 \rangle$  differs from its corresponding SM value we could conclude that the NP is most likely one of those listed immediately above. If *both*  $\langle P_{1,2} \rangle$  differ from their SM expectations while the  $\langle P_{n>2} \rangle$  remain zero the source can only be due to a spin-0 exchange in the  $s$ -channel. As we will see below only  $s$ -channel KK graviton exchange, since it is spin-2, leads to non-zero values of  $\langle P_{3,4} \rangle$  while the  $\langle P_{n>4} \rangle$  remain zero. Of course  $\langle P_{1,2} \rangle$  will also be different from their SM values in this case but as we have just observed this signal is *not* unique to gravity. This observation seems to yield a rather simple test for the exchange of graviton KK towers. It is important to note that we could not have performed this simple analysis for the case of Bhabha scattering, *i.e.*,  $e^+e^- \rightarrow e^+e^-$ , as it involves both  $s$ - and  $t$ -channel exchanges.

The real world is not so simple as the idealized case we have just discussed for several reasons. First, we have assumed that we know the cross section precisely at all values of  $z$ , *i.e.*, we have infinite statistics with no angular binning. Secondly, to use the orthonormality conditions we need to have complete angular coverage, *i.e.*, no holes for the beam pipe, *etc.* To get a feeling for how important these effects can be we consider dividing the distribution into a finite number of angular bins,  $N_{bins}$ , of common size  $\Delta z = 2/N_{bins}$ . The results of this analysis are shown in Table 1; here we see that as the number of bins grows large we rapidly recover the continuum results discussed above. Of course in any realistic experimental situation,  $N_{bins}$  remains finite but a value of order 20 is reasonable as it strikes a respectable balance between the

Table 1: Dependence on  $N_{bins}$  for the first four moments of the normalized cross section for  $e^+e^- \rightarrow f\bar{f}$  with  $m_f = 0$ . Both  $\langle P_{1,3} \rangle$  are in units of  $A_{FB}$ .

$N_{bins}$	$\langle P_2 \rangle (10^{-2})$	$\langle P_4 \rangle (10^{-3})$	$\langle P_1 \rangle$	$\langle P_3 \rangle (10^{-3})$
10	9.0040	-26.7585	0.66000	-23.1000
20	9.7503	-6.8285	0.66500	-5.8188
50	9.9600	-1.0988	0.66640	-0.9330
$\infty$	10.0	0.0	2/3	0.0

Table 2: Dependence on the cut at small scattering angles in milliradians assuming  $N_{bins} = 20$  for the first four moments of the normalized cross section as in the previous Table. Both  $\langle P_{1,3} \rangle$  are in units of  $A_{FB}$ .

Cut(mr)	$\langle P_2 \rangle (10^{-2})$	$\langle P_4 \rangle (10^{-3})$	$\langle P_1 \rangle$	$\langle P_3 \rangle (10^{-3})$
0	9.7503	-6.8285	0.66500	-5.8188
10	9.7428	-6.8981	0.66490	-5.9156
50	9.5652	-8.5590	0.66251	-8.2301
100	9.0159	-13.616	0.65508	-15.341

realistic demands of statistics and angular resolution. Now let us assume that  $N_{bins} = 20$  and examine the effects of the necessary cut at small angles due to the beam pipe, *etc.* This is straightforward and leads to the results shown in Table 2 for various values of the small angle cut. Here we observe that the ‘background contamination’ of the naive SM result increases quite rapidly as we make the angular cut stronger.

What this brief study indicates is that for a realistic detector the simple and naive expectations for the various moments will receive ‘backgrounds’ that will need to be dealt with and subtracted from the real data to obtain information on the  $\langle P_n \rangle$ . In the real world these backgrounds can be found through a detailed Monte Carlo study whose results will be influenced the detector geometry and by how well the properties of the detector are known. For our numerical analysis below we will follow a simpler approach by calculating the moments in the SM (after binning and cuts are applied) and then subtracting them from those obtained when the NP is present. Given the discussion above it is clear that we should begin by examining the process  $e^+e^- \rightarrow f\bar{f}$ . To be specific we concentrate on the model of Arkani-Hamed *et al.*, ADD, though our results are easily extended to the Randall-Sundrum model below graviton production threshold. The differential cross section for  $e^+e^- \rightarrow f\bar{f}$ , now

including graviton tower exchange, for massless fermions can be written as

$$\begin{aligned} \frac{d\sigma}{dz} = & N_c \frac{\pi\alpha^2}{s} \left\{ \tilde{P}_{ij} \left[ A_{ij}^e A_{ij}^f (2P_0 + P_2)/3 + 2B_{ij}^e B_{ij}^f P_1 \right] \right. \\ & - \frac{\lambda s^2}{2\pi\alpha\Lambda_H^4} \tilde{P}_i \left[ v_i^e v_i^f (2P_3 + 3P_1)/5 + a_i^e a_i^f P_2 \right] \\ & \left. + \frac{\lambda^2 s^4}{16\pi^2\alpha^2\Lambda_H^8} [(16P_4 + 5P_2 + 14P_0)/35] \right\}, \end{aligned} \quad (1)$$

where the indices  $i, j$  are sum over the  $\gamma$  and  $Z$  exchanges,  $\tilde{P}_{ij}$  and  $\tilde{P}_i$  are the usual dimensionless propagator factors,  $A_{ij}^f = (v_i^f v_j^f + a_i^f a_j^f)$ ,  $B_{ij}^f = (v_i^f a_j^f + v_j^f a_i^f)$ , and  $N_c$  represents the number of final state colors.  $\Lambda_H$  is the cutoff scale employed by Hewett in evaluating the summation over the tower of KK propagators and  $\lambda = 1$  will be assumed. Our results will not depend upon this choice of sign. This cross section has an explicit dependence on the  $P_{n>2}$  associated with the exchange of the graviton tower. Terms proportional to  $P_3$  occur in the interference between the SM and gravitational contributions whereas the term proportional to  $P_4$  occurs only in the pure gravity piece. This implies that for  $\sqrt{s} \ll \Lambda_H$  it will be  $\langle P_3 \rangle$  which will show the largest shifts from the expectations of the SM. With the polarized beams that we expect to have available at a linear collider, a  $z$ -dependent Left-Right Asymmetry,  $A_{LR}$ , can also be formed which provides an additional observable.

Our approach will be as follows: we consider two observables (*i*) the normalized unpolarized cross section and (*ii*) the normalized difference of the polarized cross sections  $\sim (d\sigma_L - d\sigma_R)/dz$ , which is essentially given by  $A_{LR}$ . We then calculate the first four non-trivial moments of these two observables for the  $\mu, \tau, b, c$  and  $t$  final states within the SM including the effects of Initial State Radiation (ISR). (Note that for the  $t\bar{t}$  final state we need to include finite mass effects.) Here we will assume tagging efficiencies of 100%, 100%, 80%, 60% and 60%, respectively, for the various final states and that  $N_{bins} = 20$  with  $\theta_{cut} = 50\text{mr}$ . The resulting values for the  $\langle P_n \rangle$  as calculated in the SM will be called ‘background’ values consistent with our discussion above. Next, we calculate the same moments in the ADD model by choosing a value for the parameter  $\Lambda_H$ . Combining both observables and summing over the various flavor final states we can form a  $\chi^2$  from the deviation of the  $\langle P_{3,4} \rangle$  moments from their SM ‘background’ values. For a fixed integrated luminosity this can be done using the statistical errors as well as the systematic errors associated with the precision expected on the luminosity and polarization measurements. (Here we will assume the values  $\delta L/L = 0.25\%$  and  $\delta P/P = 0.3\%$ .) Next we vary the value of the scale  $\Lambda_H$  until we obtain a  $5\sigma$  deviation from the SM; we

call this value of  $\Lambda_H$  the Identification Reach as it is the maximum value for the scale at which we observe a  $5\sigma$  deviation from the SM values of  $\langle P_{3,4} \rangle$  which we now know can only arise due to the effects of graviton exchange. Note that this value of the scale should *not* be confused with the Discovery Reach at which one observes an overall deviation from the SM. Although both the  $\langle P_{1,2} \rangle$  *also* deviate from their SM values these shifts cannot be directly attributed to a spin-2 exchange. (As noted previously, the shift in  $\langle P_1 \rangle$  results for any of the NP models listed above whereas a shift in  $\langle P_2 \rangle$  occurs whenever the new  $s$ -channel exchange is *not* spin-1, *e.g.*,  $\tilde{\nu}$  exchange.) The results of these calculations are shown in Fig.1 for several values of  $\sqrt{s}$  as a function of the integrated luminosity. Specifically, for a  $\sqrt{s} = 500$  GeV machine with an integrated luminosity of  $1 \text{ ab}^{-1}$  the ID reach with single(double) beam polarization is found to be 2.6(3.0) TeV, *i.e.*,  $(5 - 6)\sqrt{s}$ . We remind the reader that the corresponding search reach for these luminosities is in range of  $(9 - 10)\sqrt{s}$ . Note that the ID reach we obtained by is a rather respectable fraction of the corresponding search reach.

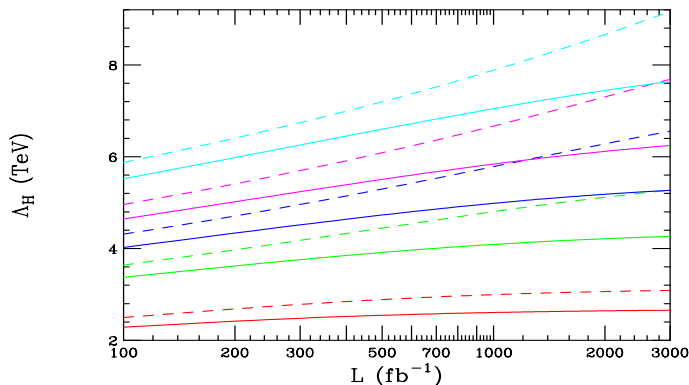


Figure 1: Identification reach in  $\Lambda_H$  as a function of the integrated luminosity for the process  $e^+e^- \rightarrow f\bar{f}$ , with  $f$  summed over  $\mu, \tau, b, c$  and  $t$ . The solid(dashed) curves are for an  $e^-$  polarization of 80%(together with a  $e^+$  polarization of 60%). From bottom to top the pairs of curves are for  $\sqrt{s} = 0.5, 0.8, 1, 1.2$  and  $1.5$  TeV, respectively.

Other SM processes with large tree-level cross sections in which gravitons can be exchanged are  $e^+e^- \rightarrow e^+e^-, \gamma\gamma, ZZ$ , and  $W^+W^-$  all of which involve  $t$ - and/or  $u$ -channel exchanges. This would apparently disqualify them from further consideration. The  $W^+W^-$  case is, however, special because the  $t$ -channel  $\nu$  exchange can be removed through the use of right-handed beam polarization. The remaining purely right-handed SM cross section is

only quadratic in  $z$  and this will not change if, *e.g.*,  $Z'$  or new  $s$ -channel scalar contributions are also present. The difficulty in this case is that the left-handed cross section is much larger than the right-handed one so that the possibility of ‘contamination’ from the wrong polarization state is difficult to eradicate unless very good control over the beam polarization is maintained. Apart from the problem of isolating the purely right-handed part of the cross section we might conclude that the non-zero  $\langle P_{3,4} \rangle$  moments will be a unique signature of graviton exchange for this process. Furthermore, since the pure gauge sector of the SM individually conserves  $C$  and  $P$ , there are no terms in the cross section linear in  $z$  and thus  $\langle P_1 \rangle$  is expected to be zero in the SM and in many NP extensions. Such terms are, however, generated by KK graviton exchange so that a non-zero value of  $\langle P_1 \rangle$  is also a gravity probe.

Apart from new particle exchanges there is another source of NP that can modify the right-handed  $W$ -pair cross section in a manner similar to gravity: anomalous gauge couplings (AGC). As is well known AGC can be  $C$  and/or  $P$  violating; one that violates both  $C$  and  $P$  but is  $CP$  conserving can produce non-zero  $\langle P_{1,3,4} \rangle$  moments. Decomposing the  $WWV$  ( $V = \gamma, Z$ ) vertex in the most general way yields 7 different anomalous couplings for each  $V$  with the corresponding form factors denoted by  $f_i^V$ . (When weighted by the sum over the  $\gamma$  and  $Z$  propagators in  $e^+e^- \rightarrow W^+W^-$  these form factors will be denoted by  $F_i$ , only two of which,  $F_{1,3}$ , are non-zero at the tree-level in the SM.) There is a single term in this general vertex expression with the correct  $C$  and  $P$  properties: that proportional to  $f_5^V \epsilon^{\mu\alpha\beta\rho} (q^- - q^+)_{\rho} \epsilon_{\alpha}(W^-) \epsilon_{\beta}(W^+)$  with  $q^{\pm}$  the outgoing  $W^{\pm}$  boson momenta, the  $\epsilon$ 's being their corresponding polarizations and  $f_5^V$  being the relevant form factor. As we will see such a term will generate non-zero values for all of  $\langle P_{1,3,4} \rangle$ . It appears that the possibility of non-zero AGC would contaminate our search for unique graviton exchange signatures. There is a way out of this dilemma; while gravity induces non-zero values for the  $\langle P_{1,3,4} \rangle$  from the angular distributions for  $e^+e^- \rightarrow W^+W^-$  *independently* of the final state  $W$  polarizations, the  $f_5^V \neq 0$  (*i.e.*,  $F_5$ ) couplings only contribute to the final state with mixed polarizations, *i.e.*, transverse plus longitudinal,  $TL + LT$ . We recall that by measuring the angular distribution of the decaying  $W$  relative to its direction of motion we can determine its state of polarization. Writing  $d\sigma_R/dz \sim \Sigma_{TT} + \Sigma_{LL} + \Sigma_{TL+LT}$ , one finds that in the SM both the  $TT$  and  $LL$  terms are proportional to  $1 - z^2$  while the  $TL + LT$  term is proportional to  $1 + z^2$ ; no terms linear in  $z$  are present. A non-zero  $F_5$  induces additional terms in the case of the  $TL + LT$  final state which now contains linear, cubic and quartic powers of  $z$  similar to that generated by gravity. However, the  $TT$  and  $LL$  final states receive no such contributions.

Thus observing non-zero values of  $\langle P_{1,3,4} \rangle$  (again, above backgrounds) for  $W$  pairs in the  $TT + LL$  final states produced by right-handed electrons *is* a signal for KK graviton exchange.

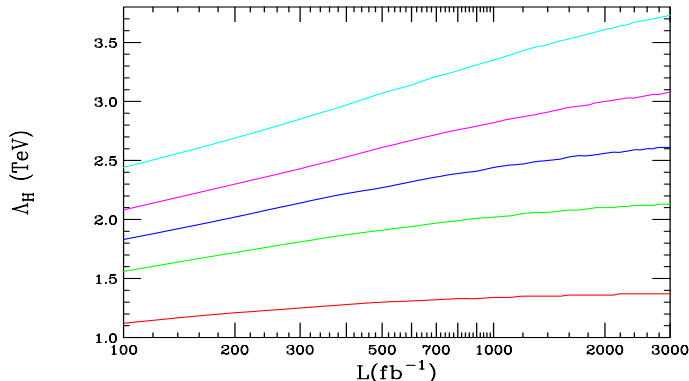


Figure 2: Same as in the previous figure but now for the  $W^+W^-$  final state using the combined analysis discussed in the text.

Unfortunately we can never obtain purely RH beams at a LC. The only possibility is to measure the  $W^+W^-$  cross section with two or more sets of different beam polarizations and then attempt to extract the purely right-handed piece from these measurements, again keeping only the  $TT + LL$  contributions. In the case of two polarized beams this is perhaps best demonstrated by examining what happens when we combine two sets of data: one with  $P(e^-) = -80\%$  and  $P(e^+) = 60\%$  and the other with both polarizations flipped. In comparison to the purely right-handed case, here we suffer from having to be able to very precisely subtract the additional large backgrounds arising from the left-handed parts of the cross section. In addition, both reduced statistics (since the luminosity is divided between both measurements) and the systematic errors associated with the polarization uncertainties will lead to further reductions in the anticipated identification reach. Fig.2 shows the results of this analysis. Here we see that the identification reach at large luminosities saturates due to the size of the systematic errors in extracting the right-handed piece of the cross section. The  $5\sigma$  identification reach is found to be roughly  $\sim 2.5\sqrt{s}$  for integrated luminosities of order  $1 ab^{-1}$  which is far below that found for fermion pairs. It is unlikely that a more judicious choice of beam polarizations could drastically increase this reach.

Many new physics scenarios predict the existence of contact interaction-like deviations from SM cross sections at high energy  $e^+e^-$  colliders. We have

found a technique that yields  $5\sigma$  ID reaches for graviton exchange of  $6(2.5)\sqrt{s}$  for the processes  $e^+e^- \rightarrow f\bar{f}(W^+W^-)$ .

### References

1. For details of the analysis and original references, see T.G. Rizzo, hep-ph/0208027.