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Leptogenesis Within A Predictive G(224)/SO(10)-Framework

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Abstract

A G(224)/SO(10)-framework has been proposed (a few years ago) that successfully describes the masses and mixings of all fermions including neutrinos. Baryogenesis via leptogenesis is considered within this framework by allowing for natural phases ($\sim$ 1/30-1/2) in the entries of the Dirac and Majorana mass-matrices. It is shown that the framework leads quite naturally to the desired magnitude for the baryon asymmetry, in full accord with the observed features of atmospheric and solar neutrino oscillations, as well as with those of quark and charged lepton masses and mixings. Hereby one obtains a *unified description* of fermion masses, neutrino oscillations and baryogenesis within a single predictive framework.

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1 Introduction

The observed matter-antimatter asymmetry of the universe [1] is an important clue to physics at truly short distances. A natural understanding of its magnitude (not to mention its sign) is thus a worthy challenge. Since the discovery of the electroweak sphaleron effect [2], baryogenesis via leptogenesis [3, 4] appears to be the most attractive and promising mechanism to generate such an asymmetry. In the context of a unified theory of quarks and leptons, leptogenesis involving decays of heavy right-handed (RH) neutrinos, is naturally linked to the masses of quarks and leptons, neutrino oscillations and, of course, CP violation.

In this regard, the route to higher unification based on an effective four-dimensional gauge symmetry of either G(224)=$SU(2)_L \times SU(2)_R \times SU(4)^C$ [5], or SO(10) [6] (that may emerge from a string theory near the string scale and breaks spontaneously to the standard model symmetry near the GUT scale [7]) offers some distinct advantages, which are directly relevant to leptogenesis. These in particular include: (a) the existence of the RH neutrinos as a compelling feature, (b) B-L as a local symmetry, and (c) quark-lepton unification through SU(4)-Color. These three features, first introduced in Ref. [5], are common to both G(224) and SO(10), though not to SU(5) [8] and $[SU(3)]^3$ [9]. They, together with the seesaw mechanism [10] and the supersymmetric unification-scale [11], help explain even quantitatively [12] the scale of ν_τ-mass [or rather of $\Delta m^2(\nu_\mu\text{-}\nu_\tau)$] as observed at SuperKamiokande [13]. Furthermore, these three features also provide just the needed ingredients - that is superheavy ν_R's and spontaneous violation of B-L at high temperatures - for implementing baryogenesis via leptogenesis.

Now, in a theory with RH neutrinos having heavy Majorana masses, the magnitude of the lepton-asymmetry is known to depend crucially on both the Dirac as well as Majorana mass matrices of the neutrinos [14]. In this regard, a predictive SO (10)/G(224) framework, describing the masses and mixings of all fermions, including neutrinos, has been proposed [15] that appears to be remarkably successful. In particular it makes seven predictions including: $m_b(m_b) \approx 4.9$ GeV, $m(\nu_L^\tau) \sim (1/20)$ eV(1/2-2), $V_{cb} \approx 0.044$, $\sin^2 2\theta^{\rm osc}_{\nu_\mu\nu_\tau} \approx 0.9$-0.99, $V_{us} \approx 0.20$-0.23, $V_{ub} \approx 0.0025$-0.0032 and $m_d \approx 8$ MeV, all in good accord with observations, to within 10% (see Sec. 2). It has been noted recently [16] that the large angle MSW solution (LMA), which is preferred by experiments [17], can arise quite plausibly within the same framework through SO(10)-invariant higher dimensional operators which can contribute directly to the Majorana masses of the left-handed neutrinos (especially to the $\nu_L^e \nu_L^\mu$ mixing mass) without involving the familiar seesaw.

As an additional point, it has been noted by Babu and myself [18] that the framework proposed in Ref. [15] can naturally accomodate CP violation by introducing complex phases in the entries of the fermion mass-matrices, which preserve the pattern of the mass-matrices suggested in Ref. [15] as well as its successes.

The purpose of the present paper is to estimate the lepton and thereby the baryon excess that would typically be expected within this realistic G(224)/SO(10)-framework for fermion masses and mixings [15, 18], by allowing for natural CP violating phases ($\sim 1/30$ to $1/2$, say) in the entries of the mass-matrices as in Ref. [18]. The goal would thus be to obtain a *unified description* of (a) fermion masses, (b) neutrino oscillations, and (c) leptogenesis within a single predictive framework [19].

It should be noted that there have in fact been several attempts in the literature [20] at estimating the lepton and baryon asymmetries, many of which have actually been carried out in the context of SO(10) [21], though (to my knowledge) without an accompanying realistic framework for the masses and mixing of quarks, charged leptons as well as neutrinos [22]. Also the results in these attempts as regards leptogenesis have not been uniformly encouraging [23].

The purpose of this letter is to note that the G(224)/SO(10) framework, proposed in Ref. [15] and [18], leads quite naturally to the desired magnitude for baryon asymmetry, in full accord with the observed features of atmospheric and solar neutrino oscillations, as well as with those of quark and charged lepton masses and mixings. To present the analysis it would be useful to recall the salient features of prior works [15, 18]. This is what is done in the next section.

2 Fermion Masses and Neutrino Oscillations in G(224)/SO(10): A Brief Review of Prior Work

The 3×3 Dirac mass matrices for the four sectors (u, d, l, ν) proposed in Ref. [15] were motivated in part by the notion that flavor symmetries [24] are responsible for the hierarchy among the elements of these matrices (i.e., for "33"$\gg$"23"$\gg$"22"$\gg$"12"$\gg$"11", etc.), and in part by the group theory of SO(10)/G(224), relevant to a minimal Higgs system (see below). Up to minor variants [25], they are as follows:

$$M_u = \begin{bmatrix} 0 & \epsilon' & 0 \\ -\epsilon' & \zeta_{22}^u & \sigma+\epsilon \\ 0 & \sigma-\epsilon & 1 \end{bmatrix} \mathcal{M}_u^0; \qquad M_d = \begin{bmatrix} 0 & \eta'+\epsilon' & 0 \\ \eta'-\epsilon' & \zeta_{22}^d & \eta+\epsilon \\ 0 & \eta-\epsilon & 1 \end{bmatrix} \mathcal{M}_d^0$$

$$M_\nu^D = \begin{bmatrix} 0 & -3\epsilon' & 0 \\ -3\epsilon' & \zeta_{22}^u & \sigma-3\epsilon \\ 0 & \sigma+3\epsilon & 1 \end{bmatrix} \mathcal{M}_u^0; \quad M_l = \begin{bmatrix} 0 & \eta'-3\epsilon' & 0 \\ \eta'+3\epsilon' & \zeta_{22}^d & \eta-3\epsilon \\ 0 & \eta+3\epsilon & 1 \end{bmatrix} \mathcal{M}_d^0 \tag{1}$$

These matrices are defined in the gauge basis and are multiplied by $\bar{\Psi}_L$ on left and Ψ_R on right. Note the group-theoretic up-down and quark-lepton correlations: the same σ occurs in M_u and M_ν^D, and the same η occurs in M_d and M_l. It will become clear that the ϵ and ϵ' entries are proportional to B-L and are antisymmetric in the family space (as shown above). Thus, the same ϵ and ϵ' occur in both (M_u and M_d) and also in (M_ν^D and M_l), but $\epsilon \to -3\epsilon$ and $\epsilon' \to -3\epsilon'$ as $q \to l$. Such correlations result in enormous reduction of parameters and thus in increased predictivity. Such a patern for the mass-matrices can be obtained, using a minimal Higgs system $\mathbf{45}_H, \mathbf{16}_H, \overline{\mathbf{16}}_H$ and $\mathbf{10}_H$ and a singlet S of SO(10), through effective couplings as follows [26]:

$$\begin{aligned}
\mathcal{L}_{\text{Yuk}} &= h_{33}\mathbf{16}_3\mathbf{16}_3\mathbf{10}_H \\
&+ \left[h_{23}\mathbf{16}_2\mathbf{16}_3\mathbf{10}_H(S/M) + a_{23}\mathbf{16}_2\mathbf{16}_3\mathbf{10}_H(\mathbf{45}_H/M')(S/M)^p + g_{23}\mathbf{16}_2\mathbf{16}_3\mathbf{16}_H^d(\mathbf{16}_H/M'')(S/M)^q\right] \\
&+ \left[h_{22}\mathbf{16}_2\mathbf{16}_2\mathbf{10}_H(S/M)^2 + g_{22}\mathbf{16}_2\mathbf{16}_2\mathbf{16}_H^d(\mathbf{16}_H/M'')(S/M)^{q+1}\right] \\
&+ \left[g_{12}\mathbf{16}_1\mathbf{16}_2\mathbf{16}_H^d(\mathbf{16}_H/M'')(S/M)^{q+2} + a_{12}\mathbf{16}_1\mathbf{16}_2\mathbf{10}_H(\mathbf{45}_H/M')(S/M)^{p+2}\right]
\end{aligned} \tag{2}$$

Typically we expect M', M'' and M to be of order $M_{\rm string}$ [27]. The VEV's of $\langle \mathbf{45}_H \rangle$ (along B-L), $\langle \mathbf{16}_H \rangle = \langle \overline{\mathbf{16}}_H \rangle$ (along standard model singlet sneutrino-like component) and of the SO(10)-singlet $\langle S \rangle$ are of the GUT-scale, while those of $\mathbf{10}_H$ and of the down type SU(2)$_L$-doublet component in $\mathbf{16}_H$ (denoted by $\mathbf{16}_H^d$) are of the electroweak scale [15,28]. Depending upon whether $M'(M'') \sim M_{\rm GUT}$ or $M_{\rm string}$ (see footnote [27]), the exponent $p(q)$ is either one or zero [29].

The entries 1 and σ arise respectively from h_{33} and h_{23} couplings, while $\hat{\eta} \equiv \eta - \sigma$ and η' arise respectively from g_{23} and g_{12}-couplings. The (B-L)-dependent antisymmetric entries ϵ and ϵ' arise respectively from the a_{23} and a_{12} couplings. [Effectively, with $\langle \mathbf{45}_H \rangle \propto$ B-L, the product $\mathbf{10}_H \times \mathbf{45}_H$ contributes as a $\mathbf{120}$, whose coupling is family-antisymmetric.] The small entry ζ_{22}^u arises from the h_{22}-coupling, while ζ_{22}^d arises from the joint contributions of h_{22} and g_{22}-couplings. As discussed in [15], using some of the observed masses as inputs, one obtains $|\hat{\eta}| \sim |\sigma| \sim |\epsilon| \sim \mathcal{O}(1/10)$, $|\eta'| \approx 4 \times 10^{-3}$ and $|\epsilon'| \sim 2 \times 10^{-4}$. The success of the framework presented in Ref. [15] (which set $\zeta_{22}^u = \zeta_{22}^d = 0$) in describing fermion masses and mixings remains essentially unaltered if $|(\zeta_{22}^u, \zeta_{22}^d)| \leq (1/3)(10^{-2})$ (say).

Such a hierarchical form of the mass-matrices, with h_{33}-term being dominant, is attributed in part to flavor gauge symmetry(ies) that distinguishes between the three families [30], and in part to higher dimensional operators involving for example $\langle \mathbf{45}_H \rangle / M'$ or $\langle \mathbf{16}_H \rangle / M''$, which are supressed by $M_{\rm GUT}/M_{\rm string} \sim 1/10$, if M' and/or $M'' \sim M_{\rm string}$.

To discuss the neutrino sector one must specify the Majorana mass-matrix of the RH neutrinos as well. These arise from the effective couplings of the form [31]:

$$\mathcal{L}_{\rm Maj} = f_{ij} \mathbf{16}_i \mathbf{16}_j \overline{\mathbf{16}}_H \overline{\mathbf{16}}_H / M \tag{3}$$

where the f_{ij}'s include appropriate powers of $\langle S \rangle / M$, in accord with flavor charge assignments of $\mathbf{16}_i$ (see [30]). For the f_{33}-term to be leading, we must assign the charge $-a$ to $\overline{\mathbf{16}}_H$. This leads to a hierarchical form for the Majorana mass-matrix [15]:

$$M_R^\nu = \begin{bmatrix} x & 0 & z \\ 0 & 0 & y \\ z & y & 1 \end{bmatrix} M_R \tag{4}$$

Following the flavor-charge assignments given in footnote [30], we expect $|y| \sim \langle S/M \rangle \sim 1/10$, $|z| \sim (\langle S/M \rangle)^2 \sim 10^{-2}(1 \text{ to } 1/2, \text{say})$, $|x| \sim (\langle S/M \rangle)^4 \sim (10^{-4}\text{-}10^{-5})$ (say). The "22" element (not shown) is $\sim (\langle S/M \rangle)^2$ and its magnitude is taken to be $< |y^2/3|$, while the "12" element (not shown) is $\sim (\langle S/M \rangle)^3$.. We expect $M_R = f_{33} \langle \overline{\mathbf{16}}_H \rangle^2 / M_{\rm string} \approx (10^{15} \text{ GeV})(1/2\text{-}2)$ for $\langle \overline{\mathbf{16}}_H \rangle \approx 2 \times 10^{16}$ GeV, $M_{\rm string} \approx 4 \times 10^{17}$ GeV [32] and $f_{33} \approx 1$. Allowing for 2-3 mixing, this value of M_R [together with the SU(4)-color relation $m(\nu_\tau^{\rm Dirac}) = m_t(M_{\rm GUT}) \approx 110$ GeV] leads to $m(\nu_L^\tau) \approx (1/20 \text{ eV})(1/2\text{-}2)$ [12, 15], in good accord with the SuperK data.

Ignoring possible phases in the parameters and thus the source of CP violation for a moment, as was done in Ref. [15], the parameters ($\sigma, \eta, \epsilon, \epsilon', \eta', \mathcal{M}_u^0, \mathcal{M}_D^0$, and y) can be determined by using, for example, $m_t^{\rm phys} = 174$ GeV, $m_c(m_c) = 1.37$ GeV, $m_S(1 \text{ GeV}) = 110$-116 MeV, $m_u(1 \text{ GeV}) = 6$ MeV, the observed masses of e, μ, and τ and $m(\nu_L^\mu)/m(\nu_L^\tau) \approx 1/15\text{-}1/8$ (as suggested by a combination of atmospheric and solar neutrino data, including SMA and LMA solutions) as inputs. One is thus led, for this CP conserving case, to the following fit for the parameters, and the associated predictions [15]. [In this fit, we drop $|\zeta_{22}^{u,d}| \lesssim (1/3)(10^{-2})$ and leave the small quatities x and z in M_R^ν undetermined and proceed by assuming that they have the magnitudes suggested by flavor symmetries (i.e., $x \sim (10^{-4}\text{-}10^{-5})$ and $z \sim 10^{-2}(1 \text{ to } 1/2)$ (see remarks below Eq. (4))]:

$$\begin{aligned} &\sigma \approx 0.110, \quad \eta \approx 0.151, \quad \epsilon \approx -0.095, \quad |\eta'| \approx 4.4 \times 10^{-3}, \\ &\epsilon' \approx 2 \times 10^{-4}, \qquad \mathcal{M}_u^0 \approx m_t(M_X) \approx 110 \text{ GeV}, \\ &\mathcal{M}_D^0 \approx m_b(M_X) \approx 1.5 \text{ GeV}, \qquad y \approx -(1/20 \text{ to } 1/17). \end{aligned} \tag{5}$$

These in turn lead to the following predictions for the quarks and light neutrinos [15]:

$$
\begin{array}{l}
m_b(m_b) \approx (4.7\text{-}4.9)\ \mathrm{GeV}, \\
V_{cb} \approx \left| \sqrt{\frac{m_s}{m_b}} \left| \frac{\eta+\epsilon}{\eta-\epsilon} \right|^{1/2} - \sqrt{\frac{m_c}{m_t}} \left| \frac{\sigma+\epsilon}{\sigma-\epsilon} \right|^{1/2} \right| \approx 0.044, \\
\left\{ \begin{array}{l}
\theta^{\rm osc}_{\nu_\mu \nu_\tau} \approx \left| \sqrt{\frac{m_\mu}{m_\tau}} \left| \frac{\eta-3\epsilon}{\eta+3\epsilon} \right|^{1/2} + \sqrt{\frac{m_{\nu_\mu}}{m_{\nu_\tau}}} \right| \approx |0.437 + (0.258 - 0.353)|, \\
\text{Thus, } \sin^2 2\theta^{\rm osc}_{\nu_\mu \nu_\tau} \approx 0.92\text{-}0.99,\ (\text{for } m(\nu_\mu)/m(\nu_\tau) \approx 1/15\text{-}1/8),
\end{array} \right. \\
V_{us} \approx \left| \sqrt{\frac{m_d}{m_s}} - \sqrt{\frac{m_u}{m_c}} \right| \approx 0.20, \\
\left| \frac{V_{ub}}{V_{cb}} \right| \approx \sqrt{\frac{m_u}{m_c}} \approx 0.07, \\
m_d(1\ \mathrm{GeV}) \approx 8\ \mathrm{MeV}, m(\nu_L^\tau) \approx (1/20\ \mathrm{eV})(1/2\text{-}2), \\
\theta^{\rm osc}_{\nu_e \nu_\mu} \approx 0.06\ (\text{ignoring non-seesaw contributions}).
\end{array} \tag{6}
$$

The Majorana masses of the RH neutrinos ($N_{iR} \equiv N_i$) are given by:

$$
\begin{array}{rcl}
M_3 & \approx & M_R \approx 10^{15}\ \mathrm{GeV}\ (1/2\text{-}2), \\
M_2 & \approx & |y^2| M_3 \approx (2.5 \times 10^{12}\ \mathrm{GeV})(1/2\text{-}2), \\
M_1 & \approx & |x - z^2| M_3 \sim (1/2\text{-}2) 10^{-5} M_3 \sim 10^{10}\ \mathrm{GeV}(1/4\text{-}4).
\end{array} \tag{7}
$$

Leaving out the ν_e-ν_μ oscillation angle for a moment, it seems remarkable that the first seven predictions in Eq. (6) agree with observations, to within 10%. Particularly intriguing is the *group-theoretic correlation* between the contribution from the first term in V_{cb} and that in $\theta^{\rm osc}_{\nu_\mu \nu_\tau}$, which explains simultaneously why one is small (V_{cb}) and the other is large ($\theta^{\rm osc}_{\nu_\mu \nu_\tau}$). That in turn provides some degree of confidence in the gross structure of the mass-matrices.

As regards ν_e-ν_μ and ν_e-ν_τ oscillations, the standard seesaw mechanism would typically lead to rather small angles as in Eq. (6), within the framework presented above [15]. It has, however, been noted recently [16] that small intrinsic (non-seesaw) masses $\sim 10^{-3}$ eV of the LH neutrinos can arise quite plausibly through higher dimensional operators of the form [33]: $W_{12} \supset \kappa_{12} \mathbf{16}_1 \mathbf{16}_2 \mathbf{16}_H \mathbf{16}_H \mathbf{10}_H \mathbf{10}_H / M^3_{\rm eff}$, without involving the standard seesaw mechanism [10]. One can verify that such a term would lead to an intrinsic Majorana mixing mass term of the form $m^0_{12} \nu^e_L \nu^\mu_L$, with a strength given by $m^0_{12} \approx \kappa_{12} \langle \mathbf{16}_H \rangle^2 (175\ \mathrm{GeV})^2 / M^3_{\rm eff} \sim (1.5\text{-}6) \times 10^{-3}$ eV, for $\langle \mathbf{16}_H \rangle \approx (1\text{-}2) M_{\rm GUT}$ and $\kappa_{12} \sim 1$, if $M_{\rm eff} \sim M_{\rm GUT} \approx 2 \times 10^{16}$ GeV [34]. Such an intrinsic Majorana and $\nu_e \nu_\mu$ mixing mass $\sim$ few$\times 10^{-3}$ eV, though small compared to $m(\nu_L^\tau)$, is still much larger than what one would generically get for the corresponding term from the standard seesaw mechanism [as in Ref. [15]]. Now, the diagonal $(\nu_L^\mu \nu_L^\mu)$ mass-term, arising from standard seesaw can naturally be $\sim$ (3-8)$\times 10^{-3}$ eV for $|y| \approx 1/20$-$1/15$, say [15]. Thus, taking the net values of $m_{22} \approx$ (6-7) $\times 10^{-3}$ eV, $m_{12}^0 \sim 3 \times 10^{-3}$ eV as above and $m_{11}^0 \ll 10^{-3}$ eV (as in [15]), which are all plausible, we obtain $m_{\nu_\mu} \approx$ (6-7) $\times 10^{-3}$ eV, $m_{\nu_e} \sim$ (1 to few) $\times 10^{-3}$ eV, so that $\Delta m_{12}^2 \approx$ (3.6-5) $\times 10^{-5}$ eV^2 and $\sin^2 2\theta_{12}^{\rm osc} \approx$ 0.6-0.7. These go well with the LMA MSW solution of the solar neutrino puzzle.

In summary, the intrinsic non-seesaw contribution to the Majorana masses of the LH neutrinos can possibly have the right magnitude for ν_e-ν_μ mixing so as to lead to the LMA solution within the G(224)/SO(10)-framework, without upsetting the successes of the first seven predictions in Eq. (6). [In contrast to the near maximality of the ν_μ-ν_τ oscillation angle, however, which emerges as a compelling prediction of the framework [15], the LMA solution, as obtained above, should, be regarded only as a consistent possibility within this framework.]

Before discussing leptogenesis, we need to discuss the origin of CP violation within the G(224)/SO(10)-framework presented above. The discussion so far has ignored, for the sake of simplicity, possible CP violating phases in the parameters (σ, η, ϵ, η', ϵ', $\zeta_{22}^{u,d}$, y, z, and x) of the Dirac and Majorana mass matrices [Eqs. (1), and (4)]. In general, however, these parameters can and generically will have phases [35]. Some combinations of these phases enter into the CKM matrix and define the Wolfenstein parameters ρ_W and η_W [36], which in turn induce CP violation by utilizing the standard model interactions. As observed in Ref. [18], an additional and potentially important source of CP and flavor violations (as in $K^0 \leftrightarrow \bar{K}^0$, $B_{d,s} \leftrightarrow \bar{B}_{d,s}$, $b \rightarrow s\bar{s}s$, etc. transitions) arise in the model through supersymmetry [37], involving squark and gluino loops (box and penguin), simply because of the embedding of MSSM within a string-unified G(224) or SO(10)-theory near the GUT-scale, and the assumption that primordial SUSY-breaking occurs near the string scale ($M_{\rm string} > M_{\rm GUT}$) [38]. It is shown that complexification of the parameters (σ, η, ϵ, η', ϵ', etc.), through introduction of phases $\sim 1/30$-$1/2$ (say) in them, still preserves the successes of the predictions as regards fermion masses and neutrino oscillations shown in Eq. (6), as long as one maintains nearly the magnitudes of the real parts of the parameters and especially their relative signs as obtained in Ref. [15] and shown in Eq. (5) [39]. Such a picture is also in accord with the observed features of CP and flavor violations in ϵ_K, Δm_{Bd}, and asymmetry parameter in $B_d \to J/\Psi + K_s$, while predicting observable new effects in processes such as $B_s \to \bar{B}_s$ and $B_d \to \Phi + K_s$ [18].

We therefore proceed to discuss leptogenesis concretely within the framework presented above by adopting the Dirac and Majorana fermion mass matrices as shown in Eqs. (1) and (4) and assuming that the parameters appearing in these matrices can have natural phases $\sim 1/30$-$1/2$ (say) with either sign up to addition of $\pm\pi$, *while their real parts have the relative signs and nearly the magnitudes given in Eq.* (6).

3 Leptogenesis

In the context of an inflationary scenario [40] with a reheat temperature $T_{RH} \sim$ (1 to few)10^9 GeV (say), one can avoid the well known gravitino problem if $m_{3/2} \sim$ (1 to 2) TeV [41] and yet produce the lightest heavy neutrino N_1 efficiently from the thermal bath for $M_1 \sim$ (3 to 5) $\times\, 10^9$ GeV [see Eq. (7)]. Given lepton number violation (through the Majorana mass of N_1) and CP violating phases in the fermion mass-matrices as mentioned above, the out-of-equilibrium decays of N_1 (produced from the thermal bath) into $l + \Phi_H$ and $\bar{l} + \bar{\Phi}_H$ systems would produce a lepton asymmetry. We will assume this commonly adopted scenario to discuss leptogenesis. (We will comment later, however, on an interesting alternative possibility proposed in Ref. [42].) The lepton asymmetry of the universe $[Y_L \equiv (n_L - n_{\bar{L}})/s]$ arising from decays of N_1 into $(l + \Phi_H)$ and $(\bar{l} + \bar{\Phi}_H)$ is given by:

$$Y_L = d\epsilon_1/g^* \qquad (8)$$

where ϵ_1 is the lepton-asymmetry produced per N_1 decay (see below), d is a dilution factor that represents washout effects due to inverse decay and lepton number violating scattering, and $g^* \approx 228$ is the number of light degrees of freedom for MSSM.

The lepton asymmetry Y_L is converted to baryon asymmetry, by the sphaleron effects, which is given by:

$$Y_B = \frac{n_B - n_{\bar{B}}}{s} \approx C\,Y_L, \tag{9}$$

where, for MSSM, $C = -8/15 \approx -1/2$. Taking into account the inteference between the tree and loop-diagrams which induce $N_1 \to (l + \Phi_H)$ and $N_1 \to (\bar{l} + \bar{\Phi}_H)$-decays, the lepton-asymmetry parameter ϵ_1 is given by [14, 43]

$$\epsilon_1 = \frac{1}{8\pi v^2 (M_D^\dagger M_D)_{11}} \sum_{j=2,3} \mathrm{Im}\left[(M_D^\dagger M_D)_{j1}\right]^2 f(M_j^2/M_1^2) \tag{10}$$

where M_D is the Dirac neutrino mass matrix evaluated in a basis in which the Majorana mass matrix of the RH neutrinos M_R^ν [see Eq. (4)] is diagonal, $v = (174\text{ GeV})\sin\beta$ and the function $f \approx -3(M_1/M_j)$ for the case of SUSY with $M_j \gg M_1$. The dilution factor appearing in Eq. (8) is obtained by solving Boltzmann equations and is approximately given by [40, 44]:

$$d \approx \begin{cases} \frac{0.3}{k(\ln k)^{0.6}} & (10 \lesssim k \lesssim 10^6) \\ \frac{1}{2k} & (1 \lesssim k \lesssim 10) \\ 1 & (0 \lesssim k \lesssim 1) \end{cases} \tag{11}$$

where $k \equiv [\Gamma(N_1)/(2H)]_{T=M_1}$ is given by:

$$k = \frac{M_{P1}}{1.66\sqrt{g^*}(8\pi v^2)} \frac{(M_D^\dagger M_D)_{11}}{M_1}. \tag{12}$$

Here M_{P1} = Planck mass $\approx 1.2 \times 10^{19}$ GeV.

Given the Dirac and Majorana mass matrices of the neutrinos [Eqs. (1) and (4)], we are now ready to evaluate lepton assymetry by using Eqs. (8)-(12).

The Majorana mass matrix [Eq. (4)] describing the mass-term $\nu_R^T C M_R^\nu \nu_R$ is diagonalized by the transformation $\nu_R = U_R^{(1)} U_R^{(2)} N_R$, where (to a good approximation)

$$U_R^{(1)} \approx \begin{bmatrix} 1 & 0 & z \\ 0 & 1 & y \\ -z & -y & 1 \end{bmatrix}, \tag{13}$$

and $U_R^{(2)} = \mathrm{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3})$ is a diagonal phase matrix that ensures real positive eigenvalues. The phases ϕ_i can of course be derived from those of the parameters in M_R^ν [see Eq.

(4)]. Applying this transformation to the neutrino Dirac mass-term $\bar{\nu}_L M_\nu^D \nu_R$ given by Eq. (1), we obtain $M_D = M_\nu^D U_R^{(1)} U_R^{(2)}$, which appears in Eqs. (10) and (12). In turn, this yields:

$$\begin{aligned}
\frac{(M_D^\dagger M_D)_{21}}{(\mathcal{M}_u^0)^2} &= e^{i(\phi_1-\phi_2)}\{\left(-3\epsilon'^* - \zeta_{13}^* y^*\right)(\zeta_{11} - z\zeta_{13}) \\
&+ \left[\zeta_{22}^{u*} - y^*(\sigma^* - 3\epsilon*)\right]\left[3\epsilon' - z(\sigma - 3\epsilon)\right] + (\zeta_{31} - z)\left[(\sigma^* + 3\epsilon^*) - y^*\right]\} \quad (14) \\
\frac{(M_D^\dagger M_D)_{11}}{(\mathcal{M}_u^0)^2} &= |3\epsilon' - z(\sigma - 3\epsilon)|^2 + |\zeta_{31} - z|^2 \quad (15)
\end{aligned}$$

In writing Eqs. (14) and (15), we have allowed, for the sake of generality, the relatively small "11", "13", and "31" elements in the Dirac mass-matrix M_ν^D, denoted by ζ_{11}, ζ_{13} and ζ_{31} respectively, which are not exibited in Eq. (4). Guided by considerations of flavor symmetry (see footnote [30]), we would expect $|\zeta_{11}| \sim (\langle S\rangle/M)^4 \sim 10^{-4}$-$10^{-5}$, and $|\zeta_{13}| \sim |\zeta_{31}| \sim (\langle S\rangle/M)^2 \sim 10^{-2}(1 \text{ to } 1/3)$ (say). These small elements (neglected in [15]) would not, of course, have any noticeable effects on the predictions of the fermion masses and mixings given in Eq. (6), except possibly on m_d.

We now proceed to make numerical estimates of lepton and baryon-asymmetries by taking the magnitudes and the relative signs of the real parts of the parameters (σ, η, ϵ, η', ϵ', and y) approximately the same as in Eq. (5), but allowing in general for natural phases in them. As mentioned before [see for example the fit given in footnote [39] and Ref. [18] (to appear)] such a procedure introduces CP violation in accord with observation, while preserving the successes of the framework as regards its predictions for fermion masses and neutrino oscillations [15, 18].

Given the magnitudes of the parameters (see Eqs. (5) and Ref. [39]), which are obtained from considerations of fermion masses and neutrino oscillations [15, 18] – that is $|\sigma| \approx |\epsilon| \approx 0.1$, $|y| \approx 0.06$, $|\epsilon'| \approx 2 \times 10^{-4}$, $|z| \sim (1/200)(1 \text{ to } 1/2)$, $|\zeta_{22}^u| \sim 10^{-3}(1 \text{ to } 3)$, $|\zeta_{13}| \sim |\zeta_{31}| \sim (1/200)(1 \text{ to } 1/2)$, with the real parts of (σ, ϵ and y) having the signs (+, -, -) respectively, we would expect the typical magnitudes of the three terms of Eq. (14) to be as follows:

$$\begin{aligned}
|1^{st}\ \text{Term}| &= \left|\left(-3\epsilon'^{*} - \zeta_{13}^{*} y^{*}\right)\left(\zeta_{11} - z\zeta_{13}\right)\right| \\
&\approx \left[(6 \text{ to } 8) \times 10^{-4}\right]\left[(2.5 \times 10^{-5})(1 \text{ to } 1/4)\right] \sim 10^{-8} \\
|2^{nd}\ \text{Term}| &= \left|\left\{\zeta_{22}^{u*} - y^{*}\left(\sigma^{*} - 3\epsilon^{*}\right)\right\}\left\{3\epsilon' - z(\sigma - 3\epsilon)\right\}\right| \\
&\approx \left(2 \times 10^{-2}\right)\left[2 \times 10^{-3}(1 \text{ to } 1/2)\right] \approx \left(4 \times 10^{-5}\right)(1 \text{ to } 1/2) \\
|3^{rd}\ \text{Term}| &= \left|\left(\zeta_{31} - z\right)\left\{\left(\sigma^{*} + 3\epsilon^{*}\right) - y^{*}\right\}\right| \\
&\approx [(1/200)(1 \text{ to } 1/3)](0.14) \approx \left(0.7 \times 10^{-3}\right)(1 \text{ to } 1/3)
\end{aligned} \tag{16}$$

Thus, assuming that the phases of the different terms are roughly comparable, the third term would clearly dominate. The RHS of Eq. (15) is similarly estimated to be:

$$\begin{aligned}
\frac{\left(M_D^{\dagger} M_D\right)_{11}}{(\mathcal{M}_u^0)^2} &= \left|3\epsilon' - z(\sigma - 3\epsilon)\right|^2 + \left|\zeta_{31} - z\right|^2 \\
&\approx \left|6 \times 10^{-4} \mp 2 \times 10^{-3}(1 \text{ to } 1/2)\right|^2 + \left|5 \times 10^{-3}(1 \text{ to } 1/3)\right|^2 \\
&\approx 2.5 \times 10^{-5}(1 \text{ to } 1/9)
\end{aligned} \tag{17}$$

Since $|\zeta_{31}|$ and $|z|$ are each expected to be of order (1/200)(1 to 1/2), we have allowed for a possible mild cancellation between their contributions to $|\zeta_{31} - z|$ by putting $|\zeta_{31} - z| \approx$ (1/200)(1 to 1/3) (say). Note that this combination enters into the dominant terms of both $(M_D^{\dagger} M_D)_{21}/(\mathcal{M}_u^0)^2$ [see the third term in Eq. (16)] and $(M_D^{\dagger} M_D)_{11}/(\mathcal{M}_u^0)^2$ [see the second term in Eq. (17)]. As a result, to a good approximation, the lepton-asymmetry parameter ϵ_1 [given by Eq. (10)] becomes independent of the magnitude of $|\zeta_{31} - z|^2$ and thereby of the uncertainty in it. It is given by:

$$\epsilon_1 \approx \frac{1}{8\pi}\left(\frac{\mathcal{M}_u^0}{v}\right)^2 |(\sigma + 3\epsilon) - y|^2 \sin\left(2\phi_{21}\right)(-3)\left(\frac{M_1}{M_2}\right) \approx -\left(2.4 \times 10^{-6}\right)\sin\left(2\phi_{21}\right), \tag{18}$$

where, $\phi_{21} = \arg[(\zeta_{31} - z)(\sigma^{*} + 3\epsilon^{*} - y^{*})] + (\phi_1 - \phi_2)$, and we have put $(\mathcal{M}_u^0/v)^2 \approx 1/2$, $|\sigma + 3\epsilon - y| \approx 0.14$ (see Eq. (5) and Ref. [39]), and for concreteness $M_1/M_2 \approx (4 \times 10^9\ \text{GeV})/(2 \times 10^{12}\ \text{GeV}) \approx 2 \times 10^{-3}$ [see Eq. (7)]. The parameter k, given by Eq. (12), is (approximately) proportional to $|\zeta_{31} - z|^2$ [see Eqs. (16) and (17)]. It is given by:

$$k \approx \frac{(M_{Pl}/M_1)}{1.66\sqrt{g^{*}}(8\pi)}\left(\frac{\mathcal{M}_u^0}{v}\right)^2 \left|\zeta_{31} - z\right|^2 \approx 60(1 \text{ to } 1/9), \tag{19}$$

where, as before, we have put $M_1 = 4 \times 10^9$ GeV and $|\zeta_{31} - z| \approx (1/200)(1 \text{ to } 1/3)$. The corresponding dilution factor d [given by Eq. (11)], lepton and baryon-asymmetries Y_L and Y_B [given by Eqs. (8) and (9)] and the requirement on the phase-parameter ϕ_{21} are listed below:

	$\lvert\zeta_{31} - z\rvert$		
	1/200	(1/200)(1/1.7)	(1/200)(1/3)
k	60	20	7
d	1/466	1/129	1/14
$Y_L/\sin(2\phi_{21})$	-2.26×10^{-11}	-8.2×10^{-11}	-7.5×10^{-10}
$Y_B/\sin(2\phi_{21})$	1.13×10^{-11}	4.1×10^{-11}	3.7×10^{-10}
ϕ_{21}	$\sim \pi/4$	$\gtrsim \pi/15$	$\sim \pi/100$-$\pi/25$

Table 1

The constraint on ϕ_{21} is obtained from considerations of Big-Bang nucleosynthesis, which requires $1.7 \times 10^{-11} \lesssim Y_B \lesssim 9 \times 10^{-11}$ [1]. We see that the first case $|\zeta_{31} - z| \approx 1/200$ leads to a baryon asymmetry Y_B that is on the borderline even for a maximal $\sin(2\phi_{21}) \approx 1$. The other cases with $|\zeta_{31} - z| \approx (1/200)(1/1.7 \text{ to } 1/3)$, which are of course perfectly plausible, lead to the desired magnitude of the baryon asymmetry for natural values of the phase parameter $\sin(2\phi_{21}) \sim (1/5 \text{ to } 1/30)$ [45].

We now comment briefly on the scenario proposed in Ref. [42], in which the inflaton decays directly into a pair of heavy RH neutrinos, which in turn decay into $l + \Phi_H$ and $\bar{l} + \bar{\Phi}_H$ and thereby generate lepton asymmetry, *during the process of reheating.* Confining to the fermion mass-pattern in Sec. 2 [Eqs. (1), (4) and (7)], a very similar conclusion as above as regards leptogenesis can be reached also within this alternative scenario. It turns out that this scenario goes well with the mass-pattern of Sec. 2 [especially Eq. (7)], in full accord with the gravitino-constraint and observed baryon-asymmetry, provided $2M_2 > m_{\text{infl}} > 2M_1$, so that the inflaton decays into $2N_1$ rather than into $2N_2$ (contrast this from the case proposed in Ref. [42]). In this case, defining the superpotential $W = \kappa S(-M^2 + \bar{\Phi}\Phi) + (\text{non-ren. terms})$, as in Ref. [42], where Φ and $\bar{\Phi}$ are the (1, 2, 4) and (1, 2, $\bar{4}$) Higgs fields and S is a singlet field, one obtains [42]: $m_{\rm infl} = \sqrt{2}\kappa M$, where $M = \langle (1,\,2,\,4)_H \rangle \approx 2 \times 10^{16}$ GeV, $\Gamma_{\rm infl} \approx [1/(8\pi)](M_1/M)^2 m_{\rm infl}$ and $T_{RH} \approx (1/7)(\Gamma_{\rm infl} M_{\rm Pl})^{1/2} \approx (1/7)(M_1/M)[m_{\rm infl} M_{\rm Pl}/(8\pi)]^{1/2}$. For concreteness, take $M_2 \approx 2 \times 10^{12}$ GeV, $M_1 \approx 10^{10}$ GeV (1 to 2) [in accord with Eq. (7)], and $m_{\rm infl} \approx 3 \times 10^{12}$ GeV (choosing $\kappa \approx 10^{-4}$). We then get: $T_{RH} \approx$ (1 to 2)(0.8×10^8 GeV), and thus (see e.g., Sec. 8 of Ref. [40]), $Y_B \approx -Y_L/2 \approx (-1/2)(\epsilon_1 T_{RH}/m_{\rm infl}) \approx (1 \text{ to } 2)^2 (8 \times 10^{-11} \sin 2\phi_{21})$, where we have used Eq. (18) with appropriate (M_1/M_2), as above. This agrees with the observed value of $Y_B \approx 4 \times 10^{-11}$ (say), again for a natural value of the phase parameter $\phi_{21} \approx (1/4)(1 \text{ to } 1/4)$, where the second factor corresponds to $M_1 \approx 10^{10}$ GeV (1 to 2) [46].

To conclude, we have considered two alternative scenarios for inflation and leptogenesis. We see that the G(224)/SO(10) framework provides a simple and *unified description* of not only fermion masses and neutrino oscillations (consistent with maximal atmospheric and large solar oscillation angles) but also of baryogenesis via leptogenesis, treated within either scenario. The existence of the right-handed neutrinos, B-L as a local symmetry, quark-lepton unification through SU(4)-color, the seesaw mechanism and the magnitude of the supersymmetric unification-scale play crucial roles in realizing this unified and successful description. These features in turn point to the relevance of either G(224) or SO(10) symmetry being effective between the string and the GUT scales, in four dimensions. While the observed magnitude of the baryon asymmetry seems to emerge naturally from within the framework, understanding its observed sign (and thus the relevant CP violating phases) remains a challenging task.

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References

[1] See e.g., K. A. Olive, G. Steigman and T. P. Walker, Phys. Rep. **333**, 389 (2000); D. E. Groom et al., Particle Data Group, Eur. Phys. J. **C15**, 1 (2000).

[2] V. Kuzmin, V. Rubakov and M. Shaposhnikov, Phys. Lett. **B155**, 36 (1985).

[3] M. Fukugita and T. Yanagida Phys. Lett. **B174**, 45 (1986); G. Lazarides and Q. Shafi, Phys. Lett, **B258**, 305 (1991); M. A. Luty, Phys. Rev. **D45**, 455 (1992). The second paper discusses leptogenesis in the context of inflationary cosmology.

[4] For an incomplete list of references on further works on leptogenesis see e.g., M. Flanz, E. A. Pascos and U. Sarkar, Phys. Lett. **B345**, 248 (1995); L. Covi, E. Roulet and F. Vissani, Phys. Lett. **B384**, 169 (1996); W. Buchmüller and M. Plümacher, Phys. Lett. **B389**, 73 (1996); ibid **B431**, 354 (1998); A. Pilaftsis, Phys. Rev. **D56**, 5431 (1997); T. Moroi and H. Murayama, hep-ph/9908223; M. S. Berger and B. Brahmachari, Phys. Rev. **D60**, 073009 (1999); J. Ellis, S. Lola and D. Nanopoulos, Phys. Lett. **B452**, 87 (1999); E. Ma, S. Sarkar and U. Sarkar, Phys. Lett. **B458**, 73 (1999); K. Kang, S. F. Kang and U. Sarkar, Phys. Lett. **B486**, 391 (2000); R. Rangarajan and H. Mishra, Phys. Rev. **D61**, 043509 (2000); H. Goldberg, Phys. Lett. **B474**, 389 (2000); R. Barbieri, P. Creminelli, A. Strumia and N. Tetradis , Nucl. Phys. **B575**, 61 (2000); D. Falcone and F. Tramontano, hep-ph/0101151; H. B. Nielsen and Y. Takanishi, Phys. Lett. **B507**, 241 (2001); J. Ellis, M. Raidal and T. Yanagida, ph/0206300; P. H. Frampton, S. L. Glashow and T. Yanagida, hep-ph/0208157; M. Fujii and T. Yanagida, hep-ph/0208191; Zhi-Zhong Xing, hep-ph/0209066; T. Endoh et al., hep-ph/0209098. For references to works based on SO(10)-models, see Ref. [21].

[5] J. C. Pati and A. Salam, Phys. Rev. Lett. **31**, 661 (1973); Phys. Rev. **D10**, 275 (1974).

[6] H. Georgi in 'Particles and Fields', Ed. by C. Carlson (AIP, NY, 1975), 575; H. Fritzsch and P. Minkowski, Ann. Phys. **93**, 193 (1975).

[7] Possible advantages of a string-unified four-dimensional G(224)-solution [initiated in I. Antoniadis, G. Leontaris and J. Rizos, Phys. Lett. **B245**, 161, (1990)] over an SO(10)-solution, and vice-versa, are discussed in J. C. Pati, hep-ph/0204240 (see Sec. 3). The G(224)-solution on the one hand is free from the problem of doublet-triplet splitting, and on the other hand can account for the observed coupling unification, if the gap between $M_{\rm st}$ and $M_{\rm GUT}$ is small ($M_{\rm GUT} \approx (1/2$ to $1/3)M_{\rm st}$, say). Despite this small gap, one would, however, have the benefits of SU(4)-color to understand fermion masses and neutrino oscillations (as reviewed in Sec. 2) and implement baryogenesis via leptogenesis, to be discussed here.

[8] H. Georgi and S. L. Glashow, Phys. Rev. Lett. **32**, 438 (1974).

[9] F. Gursey, P. Ramond and R. Slansky, Phys. Lett, **B60**, 177 (1976); Y. Achiman and B. Stech, Phys. Lett. **B77**, 389 (1978); Q. Shafi, Phys. Lett. **B79**, 301 (1978); A. deRujula, H. Georgi and S. L. Glashow, 5th Workshop on Grand Unification, edited by K. Kang et al., World Scientific, 1984, p88.

[10] M. Gell-Mann, P. Ramond and R. Slansky, in: *Supergravity*, eds. F. van Nieuwenhuizen and D. Freedman (Amsterdam, North Holland, 1979) p. 315; T. Yanagida, in: *Workshop on the Unified Theory and Baryon Number in the Universe*, eds. O. Sawada and A. Sugamoto (KEK, Tsukuba) 95 (1979); R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).

[11] For earlier works, see S. Dimopoulos, S. Raby and F. Wilczek, Phys. Rev. **D 24**, 1681 (1981); W. Marciano and G. Senjanovic, Phys. Rev. **D 25**, 3092 (1982) and M. Einhorn and D. R. T. Jones, Nucl. Phys. **B 196**, 475 (1982). For work in recent years, see P. Langacker and M. Luo, Phys. Rev. **D 44**, 817 (1991); U. Amaldi, W. de Boer and H. Furtenau, Phys. Rev. Lett. **B 260**, 131 (1991); F. Anselmo, L. Cifarelli, A. Peterman and A. Zichichi, Nuov. Cim. **A 104** 1817 (1991).

[12] J. C. Pati, "*Implications of the SuperKamiokande Result on the Nature of New Physics*", in Neutrino 98, Takayama, Japan, June 98, hep-ph/9807315; Nucl. Phys. B (Proc. Suppl.) **77**, Page 299 (1999).

[13] SuperKamiokande Collaboration, Y. Fukuda et. al., Phys. Rev. Lett. **81**, 1562 (1998).

[14] See e.g., Reviews in A. Pilaftsis, Int. J. Mod. Phys. **A14**, 1811, (1999); W. Buchmüller and M. Plümacher, Int. J. Mod. Phys. **A15**, 5047 (2000).

[15] K. S. Babu, J. C. Pati and F. Wilczek, hep-ph/9812538, Nucl. Phys. **566**, 33 (2000).

[16] J. C. Pati, *"Confronting Conventional Ideas of Grand Unification With Fermion Masses, Neutrino Oscillations and Proton Decay"*, Lectures given at the summer school, ICTP, Trieste (June, 2001), to appear in the proceedings, hep-ph/0204240.

[17] Q. R. Ahmad et al. [SNO Collaboration], nucl-ex/0204008, nucl-ex/0204009; S. Fukuda et al. [SuperK Collaboration], Phys. Rev. Lett. **86**, 5656 (2001).

[18] K. S. Babu and J. C. Pati, *"Link Between Neutrino Oscillations and CP Violation within Supersymmetric Unification"*, Abstract in Proceedings of NATO Advanced Study Institute, "Recent Developments in Particle Physics and Cosmology", June 6-July 7, 2000, Cascais, Portugal, Pages 215-217, Ed. by G. Branco, Q. Shafi and J. I. Silva-Marcos; K. S. Babu and J. C. Pati, *"Linking CP and Flavor Violations To Neutrino Oscillations Within Supersymmetric Unification"* paper to appear.

[19] By combining these results with the analysis of the forthcoming paper [18], one would incorporate CP violation into this unified picture as well.

[20] Many of these are based on phenomenological models just for neutrino masses, which are not linked to the masses and mixings of quarks and charged leptons. See e.g., S. F. King, hep-ph/0204360 for a recent analysis along these lines and references therein.

[21] For attempts within SO(10) models, see e.g., E. Nezri and J. Orloff, hep-ph/0004227; F. Bucella, D. Falcone, F. Tramontano, hep-ph/0108172; G. C. Branco, F. Gonzales Felipe, F. R. Joaquim and M. N. Rebelo, hep-ph/0202030. For a variant attempt within

left-right symmetric model, see A. Joshipura, E. A. Paschos and W. Rodejohann, hep-ph/0104228.

[22] For example, many of the attempts in [21] assume that the Dirac mass-matrix of the neutrinos is equal to that of the up-flavor quarks ($M_\nu^D = M_u$) at GUT-scale. This simple equality would be true for SO(10) if only $\mathbf{10}_H$ contributes to the fermion masses. However, the minimal Higgs system permits a (B-L)-dependent antisymmetric "23" and "32" entry [15] (as discussed later), which plays a crucial role in explaining why $m_u \neq m_s$ and why V_{cb} is so small and yet $\theta^{\rm osc}_{\nu_\mu \nu_\tau}$ is rather maximal. Such entries do not respect $M_\nu^D = M_u$.

[23] For instance, in the first paper of Ref. [21], it is found that only the solar vacuum oscillation solution gives acceptable baryon asymmetry. In the second paper, it is noted that SUSY models with full quark-lepton symmetry gives too small an asymmetry, while in the third paper it is found that the just-so and SMA solutions give viable leptogenesis, but the LMA solution is strongly disfavored [based on their assumption of $M_\nu^D = M_u$, (see comments in Ref. [22])]. In the fourth paper, it is observed that the SMA and vacuum solutions produce reasonable asymmetry, but the LMA solution produces too large an asymmetry.

[24] These have been introduced in various forms in the literature. For a sample, see e.g., C. D. Frogatt and H. B. Nielsen, Nucl. Phys. **B147**, 277 (1979); L. Hall and H. Murayama, Phys. Rev. Lett. **75**, 3985 (1995); P. Binetruy, S. Lavignac and P. Ramond, Nucl. Phys. **B477**, 353 (1996). In the string theory context, see e.g., A. Faraggi, Phys. Lett. **B278**, 131 (1992).

[25] The zeros in "11", "13" and "31" elements signify that they are relatively small quantities (specified below). While the "22" elements were set to zero in Ref. [15], because they are meant to be $<$"23""32"/"33"$\sim 10^{-2}$ (see below), and thus unimportant for purposes of Ref. [15], they are retained here, because such small ζ_{22}^u and ζ_{22}^d [$\sim (1/3) \times 10^{-2}$ (say)] can still be important for CP violation and thus baryogenesis.

[26] For G(224), one can choose the corresponding sub-multiplets – that is $(1, 1, 15)_H$, $(1, 2, \bar{4})_H$, $(1, 2, 4)_H$, $(2, 2, 1)_H$ – together with a singlet S, and write a superpotential analogous to Eq. (2).

[27] If the effective non-renormalizable operator like $\mathbf{16}_2\mathbf{16}_3\mathbf{10}_H\mathbf{45}_H/M'$ is induced through exchange of states with GUT-scale masses involving renormalizable couplings, rather than through quantum gravity, M' would, however, be of order GUT-scale. In this case $\langle \mathbf{45}_H \rangle / M' \sim 1$, rather than 1/10.

[28] While $\mathbf{16}_H$ has a GUT-scale VEV along the SM singlet, it turns out that it also has a VEV of EW scale along the "$\tilde{\nu}_L$" direction due to its mixing with $\mathbf{10}_H^d$, so that the H_d of MSSM is a mixture of $\mathbf{10}_H^d$ and $\mathbf{16}_H^d$ (See Ref. [15]).

[29] The flavor charge(s) of $\mathbf{45}_H(\mathbf{16}_H)$ would get determined depending upon whether $p(q)$ is one or zero (see below).

[30] The basic presumption here is that effective dimensionless couplings allowed by SO(10)/G(224) and flavor symmetries are of order unity [i.e., $(h_{ij}, g_{ij}, a_{ij}) \approx 1/3$-3 (say)]. The need for appropriate powers of (S/M) with $\langle S \rangle / M \sim M_{\rm GUT}/M_{\rm string} \sim (1/10\text{-}1/20)$ in the different couplings leads to a hierarchical structure. As an example, consider just one U(1)-flavor symmetry with one singlet S. The hierarchical form of the Yukawa couplings exibited in Eqs. (1) and (2) would be allowed, for the case of $p = 1$, $q = 0$, if ($\mathbf{16}_3$, $\mathbf{16}_2$, $\mathbf{16}_1$, $\mathbf{10}_H$, $\mathbf{16}_H$, $\mathbf{45}_H$ and S) are assigned U(1)-charges of (a, $a + 1$, $a + 2$, $-2a$, $-a - 1/2$, 0, -1). It is assumed that other fields are present that would make the U(1) symmetry anomaly-free. With this assignment of charges, one would expect $|\zeta_{22}^{u,d}| \sim (\langle S \rangle / M)^2$; one may thus take $|\zeta_{22}^{u,d}| \sim (1/3) \times 10^{-2}$ without upsetting the success of Ref. [15]. In the same spirit, one would expect $|\zeta_{13}, \zeta_{31}| \sim (\langle S \rangle / M)^2 \sim 10^{-2}$ and $|\zeta_{11}| \sim (\langle S \rangle / M)^4 \sim 10^{-4}$ (say). The value of "a" would get fixed by the presence of other operators (see later).

[31] These effective non-renormalizable couplings can of course arise through exchange of (for example) **45** in the string tower, involving renormalizable $\mathbf{16}_i\overline{\mathbf{16}}_H\mathbf{45}$ couplings. In this case, one would expect $M \sim M_{\rm string}$.

[32] P. Ginsparg, Phys. Lett. **B197**, 139 (1987); V. S. Kaplunovsky, Nucl. Phys. **B307**, 145 (1988); Erratum: ibid **B382**, 436 (1992).

[33] Note that such an operator would be allowed by the flavor symmetry defined in Ref. [30] if one sets $a = 1/2$. In this case, operators such as W_{23} and W_{33} that would contribute to $\nu_L^\mu \nu_L^\tau$ and $\nu_L^\tau \nu_L^\tau$ masses would be suppressed relative to W_{12} by flavor symmetry. As pointed out by other authors (see e.g., S. Weinberg, Phys. Rev. Lett. **43**, 1566 (1979) and Proc. XXVI Int'l Conf. on High Energy Physics, Dallas, TX, 1992; E. Akhmedov, Z. Berezhiani and G. Senjanovic, Phys. Rev. **D47**, 3245 (1993).), non-seesaw Majorana masses of the LH neutrinos can arise directly, even in the standard model, through operators of the form $L_i L_j \Phi_H \Phi_H / M$, by utilizing quantum gravity. [For SO(10), two $\mathbf{16}_H$'s are needed additionally to violate B-L by two units.] In the case of the standard model, ordinarily, one would expect $M \sim M_{\rm Planck}$. Thus one would still need to find a reason (in the context of the standard model) why (a) $M \sim M_{\rm GUT}$ and also (b) why $L_1 L_2 \Phi_H \Phi_H / M$ is the leading operator in its class, rather than being suppressed (due to flavor symmetries) relative to $L_3 L_3 \Phi_H \Phi_H / M$ (for example). Both (a) and (b) are needed for this direct non-seesaw mass to be relevant to the LMA MSW solution.

[34] A term like W_{12} can be induced in the presence of, for example, a singlet $\hat{S}$ and a ten-plet ($\hat{\mathbf{10}}$), possessing effective renormalizable couplings of the form $a_i \mathbf{16}_i \mathbf{16}_H \hat{\mathbf{10}}$, $b\hat{\mathbf{10}}\mathbf{10}_H \hat{S}$ and mass terms $\hat{M}_S \hat{S}\hat{S}$ and $\hat{M}_{10}\hat{\mathbf{10}}\hat{\mathbf{10}}$. In this case $\kappa_{12}/M_{\rm eff}^3 \approx a_1 a_2 b^2/(\hat{M}_{10}^2 \hat{M}_S)$. Setting the charge $a = 1/2$ (see Ref. [30] and [33]), and assigning charges (-3/2, 5/2) to ($\hat{\mathbf{10}}, \hat{S}$), the couplings a_1, and b would be flavor-symmetry allowed, while a_2 would be suppressed but so also would be the mass of $\hat{\mathbf{10}}$ compared to the GUT-scale. One can imagine that $\hat{S}$ on the other hand acquires a GUT-scale mass through for example the Dine-Seiberg-Witten mechanism, violating the U(1)-flavor symmetry. One can verify that in such a picture, one would obtain $\kappa_{12}/M_{\rm eff}^3 \sim 1/M_{\rm GUT}^3$.

[35] For instance, consider the superpotential for $\mathbf{45}_H$ only: $W(\mathbf{45}_H) = M_{45}\mathbf{45}_H^2 + \lambda\mathbf{45}_H^4/M$, which yields (setting $F_{\mathbf{45}_H} = 0$), either $\langle\mathbf{45}_H\rangle = 0$, or $\langle\mathbf{45}_H\rangle^2 = -[2M_{45}M/\lambda]$. Assuming that "other physics" would favor $\langle\mathbf{45}_H\rangle \neq 0$, we see that $\langle\mathbf{45}_H\rangle$ would be pure imaginary, if the square bracket is positive, with all parameters being real. In a coupled system, it is conceivable that $\langle\mathbf{45}_H\rangle$ in turn would induce phases (other than "0" and π) in some of the other VEV's as well, and may itself become complex rather than pure imaginary.

[36] L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1983).

[37] Within the framework developed in Ref. [18], the CP violating phases entering into the SUSY contributions (for example those entering into the squark-mixings) also arise entirely through phases in the fermion mass matrices.

[38] An intriguing feature is the prominence of the $\delta_{RR}^{23}(\tilde{b}_R \to \tilde{s}_R)$-parameter which gets enhanced in part because of the largeness of the ν_μ-ν_τ oscillation angle. This leads to large departures from the predictions of the standard model, especially in transitions such as $B_s \to \bar{B}_s$ and $B_d \to \Phi K_s\,(b \to s\bar{s}s)$ [18]. This feature has independently been noted recently by D. Chang, A. Massiero, and H. Murayama (hep-ph/0205111).

[39] As an example, one such fit with complex parameters assigns [18]: $\sigma = 0.10 - 0.012\,i$, $\eta = 0.12 - 0.05\,i$, $\epsilon = -0.095$, $\eta' = 4.0 \times 10^{-3}$, $\epsilon' = 1.54 \times 10^{-4} e^{i\pi/4}$, $\zeta_{22}^u = 1.25 \times 10^{-3} e^{i\pi/9}$ and $\zeta_{22}^d = 4 \times 10^{-3} e^{i\pi/2}$, $\mathcal{M}_u^0 \approx 110$ GeV, $\mathcal{M}_D^0 \approx 1.5$ GeV, $y \approx -1/17$ (compare with Eq. (5) for which $\zeta_{22}^u = \zeta_{22}^d = 0$). One obtains as outputs: $m_{b,s,d} \approx$ (5 GeV, 132 MeV, 8 MeV), $m_{c,u} \approx$ (1.2 GeV, 4.9 MeV), $m_{\mu,e} \approx$ (102 MeV, 0.4 MeV) with $m_{t,\tau} \approx$ (167 GeV, 1.777 GeV), $(V_{us}, V_{cb}, |V_{ub}|, |V_{td}|) \approx (0.217, 0.044, 0.0029, 0.011)$, while preserving the predictions for neutrino masses and oscillations as in Eq. (6). The above serves to demonstrate that complexification of parameters of the sort presented above can preserve the successes of Eq. (6) ([15]). This particular case leads to $\eta_W = 0.29$ and $\rho_W = -0.187$ [18], to be compared with the corresponding standard model values (obtained from ϵ_K, V_{ub} and Δm_{Bd}) of $(\eta_W)_{\rm SM} \approx 0.33$ and $(\rho_W)_{\rm SM} \approx +0.2$. The consistency of such values for η_W and ρ_W (especially reversal of the sign of ρ_W compared

to the SM value), in the light of having both standard model and SUSY-contributions to CP and flavor-violations, and their distinguishing tests, are discussed in Ref. [18].

[40] For reviews, see chapters 6 and 8 in E. W. Kolb and M. S. Turner, "The Early Universe", Addison-Wesley, 1990.

[41] J. Ellis, J. E. Kim and D. Nanopoulos, Phys. Lett. **145B**, 181 (1984); M. Yu. Khlopov and A. Linde, Phys. Lett. **138B**, 265 (1984); E. Holtmann, M. Kawasaki, K. Kohri and T. Moroi, hep-ph/9805405.

[42] For a specific scenario of inflation and leptogenesis in the context of SUSY G(224), see R. Jeannerot, S. Khalil, G. Lazarides and Q. Shafi, JHEP **010**, 012 (2000) (hep-ph/0002151), and references therein. As noted in this paper, with the VEV's of (1, 2, 4)$_H$ and (1, 2, $\bar{4}$)$_H$ breaking G(224) to the standard model, and also driving inflation, just the COBE measurement of $\delta T/T \approx 6.6 \times 10^{-6}$, interestingly enough, implies that the relevant VEV should be of order 10^{16} GeV. In this case, the inflaton made of two complex scalar fields (i.e., $\theta = (\delta\tilde{\nu}^c_H + \delta\bar{\tilde{\nu}}^c_H)/\sqrt{2}$, given by the fluctuations of the Higgs fields, and a singlet S), each with a mass $\sim 10^{12}$-10^{13} GeV, would decay *directly* into a pair of heavy RH neutrinos – that is into N_2N_2 (or N_1N_1) if $m_{\text{infl}} > 2M_2$ (or $2M_1$). The subsequent decays of N_2's (or N_1's), thus produced, into $l + \Phi_H$ and $\bar{l} + \bar{\Phi}_H$ would produce lepton-asymmetry *during the process of reheating*. I will comment later on the consistency of this possibility with the fermion mass-pattern exhibited in Sec. 2. I would like to thank Qaisar Shafi for a discussion on these issues.

[43] L. Covi, E. Roulet and F. Vissani, in Ref. [4].

[44] A variant expression [$d = 1/(2\sqrt{k^2+9})$] for the dilution factor has also been used in the literature for lower values of k ($0 \leq k \leq 10$) [see e.g., H. B. Nielsen and Y. Takanishi, Phys. Lett. **B507**, 241 (2001)]. This would yield a value for d about (2 to 6) times lower than that obtained from Eq. (11), for $k \approx$ (2 to 0.5). This should be kept in mind in viewing the results especially in the last column of Table 1.

[45] Because of supersymmetry, lepton asymmetry should of course receive contributions from out-of-equilibrium decays of heavy sneutrinos ($\tilde{N}_1$'s), and one must include the "light" sleptons in the decay and scattering processes, as well. These have not been included in our considerations for the sake of simplicity. We do not, however, expect them to alter the lepton asymmetry obtained as above by more than a factor of two.

[46] Note that for this scenario (with the inflaton decaying into $2N_1$) the gravitino constraint is very well satisfied even for $m_{3/2} \sim 300$ GeV, because the reheating temperature is rather low ($\sim 10^8$ GeV). At the same time, one can allow N_1 to be heavier (like 2×10^{10} GeV) than the case considered before (like 4×10^9 GeV) because it can be produced directly by the decay of the inflaton rather than from the thermal bath. Since $Y_B \propto \epsilon_1 T_{RH}$, Y_B increases as M_1^2 for a given M_2. Thus, somewhat higher values of M_1 compatible with the range shown in Eq. (7), are prefered.