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NONLINEAR DYNAMICS IN SPEAR WIGGLERS

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Abstract

BL11, the most recently installed wiggler in the SPEAR storage ring at SSRL, produces a large nonlinear perturbation of the electron beam dynamics, which was not directly evident in the integrated magnetic field measurements. Measurements of tune shifts with betatron oscillation amplitude and with closed orbit shifts were used to characterise the nonlinear fields of the SPEAR insertion devices (IDs). Because of the narrow pole width in BL11, the nonlinear fields seen along the wiggling electron trajectory are dramatically different than the flip coil measurements made along a straight line. This difference explains the tune shift measurements and the observed degradation in dynamic aperture. Corrector magnets to cancel the BL11 nonlinear fields are presently under construction.

1 INTRODUCTION

When the BL11 wiggler was installed in SPEAR in 1998, it was discovered that beam could not be stored at the 2.3 GeV injection energy when the wiggler gap was fully closed. Once the beam was ramped to the standard operational energy of 3 GeV, the effect was less severe. Closing the gap reduced the lifetime from 48 to 33 hours, provided the orbit was centered in the wiggler. If the horizontal orbit was off-center by more than 3 mm, the lifetime dropped to minutes.

The lifetime degradation was determined to be the result of nonlinear fields associated with the finite wiggler pole width. The lessons learned from BL11 could prove useful when building future wigglers for light sources, storage ring colliders and damping rings.

Danfysik met or exceeded all specifications with BL11. The specification they were given for transverse field roll-off in a single pole should have been tighter.

2 ELECTRON BEAM MEASUREMENTS

Measurements were made to characterise the effect of BL11 on SPEAR. Fig. 1 shows the tune shift with betatron oscillation amplitude measured with the bunch motion monitor [2]. The large change in the linear term of ν_x vs. x_β^2 from BL11 indicates a strong octupole-like x^3 component in the horizontal equation of motion. Also note the limited amplitude to which the beam could be kicked with BL11 closed, indicating reduced dynamic aperture. These two measurements were made with all other IDs closed. With all IDs open, the beam could be kicked to x_β^2 = 245 mm^2, so the dynamic aperture had already been compromised before BL11 was installed.

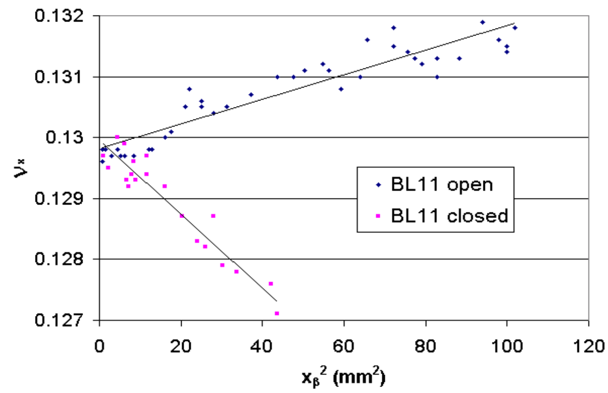

Figure 1: Tune with betatron oscillation amplitude.

This octupole-like term was confirmed with measurements of the horizontal tune with horizontal closed orbit bump shown in Fig. 2. With the BL11 gap open, the size of the closed orbit bump was limited only by the vacuum chamber. The bump size with BL11 closed was considerably smaller and limited by the lifetime dropping to minutes.

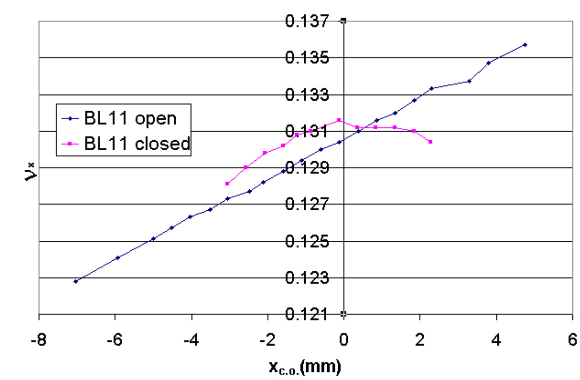

Figure 2: Tune vs. closed orbit bump in BL11.

The tune shift with closed orbit in the wiggler is a measure of the integrated field gradient vs. *x*. Fig. 3 shows that the field integral variation predicted by the tune measurements is much larger than that indicated by magnetic measurements of BL11 made using a flip coil. To check that there was not some systematic error in the tune measurement method, ν_x vs. horizontal closed orbit was measured in the other SPEAR insertion devices. Fig. 4 shows quite good agreement for BL9.

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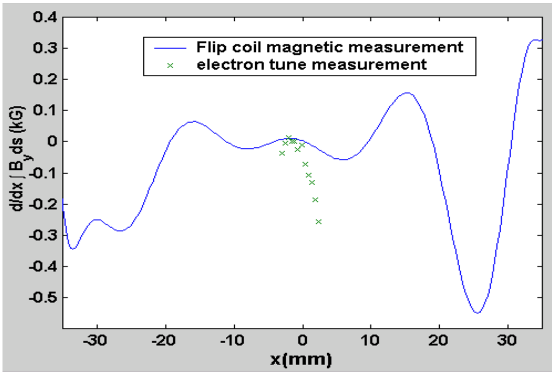

Figure 3: Derivative of BL11 field integral for y = 0.

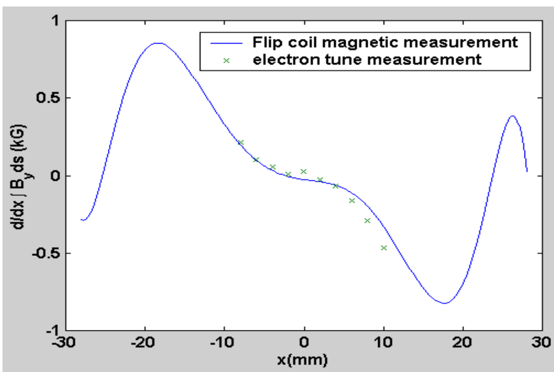

Figure 4: Derivative of BL9 field integral for y = 0.

3 BEAM DYNAMICS IN WIGGLERS

There are two contributions to the field integrals in the y = 0 plane – random errors from construction tolerances and systematic field integrals associated with finite pole width. To a good approximation, the random errors produce field integrals that are the same in the coordinate system fixed to the wiggler (x,y,z) as in the curvilinear coordinates that move along the electrons wiggling trajectory (x_w,y_w,s), so the random errors show up in the flip coil measurements. The systematic integrals are nonzero only in the curvilinear coordinates, so they do not show up with flip coils. This explains the discrepancy in Fig. 3.

A simple analytical calculation illustrates the systematic integrals. Assume the wiggler fields are ideal (no construction errors) and have only the first longitudinal harmonic in the wiggler period. In the mid-plane (y=0), the wiggler fields are

$$B_y(x,z) = B_y(x)\cos kz \qquad B_{x,z}(x,z) = 0. \tag{1}$$

Once the fields are specified in the mid-plane, they are fixed everywhere by Maxwell's equations. The real fields in BL11 have significant 3[rd] and 5[th] harmonics, but this approximation still gives useful qualitative results.

For Eq. 1, $\int B_y dz$, is zero for fixed x over an integer number of periods. The integrated field, however, is nonzero along the wiggling electron trajectory. If a particle is launched at the entrance to the wiggler with $(x,x') = (x_i,0)$, then it will follow a wiggling trajectory of

$$x_w = x_i - x_p\cos kz \quad x_p = B_y(x)/(k^2 B\rho), \tag{2}$$

neglecting the small curve in the trajectory from the wiggler focusing. For BL11, the wiggler period $2\pi/k$ is 17.5 cm and the peak field is 2 Tesla, so x_p is 155 µm. The integrated field seen along the wiggling trajectory is

$$\int B_y ds = \int B_y(x_i - x_p\cos kz)\cos kz ds$$

$$= -1/2\ L x_p(x_i)\ dB_y(x_i)/dx, \tag{3}$$

where L is the wiggler length. Integrated field scales as the derivative of transverse field roll-off in a single pole as sampled by the wiggling trajectory. The poles are 50 mm wide in BL11 and 95 mm in BL9. For this reason, the systematic integrals are particularly strong in BL11 and explain the discrepancy seen in Fig. 3. The integral also scales as the wiggler period squared, so wigglers with longer period generate larger nonlinear fields. The field integral scales as $1/B\rho$, so the perturbation in the electron equation of motion scales as $1/(B\rho)^2$, in contrast to standard multipoles which scale as $1/B\rho$.

Eq. 3 gives a useful qualitative description of the nonlinear field integral generated by the transverse roll-off of the single-pole fields. A more accurate derivation of the field integral, including all longitudinal harmonics, was generated by numerically integrating trajectories through a single period of the TOSCA field model of $B_y(x,z)$. Fig. 5 shows the field integrals from Eq. 3 compared to those from numerical integration. Note the different scales on Figs. 3 and 5; the small wiggle generates a large change in field integral.

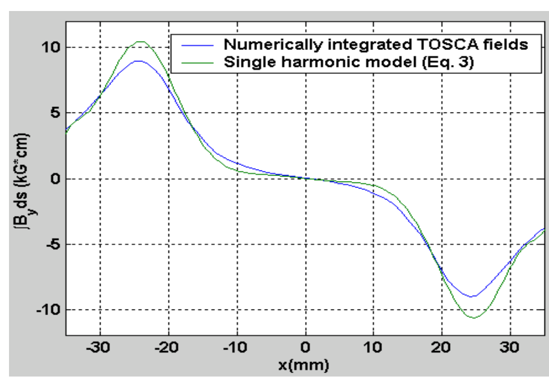

Figure 5: Field integral along wiggling trajectory.

Because the pole design in BL11 is symmetric in x, $dB_y(x)/dx$ is an odd function of x, so the horizontal equation of motion has only terms that look like the odd multipoles – quadrupole, octupole, etc. The model of BL11 in Fig. 5 reasonably predicts the electron beam measurements. The horizontal tune shift when closing the BL11 magnet gap indicated an integrated field gradient of -.084 Tesla, while TOSCA field model in Fig. 5 has an integrated gradient of -.069 Tesla. Table 1 compares the octupole-like cubic term from the TOSCA field model to that from three different electron beam measurements described in section 2.

The quadrupole-like focusing generates a horizontal beta-beat of 12 and 21 % at 3 and 2.3 GeV respectively.

This is not enough to explain the reduction in dynamic aperture; the reason for the dynamic aperture reduction is the nonlinear fields.

Table 1: Cubic term in horizontal Eq. of motion.

TOSCA model	-0.38 kG/cm^2
Tune vs. β-amplitude	-0.59
Tune vs. x-bump	-0.54
Tune vs. rf frequency	-0.43

A nonlinear map of BL11 was generated using the 3D RADIA code [3] and studied using BETA [4,5]. Fig. 6 dramatically demonstrates the strength of the nonlinear fields perturbing the SPEAR dynamics. The optics in SPEAR have a 1.2 m dispersion at BL11. Without BL11, the closed orbit at BL11 vs. energy is close to a straight line with slope 1.2 m. With BL11 it is discontinuous when x at BL11 approaches the half pole width, because of the large d/dx($\int B_y ds$) (see Fig. 5).

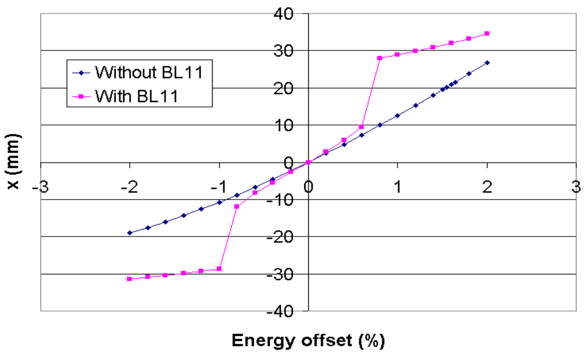

Figure 6: BETA simulation - η_x at BL11.

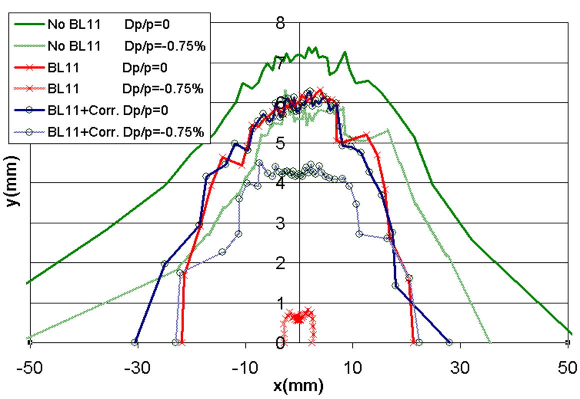

Figure 7: Dynamic aperture tracking with BETA. (Corr. means with nonlinear corrector magnets.)

Fig. 7 shows the dynamic aperture at 3 GeV with and without BL11. As could be guessed from Fig. 6, BL11 particularly reduces the off-energy dynamic aperture.

4 NONLINEAR FIELD CORRECTORS

Correction magnets are presently being built to compensate for the BL11 nonlinear fields. A correction magnet will be attached to each end of the wiggler. The correction will not perfectly restore the beam dynamics for a number of reasons:

1. The wiggler fields are not standard multipoles, while the correctors must be. (More on this below.)
2. The wiggler errors scale as the square of the electron energy, E^2, (electron rigidity and wiggle amplitude), the correctors as E (rigidity only).
3. The wiggler errors and correctors scale differently with magnet gap.
4. The correction is not perfectly local; the correctors will be on either end of the 2.3 m wiggler.
5. Given the small longitudinal space available for the correctors, it was impossible to perfectly cancel $\int B_y ds$ shown in Fig. 5.

The correctors will be thin lens multipoles (There is only 2 cm of longitudinal space for the correctors.), so the integrated fields will have to be of the form

$$\int(B_y + iB_x)ds = -B\rho\Sigma_n(b_n+ia_n)(x+iy)^{n-1} \quad (4)$$

It can be shown that the field integrals must satisfy Eq. 4, if the change in x and y is negligible throughout the magnet (i.e. ds=dz). For example, the field integrals measured with a straight flip coil shown in Fig. 3 satisfy Eq. 4. The field integrals along the wiggling trajectory do not [1]. It is theoretically possible to make correctors that cancel the $\int B_y(x,y=0,z)ds$ shown in Fig. 5, but the corrector will only correct the fields for y = 0. The correction for $y \neq 0$ will be wrong.

SPEAR has relatively large β_x and η_x at BL11 (β_x=21m, η_x=1.2m, β_y =1.8m), so the optics perturbation is largest in x and energy. The correctors were designed for y = 0, and are not expected to be effective off mid-plane. Fig. 7 includes the dynamic aperture with the correctors. The correctors are particularly effective for improving the off-energy dynamic aperture.

The correctors will be tested in SPEAR in fall, 2000.

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