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**Background Effects in  $B \rightarrow \phi K$ ,**

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## Abstract

Background Effects in  $B \rightarrow \phi K_s$ . PABLO GOLDENZWEIG (University of Cincinnati, Cincinnati, Ohio 45221). HELEN QUINN (Stanford Linear Accelerator Center, Menlo Park, Ca. 94025).

The recent measurement  $\mathcal{C}F \sin(2\beta(\phi K_s)) = -0.19_{-0.50}^{+0.52}(\text{stat}) \pm 0.09(\text{syst})$  by BABAR and  $\sin(2\beta(\phi K_s)) = -0.73 \pm 0.64 \pm 0.18$  by Belle have challenged the limits of the SM. We present a method to isolate the  $\sin(2\beta(\phi K_s))$  measurement from possible background S-wave decays  $B \rightarrow K^+ K^- K_s$  that may be polluting the signal  $B \rightarrow (\phi \rightarrow K^+ K^-) K_s$ . By introducing an additional term into the time dependent decay amplitude, the time dependent CP asymmetry is expressed as a function of CP(+) (background) and CP(-) (signal) constituents. An expression relating the CP violating parameter  $\sin(2\beta_{\text{eff}})$  as a function of varying background strength is obtained. This quantity can be directly related to the experimental value.

## Introduction

Charge parity reversal (CP) is an operation, by which a particle is converted into its corresponding antiparticle and all spatial directions inverted. CP is a symmetry which appears to be exact for the strong, electromagnetic and gravitational interactions, but not for the weak interaction<sup>1</sup>. This asymmetry indicates fundamental differences in the behavior of matter and antimatter.

According to the standard model (SM), CP violation can occur via the interactions that transform a quark into another quark with a different charge. These interactions are described by the 3 X 3, Cabibbo-Kobayashi-Maskawa (CKM) matrix, where each matrix element represents the strength of one of the nine transitions. The CKM matrix can be written as

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}.$$

The strength of each of the nine transitions is governed by a complex quantity known as  $V_{ij}^*$ , where  $i,j$  denote the flavor of the initial and final quark. CP violation arises due to the differences in the phases of the nine terms in the matrix. Direct CP violation occurs when the decay rate of a particle differs from that of its CP conjugate (ie.  $A(B \rightarrow X) \neq A(\bar{B} \rightarrow \bar{X})$ ). CP violation from the interference between mixing and decay is due to the interference between the different processes by which the particle can decay to the same final state. The asymmetry arises from the direct interference between the various terms in the decay amplitude. Indirect CP violation, on

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<sup>1</sup> We neglect strong CP effects, which are small  $\vartheta < 10^{-10}$ .

the other hand, is due to the mixing of the particles and their corresponding antiparticles. It results from the mass eigenstates being different from the CP eigenstates.

Unfortunately, physicists do not know whether or not the SM provides the sole source of CP violation; there may be additional sources beyond the standard model, which contribute to CP violation. The strongest evidence for this uncertainty lies in the fact that the amount of CP violation predicted by the SM is not enough to account for the dominance of matter over antimatter in the universe. As a result, further research is necessary to search for violations in the standard model pattern of CP violations.

CP violation was first observed in the decay of the neutral kaon system, where it was found that the final decay products contained 0.3% more of the neutral kaon than its antiparticle [1]. These observations place constraints on the parameters of the CKM mixing matrix. However, further evidence of CP violation is needed to test these results.

The asymmetric B factory at SLAC was designed to test for CP violation in the decays of certain B mesons. By comparing measurements of similar decay channels, one can extract CKM parameters with controlled uncertainties. The fact that the CP asymmetries in B decays are sizeable helps, in that systematic errors will not obscure the signal. Charmed B decays provide the cleanest measurement of  $\sin(2\beta)$ , where  $\beta$  is one of the phases that quantifies CP violation. The  $B \rightarrow \psi K_s$  decays have been named the “golden modes” for studying CP violation, since the hadronic uncertainties are below the 1% level [2]. The updated measurement of  $\sin(2\beta) = 0.741$

$\pm 0.067$  (*stat*)  $\pm 0.033$  (*syst*) by BABAR [3] is consistent with the SM expectation, based on measurements of the CKM quark-mixing matrix.

However, recent measurements of the time dependent CP violating asymmetry from the decay

$B \rightarrow \phi K_s$ , yields  $\sin(2\beta) = -0.19_{-0.50}^{+0.52}$  (*stat*)  $\pm 0.09$  (*syst*) by BaBar [4] and

$\sin(2\beta) = -0.73 \pm 0.64 \pm 0.18$  by Belle [5] have challenged the limits of the SM. The resulting

error weighted average  $\sin(2\beta(\phi K_s)) = -0.39 \pm 0.41$  [6] differs significantly from the value

obtained from  $B \rightarrow \psi K_s$ . This is due to the fact that  $|\sin(2\beta(\phi K_s)) - \sin(2\beta(\psi K_s))| < \lambda^2$ , where

$\lambda \approx 0.22$  is the expansion parameter in the CKM matrix (see below). In the following we attempt to

refine the  $\sin(2\beta(\phi K_s))$  measurement by accounting for the interference effects of S-wave

decays  $B \rightarrow K^+ K^- K_s$  with CP = +1 in the vicinity of the CP = -1 decay,  $B \rightarrow \phi K_s$ . By factoring

in an interference term into the time dependent decay amplitude of  $B \rightarrow \phi K_s$ , the time dependent

CP asymmetry is obtained as a function of both CP = +1 and CP = -1 constituents.

## FORMALISM

The neutral B meson mass eigenstates are defined in terms of flavor eigenstates as

$$|B_{L,H}\rangle = p_B |B^0\rangle \pm q_B |\bar{B}^0\rangle. \quad (1)$$

Neglecting the difference in lifetimes, the amplitudes for  $|B_{L,H}(t)\rangle$  can be written as

$$a_{L,H}(t) = a_{L,H}(0) e^{-\left(\frac{\Gamma}{2} + iM_L t\right)}, \quad (2)$$

where we have defined

$$M \equiv \frac{M_H + M_L}{2}, \quad \Delta M \equiv M_H - M_L. \quad (3)$$

The proper time evolution of an initially pure  $B^o$  or  $\bar{B}^o$  is given by

$$\begin{aligned} |B_{phys}^o(t)\rangle &= g_+(t)|B^o\rangle + \frac{q}{p}g_-(t)|\bar{B}^o\rangle, \\ |\bar{B}_{phys}^o(t)\rangle &= \frac{p}{q}g_-(t)|B^o\rangle + g_+(t)|\bar{B}^o\rangle, \end{aligned} \quad (4)$$

where,

$$\begin{aligned} g_+(t) &= e^{-\frac{\Gamma}{2}t} e^{-iMt} \cos\left(\frac{\Delta M}{2}t\right), \\ g_-(t) &= e^{-\frac{\Gamma}{2}t} e^{-iMt} i \sin\left(\frac{\Delta M}{2}t\right). \end{aligned} \quad (5)$$

We define the time independent amplitudes for the decay of a neutral  $B$  or  $\bar{B}$  meson into a final CP eigenstate as

$$\begin{aligned} A &= \langle f_{cp} | H | B_{phys}^o \rangle = \sum_i A_i e^{i\delta_i} e^{i\phi_i}, \\ \bar{A} &= \langle f_{cp} | H | \bar{B}_{phys}^o \rangle = \sum_i A_i e^{i\delta_i} e^{-i\phi_i} \end{aligned} \quad (6)$$

where  $\delta_i$  and  $\phi_i$  denote the strong and weak phases, respectively.

Time dependence yields

$$\begin{aligned} A(t) &\equiv \langle f_{cp} | H | B_{phys}^o(t) \rangle = A(g_+(t) + \lambda g_-(t)), \\ \bar{A}(t) &\equiv \langle f_{cp} | H | \bar{B}_{phys}^o(t) \rangle = A \frac{p}{q} (g_-(t) + \lambda g_+(t)), \end{aligned} \quad (7)$$

where we have defined

$$\lambda = \left( \frac{q}{p} \right) \left( \frac{\bar{A}}{A} \right). \quad (8)$$

The time dependent rates for initially pure  $B$  or  $\bar{B}$  states to decay into a final CP eigenstate are

given by

$$\begin{aligned}\Gamma(B_{phys}^{\circ}(t) \rightarrow f_{cp}) &= |A(t)|^2 = |A|^2 e^{-\Gamma t} \left( \frac{1+|\lambda|^2}{2} + \frac{1-|\lambda|^2}{2} \cos(\Delta Mt) - \text{Im } \lambda \sin(\Delta Mt) \right) \\ \Gamma(\bar{B}_{phys}^{\circ}(t) \rightarrow f_{cp}) &= |\bar{A}(t)|^2 = |A|^2 e^{-\Gamma t} \left( \frac{1+|\lambda|^2}{2} - \frac{1-|\lambda|^2}{2} \cos(\Delta Mt) + \text{Im } \lambda \sin(\Delta Mt) \right).\end{aligned}\tag{9}$$

The time dependent CP asymmetry in  $B \rightarrow f_{cp}$  is defined by

$$\begin{aligned}a_{f_{cp}}(t) &= \frac{\Gamma(B_{phys}^{\circ}(t) \rightarrow f_{cp}) - \Gamma(\bar{B}_{phys}^{\circ}(t) \rightarrow f_{cp})}{\Gamma(B_{phys}^{\circ}(t) \rightarrow f_{cp}) + \Gamma(\bar{B}_{phys}^{\circ}(t) \rightarrow f_{cp})} \\ &= \frac{(1-|\lambda|^2) \cos(\Delta Mt) - 2 \text{Im } \lambda \sin(\Delta Mt)}{1+|\lambda|^2}.\end{aligned}\tag{10}$$

The cosine term is due to the interference between two or more decay amplitudes with different weak phases, while the sine term is due to the interference between direct decay and  $B - \bar{B}$  mixing [4]. In the  $|\lambda|=1$  limit, which applies to the decay  $B \rightarrow \psi K_s$  in the SM, the asymmetries reduce to the simple form

$$a_{f_{cp}}(t) = -\text{Im } \lambda \sin(\Delta Mt).\tag{11}$$

The quantity  $\text{Im } \lambda$  is the CP violating parameter. It is directly related to the CKM matrix elements in the SM.

### The CKM Quark-Mixing Matrix

The CKM matrix describes the interactions by which a quark can transform into another quark with a different charge. CP violation arises due to the differences in the phases of the nine terms



in the matrix. The CKM matrix can be expressed in terms of the Wolfenstein parameters ( $\lambda$ ,  $\rho$ ,  $\eta$ ,  $A$ ) as

$$V = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}, \quad (12)$$

where  $\lambda = 0.22$  is the expansion parameter and  $\eta$  represents the CP-violating phase. If there are only three generations of quarks then this matrix must be unitary:  $V^\dagger V = VV^\dagger = 1$ . This implies that  $V_{ij}^* V_{ik} = \delta_{jk}$  and  $V_{ij}^* V_{kj} = \delta_{ik}$ . [7]. For example, we have  $V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$ . By choosing phase conventions such that  $V_{cb}^* V_{cd}$  is real and dividing the lengths of all sides by  $|V_{cb}^* V_{cd}|$ , this relation can be viewed as a triangle in the complex plane, rescaled so that the base is real and one unit long (see figure 1) [1]. The three angles of this triangle are defined by,

$$\alpha \equiv \arg\left(-\frac{V_{td} V_{tb}^*}{V_{ud} V_{ub}^*}\right), \quad \beta \equiv \arg\left(-\frac{V_{cd} V_{cb}^*}{V_{td} V_{tb}^*}\right), \quad \gamma \equiv \arg\left(-\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*}\right). \quad (13)$$

The three angles can be measured independently of each other by CP asymmetries in various B decays.

### Measuring $\sin(2\beta)$ in $B \rightarrow \psi K_s$

The decay  $B \rightarrow \psi K_s$  provides the cleanest mechanism for measuring the quantity  $\sin(2\beta)$  since the decay is dominated by tree level processes. Furthermore, the penguin contribution has the same phase, since  $V_{cb} V_{cs}^* \cong -V_{tb} V_{ts}^*$ . For this mode,

$$\lambda = -\left(\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}\right) \left(\frac{V_{cs}^* V_{cb}}{V_{cs} V_{cb}^*}\right) \left(\frac{V_{cd}^* V_{cs}}{V_{cd} V_{cs}^*}\right), \quad (14)$$

$$\text{Im}\lambda = \sin(2\beta). \quad (15)$$

We assume that  $\beta$  is measured by the  $B \rightarrow \psi K_s$  decay and use it as an input in the following calculation.

### Measuring $\sin(2\beta)$ in $B \rightarrow \phi K_s$

The decay  $B \rightarrow \phi K_s$  is more susceptible to physics beyond the standard model because the process is dominated by penguin loops. Furthermore, there are CP = +1 decays that may be polluting the signal, namely  $B \rightarrow a_0 K_s$  and  $B \rightarrow f_0 K_s$ , with masses,  $m_a = 980 \pm 10$  MeV and  $m_f = 983.4 \pm 9$  MeV respectively, which decay into  $K^+ K^-$  in an S-wave. In this analysis, we attempt to account for their presence by introducing a CP = +1 term in the time dependent decay amplitude. The corrected time dependent decay amplitude is

$$A(t) = A^+(t) + A^-(t), \quad \bar{A}(t) = \bar{A}^+(t) + \bar{A}^-(t), \quad (16)$$

where we have defined the CP = +1 and CP = -1 components as

$$A^\pm(t) = A^\pm(g_+(t) + \lambda^\pm g_-(t)), \quad \bar{A}^\pm(t) = A^\pm \frac{p}{q} (g_-(t) + \lambda^\pm g_+(t)). \quad (17)$$

Thus,

$$\text{Im}\lambda^\pm = -\frac{|\bar{A}^\pm|}{|A^\pm|} \sin(2\beta_{\text{eff}}^\pm), \quad (18)$$

In equation (18), we have defined  $\beta_{\text{eff}}^\pm \equiv \beta + \theta_{\text{Decay}}^\pm$ , where  $\theta_{\text{Decay}}^\pm$  represents the phase difference in the decay amplitudes for the background (CP+) and signal (CP-). The time dependent decay rates are given by

$$\Gamma(B_{\text{phys}}^\circ(t) \rightarrow f_{\text{CP}}) = |A(t)|^2 = |A^+(t)|^2 + |A^-(t)|^2 + 2\text{Re}[A^+(t)A^{-*}(t)], \quad (19)$$

$$\Gamma(\bar{B}_{phys}^0(t) \rightarrow f_{cp}) = |\bar{A}(t)|^2 = |\bar{A}^+(t)|^2 + |\bar{A}^-(t)|^2 + 2 \operatorname{Re}[\bar{A}^+(t)\bar{A}^{-*}(t)].$$

The time dependent CP asymmetry yields,

$$a_{f_{cp}} = \frac{r(1-|\lambda^+|^2) + (1-|\lambda^-|^2)}{r(1+|\lambda^+|^2) + (1+|\lambda^-|^2)} \cos(\Delta Mt) - 2 \frac{r \operatorname{Im} \lambda^+ + \operatorname{Im} \lambda^-}{r(1+|\lambda^+|^2) + (1+|\lambda^-|^2)} \sin(\Delta Mt) \quad (20)$$

with the background versus signal ratio

$$r \equiv \left| \frac{A^+}{A^-} \right|^2. \quad (21)$$

In equation (21) we have integrated over all helicity angles, which eliminates the interference terms<sup>2</sup>. The angular analysis necessary to separate out the various components of the interference terms is not included in this discussion. The experimental program is yet to be performed. The simplest case, which involves setting  $\lambda^+ = -\lambda^-$  and  $|\lambda^\pm| = 1$ , reduces equation (21) to

$$a_{f_{cp}} = \frac{r-1}{r+1} \sin(2\beta_{eff}^-) \sin(\Delta Mt). \quad (22)$$

The factor  $\frac{r-1}{r+1} \sin(2\beta_{eff}^-)$  of equation (23) is very interesting, since it can be directly related to the experimentally measured value  $\sin(2\beta(\phi_{K_s}))_{ave} = -0.39 \pm 0.41$ . Varying  $r$  will allow for better approximation of the strength of the background decay of  $B \rightarrow \phi K_s$ . A plot of  $\sin(2\beta)$  versus  $r$  (see figure 3) indicates that increasing the value of  $r$  (ie, increasing the strength of the background) increases the value of  $\sin(2\beta_{eff}^-)$  (e.g. for a 5% and 10% background,  $\sin(2\beta_{eff}^-) = -.43, -.48$ , respectively, assuming central value  $\sin(2\beta(\phi_{K_s}))_{ave} = -0.39 \pm 0.41$ ).

<sup>2</sup> For the interference terms, see figure 2.

## Conclusions

The recent measurement of  $\sin(2\beta(\phi K_s))_{\text{ave}}$  has challenged the limits of the SM. We studied the effects that the  $B \rightarrow a_0 K_s$  and  $B \rightarrow f_0 K_s$  decays produce on the background of the decay  $B \rightarrow \phi K_s$ .

The quantity  $\sin(2\beta)$  was plotted as a function of the magnitude of the background decays. It was observed that increasing the background versus signal ratio  $r$ , increases the fitted value of  $\sin(2\beta_{\text{eff}}^-)$ .

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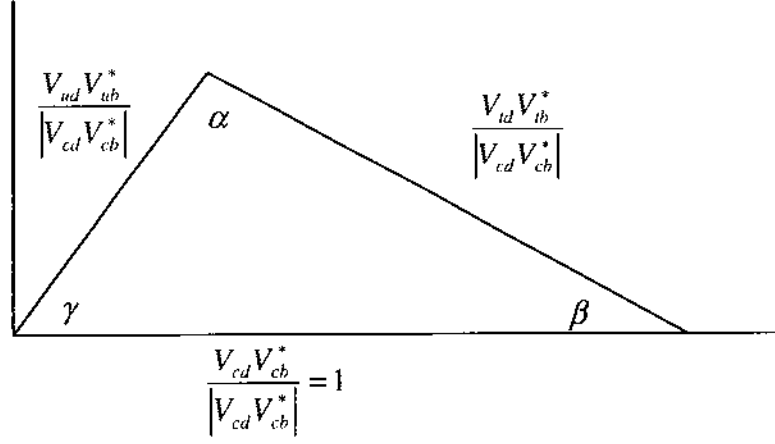


Figure 1. The unitary triangle, rescaled so that the base is real and one unit long.

$$2 \operatorname{Re} \left[ A^+(t) A^{-*}(t) - \bar{A}^+(t) \bar{A}^{-*}(t) \right] = 2 \operatorname{Re} \left[ A^+ A^{-*} (1 - \lambda^+ \lambda^{-*}) \right] \cos(\Delta M t) + 2 \operatorname{Re} \left[ i A^+ A^{-*} (\lambda^+ - \lambda^{-*}) \right] \sin(\Delta M t)$$

$$2 \operatorname{Re} \left[ A^+(t) A^{-*}(t) + \bar{A}^+(t) \bar{A}^{-*}(t) \right] = 2 \operatorname{Re} \left[ A^+ A^{-*} (1 + \lambda^+ \lambda^{-*}) \right]$$

Figure 2. The interference terms for equation (20)

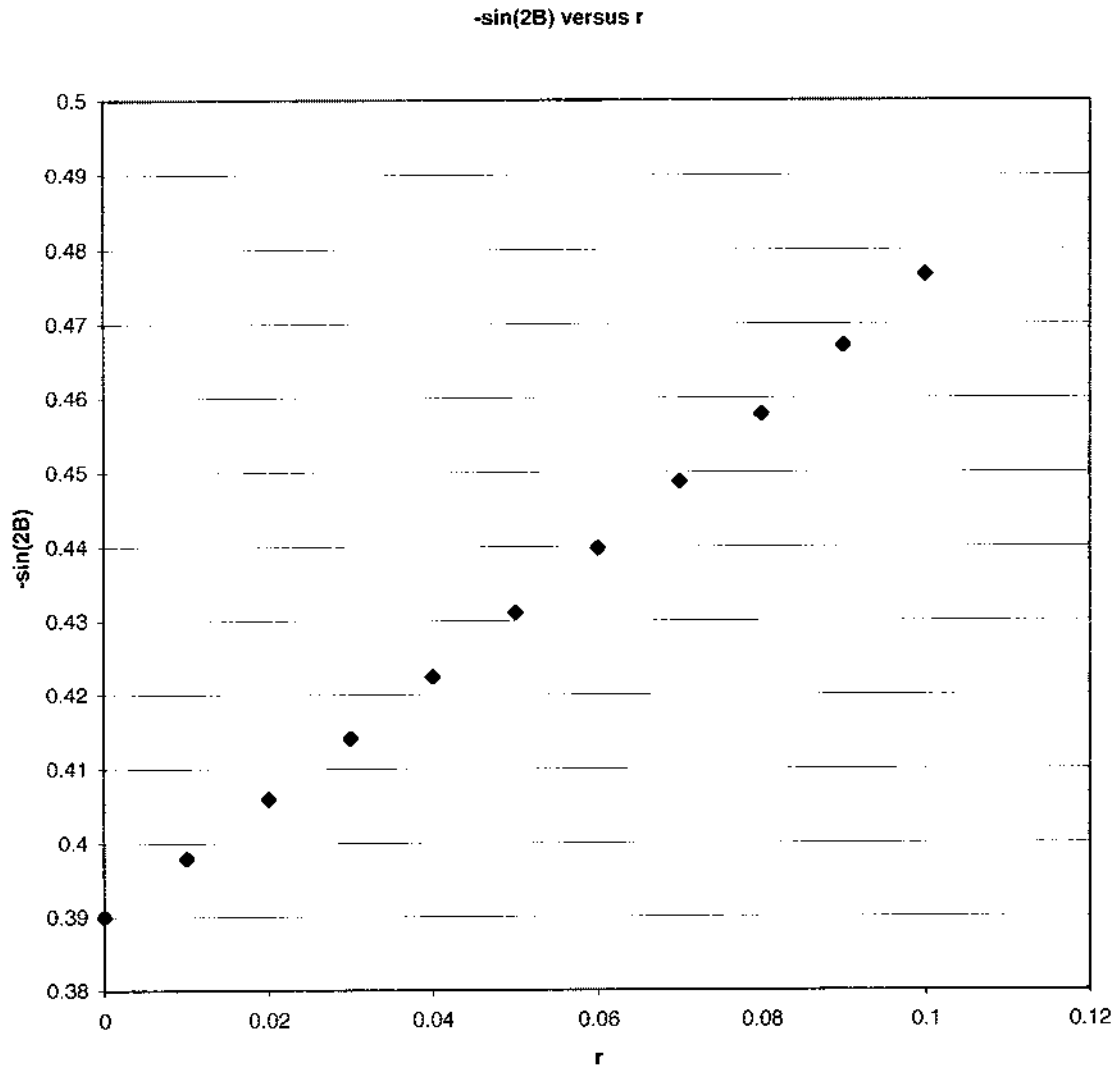


Figure 3. Plot of  $-\sin(2\beta_{eff}^-)$  versus  $r$ . The value  $\sin(2\beta(\phi K_s))_{ave} = -0.39$  is used as the experimental value.