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BLACK HOLE REMNANTS AND DARK MATTER

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We argue that, when the gravity effect is included, the generalized uncertainty principle (GUP) may prevent black holes from total evaporation in a similar way that the standard uncertainty principle prevents the hydrogen atom from total collapse. Specifically we invoke the GUP to obtain a modified Hawking temperature, which indicates that there should exist non-radiating remnants (BHR) of about Planck mass. BHRs are an attractive candidate for cold dark matter. We investigate an alternative cosmology in which primordial BHRs are the primary source of dark matter.

1. INTRODUCTION

In the standard view of black hole thermodynamics, based on the entropy expression of Bekenstein[1] and the temperature expression of Hawking[2], a small black hole should emit black body radiation, thereby becoming lighter and hotter, leading to an explosive end when the mass approaches zero. However Hawking's calculation assumes a classical background metric and ignores the radiation reaction, assumptions which must break down as the black hole becomes very small and light. Thus it does not provide an answer as to whether a small black hole should evaporate entirely, or leave something else behind, which we refer to as a black hole remnant (BHR).

Numerous calculations of black hole radiation properties have been made from different points of view[3], and some hint at the existence of remnants, but in the absence of a well-defined quantum gravity theory none appears to give a definitive answer.

A cogent argument against the existence of BHRs can be made[4]: since there is no evident symmetry or quantum number preventing it, a black hole should radiate entirely away to photons and other ordinary stable particles and vacuum, just like any unstable quantum system.

In a recent paper[5], we invoked the generalized uncertainty principle (GUP)[6–8] and argued the contrary, that the total collapse of a black hole may be prevented by dynamics and not by symmetry, just like the prevention of hydrogen atom from collapse by the uncertainty principle[9]. Our arguments then lead to a modified black hole entropy and temperature, and as a consequence the existence of a BHR at around the Planck mass. Here we first repeat these arguments and derivations. We then investigate a cosmology in which primordial BHRs serve as the primary source for dark matter.

2. GENERALIZED UNCERTAINTY PRINCIPLE

The uncertainty principle argument for the stability of hydrogen atom can be stated very briefly. The energy of the electron is $p^2/2m - e^2/r$, so the classical minimum energy is very large and negative, corresponding to the configuration $p = r = 0$, which is not compatible with the uncertainty principle. If we impose as a minimum condition that $p \approx \hbar/r$, we see that $E = \hbar^2/2mr^2 - e^2/r$,

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thus we find

$$r_{min} = \frac{\hbar^2}{me^2} \ , \quad \text{and} \quad E_{min} = -\frac{me^4}{2\hbar^2} \ . \qquad (1)$$

That is the energy has a minimum, the correct Rydberg energy, when r is the Bohr radius, so the atom is stablized by the uncertainty principle.

As a result of string theory[6] or more general considerations of quantum mechanics and gravity[7,8], the GUP gives the position uncertainty as

$$\Delta x \geq \frac{\hbar}{\Delta p} + L_p^2 \frac{\Delta p}{\hbar} \qquad \left(L_p = \sqrt{\frac{G\hbar}{c^3}} \right) \ . \qquad (2)$$

A heuristic derivation may also be made on dimensional grounds. We think of a particle such as an electron being observed by means of a photon of momentum p. The usual Heisenberg argument leads to an electron position uncertainty given by the first term in Eq.(2). But we should add to this a term due to the gravitational interaction of the electron with the photon, and that term must be proportional to G times the photon energy, or Gpc. Since the electron momentum uncertainty Δp will be of order of p, we see that on dimensional grounds the extra term must be of order $G\Delta p/c^3$, as given in Eq.(2). Note that there is no $\hbar$ in the extra term when expressed in this way. The position uncertainty has a minimum value of $\Delta x = 2L_p$, so the Planck distance plays the role of a fundamental distance.

3. STANDARD HAWKING EFFECT

The Hawking temperature for a spherically symmetric black hole may be obtained in a heuristic way with the use of the standard uncertainty principle and general properties of black holes. We picture the quantum vacuum as a fluctuating sea of virtual particles; the virtual particles cannot normally be observed without violating energy conservation. However near the surface of a black hole there is an effective potential energy that is strong enough to negate the rest energy of a particle and give it zero total energy; of course the surface itself is a one-way membrane which can swallow particles so that they are henceforth not observable from outside. The net effect is that for a pair of photons one photon may be absorbed by the black hole with effective negative energy $-E$, and the other may be emitted to asymptotic distances with positive energy $+E$.

The characteristic energy E of the emitted photons may be estimated from the uncertainty principle. In the vicinity of the black hole surface there is an intrinsic uncertainty in the position of any particle of about the Schwarzschild radius, $\Delta x = r_s$, due to the behavior of its field lines[10] - as well as on dimensional grounds. This leads to a momentum uncertainty

$$\Delta p \approx \frac{\hbar}{\Delta x} = \frac{\hbar}{2r_s} = \frac{\hbar c^2}{4GM} \ , \qquad (3)$$

and hence to an energy uncertainty of $\Delta pc = \hbar c^3/4GM$. We identify this as the characteristic energy of the emitted photon, and thus as a characteristic temperature; it agrees with the Hawking temperature up to a factor 2π, which we will henceforth include as a "calibration factor" and write (with $k_B = 1$),

$$T_{\rm H} \approx \frac{\hbar c^3}{8\pi GM} = \frac{M_p^2 c^2}{8\pi M} \qquad \left(M_p = \sqrt{\frac{\hbar c}{G}} \right) \ . \qquad (4)$$

We know of no way to show heuristically that the emitted photons should have a thermal black body spectrum except on the basis of thermodynamic consistency.

If the energy loss is dominated by photons we may use the Stefan-Boltzmann law to estimate the mass and energy output as functions of time. With use of the Hawking temperature and a mass in units of the Planck mass, $x = M/M_p$, the rate of energy loss is

$$\frac{dx}{dt} = -\frac{1}{60(16)^2\pi T_p}\frac{1}{x^2} = -\frac{1}{t_{ch}}\frac{1}{x^2} \ , \qquad (5)$$

where $T_p = (\hbar G/c^5)^{1/2}$ is the Planck time and $t_{ch} = 60(16)^2\pi T_p \approx 4.8 \times 10^4 T_p$ is a characteristic time for BH evaporation. It follows that the mass and the energy output rate are given by

$$x(t) = \left[x_i^3 - \frac{3t}{t_{ch}} \right]^{1/3} \ , \qquad (6)$$

$$\frac{dx}{dt} = \frac{1}{t_{ch}(x_i^3 - 3t/t_{ch})^{2/3}} \ , \qquad (7)$$

where x_i refers to the initial mass of the hole. The black hole thus evaporates to zero mass in a time given by $t/t_{ch} = (M_i/M_p)^3/3$, and the rate of radiation has an infinite spike at the end of the process.

4. BLACK HOLE REMNANTS

We may use the GUP to derive a black hole temperature exactly as in the previous section. This gives

$$\frac{\Delta p}{\hbar} = \frac{r_s}{2L_p^2}\left[1 \mp \sqrt{1 - 2L_p^2/\Delta x^2}\right] , \tag{8}$$

and therefore

$$T_{\rm GUP} = \frac{Mc^2}{4\pi}\left[1 \mp \sqrt{1 - M_p^2/M^2}\right] . \tag{9}$$

This agrees with the Hawking result for large mass if the negative sign is chosen, whereas the positive sign has no evident physical meaning. Note that the temperature becomes complex and unphysical for mass less than the Planck mass and Schwarzschild radius less than $2L_p$, the minimum size allowed by the GUP. At the Planck mass the slope is infinite, which corresponds to zero heat capacity of the black hole.

The BH evaporation rate is

$$\frac{dx}{dt} = -\frac{16x^6}{t_{ch}}\left[1 - \sqrt{1 - \frac{1}{x^2}}\right]^4 . \tag{10}$$

Thus the hole evaporates to a Planck mass remnant in a time given by

$$\begin{aligned}\frac{t}{t_{ch}} \;=\; & - \;\frac{1}{16}\Big[\frac{8}{3}x_i^3 - 8x_i - \frac{1}{x_i} + \frac{8}{3}(x_i^2 - 1)^{3/2} \\ & - \;4\sqrt{x_i^2 - 1} + 4\cos^{-1}\frac{1}{x_i} + \frac{19}{3}\Big] . \end{aligned} \tag{11}$$

The energy output given by Eq.(10) is finite at the end point when $x = 1$ and is given by $dx/dt = -16/t_{ch}$, whereas for the Hawking case it is infinite at the endpoint when $x = 0$. The present results thus appear to be more physically reasonable.

5. BHRs AS DARK MATTER

Black hole remnants (BHRs) are a natural candidate for cold dark matter[11] since they are a form of weakly interacting massive particles (WIMPs)[12].

The possible source and abundance of BHRs are of interest. The most natural source is in primordial geometric fluctuations, which would be sufficiently large only in the Planck era, at about the Planck temperature. Rigorous derivations[13] as well as simple thermodynamic arguments[14] imply that random fluctuations would produce a Boltzmann distribution of black holes, down to Planck mass, with a number density of $\sim 1/L_p^3$. In one version of standard inflationary cosmology[15] the scale function increases by a factor of about 10^{74} from the Planck era to the present, and since the number density of matter scales as the cube of this we obtain a number density that is down by about 10^{223}, or $10^{-118}/\mathrm{m}^3$. In comparison, the large scale density of dark matter is roughly equal to the critical density, about $2 \times 10^{-26}\mathrm{kg/m}^3$, which implies a BHR number density $\sim 10^{-18}/\mathrm{m}^3$. These are evidently incompatible.

We have made some preliminary considerations of an alternative cosmology containing primordial black holes. In the spirit of Hartle and Hawking[16] we suppose the universe was initially a truly chaotic quantum foam system without ordinary spacetime, and in particular without a time direction in that the signature was (1,1,1,1). A fluctuation in the signature to (-1, 1,1,1) would then produce a time direction and turn it into an exponentially expanding de Sitter space with heavy vacuum density probably equal to the Planck density. At the very beginning of time a thermal distribution of black holes would be produced[13], and during the expansion would decay to Planck size remnants and radiation. The presence of the black holes and radiation (photons, gravitons, etc.) would change the equation of state from that of heavy vacuum with $p = -\rho$, to a mixture of BHR matter with $p = 0$ and radiation with $p = \rho/3$, and presumably very very little residual vacuum energy. This would change the scale function from an exponential to a power-law, and apparently not involve a horizon paradox. If the transition involves a continuous energy density change, the scale function would also be continuous and have a continuous derivative, so it would change according to

$$a(t) = e^{t/T_p} \qquad \rightarrow \qquad a(t) = \left(\frac{e}{n}\right)^n \frac{t^n}{T_p^n} \tag{12}$$

at $t = nT_p$, where n should be between 1/2 for radiation and 2/3 for matter. The duration of exponential expansion would thus be quite short. Very roughly the decrease in number density of BHRs from the beginning of time, $t = 0$, to the present time, $t = t_0$, would then be

$$\left[\frac{a(t_0)}{a(0)}\right]^3 \approx \left(\frac{t_0}{T_p}\right)^{3n} . \tag{13}$$

If n were about 2/3, appropriate to matter, then the scale function would decrease by about 10^{41} and the density factor by about 10^{123}, which implies a present number density of remnants of

$$\rho_{\rm BHR} \approx 10^{-18}{\rm m}^{-3} , \tag{14}$$

which is the value needed. However if a radiation dominated period of expansion is included, as it apparently must, then n should be about 1/2 until the decoupling time t_d, and the present density would be about $10^{10}/{\rm m}^3$, which is far too large. We therefore need to include an ad hoc period of inflation to obtain a reasonable density. Specifically, if we extend the period of inflation from nT_p to ηT_p, followed by a period of radiation dominance to t_d, and then matter dominance to the present (but do not ask that the scale function have a continuous derivative), we obtain roughly

$$\frac{a(t_0)}{a(0)} \approx \frac{e^\eta}{\sqrt{\eta}}\left(\frac{t_0^{2/3}}{T_p^{1/2} t_d^{1/6}}\right) . \tag{15}$$

This gives the desired density if the number of e-folding times is chosen to be about ~ 27, roughly half the number usually used in standard inflation. Thus this scenario shares features with the standard inflation scenario, that the period of inflation and the manner in which the vacuum energy is converted to radiation and particles are ad hoc.

6. CONCLUSION

We have argued the logical existence of black hole remnants based on the generalized uncertainty principle. We then investigated an alternative cosmology in which BHRs are the primary source of dark matter. The initial study indicates that our scenario is not inconsistent with basic cosmological facts, but more scrutiny is required before it can become a viable option.

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