

CERN-TH/2002-143  
PNPI-2002-2484  
SLAC-PUB-9318  
hep-ph/0207297  
July 2002

## High-Energy QCD Asymptotics of Photon–Photon Collisions

Stanley J. Brodsky<sup>§</sup>, Victor S. Fadin<sup>†</sup>, Victor T. Kim<sup>&‡</sup>,  
Lev N. Lipatov<sup>‡</sup> and Grigorii B. Pivovarov<sup>§</sup>

<sup>§</sup> SLAC, Stanford, CA 94309, USA

<sup>†</sup> Budker Institute for Nuclear Physics, Novosibirsk 630090, Russia  
& CERN, CH-1211, Geneva 23, Switzerland

<sup>‡</sup> St. Petersburg Nuclear Physics Institute, Gatchina 188300, Russia

<sup>§</sup> Institute for Nuclear Research, Moscow 117312, Russia

### Abstract

The high-energy behaviour of the total cross section for highly virtual photons, as predicted by the BFKL equation at next-to-leading order (NLO) in QCD, is discussed. The NLO BFKL predictions, improved by the BLM optimal scale setting, are in good agreement with recent OPAL and L3 data at CERN LEP2. NLO BFKL predictions for future linear colliders are presented.

Photon–photon collisions, particularly  $\gamma^*\gamma^*$  processes, play a special role in QCD [1], since their analysis is under much better control than the calculation of hadronic processes, which require the input of non-perturbative hadronic structure functions or wave functions. In addition, unitarization (screening) corrections due to multiple Pomeron exchange should be less important for the scattering of  $\gamma^*$  of high virtuality than for hadronic collisions.

The high-energy asymptotic behaviour of the  $\gamma\gamma$  total cross section in QED can be calculated [2] by an all-orders resummation of the leading terms:  $\sigma \sim \alpha^4 s^\omega$ ,  $\omega = \frac{11}{32}\pi\alpha^2 \simeq 6 \times 10^{-5}$  (Fig. 1). However, the slowly rising asymptotic behaviour of the QED cross section is not apparent since large contributions come from other sources, such as the cut of the fermion-box contribution:  $\sigma \sim \alpha^2(\log s)/s$  [1] (which although subleading in energy dependence, dominates the rising contributions by powers of the QED coupling constant) and QCD-driven processes (Fig. 2).

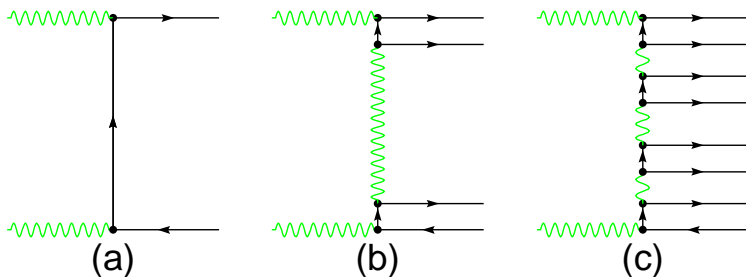


Figure 1: Photon–Photon collisions in QED: (a) electron–box diagram:  $\sigma \sim \alpha^2(\log s)/s$ ; (b) one–photon exchange diagram:  $\sigma \sim \alpha^4 s^0$ ; (c) a typical higher–order diagram; its resummation leads to  $\sigma \sim \alpha^4 s^\omega$ ,  $\omega = \frac{11}{32}\pi\alpha^2$  [2].

The high-energy asymptotic behaviour of hard QCD processes is governed by the Balitsky–Fadin–Kuraev–Lipatov (BFKL) formalism [3, 4]. The highest eigenvalue,  $\omega$ , of the BFKL equation [3] is related to the intercept of the QCD BFKL Pomeron, which in turn governs the high-energy asymptotics of the cross sections:  $\sigma \sim s^{\alpha_{IP}-1} = s^\omega$ . The BFKL Pomeron intercept in the leading order (LO) turns out to be rather large:  $\alpha_{IP}-1 = \omega_{LO} = 12 \ln 2 (\alpha_S/\pi) \simeq 0.55$  for  $\alpha_S = 0.2$  [3]. The next-to-leading order (NLO) corrections to the BFKL intercept have recently been calculated [5], but the results in the  $\overline{\text{MS}}$  scheme have a strong renormalization scale dependence. In Ref. [6] we used the Brodsky–Lepage–Mackenzie (BLM) optimal scale setting procedure [7] to eliminate the renormalization scale ambiguity. (For other approaches to the NLO BFKL predictions, see Refs. [8, 6] and references therein.) The BLM optimal scale setting resums the conformal-violating  $\beta_0$ -terms into the running coupling in all orders of perturbation theory, thus preserving the conformal properties of the theory. The NLO BFKL predictions, as improved by the BLM scale setting, yields  $\alpha_{IP}-1 = \omega_{NLO} = 0.13\text{--}0.18$  [6]. Strictly speaking the integral kernel of the BFKL equation at NLO is not conformally invariant and, hence, one should use a more accurate method for its solution (see Ref. [9]). But in the BLM approach the dependence of the eigenvalue of the kernel from the gluon virtuality is extremely weak [6] and, therefore,  $\omega_{NLO}$  coincides basically with the eigenvalue.

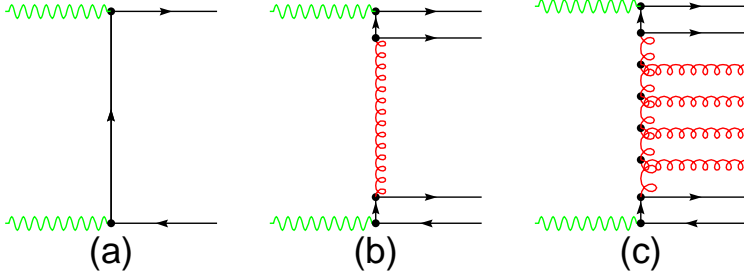


Figure 2: High-energy photon-photon collisions in QCD: (a) quark-box diagram:  $\sigma \sim \alpha^2(\log s)/s$ ; (b) one-gluon exchange diagram:  $\sigma \sim \alpha^2\alpha_S^2s^0$ ; (c) a typical higher-order diagram; its resummation leads to  $\sigma \sim \alpha^2\alpha_S^2s^\omega$ ,  $\omega_{LO} = 12 \ln 2 (\alpha_S/\pi) \simeq 0.55$  [3] and  $\omega_{NLO} = 0.13-0.18$  [6].

The photon-photon cross sections with LO BFKL resummation was considered in Refs. [4, 10, 11, 12]. The total cross section of two unpolarized gammas with virtualities  $Q_A$  and  $Q_B$  in the LO BFKL [11, 4] reads as follows:

$$\sigma(s, Q_A^2, Q_B^2) = \sum_{i,k=T,L} \frac{1}{\pi \sqrt{Q_A^2 Q_B^2}} \int_0^\infty \frac{d\nu}{2\pi} \cos\left(\nu \ln\left(\frac{Q_A^2}{Q_B^2}\right)\right) F_i(\nu) F_k(-\nu) \left(\frac{s}{s_0}\right)^{\omega(Q^2, \nu)}, \quad (1)$$

with the gamma impact factors in the LO for the transverse and longitudinal polarizations:

$$F_T(\nu) = F_T(-\nu) = \alpha \alpha_S \left( \sum_q e_q^2 \right) \frac{\pi}{2} \frac{\left[\frac{3}{2} - i\nu\right] \left[\frac{3}{2} + i\nu\right] \Gamma\left(\frac{1}{2} - i\nu\right)^2 \Gamma\left(\frac{1}{2} + i\nu\right)^2}{\Gamma(2 - i\nu) \Gamma(2 + i\nu)}$$

$$F_L(\nu) = F_L(-\nu) = \alpha \alpha_S \left( \sum_q e_q^2 \right) \pi \frac{\Gamma\left(\frac{3}{2} - i\nu\right) \Gamma\left(\frac{3}{2} + i\nu\right) \Gamma\left(\frac{1}{2} - i\nu\right) \Gamma\left(\frac{1}{2} + i\nu\right)}{\Gamma(2 - i\nu) \Gamma(2 + i\nu)},$$

where a Regge scale parameter  $s_0$  is proportional to a hard scale  $Q^2 \sim Q_A^2, Q_B^2$ ;  $\Gamma$  is the Euler  $\Gamma$ -function and  $e_q$  is the quark electric charge.

Although the NLO impact factor of the virtual photon is not known [13], one can use the LO impact factor of Refs. [2, 11], assuming that the main energy-dependent NLO corrections come from the NLO BFKL subprocess rather than from the photon impact factors [14, 15].

Fig. 3 compares the LO and BLM scale-fixed NLO BFKL predictions  $\sigma \sim \alpha^2\alpha_S^2s^\omega$  [6, 14, 15] with recent CERN LEP2 data from OPAL [16] and L3 [17]. The spread in the curves reflects the uncertainty in the choice of the Regge scale parameter, which defines the beginning of the asymptotic regime:  $s_0 = Q^2$  to  $4Q^2$  for LO and NLO BFKL, where  $Q^2$  is the mean virtuality of the colliding photons. One can see from Fig. 3 that the agreement of the NLO BFKL predictions [14, 15, 6] with the data is quite good. The sensitivity of the NLO BFKL results to the Regge parameter  $s_0$  is much smaller than in the case of the LO BFKL. The variation of the predictions in the value of  $s_0$  reflects uncertainties from uncalculated

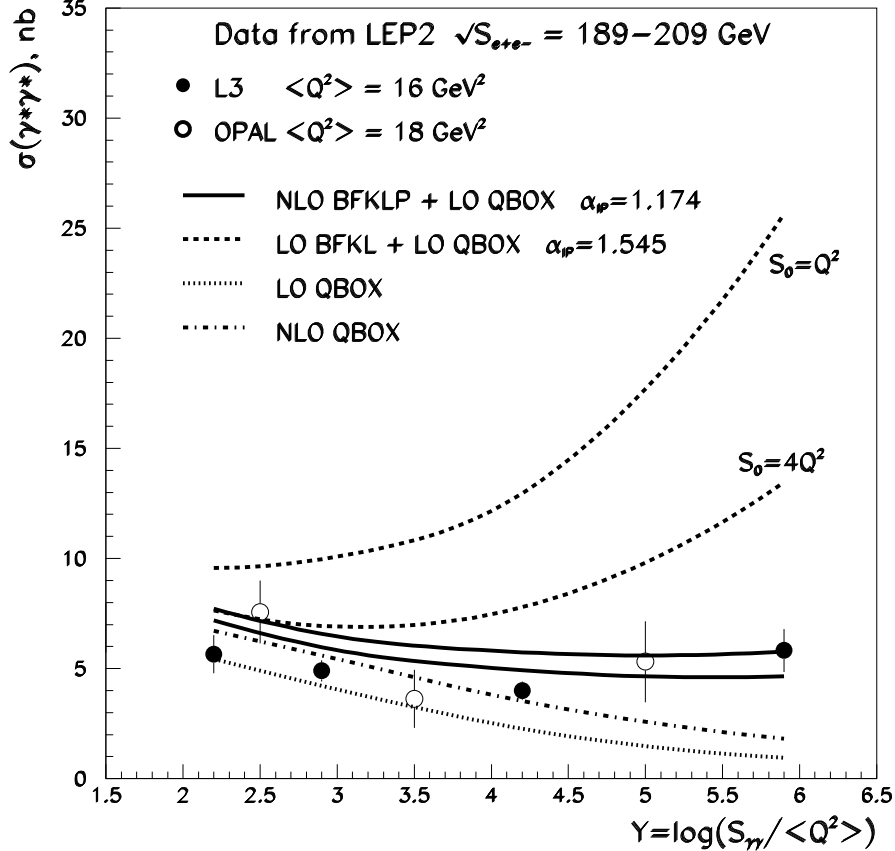


Figure 3: The energy dependence of the total cross section for highly virtual photon–photon collisions predicted by the BLM scale-fixed NLO BFKL [14, 15, 6] compared with OPAL [16] and L3 [17] data from LEP2 at CERN. The (solid) dashed curves correspond to the (N)LO BFKL predictions for two different choices of the Regge scale:  $s_0 = Q^2$  for upper curves and  $s_0 = 4Q^2$  for lower curves.

subleading terms. The parametric variation of the LO BFKL predictions is so large that it can be, in fact, neither ruled out nor confirmed at the energy range of CERN LEP2.

The double-logarithmic DGLAP asymptotics related with  $\log(Q_A^2/Q_B^2)$ -terms for the total photon–photon cross section was considered in Ref. [12] and found to be small for the CERN LEP2 kinematical region. The point is that most of the CERN LEP2 data [16, 17, 18] are collected at the approximately equal virtualities of the colliding photons:  $1/2 < Q_A^2/Q_B^2 < 2$ . It should be stressed that the soft Pomeron contribution to the  $\gamma^*\gamma^*$  total cross section, if estimated within the vector-dominance model, is proportional to  $\sigma_{\gamma^*\gamma^*} \sim (m_V^2/Q^2)^4 \sigma_{\gamma\gamma}$  and therefore suppressed for such highly virtual photons as those under consideration.

We also note that the NLO BFKL predictions are consistent [15] with data recently presented by ALEPH [18]. In contrast, the NLO quark-box contribution [19] underestimates the L3 data point at  $Y \equiv \log(s_{\gamma\gamma}/\langle Q^2 \rangle) = 6$  by 4 standard deviations. Indeed, the NLO quark-box contribution [19], calculated in massless approximation, can be scaled down from general considerations with the quark masses. For example, at leading order, the inclusion of masses to the quark-box diagram reduces its contribution by 10-15% [19]. Also, the one–

gluon exchange added to the (N)LO quark-box contribution is not sufficient to describe the data at  $Y = 6$  within (3) 4 standard deviations (see also Fig. 4).

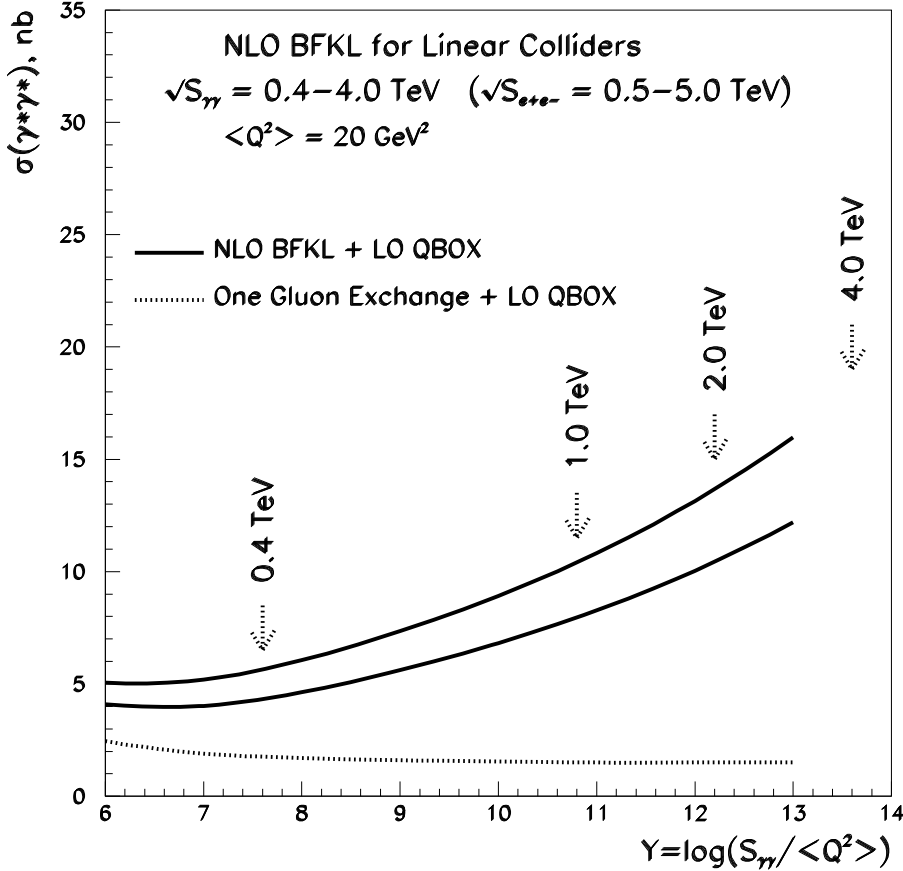


Figure 4: The energy dependence of the total cross section for virtual photon–photon collisions predicted by the NLO BFKL for future linear colliders. The solid curves correspond to the BLM scale-fixed NLO BFKL predictions with  $s_0 = Q^2$  (upper curve) and  $s_0 = 4Q^2$  (lower curve). The dotted curve shows the one-gluon exchange contribution.

In Fig. 4 the BLM fixed-scale NLO BFKL predictions for a future linear collider with the photon-photon collider option ( $\sqrt{s_{\gamma\gamma}} \leq 0.8\sqrt{s_{e+e-}}$ ) under discussion [22] are shown.

The NLO BFKL phenomenology is consistent with the assumption of small unitarization corrections in the photon–photon scattering at large  $Q^2$ . Thus one can accommodate the NLO BFKL Pomeron intercept value 1.13–1.18 [6] predicted by the BLM optimal scale setting. In the case of hadron scattering, the larger unitarization corrections [20] lead to a smaller effective Pomeron intercept value, about 1.10 [21].

In summary, highly virtual photon–photon collisions provide a very unique opportunity to test high-energy asymptotics of QCD. The NLO BFKL predictions for the  $\gamma^*\gamma^*$  total cross section, with the renormalization scale fixed by the BLM procedure, show good agreement

with the recent data from OPAL [16] and L3 [17] at CERN LEP2. The obtained results can be very important for future lepton and photon colliders.

The authors thank V. A. Andreev, A. Bohrer, A. De Roeck, J. R. Ellis, J. H. Field, I. M. Ginzburg, A. B. Kaidalov, V. A. Khoze, M. Kienzle-Focacci, M. Krawczyk, C.-N. Lin, V. A. Schegelsky, V. G. Serbo, M. Przybycień, A. A. Vorobyov and M. Wadhwa for helpful discussions. This work was supported in part by the Russian Foundation for Basic Research (RFBR), the INTAS Foundation, the U.S. National Science Foundation, and the U.S. Dept. of Energy under contract No. DE-AC03-76SF00515.

## References

- [1] For a review, see V. M. Budnev, I. F. Ginzburg, G. V. Meledin and V. G. Serbo, *Phys. Rep.* **C15**, 181 (1975).
- [2] V. N. Gribov, L. N. Lipatov and G. V. Frolov, *Phys. Lett.* **B31**, 34 (1970); *Yad. Fiz.* **12**, 994 (1970) [*Sov. J. Nucl. Phys.* **12**, 543 (1971)]; H. Cheng and T. T. Wu, *Phys. Rev.* **D1**, 2775 (1970).
- [3] V. S. Fadin, L. N. Lipatov and E. A. Kuraev, *Phys. Lett.* **B60**, 50 (1975); *Zh. Eksp. Teor. Fiz.* **71**, 840 (1976) [*Sov. Phys. - JETP* **44**, 443 (1976)]; *ibid.* **72**, 377 (1977) [**45**, 199 (1977)].
- [4] I. I. Balitsky and L. N. Lipatov, *Yad. Fiz.* **28**, 1597 (1978) [*Sov. J. Nucl. Phys.* **28**, 822 (1978)].
- [5] V. S. Fadin and L. N. Lipatov, *Phys. Lett.* **B429**, 127 (1998); G. Camici and M. Ciafaloni, *Phys. Lett.* **B430**, 349 (1998).
- [6] S. J. Brodsky, V. S. Fadin, V. T. Kim, L. N. Lipatov and G. B. Pivovarov, *Pis'ma ZhETF* **70**, 161 (1999) [*JETP Lett.* **70**, 155 (1999)].
- [7] S. J. Brodsky, G. P. Lepage and P. B. Mackenzie, *Phys. Rev.* **D28**, 228 (1983).
- [8] B. Andersson, G. Gustafson and J. Samuelsson, *Nucl. Phys.* **B467** 443, 1996; M. Ciafaloni, D. Colferai and G. P. Salam, *Phys. Rev.* **D60**, 114036 (1999); R. S. Thorne, *Phys. Rev.* **D60**, 054031 (1999); G. Altarelli, R. D. Ball and S. Forte, *Nucl. Phys.* **B599**, 383 (2001).
- [9] L. N. Lipatov, *Zh. Eksp. Teor. Fiz.* **90**, 1536 (1986) [*Sov. Phys. - JETP* **63**, 904 (1986)]; *Phys. Rep.* **C286**, 131 (1997); L. V. Gribov, E. M. Levin and M. G. Ryskin, *Phys. Rep.* **C100**, 1 (1983).
- [10] J. Bartels, A. De Roeck and H. Lotter, *Phys. Lett.* **B389**, 742 (1996); A. Białas, W. Czyż and W. Florkowski, *Eur. Phys. J.* **C2**, 683 (1998); J. Kwieciński and L. Motyka, *Acta Phys. Pol.* **B30**, 1817 (1999); *Eur. Phys. J.* **C18**, 343 (2000); J. Bartels, C. Ewerz and R. Staritzbichler, *Phys. Lett.* **B492**, 56 (2000); N. N. Nikolaev, J. Speth and V. R. Zoller, *Zh. Eksp. Teor. Fiz.* **93**, 1104 (2001) [*JETP* **93**, 957 (2001)].

- [11] S. J. Brodsky, F. Hautmann and D. E. Soper, Phys. Rev. **D56**, 6957 (1997); Phys. Rev. Lett. **78**, 803 (1997), (E) **79**, 3544 (1997).
- [12] M. Boonekamp, A. De Roeck, C. Royon and S. Wallon, Nucl. Phys. **B555**, 540 (1999).
- [13] V. S. Fadin, D. Yu. Ivanov and M. I. Kotsky, BUDKER-INP-2001-33, DFCAL-TH-01-2 (2001), hep-ph/0106099; J. Bartels, S. Gieseke and C. F. Qiao, Phys. Rev. **D63**, 056014 (2001), (E) **D65**, 079902 (2002); J. Bartels, S. Gieseke and A. Kyrieleis, Phys. Rev. **D65**, 014006, (2002).
- [14] V. T. Kim, L. N. Lipatov and G. B. Pivovarov, Proc. 29th Int. Symposium on Multi-particle Dynamics (ISMD99), Providence, USA, 1999, hep-ph/9911242; Proc. 8th Blois Workshop (EDS99), Protvino, Russia, 1999, hep-ph/9911228; V. S. Fadin, V. T. Kim, L. N. Lipatov and G. B. Pivovarov, Proc. XXXV PNPI Winter School, ed. V. A. Gordeev *et al.*, pp. 259-278 (St. Petersburg, 2001), hep-ph/0207296.
- [15] S. J. Brodsky, V. S. Fadin, V. T. Kim, L. N. Lipatov and G. B. Pivovarov, presented at PHOTON2001, Ascona, Switzerland, 2001, SLAC-PUB-9069, CERN-TH/2001-341, hep-ph/0111390, to appear in the Proc.
- [16] OPAL, G. Abbiendi *et al.*, Eur. Phys. J. **C24**, 17 (2002).
- [17] L3, P. Achard *et al.*, Phys. Lett. **B531**, 39 (2002).
- [18] ALEPH, presented by G. Prange at PHOTON2001, Ascona, Switzerland, 2001, to appear in the Proc.
- [19] M. Cacciari, V. Del Duca, S. Frixione and Z. Trocsanyi, JHEP **0102**, 029 (2001).
- [20] A. B. Kaidalov, L. A. Ponomarev and K. A. Ter-Martirosyan, Yad. Fiz. **44**, 722 (1986) [Sov. J. Nucl. Phys. **44**, 468 (1986)]; M. S. Dubovikov, B. Z. Kopeliovich, L. I. Lapidus and K. A. Ter-Martirosyan, Nucl. Phys. **B123**, 147 (1977); B. Z. Kopeliovich and L. I. Lapidus, Zh. Eksp. Teor. Fiz. **71**, 61 (1976) [Sov. Phys. - JETP **44**, 31 (1976)].
- [21] J. R. Cudell, V. Ezhela, K. Kang, S. Lugovsky and N. Tkachenko, Phys. Rev. **D61**, 034019 (2000), (E) **D63**, 059901 (2001); J. R. Cudell, A. Donnachie and P. V. Landshoff, Phys. Lett. **B448**, 281 (1999); M. M. Block, E. M. Gregores, F. Halzen and G. Pancheri, Phys. Rev. **D58**, 017503 (1998); P. Gauron and B. Nicolescu, Phys. Lett. **B486**, 71 (2000).
- [22] M. M. Velasco *et al.*, APS/DPF/DPB Summer Study on the Future of Particle Physics (Snowmass 2001), Snowmass, USA, 2001, hep-ex/0111055; ECFA/DESY Photon Collider Working Group, B. Badelek *et al.*, DESY-TESLA-2001-23, DESY-TESLA-FEL-2001-05 (2001), hep-ex/0108012; CLIC Study Group, J.-P. Delahaye *et al.*, CERN-PS/98-009-LP (1999), Acta Phys. Pol. **B30**, 2029 (1999); ACFA Linear Collider Working Group, K. Abe *et al.*, KEK-REPORT-2001-11 (2001), hep-ph/0109166.