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Lifetime differences in heavy mesons with time independent measurements*

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Abstract

Heavy meson pairs produced in the decays of heavy quarkonium resonances at e^+e^- machines (beauty and tau-charm factories) have the useful property that the two mesons are in the CP-correlated states. By tagging one of the mesons as a CP eigenstate, a lifetime difference of heavy neutral meson mass eigenstates $\Delta\Gamma$ may be determined by measuring the leptonic branching ratio of the other meson. We discuss the use of this and related methods both in the case where time dependent mixing is small and when it is significant. We consider the impact of possible CP-violating effects and present the complete results for CP-entangled decay rates with CP-violation taken into account.

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1 Introduction

One of the most important motivations for studies of heavy meson mixing is the possibility of observing a signal from new physics which can be separated from the one generated by the Standard Model (SM) interactions. If H^0 is a neutral heavy meson, $\Delta Q = 2$ transitions, occurring only at one loop in the Standard Model, as well as possible new interactions, generate contributions to the effective operators that change H^0 state into $\overline{H}^0$ state leading to the mass eigenstates

$$|H_{\frac{1}{2}}\rangle = p|H^0\rangle \pm q|\bar{H}^0\rangle, \tag{1}$$

where the complex parameters p and q are obtained from diagonalizing the $H^0 - \overline{H}^0$ mass matrix [1]. It is conventional to parameterize the mass and width splittings between these eigenstates by

$$x \equiv \frac{m_2 - m_1}{\Gamma}, \quad y \equiv \frac{\Gamma_2 - \Gamma_1}{2\Gamma}, \tag{2}$$

where $m_{1,2}$ and $\Gamma_{1,2}$ are the masses and widths of $H_{1,2}$ and the mean width and mass are $\Gamma = (\Gamma_1 + \Gamma_2)/2$ and $m = (m_1 + m_2)/2$. Since y is constructed from the decays of H into physical states, it should be dominated by the Standard Model contributions, unless new physics significantly modifies $\Delta Q = 1$ interactions.

Our goal in this paper is to discuss time independent methods to determine y for the various oscillating hadron species (i.e. $H = D^0$, B^0 and B_s) at electron-positron colliders where an $H\overline{H}^0$ pair can be produced in a correlated initial state. In particular we will consider methods which are time independent and therefore may be applied in experiments where the time history is difficult to obtain. Results which are linear, rather then quadratic, in y are of particular interest because y is generically small for B_d^0 and D^0 mesons.

Indeed, in the case of D^0, such studies may be carried out at tau-charm factories running on the $\psi(3370)$ resonance. In this case x is small and it could well be that $y \gg x$. It has been argued that the Standard Model $y > x$ [2] with $y \sim O(1\%)$. Thus, lifetime differences may be the dominant form of mixing in D^0 and their study is perhaps within the reach of proposed experiments.

In the case of B_d^0, $x = 0.730 \pm 0.029$ [3] and y is small, of the order of 0.3% [4]. It is possible that at future B-factories there would be enough statistics to measure y. In addition, if e^+e^- machines are run at the $\Upsilon(5s)$ resonance, these methods could be used to investigate y in the B_s. In the B_s system, it has been estimated that y may be particularly large (5-15%) [5] and indeed similar methods have been previously discussed in [6].

In the cases of B_s and D^0, it is thought that CP violation in the mixing is small while in B^0, mixing is known to produce large CP violating effects [7, 8]. In our methods, CP violation in mixing will impact the signal, generally reducing it, so we will derive the formalism in a framework which includes CP violation.

The paper will proceed as follows. In Section 2 we will discuss the formalism which applies in the case where CP violation is negligible. In Section 3 we will discuss the application to $D^0\overline{D}^0$ production at a charm factory assuming there is no CP violation in D^0 oscillation or decay. In Section 4 we will generalize the formalism to the case where CP violation is present and consider the application to B and D mesons. In Section 5 we will give our conclusions. In the appendix we give the time integrated correlated decay rates for H^0H^0 decaying to various combinations of final states where indirect CP violation is present.

2 Formalism if CP is Conserved

The essential point of our method is best illustrated in cases where CP violation may be neglected. In such cases, $p = q$, so mass eigenstates also become eigenstates of CP, which we denote:

$$|H_{\pm}\rangle = \frac{1}{\sqrt{2}}\left[|H^0\rangle \pm |\bar{H}^0\rangle\right]. \tag{3}$$

The crucial point here is that these CP eigenstates $|H_{\pm}\rangle$ do not evolve with time. We can take advantage of this fact at threshold e^+e^- machines, such as BaBar or CLEO-c. $H^0\bar{H}^0$ production at these machines is through resonances leading to HH pairs being in a quantum mechanically coherent state. Thus, if the production resonance has angular momentum L, the quantum mechanical state at the time of $H\overline{H}^0$ production is

$$\Psi_L \equiv |H^0\overline{H}^0\rangle_L = \frac{1}{\sqrt{2}}\left\{|H^0(k_1)\overline{H}^0(k_2)\rangle + (-1)^L|H^0(k_2)\overline{H}^0(k_1)\rangle\right\}. \tag{4}$$

where k_1 and k_2 are the momenta of the mesons. Rewriting this in terms of the CP basis we arrive at

$$\begin{aligned}\Psi_{2n} &= \frac{1}{\sqrt{2}}\left\{|H_+(k_1)H_+(k_2)\rangle - |H_-(k_1)H_-(k_2)\rangle\right\} \\ \Psi_{2n+1} &= \frac{1}{\sqrt{2}}\left\{|H_-(k_1)H_+(k_2)\rangle + |H_+(k_1)H_-(k_2)\rangle\right\}\end{aligned} \tag{5}$$

Thus in the $L =$ odd case, which would apply to the experimentally important $\psi(3770)$ and $\Upsilon(4S)$ resonances, the CP eigenstates of the H mesons are anti-correlated while if $L =$ even the eigenstates are correlated[1]. In either case the correlation between the eigenstates is independent of when they decay. In this way, if meson $H(k_1)$ decays to the final state which is also a CP eigenstate, then the CP eigenvalue of the meson $H(k_2)$ can be determined.

[1]While $L =$ even resonances are not directly produced in e^+e^- collisions, quantum-mechanically *symmetric* states can be produced in the decays, such as $\psi(4140) \to D\overline{D}\gamma$. In the following, $L =$ even case can also refer to this situation.

Using this eigenstate correlation as a tool to investigate CP violation has been suggested by [9]. In this paper we will take advantage of such correlations for the experimental investigation of lifetime differences. The idea is fairly straightforward: we look at decays of the form $\psi_L \to (H \to S_\sigma)(H \to Xl\nu)$ where S_σ is a CP eigenstate of eigenvalue $\sigma = \pm 1$ and ψ_L generically means any resonance of angular momentum L that decays to $H\overline{H}^0$ (see also a footnote after the Eqn. (5).

In this scenario, the CP quantum numbers of the $H(k_2)$ is thus determined. The semi-leptonic *width* of this meson should be independent of the CP quantum number since it is flavor specific. It follows that the semi-leptonic *branching ratio* of $H(k_2)$ will be inversely proportional to the total width of that meson. Since we know whether $H(k_2)$ is a H_+ or and H_- from the decay of $H(k_1)$, we can easily determine y in terms of the semileptonic branching ratios of $H_\pm$.

This can be expressed simply by introducing the ratio

$$R^L_\sigma = \frac{\Gamma[\psi_L \to (H \to S_\sigma)(H \to Xl^\pm \nu)]}{\Gamma[\psi_L \to (H \to S_\sigma)(H \to X)]\ Br(H^0 \to Xl\nu)}, \tag{6}$$

where X in $H \to X$ stands for an inclusive set of all final states. A deviation from $R^L_\sigma = 1$ implies a lifetime difference. From this experimentally obtained quantity, we extract y by

$$R^L_\sigma = \frac{1}{1+(-1)^L \sigma y}, \qquad y = \sigma(-1)^L \frac{R^L_\sigma - 1}{R^L_\sigma} \tag{7}$$

3 Charmed Mesons if CP is Conserved

Let us consider now the production of $D^0\overline{D}^0$ mesons at an electron positron collider. In the Standard Model, CP violation is expected to be small in D^0 hence the above formalism should apply directly to this case. For instance, the new tau-charm factory, under construction at CESR, will allow for simple and effective CP tagging in the case of $\psi(3770) \to D\overline{D}$ because there are numerous candidates for S_σ in D^0 decay which have branching ratios in the few percent range, for instance $K_S\pi^0$ (1.05%); $K_S\omega$ (1.05%); $K_S\eta'$ (0.85%); $\pi^+\pi^-$ (0.15%). In additon, the modes $K^{*0}\pi^0$ and $K^{*0}\rho^0$ may be used provided the K^{*0} itself decays to a CP eigenstate, $K_{S,L}\pi^0$ and one can separate the main amplitude from cross channel processes. Similar comments apply to analogous states containing higher neutral kaonic resonances.

These modes are thus candidates for S_σ in Eqn. (6). If we write Eqn. (6) in terms of the semi-leptonic branching ratio of $D_\pm$, $\mathcal{B}^\ell_\pm$, then equation Eqn. (7) becomes:

$$\left(\frac{\mathcal{B}^\ell_+(D)}{\mathcal{B}^\ell_-(D)} - \frac{\mathcal{B}^\ell_-(D)}{\mathcal{B}^\ell_+(D)} \right) = 4y. \tag{8}$$

In either case the statistical uncertainty in y is given by

$$\Delta y = \left(2N_0 \mathcal{A}^\ell \mathcal{B}^\ell \mathcal{A}^\sigma \mathcal{B}^\sigma\right)^{-\frac{1}{2}} \tag{9}$$

where N_0 is the initial number of ψ's, $\mathcal{A}^\sigma$ and $\mathcal{A}^\ell$ are the acceptances for the CP eigenstate modes and the semileptonic modes respectively while $\mathcal{B}^\sigma$ and $\mathcal{B}^\ell$ are the branching ratios for those modes. In general, of course, we can combine the statistics for a number of modes so, as an example, if we assume that $\mathcal{B}^\ell = 12\%$, $\mathcal{B}^\sigma = 2\%$, with $\mathcal{A}^\ell \mathcal{A}^\sigma = 0.1$ then $N_0 = 10^8$ gives $\Delta y = 0.5\%$.

At present, the information about the $D^0 - \overline{D}^0$ mixing parameters x and y comes from the time-dependent analyses that can roughly be divided into two categories. First, more traditional analyses study time dependence of $D \to f$ decays, where f is the final state that can be used to tag the flavor of the decayed meson. The most popular is the non-leptonic doubly Cabibbo suppressed (DCS) decay $D \to K^+\pi^-$. Time-dependent studies allow one to separate the doubly Cabibbo suppressed decay from the mixing contribution,

$$\Gamma[D^0(t) \to K^+\pi^-] = e^{-\Gamma t}|A_{K^-\pi^+}|^2 \times \left[R + \sqrt{R}R_m(y'\cos\phi - x'\sin\phi)\Gamma t + \frac{R_m^2}{4}(y^2+x^2)(\Gamma t)^2\right], \quad (10)$$

where R is the ratio of Cabibbo favored (CF) and doubly Cabibbo suppressed decay rates and $R_m = |p/q|$ while $\phi = arg(p/q)$. Since x and y are small, the best constraint comes from the linear terms in t that are also linear in x and y. Using this method, direct extraction of x and y is not possible from Eq. (10) due to unknown relative strong phase δ of DCS and CF amplitudes (see [10] for extensive discussion), as $x' = x\cos\delta + y\sin\delta$, $y' = y\cos\delta - x\sin\delta$. This phase, however, can be measured independently [11]. The corresponding formula can also be written for $\overline{D}^0$ decay with $x' \to -x'$ and $R_m \to R_m^{-1}$ [12].

Another method to measure D^0 mixing is to compare the lifetimes extracted from the analysis of D decays into the CP-even and CP-odd final states. This study is also sensitive to a linear function of y, via

$$\frac{\tau(D \to K^-\pi^+)}{\tau(D \to K^+K^-)} - 1 = y\cos\phi + x\sin\phi\left[\frac{1-R_m^2}{2}\right]. \quad (11)$$

Time-integrated studies of the semileptonic transitions are sensitive to the quadratic form $x^2 + y^2$ and at the moment are not competitive with the analyses discussed above.

4 Formalism if CP is Violated in $H^0\overline{H}^0$ Oscillations

In the case where CP violation is present in the $H^0\overline{H}^0$ mixing, it is necessary to consider general time dependent entangled states of the $H^0\overline{H}^0$ pair. Following the notation of [1], we will denote the wave function $|H(t)\rangle$ at a given moment in time, t, by a two element vector:

$$|H(t)\rangle = a|H^0\rangle + \overline{a}|\overline{H}^0\rangle \equiv \begin{pmatrix} a \\ \overline{a} \end{pmatrix} \quad (12)$$

CPT conservation forces the general mass matrix in the following form

$$\mathcal{M} \equiv \hat{M} + i\hat{\Gamma}/2 = \begin{pmatrix} A & p^2 \\ q^2 & A \end{pmatrix} \tag{13}$$

where $\hat{M}$ and $\hat{\Gamma}$ are Hermitian while A, p and q are in general complex numbers. The effects of CP violation in the system are usually parameterized in terms of the ratio:

$$\frac{p}{q} \equiv \frac{1+\epsilon}{1-\epsilon} \equiv R_m e^{i\phi} \tag{14}$$

In the limit of CP conservation in mixing matrix, $R_m = 1$. Even if CP is violated, in the case of heavy neutral mesons, it is expected that $R_m \approx 1$. The phase ϕ, of course, depends on the convention one uses for weak phases that can be traded off against the weak phase in the decay in the usual way. In our discussion it will be useful to assume that we are using a convention where we have absorbed any weak phase from the decay into the the mixing.

For an isolated H meson, the wave function at time t is related to the the wave function at $t = 0$ by:

$$|H(t)\rangle = U_t |H(0)\rangle, \tag{15}$$

where the time evolution operator U_t satisfies the equation

$$i\frac{dU_t}{dt} = \mathcal{M} U_t, \qquad U_0 = 1. \tag{16}$$

This Schrödinger-like equation can be solved to yield the familiar result

$$U_t = \begin{pmatrix} g_+(t) & (p/q)\ g_-(t) \\ (q/p)\ g_-(t) & g_+(t) \end{pmatrix}. \tag{17}$$

Here, the time dependence of D^0 and $\overline{D}^0$ is driven by

$$\begin{aligned} g_+(t) &= (\cosh y\tau/2 \cos x\tau/2 - i \sinh y\tau/2 \sin x\tau/2)\, e^{-\mu\tau/2} \\ g_-(t) &= (-\sinh y\tau/2 \cos x\tau/2 + i \cosh y\tau/2 \sin x\tau/2)\, e^{-\mu\tau/2} \end{aligned} \tag{18}$$

with $\tau = \Gamma t$ and $\mu = 1 + 2im/\Gamma$.

Let us now consider the time integrated decay rate for a single H to a final state f. If a and $\overline{a}$ are the amplitudes for H^0 and $\overline{H}^0$ to decay to f respectively and $|\psi_0\rangle$ is the initial wave function for the meson, then the time integrated decay rate is

$$2\Gamma_f(\rho_0) = (Q+P)tr[\rho_f\rho_0] + (Q-P)tr[v^\dagger \rho_f \rho_0] - 2Re\left[(yQ - ixP)tr[\rho_f v \rho_0]\right], \tag{19}$$

where

$$\rho_0 = |\psi_0\rangle\langle\psi_0| \qquad \rho_f = \begin{pmatrix} |a|^2 & a^*\overline{a} \\ \overline{a}^* a & |\overline{a}| \end{pmatrix} \qquad v = \begin{pmatrix} 0 & p/q \\ q/p & 0 \end{pmatrix} \tag{20}$$

and

$$P = 1/(1+x^2) \qquad Q = 1/(1-y^2). \tag{21}$$

Of particular interest is the case case where f is a CP eigenstate with CP=$\sigma = \pm 1$. If we assume $R_m = 1$, as would be the case for $B_d \to \psi K_s$ in the Standard Model, then

$$\frac{1}{2}\left(\Gamma_f(H^0) + \Gamma_f(\overline{H}^0)\right) = \frac{1 - \sigma y \cos\phi}{1 - y^2}\Gamma_0 \tag{22}$$

$$\frac{1}{2}\left(\Gamma_f(H^0) - \Gamma_f(\overline{H}^0)\right) = \sigma\frac{x \sin\phi}{1 + x^2}\Gamma_0, \tag{23}$$

where Γ_0 is the decay rate for H to f.

This can be easily generalized to the case of the entangled initial state which presents itself in the creation of $H^0\overline{H}^0$ pairs from a Ψ_L resonance. As was shown in Eqns. (4,5), the states of interest can be decomposed into the coherent sum of products of flavor (or CP) eigenstates. Using Eqn. (4) we can write the time integrated correlated decay rate for $\Psi_L \to (H \to f_1)(H \to f_2)$ is:

$$\begin{aligned} \Gamma^{f_1 f_2}(\Psi_L) &= \Gamma_{f_1}(H^0)\Gamma_{f_2}(\overline{H}^0) + \Gamma_{f_1}(\overline{H}^0)\Gamma_{f_2}(H^0) \\ &+(-1)^L\left[\Gamma_{f_1}(\rho_{+-})\Gamma_{f_2}(\rho_{-+}) + \Gamma_{f_1}(\rho_{-+})\Gamma_{f_2}(\rho_{+-})\right] \end{aligned} \tag{24}$$

where ρ_{ik} are the matrices:

$$\begin{aligned} \rho_{++} &= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \qquad \rho_{--} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \\ \rho_{+-} &= \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \qquad \rho_{-+} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \end{aligned} \tag{25}$$

Let us return to the calculation of y through the determination of R^L_σ defined in Eqn. (6). In the case $L = 1$ the numerator is $\Gamma^{S_\sigma X_l}(\Psi_1)$ and so Eqn. (24) implies:

$$\Gamma^{S_\sigma X_l}(\Psi_L) \propto \frac{2 + x^2(R_m^{\pm 2} + 1) + y^2(R_m^{\pm 2} - 1)}{2(1+x^2)(1-y^2)} \tag{26}$$

where we assume there is no further CP violation in the decay amplitude, and the $\pm$ signs are for the positively and negatively charged leptons respectively.

The denominator of Eqn. (6) is given by Eqn. (24) where f_1 is S_σ and all possible values of f_2 are summed over. In this case the L dependent terms vanish and the rest simplifies to:

$$\Gamma^{f_1 X}(\Psi_L) \equiv \sum_{f_2} \Gamma^{f_1 f_2}(\Psi_L) = \Gamma_{f_1}(\mathbf{1})/2, \tag{27}$$

where $\mathbf{1}$ is the identity matrix. This is proportional to the average between the time integrated decay rate of H^0 and $\overline{H}^0$ to the final state f_1. It is easiest to see that in the CP-eigenstate basis spanned by the states of Eq. (5). Indeed, in the particular example of $L =$ odd the time-integrated decay rate is

$$\begin{aligned}\Gamma^{f_1 X}(\Psi_1) = \int dt_1 dt_2 \, \frac{1}{2} \Big[& Tr \left[U_{t2}^\dagger \, \rho_X \, U_{t2} \, \rho_{++} \right] Tr \left[U_{t1}^\dagger \, \rho_{f1} \, U_{t1} \, \rho_{--} \right] \\ & + Tr \left[U_{t2}^\dagger \, \rho_X \, U_{t2} \, \rho_{--} \right] Tr \left[U_{t1}^\dagger \, \rho_{f1} \, U_{t1} \, \rho_{++} \right] \\ & - Tr \left[U_{t2}^\dagger \, \rho_X \, U_{t2} \, \rho_{+-} \right] Tr \left[U_{t1}^\dagger \, \rho_{f1} \, U_{t1} \, \rho_{-+} \right] \\ & - Tr \left[U_{t2}^\dagger \, \rho_X \, U_{t2} \, \rho_{-+} \right] Tr \left[U_{t1}^\dagger \, \rho_{f1} \, U_{t1} \, \rho_{+-} \right] \Big],\end{aligned} \tag{28}$$

where ρ_X and ρ_{f1} are the matrices for $H \to X$ and $H \to f_1$ decay amplitudes respectively (see Eqn (20)). It is easy to see that in the *mass eigenstate* basis $U_{t2}^\dagger \, \rho_X \, U_{t2}|_{mass} = diag(\Gamma_1 e^{-\Gamma_1 t_2}, \Gamma_2 e^{-\Gamma_2 t_2})$. In principle, this needs to be translated back to the CP eigenstate basis. However, integration with respect to t_2 yields a unit matrix, which is invariant under the change of basis. This simplifies the Eq. (28) considerably, which, after taking the corresponding traces transforms into

$$\Gamma^{f_1 X}(\Psi_1) = \frac{1}{2} \int dt \; Tr \left[\rho_{f1} \, U_t \, U_t^\dagger \right],$$

which, in the limit $R_m = 1$, becomes for the semileptonic final state (a complete expression is available in the Appendix)

$$\Gamma^{X X_l}(\Psi_L) \propto \frac{1 + (-1)^L \cos\phi}{(1 - y^2).} \tag{29}$$

In the limit $R_m = 1$ the ratio of Eqs. (26) and (29) becomes:

$$R_\sigma^L = \frac{1}{1 + (-1)^L \sigma y \cos\phi} \tag{30}$$

In which case the generalization of Eqn. (7) is

$$y \cos\phi = (-1)^L \sigma \frac{R_\sigma^L - 1}{R_\sigma^L}. \tag{31}$$

So we can regard the measurement of R^L_σ as leading to a determination of $y\cos\phi$. A similar result holds for the non-leptonic final state (such as $D \to K\pi$, with corrections proportional to R). In the case where $R_m \neq 1$ the corresponding expression depends on x as well as y. For instance, for $L = 1$

$$R^L_\sigma = \frac{PQ(1 + R_m x^2 \cosh a_m + R_m y^2 \sinh a_m)}{(Q\cosh^2 a_m - P\sinh^2 a_m - xP\sinh a_m \sin\phi - yQ\cosh a_m \cos\phi)} \tag{32}$$

where $a_m = \log(R_m) = \log\sqrt{1 + A_m} \approx A_m/2$ [12]. Expanding this to first order in a_m we obtain:

$$R^L_\sigma = \frac{1}{1 - \sigma y\cos\phi} + \frac{(x^2 + y^2)(1 - y\cos\phi) + x(1 - y^2)\sin\phi}{(1 - y\cos\phi)^2(1 + x^2)} a_m + O(a_m^2) \tag{33}$$

Thus, if we define $\hat{y}$ by

$$\hat{y}\cos\phi = \sigma\frac{R^L_\sigma - 1}{R^L_\sigma} \tag{34}$$

If we expand $\hat{y}$ to first order in a_m we obtain:

$$\hat{y} = y - a_m\left[\frac{(x^2 + y^2)(1 - y\cos\phi) + (1 - y^2)x\sin\phi)}{(1 + x^2)\cos\phi}\right] \tag{35}$$

Clearly then, Eqn. (31) gives y only if a_m is known to be small. The actual value of a_m can be experimentally obtained from the semileptonic decay asymmetry [3].

In our discussion we will now assume that $R_m \approx 1$ and so the ratio R^L_σ gives us $y\cos\phi$ through Eqn. (31). The error in determining y is thus given by the generalization of Eqn. (9)

$$\Delta y\cos\phi = \left(2N_0\mathcal{A}^\ell\mathcal{B}^\ell\mathcal{A}^\sigma\mathcal{B}^\sigma\right)^{-\frac{1}{2}} \tag{36}$$

In the case of D^0, the systematics for $\Delta y\cos\phi$ is the same as the systematics for Δy in the CP conserving case discussed above.

In the case of B^0, ϕ which is equal to 2β in the Standard Model has been measured at the BaBar and BELLE experiments [7, 8] The average of these two results is currently $\sin 2\beta = 0.78\pm0.08$ thus $\cos 2\beta \approx 0.6$. Thus, if we take $N_0 = 10^8$ and use only ψK_S decay mode with $\psi \to l^+l^-$ and assume that $\mathcal{A}_\sigma\mathcal{A}_l \approx 1/4$ then $\Delta y\cos 2\beta = 0.06$ corresponding to $\Delta y = 0.1$. Clearly bringing in additional S_σ modes will improve determination of Δy. We can also improve the statistics by using flavor specific decays of the B^0 other than pure leptonic decays. The BaBaR and Belle experiments have made considerable progress in their ability to accomplish this and obtain an effective value of $\mathcal{A}_l B_l \approx 0.7$. Using this result, the above gives $\Delta y \approx 0.026$.

To produce correlated B_s pairs one needs to run an electron-positron machine at the $\Upsilon(5s)$ resonance. This state can decay into $B_s B_s$, $B_s^* B_s$ and $B_s^* B_s^*$ where the B_s^* decays to $B_s\gamma$. As discussed in [6] if there are 0 or 2 photons in the final state (i.e. the decay was to $B_s B_s$ or $B_s^* B_s^*$) then the $B_s B_s$ is in an $L =$ odd state while if there is one photon in the final state (i.e. $B_s^* B_s$) then the final $B_s B_s$ state is $L =$ even.

The branching ratio to S_σ states in the case of B_s is in principle much larger than in the case of B_0. For instance, the branching ratio for $B_s \to D_s^+ D_s^-$ should be similar to the measured branching ratio for $B^0 \to D^- D_s^+$ which is about 0.8%. likewise one can also estimate the branching ratio of $B_s \to J/\psi\eta^{(\prime)}$ at about 0.15% in addition analogous states such as $D_s^* D_s^*$ etc should have branching ratios on the order of 0.1% at least. The acceptance for such states may be lower than for ψK_s so we will assume that $\mathcal{A}_\sigma \mathcal{A}_l \approx 0.1$ with a branching ratio to CP states of about 0.8%. Using these assumptions, if one had a high luminosity $\Upsilon(5s)$ machine that was able to produce 10^8 B_s pairs then $\Delta y \cos\phi = 0.7\%$ which would be the same as Δy if the Standard Model expectation that $\phi = 0$ was correct. In [6] it was shown that using a method of generalizing the identification of CP eigenstate decays of B_s to include all states with a final quark content of $c\bar{c}s\bar{s}$ at the expense of the efficiency of such tagging may allow the determination of Δy to a precision of about 0.28% under the same assumption although in this case the precision also depends on the value of y.

5 Conclusions

In summary, we discussed the possibility of time-independent measurements of lifetime differences in D and B systems. It is important to reiterate that time-dependent measurements are quite difficult at the symmetric e^+e^- threshold machines due to the fact that the pair-produced heavy mesons are almost at rest [13]. The techniques described above will provide a *time integrated* quantity that is separately sensitive to the lifetime difference y.

This will be particularly useful in the case of D^0 where a charm factory running at the $\psi(3770)$ resonance can yield the measurement with precision of $\Delta y \cos\phi \approx 0.5\%$ which is in the range of some standard model predictions. At a $\Upsilon(5S)$ B factory with a luminosity sufficient to produce 10^8 B_s pairs, a precision of $\Delta y \cos\phi \approx 0.7\%$ should be achievable which is much smaller that the standard model prediction of 5-15%. Thus, even if only $\sim 10^6$ B_s pairs are produced, precision on the order of the Standard Model prediction can be obtained. In the case of the $\Upsilon(4S)$ B factory with 10^8 BB pairs, $\Delta y \cos 2\beta \approx 3\%$ which does not probe to the level of the Standard Model estimate in this case. Yet, a new high luminosity B factory (such as discussed SuperBaBar) will be able to measure y in B_d system.

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Appendix: Correlated Decays with CP Violation

In this appendix we provide the expressions for the time integrated decay of a correlated $H^0 - \overline{H^0}$ state to various pairs of final states using the formalism discussed in the text.

The final states we consider are:

1. $S_\pm$, a CP eigenstate such as in $D^0 \to K_s\pi^0$ or $B_{d(s)} \to J/\psi K_S(\phi)$.

2. $L^\pm$, a flavor specific semi-leptonic decay to a final state containing $\ell^\pm$.

3. G, a hadronic final state such that both H^0 and $\overline{H^0}$ can decay to it. For example, in charmed mesons, $D^0 \to G^+$ is Cabibbo Favored (CF) and $D^0 \to G^-$ is doubly Cabibbo suppressed, as in $D^0 \to K^-\pi^+$. This implies that the ratio of DCS to CA decay rates R is small and the results can be expanded in terms of this ratio. Alternatively, the ratio of amplitudes can be of order one in B decay, as in the example of $B_s \to D_s^+ K^-$, so all powers of R must be kept.

4. X is an inclusive set of all final states.

It is now easy to construct all possible combinations of the above final states. For the case of antisymmetric initial state (L =odd), we have for $\Gamma_{odd}^{f_1,f_2}$

$$\Gamma_{odd}^{S_\sigma L^\pm} = \left[2 + (1+R_m^{\pm 2})x^2 - (1-R_m^{\pm 2})y^2\right]\left[\frac{\Gamma_0(S_\sigma)\Gamma_0(L^+)}{2(1-y^2)(1+x^2)}\right] \quad (37)$$

$$\begin{aligned}\Gamma_{odd}^{S_+S_-} &= \Big[8R_m^2 + (1+R_m^4)(x^2+y^2) \\ &\quad +2R_m^2((1+2\cos^2\phi)x^2 + (1+2\sin^2\phi)y^2)\Big] \times \\ &\quad \times \left[\frac{\Gamma_0(S_+)\Gamma_0(S_-)}{2(1-y^2)(1+x^2)}\right]\end{aligned} \quad (38)$$

$$\Gamma_{odd}^{S_+S_+} = \left[(x^2+y^2)(\cosh^2 a_m - \cos^2\phi)\right]\left[\frac{\Gamma_0^2(S_\sigma)}{(1-y^2)(1+x^2)}\right] \quad (39)$$

$$\Gamma_{odd}^{L^\pm L^\pm} = \left[R_m^{\mp 2}(x^2+y^2)\right]\left[\frac{\Gamma_0^2(L^+)}{2(1-y^2)(1+x^2)}\right] \tag{40}$$

$$\Gamma_{odd}^{L^\pm L^\mp} = \left[2+x^2-y^2\right]\left[\frac{\Gamma_0^2(L^+)}{2(1-y^2)(1+x^2)}\right] \tag{41}$$

$$\begin{aligned}\Gamma_{odd}^{S_\sigma X} = & \Big[1+y^2\sinh^2 a_m + x^2\cosh^2 a_m \\ & -\sigma\Big(y(1+x^2)\cosh a_m\cos\phi + x(1-y^2)\sinh a_m\sin\phi\Big)\Big]\times \\ & \times\left[\frac{2\Gamma_D\Gamma_0(S_\sigma)}{(1-y^2)(1+x^2)}\right]\end{aligned} \tag{42}$$

$$\begin{aligned}\Gamma_{odd}^{L^\sigma X} = & \left[1+x^2\cosh^2 a_m + y^2\sinh^2 a_m + \frac{\sigma}{2}(x^2+y^2)\sinh 2a_m\right]\times \\ & \times\left[\frac{2\Gamma_D\Gamma_0(L^+)}{(1-y^2)(1+x^2)}\right]\end{aligned} \tag{43}$$

$$\begin{aligned}\Gamma_{odd}^{G^+X} = & \Big[x^2+y^2+\left(1+R^2\right)\left(2+x^2-y^2\right)R_m^2 + R^2\left(x^2+y^2\right)R_m^4 \\ & +2Rx\left(1-y^2\right)\sin(\delta+\phi)R_m\left(1-R_m^2\right) \\ & -2Ry\left(1+x^2\right)\cos(\delta+\phi)R_m\left(1+R_m^2\right)\Big]\times \\ & \times\left[\frac{\Gamma(G^+)\Gamma_D}{4(1-y^2)(1+x^2)R_m^2}\right]\end{aligned} \tag{44}$$

$$\begin{aligned}\Gamma_{odd}^{GS_\sigma} = & \Big[R_m^2(1+R^2)+(R^2+R_m^2)(1+R_m^2)x^2+(R^2-R_m^2)(1-R_m^2)y^2 \\ & +4rR_m\Big(y^2(\cos\phi\sin\phi\sin\delta - \sin^2\phi\cos\delta) \\ & +x^2(\cos\phi\sin\phi\sin\delta+\cos^2\phi\cos\delta)+\cos\delta\Big)\Big]\times \\ & \times\left[\frac{2\Gamma(G^+)\Gamma(S_\sigma)}{(1-y^2)(1+x^2)R_m^2}\right]\end{aligned} \tag{45}$$

$$\Gamma_{odd}^{G^\pm L^\pm} = \left[2R^2 + (R_m^{\mp 2} + R^2)x^2 + (R_m^{\mp 2} - R^2)y^2\right]\left[\frac{\Gamma(G)\Gamma(L^+)}{2(1-y^2)(1+x^2)}\right] \tag{46}$$

$$\Gamma_{odd}^{G^\pm L^\mp} = \left[2 + (R^2 R_m^{\pm 2} + 1)x^2 + (R^2 R_m^{\pm 2} - 1)y^2\right]\left[\frac{\Gamma(G)\Gamma(L^+)}{2(1-y^2)(1+x^2)}\right] \tag{47}$$

$$\begin{aligned}\Gamma_{odd}^{G^\pm G^\mp} = &\left[\left(1+R^4\right) R_m^2 \left(-2 - x^2 + y^2\right) - R^2\left(x^2+y^2\right) - R^2 R_m^4 \left(x^2+y^2\right)\right.\\ &\left. + \ 2R^2R_m^2\left[\left(2+x^2-y^2\right)\cos 2\delta + \left(x^2+y^2\right)\cos 2\phi\right]\right] \times \\ &\times \left[\frac{\Gamma^2(G)}{2(y^2-1)(1+x^2)R_m^2}\right]\end{aligned} \tag{48}$$

$$\begin{aligned}\Gamma_{odd}^{G^\pm G^\pm} = &\left[(x^2+y^2)(R^4R_m^{\pm 2} - 2R^2\cos 2(\delta \pm \phi) + R_m^{\mp 2})\right] \times \\ &\times \left[\frac{\Gamma^2(G^+)}{2(1-y^2)(1+x^2)}\right]\end{aligned} \tag{49}$$

In the case of charmed mesons, $R \ll 1$. Neglecting possible CP-violating effects and taking the ratio of Eqs. (49) and (48) simultaneously expanding numerator and denominator in R, x, and y we reproduce the well-known result that DCS/CF interference cancels out in the ratio for L =odd [14] and gives the result, identical to the semileptonic final state, $(x^2+y^2)/2$.

The results for L =even are more cumbersome, so we present only a few of $\Gamma_{even}^{f_1,f_2}$:

$$\begin{aligned}\Gamma_{even}^{S_\sigma L^\pm} = &\left[\left(x^2+y^2\right)\left(3 + x^2 - \left(1-x^2\right)y^2\right)\right.\\ &+R_m^{\pm 2}\left(2 + x^2 + x^4 - \left(1 - 4x^2 - x^4\right)y^2 + \left(1-x^2\right)y^4\right)\\ &\left.-4R_m^{\pm 1}\left(1+x^2\right)^2 y\cos\phi + 4R_m^{\pm 1}x\left(1-y^2\right)^2\sin\phi\right]\left[\frac{\Gamma_0(S_\sigma)\Gamma_0(L^+)}{2(1-y^2)^2(1+x^2)^2}\right]\end{aligned} \tag{50}$$

$$\Gamma_{even}^{S_+S_-} = \left[(x^2+y^2)(x^2+(x^2-1)y^2+3)(\cosh 2a_m - \cos 2\phi)\right]\left[\frac{\Gamma_0^2(S_\sigma)}{(1-y^2)^2(1+x^2)^2}\right] \tag{51}$$

$$\Gamma_{even}^{L^\pm L^\pm} = \left[R_m^{\mp 2}(x^2+y^2)(x^2+(x^2-1)y^2+3)\right]\left[\frac{\Gamma_0^2(L^+)}{2(1-y^2)^2(1+x^2)^2}\right] \tag{52}$$

$$\Gamma_{even}^{L^{\pm}L^{\mp}} = \left[x^4 + x^2 - (x^2-1)y^4 + (x^4+4x^2-1)y^2 + 2)\right]\left[\frac{\Gamma_0^2(L^+)}{2(1-y^2)^2(1+x^2)^2}\right] \quad (53)$$

Taking the ratios of the decay rates presented above, one can easily generalize the results of [14] to the case of CP-nonconservation.

References

[1] See, for example, J. F. Donoghue, E. Golowich and B. R. Holstein, "Dynamics of the Standard Model," *Cambridge Univ. Press (1992).*

[2] A. F. Falk, Y. Grossman, Z. Ligeti and A. A. Petrov, Phys. Rev. D **65**, 054034 (2002).

[3] Particle Data Group, *Review of Particle Physics*, Eur. Phys. J., **C15**, 1 (2000).

[4] A. S. Dighe, T. Hurth, C. S. Kim and T. Yoshikawa, Nucl. Phys. B **624**, 377 (2002). T. Altomari, L. Wolfenstein and J. D. Bjorken, Phys. Rev. D **37**, 1860 (1988).

[5] A. Datta, E. A. Paschos and U. Turke, Phys. Lett. B **196**, 382 (1987); M. Beneke, G. Buchalla and I. Dunietz, Phys. Rev. **D54**, 4419 (1996); M. Beneke, G. Buchalla, C. Greub, A. Lenz and U. Nierste, Phys. Lett. **B459**, 631 (1999); S. Hashimoto, K. I. Ishikawa, T. Onogi and N. Yamada, Phys. Rev. D **62**, 034504 (2000); D. Becirevic *et. al.* Eur. Phys. J. C **18**, 157 (2000); M. Beneke and A. Lenz, J. Phys. G **27**, 1219 (2001).

[6] D. Atwood and A. Soni, Phys. Lett. B **533**, 37 (2002).

[7] B. Aubert *et al.* [BABAR Collaboration], arXiv:hep-ex/0203007.

[8] K. Abe *et al.* [Belle Collaboration], arXiv:hep-ex/0204002.

[9] A. F. Falk and A. A. Petrov, Phys. Rev. Lett. **85**, 252 (2000).

[10] A. F. Falk, Y. Nir and A. A. Petrov, JHEP **9912**, 019 (1999).

[11] M. Gronau, Y. Grossman and J. L. Rosner, Phys. Lett. B **508**, 37 (2001); J. P. Silva and A. Soffer, Phys. Rev. D **61**, 112001 (2000). E. Golowich and S. Pakvasa, Phys. Lett. B **505**, 94 (2001).

[12] S. Bergmann, Y. Grossman, Z. Ligeti, Y. Nir and A. A. Petrov, Phys. Lett. **B486**, 418 (2000). For a recent review, see A. A. Petrov, arXiv:hep-ph/0009160.

[13] A. D. Foland, Phys. Rev. D **60**, 111301 (1999).

[14] I. I. Bigi and A. I. Sanda, Phys. Lett. B **171**, 320 (1986).