A Half-Composite Standard Model at a TeV and $\text{Sin}^2\theta_{\text{W}}$

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We apply a recently proposed mechanism – in which an $SU(3)$ symmetry predicts the weak mixing angle – to construct realistic theories with composite quarks and leptons at a TeV. Although the models are strongly coupled, they are reliably analyzed using complementarity and 't Hooft's anomaly matching. In the simplest models the right-handed fermions are composite, while the left-handed are elementary. Strong $SU(2)_R$ forces give rise to 12-particle instanton-mediated processes. They violate baryon and lepton numbers by three units and result in spectacular multilepton and multijet events at the LHC. Models in which the leptons are in an $SU(3)$-triplet can be directly tested in muonium-antimuonium conversion experiments.

1. Introduction

The one quantitative success of physics beyond the standard model (SM) is the prediction of the weak mixing angle by supersymmetric grand unified theories (GUTs) [1]. Running the $SU(5)$ prediction $\sin^2\theta_W = 3/8$ [2,3] from the GUT scale to the weak scale in supersymmetric theories produces the measured value of 0.231 within theoretical uncertainties. This requires the existence of a large energy desert above the weak scale. For a theory with a low cutoff, a different approach is necessary.

Recently, a new mechanism for predicting the weak mixing angle with TeV-physics was proposed [4]. It leads to the unification of the two electroweak gauge couplings into their $SU(3)$-symmetric value, giving $\sin^2\theta_W = .25$ at tree level. Since this value is close to the experimental value of $\sin^2\theta_W = .231$ at M_Z, "$SU(3)$-unification" occurs at a few TeV [4].

In this paper we show how this mechanism can be incorporated in models with composite quarks and leptons. Specifically, we consider models with an $SU(2)_R$ gauge group that becomes strong and confines at around a TeV. The resulting composite fermions are all the right handed quarks and leptons of the standard model. All the models predict the correct value of $\sin^2\theta_W$ to within

a few percent, provided that the compositeness scale is near a TeV – as motivated by the hierarchy problem. In general, it is difficult to analyze the spectrum of strongly coupled field theories, let alone predict any quantities to within a few percent. Yet, in the models we propose all the properties of interest can be reliably analyzed. One reason is that the strongly coupled phase of our models is "complementary" to the weakly coupled Higgs phase which can be studied within perturbation theory [5].

The crucial ingredient guaranteeing complementarity is that all the Higgs fields that develop VEVs are in the fundamental representation of the gauge group. As a result, the Wilson line is always screened by the Higgs field and does not discriminate the confinement and Higgs phases – leading to the conjecture that there is no essential difference between the two [6,7]. In particular, the spectrum of massless composite fermions produced by the strong confinement dynamics is deduced by the much easier task of reading-off the massless fermions in the Higgs phase. An important check of this strategy is 't Hooft's anomaly matching condition. It states that any exact global symmetry spontaneously unbroken by the strong confinement dynamics, must have the same global anomalies in the infrared as it does in the ultraviolet. In others

words, the anomalies of the of the unbroken global currents must be the same weather evaluated by using the elementary fermions or the massless composite fermions produced by the strong dynamics. Complementarity is a good strategy for looking for theories satisfying 't Hooft's matching conditions. We will use both of these tools to ensure that we indeed get the spectrum of the standard model in our theories.

In the section 2 we will discuss a minimal realistic composite model predicting $\sin^2 \theta_W$. In section 3 we will generalize it and we will conclude with experimental signatures in section 4.

2. The Strong Left-Right Model

Before we describe the model, we review the mechanism which predicts the correct low energy value of $\sin^2 \theta_W$ with relatively small theoretical uncertainties.

Start with the SM, $SU(3)_c \times SU(2) \times U(1)$ with all matter and Higgs in their normal representations, and add an $SU(3)_W$ gauge symmetry. A single scalar representation Σ connects the $SU(3)_W$ to $SU(2) \times U(1)$ – it is a triplet of $SU(3)_W$ and has the Higgs quantum numbers $(2, +1/2)$ under the $SU(2) \times U(1)$. A generic potential allows Σ to get a VEV which breaks $SU(3)_W \times SU(2) \times U(1)$ to the diagonal $SU(2)_L \times U(1)_Y$. If the gauge couplings of the original $SU(2) \times U(1)$ are at least somewhat bigger than the $SU(3)_W$ coupling, then there is a large region in parameter space in which the low energy couplings reflect the $SU(3)$ symmetry up to a few percent correction [4]. The symmetry insures that at the breaking scale M, the low energy gauge couplings approximately satisfy the $SU(3)$ relation $g/g' = \sqrt{3}$ or $\sin^2 \theta_W = .25$ [8].

Now we gauge $SU(2)_R$. The full gauge sector of the model contains an $SU(3)_W \times SU(3)_c \times SU(2)_{\tilde{L}} \times SU(2)_R \times U(1)_X$ symmetry. The fermion content consists of three generations of quarks $q : (1, 3, 2, 1, +1/6)$ and $q^c : (1, \bar{3}, 1, 2, -1/6)$ and leptons $\ell : (1, 1, 2, 1, -1/2)$ and $\ell^c : (1, 1, 1, 2, +1/2)$, which become the standard model content plus right handed neutrinos after left-right breaking.

We now describe the scalars and their roles

in the breaking to the standard model when the theory is in the Higgs phase. First, it includes $\phi : (1, 1, 1, 2, +1/2)$, responsible for breaking $SU(2)_R \times U(1)_X \rightarrow U(1)_{\tilde{Y}}$. A field $\Sigma : (3, 1, 2, 2, 0)$ is responsible for breaking $SU(3)_W \times SU(2)_{\tilde{L}} \times U(1)_{\tilde{Y}} \rightarrow SU(2)_L \times U(1)_Y$, the electroweak sector of the standard model. This breaking results in the value $\sin^2 \theta_W \simeq .25$ when the Left-Right and x couplings are somewhat large [4]. Finally, the standard model Higgs field is contained in $h : (1, 1, 2, 2, 0)$.

Next we switch to the confinement picture by taking the $SU(2)_R$ gauge coupling strong. Since all scalars are in the fundamental representation of $SU(2)_R$ we expect complementarity between the Higgs and confining descriptions to hold. Physics below the confining scale looks like the standard model; this matches the physics of the Higgs picture below the breaking scale.

't Hooft Anomaly Matching: While the tools of complementarity allow us to construct the theory in the infrared, we perform a non-trivial check that the low energy spectrum is correct via 't Hooft anomaly matching [6]. To do so, we find all of the global anomalies in the ultraviolet (UV) and infrared (IR) and see if they are equal for the symmetry not spontaneously broken by the strong dynamics.

When the $SU(3)_c \times SU(2)_{\tilde{L}} \times U(1)_X$ couplings and all Yukawa couplings are turned off, there exists a global $SU(12)_f$ symmetry which transforms the 12 fermion $SU(2)_R$ doublets – three quarks and one lepton per generation – as the fundamental representation[1]. This symmetry has a global $SU(12)_f^3$ anomaly with two **12**s contributing and no **$\overline{12}$**s.

Turning off the gauge coupling of the $SU(3)_W$ along with any scalar potential couplings leaves a global $U(9)_s$ symmetry under which Σ, ϕ and h together form a nine-dimensional representation. We will number the $SU(2)_R$ doublets in

---

[1]Classically the symmetry is $U(12)_f$ but the $U(1)$ is anomalous.

these scalars as follows:

$$
\Sigma \;=\;
\begin{array}{c}
\\
\uparrow \\
\mathbf{3_W} \\
\downarrow
\end{array}
\begin{array}{c}
\mathbf{2_L} \;\rightarrow \\
\begin{pmatrix}
1 & 2 \\
3 & 4 \\
5 & 6
\end{pmatrix}
\end{array}
$$

$$
h \;=\;
\begin{array}{c}
\mathbf{2_L} \\
\downarrow
\end{array}
\begin{pmatrix}
7 \\
8
\end{pmatrix}
\tag{1}
$$

$$
\phi \;=\; (9)
$$

Because these global symmetries only rotate scalars, there are trivially no additional global anomalies beyond $SU(12)_f^3$ mentioned above.

Thus the full global symmetry of the theory at high energies is $SU(12)_f \times SU(9)_s \times U(1)_s$ and the matter content is the fermion doublets $\{q^c, \ell^c\} \equiv \psi^a$ in the fundamental representation of $SU(12)_f$ and scalar doublets $\{\Sigma, h, \phi\} \equiv H^i$ in the fundamental representation of $SU(9)_s$ and carrying a unit of $U(1)_s$ charge.

We postulate the existence of a pair of condensates due to the $SU(2)_R$ gauge coupling getting strong which breaks the weakly coupled gauge groups $SU(3)_W \times SU(2)_{\tilde{L}} \times U(1)_X$ to the electroweak group $SU(2)_L \times U(1)_Y$. This breaking is accomplished by

$$
\langle H^\dagger H \rangle \;\sim\;
\begin{array}{c}
\\
1 \\
4 \\
\\
9
\end{array}
\begin{array}{c}
\begin{array}{ccc}
1 & 4 & 9
\end{array} \\
\begin{pmatrix}
M^2 & M^2 & \\
M^2 & M^2 & \\
& & \mathcal{V}^2
\end{pmatrix}
\end{array}
\tag{2}
$$

$$
\langle H \epsilon H \rangle \;\sim\;
\begin{array}{c}
\\
1 \\
4 \\
\\
9
\end{array}
\begin{array}{c}
\begin{array}{ccc}
1 & 4 & 9
\end{array} \\
\begin{pmatrix}
& & -M\mathcal{V} \\
& & -M\mathcal{V} \\
M\mathcal{V} & M\mathcal{V} &
\end{pmatrix}
\end{array}
\tag{3}
$$

where the $SU(2)_R$ indices are contracted and $M \sim \mathcal{V}$. This is the vev that would arise by plugging in the vev for Σ and ϕ from the weakly-coupled version of this story.

The first of the condensates above break $SU(9)_s \times U(1)_s$ to $SU(7)_s \times U(1)_s^3$. The second breaks one linear combination of the $U(1)$s so the remaining global symmetry group is $SU(7)_s \times U(1)_s^2$. The low energy theory will contain the light scalars which are the goldstone bosons as a result of the symmetry breaking (though some will be eaten since a portion of these symmetries are gauged), and light fermions which will reproduce the global anomalies of the UV theory as required by anomaly matching [6,9,10].

In the IR we need to find composite fermions which produce the same anomalies in the global symmetries as those in the UV theory. The simplest set of composites, and their representations under the global $SU(12)_s \times SU(7)_s \times U(1)_s^2$ symmetry are

$$
\begin{aligned}
(H^i \epsilon \psi^a): \quad & (12, 1, a, b) \\
+ \quad & (12, 7, 0, c) + (12, 1, -a, -b) \\
(H_i^\dagger \psi^a): \quad & (12, 1, -a, -b) \\
+ \quad & (12, \bar{7}, 0, c) + (12, 1, a, b)
\end{aligned}
\tag{4}
$$

where $a, b, c \neq 0$.

The $U(1)$ charges can be understood as follows: The two unbroken $U(1)$s must act trivially on the non-zero elements in (3). If we diagonalize the matrix in eq. (2) using $SU(9)_s$ transformations making only the (1,1) and (9,9) entries non-zero, the other condensate (3) under the same rotation becomes:

$$
\langle H \epsilon H \rangle \;\sim\;
\begin{array}{c}
\\
1 \\
\\
9
\end{array}
\begin{array}{c}
\begin{array}{cc}
1 & \quad 9
\end{array} \\
\begin{pmatrix}
& -M\mathcal{V} \\
& \\
M\mathcal{V} &
\end{pmatrix}
\end{array}
\tag{5}
$$

Thus the unbroken $U(1)_s$ charges of the first and ninth components of H (the $SU(7)_s$ singlets) must be equal and opposite. One $U(1)_s$ can be taken as a subgroup of the full $SU(9)_s$, and is therefore traceless. There exists a basis for the two $U(1)$s such that the charges chosen in eq. (4) are the most general set.

Now, to reproduce the $SU(12)_s^3$ anomaly of the UV theory, we need two and only two 12-dimensional representations of fermions. Therefore, anomaly matching requires that only the

$SU(7)_s$ singlets can be the chiral matter in the low-energy theory. The remaining global anomalies must all vanish. This is accomplished by choosing two **12**s which are vector-like under the remaining quantum numbers. The choice is then just one of each linear combination of $SU(7)_s$ singlets with the same quantum numbers.

Finally, we need to indentify the quantum numbers of the composite fermions under the unbroken gauge symmetries. Since the unbroken $SU(2)_L$ acts trivially on the original UV fermions ψ^a, then the composite fermions which are $SU(7)_s$ singlets will be $SU(2)_L$ singlets. The unbroken $U(1)_Y$ is a direct sum of the $U(1)_X$ generator and the T^8 generator of $SU(3)_W$ with the normalization $T^8 = diag\{-1/2, -1/2, +1\}$ as it acts on triplets. The action of $U(1)_Y$ on the fermions ψ and scalars H is via the following representations of the generator, respectively:

$$diag\{(-\tfrac{1}{6}, -\tfrac{1}{6}, -\tfrac{1}{6}, \tfrac{1}{2}), (\cdots), (\cdots)\}$$
$$diag\{-\tfrac{1}{2}, -\tfrac{1}{2}, 1, -\tfrac{1}{2}, -\tfrac{1}{2}, 1, 0, 0, \tfrac{1}{2}\}$$

where we construct the fermion multiplet as $\psi = \{q_1^{\alpha_1}, \ell_1, q_2^{\alpha_2}, \ell_2, q_3^{\alpha_3}, \ell_3\}$; the subscripts indicate generation number and the α_i are color indicies. We see that the $SU(7)_s$-singlet parts of H (the first and last components) have hypercharges $\pm 1/2$ and thus the composite quarks have hypercharges $-1/2 - 1/6 = -2/3$ and $1/2 - 1/6 = 1/3$ and composite leptons have hypercharges $-1/2 + 1/2 = 0$ and $1/2 + 1/2 = 1$ giving three generations of u^c, d^c, ν_R^c and e^c respectively. The low energy theory has exactly the fermion content of the SM plus three generations of right-handed neutrinos.

Fermion Masses: The light scalars will be the goldstone bosons associated with the breaking of approximate global symmetries. They can be defined as the variations around the condenstates (2) and (3). Nearly all of them can be described as variations around the first condensate:

$$\delta\langle H^\dagger H\rangle \equiv U = e^{i\pi/M} \tag{6}$$

where $\pi = \pi^a t^a$ is sum over only the broken generators of the original $SU(9)_s$ global symmetry and contains two complex **7** representations of $SU(7)_s$ and a complex singlet. The **7** contains

the Higgs and components of Σ, some of which are eaten by $SU(3)_W$ breaking.

Without Yukawa couplings in the UV theory, Yukawas do not appear in the IR and an exact chiral symmetry leaves the fermions massless. If we introduce Yukawa couplings $\lambda^u q h q^c$ and $\lambda^d q h^\dagger q^c$ in the UV theory, we can treat $\lambda^q q$ as spurions for global symmetry breaking. The global-symmetry-invariant operator becomes $(\lambda^q q) H \psi$, where $(\lambda^q q)$ transforms as a $(\overline{\textbf{12}}, \overline{\textbf{9}}, -1)$ under $SU(12)_s \times SU(9)_s \times U(1)_s$. Using naïve dimensional analysis (NDA) [11–13], we estimate the size of the operators to be

$$\frac{\Lambda^4}{g^2}\left[\frac{\lambda^q q}{\Lambda^{3/2}} U \frac{g\Psi}{\Lambda^{3/2}}\right] \tag{7}$$

where we take $\Lambda/g = \Lambda/4\pi = f_M \equiv M$. Thus when U is expanded to first order, NDA gives a Yukawa strength in the IR equal to that in the UV – a result which agrees with what one would have expected from complementarity.

The theory is a two-Higgs-doublet version of the SM. If only the "up-type" Higgs gets a VEV, then the couplings $\lambda^u q h q^c$, $\lambda^d q h^\dagger q^c$ and $\lambda^e \ell h^\dagger \ell^c$ in the UV theory produce the necessary charged fermion masses in the low energy theory. The neutrino masses would come from the coupling $\lambda^\nu \ell h \ell^c$ thus requiring λ^ν to be extremely small. This can be ameliorated if a singlet neutrino is added which couples as $\lambda^s \ell^c \phi \nu_s$ to the right-handed neutrino. With only this and the above Yukawa terms, there exists a massless neutrino state. A small Majorana mass m_s for the singlet produces a light neutrino with mass $m_s(\lambda^\nu h/\lambda^s \phi)^2$. This gives a symmetry reason for the existence of a small quantity – the fact that non-zero m_s violates fermion number. This mechanism becomes more important for the case described below.

The resulting model in the IR is simply the SM in which all right-handed fermions are composite with a "pion decay constant" of $M \sim 3 - 4$ TeV and in which $\sin^2\theta_W$ is acurately predicted to order a few percent.

Theoretical Uncertainty: The main contributors to the uncertainty in the prediction of $\sin^2\theta_W$ come from the, in principle, uncorrelated values of the $SU(2)_{\tilde{L}} \times U(1)_X$ gauge couplings.

As discussed in a previous paper [4], these contributions are of order a few percent over a large region of parameter space in which their couplings are greater than unity. Another source is the unknown value of the $SU(3)_W$-breaking scale. If we assume it is in the few TeV range, motivated by theories which explain the hierarchy problem with a low cutoff, the uncertainty is again reduced to the few percent level.

The corrections to the value of the weak mixing angle from strong dynamics can be estimated by NDA and come in the form of operators such as

$$\frac{\Lambda^4}{g^2}\left[U\frac{g_i^2 F^{\mu\nu}F_{\mu\nu}}{\Lambda^4}\right] \tag{8}$$

where the gauge fields are $SU(3)_W$ or $SU(2)_L \times U(1)_X$ fields, g_i are their gauge couplings and U breaks the gauge symmetry. From this operator, we estimate fractional contributions to $\sin^2\theta_W$ to be of order $(\alpha_i/4\pi)/g_W^2 \sim few\%$. Thus, although this is a strongly coupled theory, a measured observable can be accurately predicted.

3. Variations

One obvious question is why not take the minimal module of $SU(3)_W \times SU(2)_{\tilde{L}} \times U(1)_{\tilde{Y}}$ and let $SU(2)_{\tilde{L}}$ get strong. This model does in fact support complementarity and it can be shown via 't Hooft anomaly matching that the gauge symmetry *must* be broken in the IR. The low energy theory is just the SM with composite left-handed fermions. In addition, there is one less theoretical uncertainty as the $SU(2)_{\tilde{L}}$ coupling has been removed and only the extra $U(1)_{\tilde{Y}}$ remains.

However, this model is ruled out. If we make the very reasonable restriction that the $U(1)$ Landau pole is at least a factor of 4π larger than the breaking scale M, then the coupling $\tilde{g}' < 2$. These values of the coupling decrease the value of $\sin^2\theta_W$ at the breaking scale M thus requiring a smaller M to reproduce the measured result at M_z. For $\tilde{g}' = 2$, the measured value of $\sin^2\theta_W$ predicts a breaking scale of 1.1 TeV. The bounds on four-fermion operators from atomic parity violation measurements require $M > 3-4$ TeV, with a slightly weaker bound coming from other operators. A more concrete number comes from LEP bounds on a SM-like Z' gauge boson. The bound is from the elecroweak fit and is about $M_{z'} > 900$ GeV. This limit, though, comes from virtual effects and so the coupling-independent bound on the $U(1)$-breaking scale is 900 GeV/$g_z \sim 2$ TeV.

These limits can be avoided by noticing that the leptons make up complete $SU(3)_W$ triplets. If we introduce three generations of vector-like leptons in triplet representations, L_i and $\bar{L}_i$ with a technically natural small mass term and promote Σ to the $(3, 2, 1/2) + (3, 1, -1)$ representation, then the couplings $y\bar{L}_i\Sigma(\ell, e^c)$ produces a mass for the leptons in the $SU(2) \times U(1)$ sector. The SM leptons now do not feel the strong dynamics or the coupling to a single Z'. Bounds on charged extra gauge bosons from LEP put a limit on M of about 1 TeV due to the leptons coupling to the rest of the $SU(3)$ gauge multiplet. The strongest bound, however, comes from non-observation of muonium-antimuonium conversion which puts a limit of $M > 1.4$ TeV and $M > 850$ GeV at 90% and 95% C.L. respectively [14]. Versions of the model where only one generation lives in a triplet may avoid these bounds. Four quark operators also put a bound on the breaking scale at ~ 1 TeV.

Another possible model is to put $SU(2)_L \times U(1)_{\tilde{Y}}$ into the Pati-Salam group [15]. The full gauge group is $SU(3)_W \times SU(4)_c \times SU(2)_L \times SU(2)_R$, a semi-simple group in which case charge quantization is guaranteed. The fermions are three generations of $Q : (1, 4, 2, 1)$, $Q^c : (1, \bar{4}, 1, 2)$ and $\nu_s : (1, 1, 1, 1)$. These multiplets become, after Higgsing to the SM, the normal fermion content plus three generations of right-handed and sterile neutrinos.

The scalars are $\Sigma : (3, 1, 2, 2)$, $h : (1, 1, 2, 2)$ and $\phi : (1, 4, 1, 2)$, where ϕ is responsible for breaking Pati-Salam to the SM-like groups. The relevant global symmetries are $SU(12)_f \times SU(12)_s \times U(1)_s$ analogous to those in the left-right model. We postulate the existence of condensates similar to Eqs. (2) and (3) which break the global symmetries to $SU(12)_f \times SU(10)_s \times U(1)_s^2$. Again the SM is the IR field content (with extra singlet neutrinos) and the set of goldstones contains the Higgs scalar.

A new constraint due to the Pati-Salam group

is on the Yukawa couplings. The couplings $\lambda^u Q h Q^c$ and $\lambda^d Q h^\dagger Q^c$ will eventually give Dirac masses to all the fermions in the IR, including the neutrinos. The neutrino masses are equal to the up-type quark masses with these Yukawas alone. Again, as in the left-right model, we add the coupling $\lambda^s Q^c \phi \nu_s$. If we don't add Majorana masses for the singlets ν_s, then the IR theory has a set of left-handed massless neutrinos. A small Majorana mass m_s produces a small Majorana mass for the low-energy neutrinos of size $\sim m_s (m_q/M)^2$.

The down-type quarks and charged leptons also have equal Yukawa couplings in the UV. However, at the scale $M \sim few$ TeV, assuming just the SM particle content up to that scale, the bottom Yukawa is expected to be about 50% larger than that of the τ. This discrepancy can be understood in the case when the theory has a low cutoff. Non-renormalizable operators of the form:

$$\frac{\lambda'}{\Lambda^2} Q^c \phi \phi^\dagger h^\dagger Q \qquad (9)$$

contribute to Yukawa couplings at the Pati-Salam-breaking scale M. If the $SU(4)_c$ indices on the ϕs are contracted with the Qs, then this operator only contributes to the leptons. Otherwise, when Qs and ϕs are contracted with themselves, the contribution is quark-lepton universal. If the cutoff Λ is around 30 TeV, these contributions can be as large as 10^{-2}, easily enough to explain the dicrepancy. As complementarity suggests, this should be true in the strongly coupled picture and we leave this as an exercize for the reader.

Besides charge quantization, a nice feature of this model is the reduction of theoretical inputs. Since the gauge coupling of $SU(2)_R$ gets strong and the $SU(4)_c$ coupling is fixed by the measured value of the QCD coupling, only the $SU(2)_L$ can be a significant unknown contribution to $\sin^2 \theta_W$ at the scale M. There is a large region in this coupling's parameter space for which the contribution to $\sin^2 \theta_W$ is order a few percent. This can be seen by looking at the right-most boundary of the plot in Figure 2 of [4].

The examples of this section have fundamental scalars whose masses are quadratically sensitive to the cutoff. One option for avoiding this is a $\sim$ TeV-scale cutoff. Others are the possibility that the scalars are composite, or TeV-scale supersymmetry.

4. Experimental Signatures and Bounds

The models presented here have two ingredients: those responsible for the prediction of the weak mixing angle (such as SU(3)), and the strong $SU(2)_R$ sector. The experimental consequences associated with the first are the presence of a weakly coupled SU(3), an $SU(2) \times U(1)$ with intermediate-strength coupling, and Σ which bridges the two. These have already been introduced in reference [4]. The new physics introduced in this paper is the $SU(2)_R$ force, which becomes strong at a few TeV and leads to composite right handed particles.

The main experimental bounds to compositeness come from limits on 4-fermi interactions. Parametrizing their coefficients as $\frac{1}{M_c^2}$, we obtain the following limits to M_c: Atomic parity violation limits the coefficient of the operator llqq, $M_c \geq 4$ TeV. The strongest bound on the four-lepton operator implies $M_c \geq 3$ TeV, whereas the limit on the four-quark operator gives $M_c \geq 1$ TeV (assuming left-left and right-right bounds are about equal). Naive dimensional analysis gives coefficients of the four-fermion composite operators that are of order $\frac{1}{M^2}$, where M is the analogue of f_π, leading to the identification $M \sim M_c$. This shows that these constraints are consistent with the range of M suggested by the weak mixing angle as well as the hierarchy problem.

In models where the leptons are in $SU(3)$ triplets they couple to doubly-charged gauge bosons and therefore are most sensitively probed by experiments which look for muonium-antimuonium conversion ($\mu^+ e^- \rightarrow \mu^- e^+$). The current limits on V-A and V+A interactions are [14]

$$G_{\mathrm{M\overline{M}}} \leq 3.0 \times 10^{-3} \, \mathrm{G_F} \quad (90\% \mathrm{C.L.}) \qquad (10)$$

These bounds already cover an interesting regions in parameter space. If higher muonium production efficiency can be achieved, the entire parameter space of most models could be probed.

Perhaps the most exciting signature of the $SU(2)_R$ force comes from its instantons. Since $SU(2)_R$ gets strong, the instanton amplitude is large – just as in QCD. It emits twelve fermions (nine quarks and three leptons), one for each right-handed doublet. At the LHC this will result in spectacular events with many jets and three leptons. The $SU(2)_R$ instantons can also mediate $\Delta B = \Delta L = 3$ nuclear transitions, such as the decay of tritium or helium-3 to three leptons. The strongest bound comes from the decay of oxygen into a state which contains thirteen nucleons and three leptons; this is tested in the Super-Kamiokande water-stability experiment. Since the instanton is a dimension 18 operator, the rate is supressed by 28 powers of M and is adequately small.

The simplest theory with composite right-handed particles is the three-family $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_x$ model, with $SU(2)_R$ getting strong near a TeV. This model does not have the new $SU(3)_W$ symmetry (so, it does not predict $\sin^2\theta_W$), nevertheless it has all the signatures associated with composite right-handed particles –including $SU(2)_R$ instantons. The model is complementary, and it is straightforward to verify 't Hooft's matching conditions.

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