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Radiative Processes ($\tau \to \mu\gamma$, $\mu \to e\gamma$ and $(g-2)_\mu$) as Probes of ESSM/SO(10)

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Abstract

The Extended Supersymmetric Standard Model (ESSM), motivated on several grounds, introduces two vectorlike families ($\mathbf{16}+\bar{\mathbf{16}}$) of $SO(10)$) with masses of order one TeV. It is noted that the successful predictions of prior work on fermion masses and mixings, based on MSSM embedded in $SO(10)$, can be retained rather simply within the ESSM extension. These include an understanding of the smallness of $V_{cb} \approx 0.04$ and the largeness of $\nu_\mu - \nu_\tau$ oscillation angle, $\sin^2 2\theta^{\rm osc}_{\nu_\mu\nu_\tau} \approx 1$. We analyze the new contributions arising through the exchange of the vectorlike families of ESSM to radiative processes including $\tau \to \mu\gamma$, $\mu \to e\gamma$, $b \to s\gamma$, EDM of the muon and the muon $(g-2)$. We show that ESSM makes significant contributions especially to the decays $\tau \to \mu\gamma$ and $\mu \to e\gamma$ and simultaneously to muon $(g-2)$. For a large and plausible range of relevant parameters, we obtain: $a_\mu^{ESSM} \approx +(10-40)\times 10^{-10}$, with a correlated prediction that $\tau \to \mu\gamma$ should be discovered with an improvement in its current limit by a factor of 3-20. The implications for $\mu \to e\gamma$ are very similar. The muon EDM is within reach of the next generation experiments. Thus, ESSM with heavy leptons being lighter than about 700 GeV (say) can be probed effectively by radiative processes before a direct search for these vectorlike leptons and quarks is feasible at the LHC.

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1 Introduction

A variant of MSSM – the so-called Extended Supersymmetric Standard Model (ESSM) – has been motivated sometime ago on several grounds [1, 2]. Briefly speaking, in addition to the three chiral families, ESSM introduces two vectorlike families of quarks and leptons (together with their superpartners) that transform as $\mathbf{16}+\bar{\mathbf{16}}$ of SO(10), and possess an SO(10)-invariant mass of order one TeV. It assumes that the three chiral families acquire their masses primarily (barring small corrections of order one MeV) through their mixings with the two vectorlike families. As we will explain, this mechanism of mass-generation for the three chiral families has the advantage that it provides a simple understanding of the interfamily mass-hierarchy ($m_{u,d,e} \ll m_{c,s,\mu} \ll m_{t,b,\tau}$) [1, 2]. In particular, it automatically renders the electron family massless (barring small corrections $\sim$ 1 MeV) and also naturally accounts for the μ/τ mass-hierarchy, even if no small numbers are introduced from the start.

In the sequel we will list other theoretical motivations for the ESSM tied to issues that arise in the context of unification, and also the reason for its consistency with LEP neutrino counting as well as precision electroweak tests. No doubt the vectorlike quarks and leptons, if they exist with masses $\lesssim$ 1-2 TeV, as ESSM proposes, would be visible prominently at the LHC. Recently it has been noted [3] that ESSM with the heavy lepton members having masses $\lesssim$ 500 GeV (say), would provide a simple explanation of the anomaly in νN-scattering that has been recently reported by the NuTeV group [4] and simultaneously of the LEP neutrino counting that is presently at 2σ below the standard model value of 3 [5].

The purpose of this note is to stress that radiative processes – in particular $\tau \to \mu\gamma$, $\mu \to e\gamma$ and muon $(g-2)$ associated with the vertex $\mu \to \mu\gamma$ – can provide additional sensitive probes of ESSM. Of these three processes, the measurement of muon $(g-2)$ has drawn special attention over the last year. This is because, the BNL result based on its 1999-data [6], in spite of the realization of the reversal of sign of the hadronic light-by-light scattering contribution to $(g-2)_\mu$ [7], points to a possible anomaly in $(g-2)_\mu$, given by $\delta a_\mu \equiv a_\mu^{\rm expt} - a_\mu^{SM} \approx (25\pm16)\times10^{-10}$. This result by itself would suggest that δa_μ could quite possibly lie in the range of (10-40)$\times10^{-10}$. Such a view has recently been called to question, however, in Ref. [8], where it is noted that the hadronic light-by-light scattering contribution to $(g-2)_\mu$ has a rather large uncertainty given by $(\pm6+3\tilde{c})\times10^{-10}$. While model calculations yield $\tilde{c} \approx 1$ [9], in general $\tilde{c}$ is expected to be of order unity with either sign. In the presence of such uncertainty, a definitive conclusion as to whether there exists an anomaly [$a_\mu \gtrsim \times10^{-10}$

(say)] would have to await a further reduction of experimental error (which is due soon), as well as (depending upon the central value) a reduction in the theoretical uncertainty of hadronic effects. Meanwhile, anticipating either outcome, it seems worthwhile to explore possible new physics which would contribute to a_μ in the range mentioned above, especially if such physics is motivated on other grounds. Theoretical exploration of this kind could eventually help constrain new physics regardless of whether the final verdict confirms or denies an anomaly in $(g-2)_\mu$.

It has been noted by several authors [10] that low energy-supersymmetry [11] arising in the context of MSSM is a natural source of the new contribution to a_μ. As we will show in this paper, ESSM would provide an additional source of new contribution to a_μ, which can naturally be in the range of (10-40)$\times 10^{-10}$, provided the heavy leptons are relatively light ($m_{E,E'} \approx M_{N,N'} \approx$ 250-500 GeV, say). The intriguing feature of ESSM with such moderately light heavy leptons is that it leads to crucial predictions as regards observability of especially $\tau \to \mu\gamma$ and also $\mu \to e\gamma$. In this sense, ESSM with a moderately light spectrum would be testable even before LHC turns on.

We recall some salient features of ESSM and theoretical motivations in its favor in the next section. In Sec. 3 we discuss the Yukawa couplings and fermion mass matrices for the case of ESSM to indicate how one can essentially reproduce in this case the successes of the G(224)/SO(10)-framework for fermion masses and mixings that was presented in Ref. [12], for the case of MSSM. In Sec. 4 we use this realistic framework to discuss the contributions of ESSM to $(g-2)_\mu$, $\tau \to \mu\gamma$, $\mu \to e\gamma$, the muon electric dipole moment and $b \to s\gamma$. In Sec. 5 we present a summary and concluding remarks.

2 Salient Features of ESSM

The so called "Extended Supersymmetric Standard Model" (ESSM), which introduces *two complete "vectorlike" families of quarks and leptons* – denoted by $Q_{L,R} = (U, D, N, E)_{L,R}$ and $Q'_{L,R} = (U', D', N', E')_{L,R}$ – with relatively light masses of order one TeV. Both Q_L *and* Q_R transform as (2,1,4), while Q'_L and Q'_R transform as (1,2,4) of the symmetry group G(224)=SU(2)$_L\times$SU(2)$_R\times$SU(4)^C. Thus together they have the quantum numbers of a pair of $\mathbf{16}+\bar{\mathbf{16}}$ of SO(10), to be denoted by $\mathbf{16_V} = (Q_L|\bar{Q}'_R)$ and $\bar{\mathbf{16}}_\mathbf{V} = (\bar{Q}_R|Q'_L)$. The subscript "V" signifies two features: (a) $\bar{\mathbf{16}}_\mathbf{V}$ combines primarily with $\mathbf{16_V}$, so that the pair gets a (dominant) SO(10)-invariant mass term of the form $M_V \mathbf{16_V} \cdot \bar{\mathbf{16}}_\mathbf{V} + h.c. = M_V(\bar{Q}_R Q_L +$

$\bar{Q}'_R Q'_L) + h.c.$, at the GUT scale, presumably utilizing the VEV of an SO(10)-singlet (see below). (b) Since Q_L and Q_R are doublets of $\mathrm{SU}(2)_L$, the massive four-component object $(Q_L \oplus Q_R)$ couples vectorially to W_L's; likewise $(Q'_L \oplus Q'_R)$ couples vectorially to W_R's. Hence the name "vectorlike" families. The three chiral families are denoted by (16_i), $i = 1, 2, 3$.

It is assumed (see e.g., Ref. [1] and [2]) that the mass term M_V of the two vectorlike families is protected by some local generalized "flavor" or discrete symmetries (presumably of string origin), so that it is of order TeV, rather than the GUT-scale, just like the μ-term of MSSM. It is furthermore assumed that the same set of "flavor" symmetries dictate that the direct mass term of the three chiral families which could arise from couplings of the form $h^{(0)}_{ij}\mathbf{16_i 16_j}\Sigma_H$ (where $\Sigma_H = \mathbf{10_H}$ or $\mathbf{10_H} \times \mathbf{45_H}/M$, etc.), are strongly suppressed, up to small corrections $\lesssim$ a few MeV (see remarks later). Thus the chiral families get their masses (barring corrections $\lesssim$ a few MeV) primarily through their mixings with the two vectorlike families. It is shown in the next section that such a pattern of the 5×5 mass matrix, involving the three chiral and the two vectorlike families, would naturally yield an exactly massless family (barring corrections $\lesssim$ a few MeV) and an inter-family mass-hierarchy $(m_{u,d,e} \ll m_{c,s,\mu} \ll m_{t,b,\tau})$, even if such large hierarchy ratios were not present to begin with in the mass-elements that mix the three chiral with the two vectorlike families [1, 2, 13].

A few general comments about ESSM are in order. Note that it of course preserves all the merits of MSSM as regards gauge coupling unification and protection of the Higgs masses against large quantum corrections. From the point of view of adding extra families of quarks and leptons, ESSM in fact seems to be the minimal as well as the maximal extension of the MSSM that is allowed by (a) LEP neutrino counting, (b) precision measurements of the oblique electroweak parameters [14, 15] as well as (c) the demand of a perturbative or semi-perturbative [2, 16] as opposed to a nonperturbative gauge coupling unification [e.g., addition of a fourth chiral family, as opposed to two vectorlike families of ESSM, would in general be incompatible with (b)].

Theoretical motivations for the case of ESSM arise on several grounds: (a) It raises α_{unif} to a semiperturbative value of 0.25 to 0.3 and therefore provides a much better chance to stabilize the dilaton than the case of MSSM, for which α_{unif} is rather weak (only 0.04) [2]; (b) Owing to increased two-loop effects, ESSM raises the unification scale M_X to about (0.5-2)$\times 10^{17}$ GeV [2, 16] and thereby considerably reduces the problem of a mismatch between the MSSM and the string unification scales [17, 18]; (c) It lowers the GUT-prediction for $\alpha_3(m_Z)$ compared to that for MSSM [2], as needed by the data [19, 20]; (d) It naturally enhances the GUT-prediction for proton lifetime [21] compared to that for MSSM embedded in a GUT, also as needed by the data (i.e., by the SuperK limit); and finally (e) as noted above, ESSM provides a simple reason for inter-family mass hierarchy [1, 2, 13].

In this sense, ESSM, though less economical than MSSM, offers some distinct advantages over MSSM. The main purpose of this paper is to point out that ESSM can also offer a simple explanation of the muon $(g-2)$ anomaly, should it eventually persist, without requiring a light SUSY threshold. Simultaneously, it would offer a set of crucial tests, involving radiative processes, especially $\tau \to \mu\gamma$, and also $\mu \to e\gamma$, and edm of the muon and last but not least a clear potential for the discovery of a host of vectorlike quarks and leptons, in addition to the SUSY particles, at the LHC and possibly the NLC.

In the next section we discuss the Yukawa coupling and fermion mass matrices for the case of ESSM to indicate how one can essentially reproduce in this case the successful SO(10)-framework for fermion masses and mixings that was presented in Ref. [12] for the case of MSSM. In section 4 we use this realistic framework to discuss the contributions of ESSM to $(g-2)_\mu$ and to the radiative transitions $\tau \to \mu\gamma$ and $\mu \to e\gamma$. We will see that ESSM can naturally account for the indicated anomaly in $(g-2)_\mu$, but in this case vectorlike leptons and quarks would have to be discovered at the LHC and possibly NLC and quite certainly $\tau \to \mu\gamma$ and very likely also $\mu \to e\gamma$ should be discovered with modest improvements in the current limits.

3 The Yukawa Coupling Matrix in ESSM

Following the discussion in the introduction (see Ref. [2] and [1] for details and notation), the 5×5 Yukawa coupling matrix involving the 3 chiral ($q^i_{L,R}$) and the two vectorlike families ($Q_{L,R}$ and $Q'_{L,R}$) is assumed to have the simple form:

$$h^{(0)}_{f,c} = \begin{array}{c} \\ \bar{q}^i_R \\ \bar{Q}_R \\ \bar{Q}'_R \end{array} \begin{array}{c} \begin{array}{ccc} q^i_L & Q_L & Q'_L \end{array} \\ \begin{pmatrix} 0_{3\times 3} & X_f H_f & Y_c H_s \\ Y'^{\dagger}_c H_s & z_c H_V & 0 \\ X'^{\dagger}_f H_f & 0 & z'_f H_V \end{pmatrix} \end{array} . \tag{1}$$

Here the symbols q, Q and Q' stand for quarks *as well as* leptons; $i = 1, 2, 3$ corresponds to the chiral families. The subscript f for the Yukawa couplings X_f and X'_f denotes the four sectors u, d, l or ν, while $c = q$ or l denotes quark or lepton color. The fields H_f

with $f = u, d$ denote the familiar two Higgs doublets, while H_s and H_V are Higgs Standard Model singlets, which can effectively be admixtures of for example a dominant SO(10)-singlet and a sub-dominant SO(10) 45-plet with a VEV along the B-L direction (see below). The zeros appearing in Eq. (1), especially the direct coupling terms in the upper 3×3 block, are expected to be corrected so as to lead to masses $\lesssim$ a few MeV, through VEVs inserted into higher dimensional operators. The Higgs fields are assumed to acquire VEVs so that $\langle H_V\rangle \sim \langle H_s\rangle \sim 1$ TeV $\gtrsim \langle H_u\rangle \sim 200$ GeV $\gg \langle H_d\rangle$.

The parametrization in Eq. (1) anticipates that with SO(10) intact, even if $z_c = z'_f$, $X_f = X'_f$ and $Y_c = Y'_c$ at the GUT-scale, renormalization effects would introduce differences between these Yukawa couplings at the electroweak scale, because $Q_{L,R}$ are SU(2)$_L$-doublets, while $Q'_{L,R}$ are SU(2)$_L$-singlets [see Eq. (10) of Ref. [2]]. Denoting $X_f^T = (x_1, x_2, x_3)_f$, and $Y_c^T = (y_1, y_2, y_3)_c$, it is easy to see that regardless of the values of these Yukawa couplings, one can always rotate the basis vectors so that Y_c^T is transformed into $\hat{Y}_c^T = (0, 0, 1)y_c$, $X_{f,c}^T$ simultaneously into the form $\hat{X}_f^T = (0, p_f, 1)x_f$, X'_f into $\hat{X}'_f = (0, p'_f, 1)x'_f$ and Y'_c into $\hat{Y}'_c = (0, 0, 1)y'_c$. It is thus apparent why one family remains massless (barring corrections $\lesssim$ a few MeV), even if there is no hierarchy in the original Yukawa couplings $(x_i)_f$ and $(x_i)_c$, etc., defined in the gauge basis; this one is naturally identified with the electron family. If, for simplicity, one puts $x_f = x'_f$, $y_c = y'_c$ and $z = z'$ at the unification scale, one would obtain $m^{(0)}_{t,b,\tau} \approx (2x_f y_c)(\langle H_s\rangle\langle H_f\rangle)/(z\langle H_V\rangle)$, and $m^{(0)}_{c,s,\mu} \approx m^{(0)}_{t,b,\tau}(p_f p'_f/4)$. Note, even if p_f and p'_f are not very small compared to unity, their product divided by four can still be pretty small (e.g., suppose $p_f \sim p'_f \sim 1/2$ to $1/7$, then $p_f p'_f/4 \sim 1/16$ to $1/200$). One can thus naturally get a large hierarchy between the masses of the muon and the tau families as well, without introducing very small numbers from the beginning.

We stress that the parameters of the mass-matrices of the four sectors u, d, l and ν, and also those entering into X versus X' or Y versus Y' in a given sector, are of course not all independent, because a large number of them are related to each other at the GUT-scale by the group theory of SO(10) and the representations of the relevant Higgs multiplets. [For most purposes the group theory of G(224) suffices.] This results in an enormous reduction of parameters. [For example, with $(h_V \mathbf{16_V}\mathbf{\bar{16}_V} H_V + h_{3V}\mathbf{16_3}\mathbf{16_V}\mathbf{10_H} + h_{3\bar{V}}\mathbf{16_3}\mathbf{\bar{16}_V} H_V)$ being the leading terms of the effective superpotential, one would get the following relations at the GUT-scale for either SO(10) or G(224): $x_u = x_d = x_l = x_\nu = x'_u = x'_d = x'_l = x'_\nu$; $y_q = y_l = y'_q = y'_l$ and $z_f = z'_c$. In this case, the entries denoted by "1" in the matrices $\hat{X}_f^T$, $\hat{X}'_f$, $\hat{Y}_c^T$ and Y'_c will be given by just three parameters (including $\langle H_u\rangle/\langle H_d\rangle = \tan\beta$)

instead of sixteen - at the GUT-scale. Similar economy arises for the (p_f, p'_f) parameters and the masses of the vectorlike fermions (see below). At this point, it is worth noting that a successful framework, based on MSSM embedded in SO(10), has recently been proposed [12] that introduces only the three chiral families but no vectorlike families (appropriate for MSSM) and a minimal Higgs system – i.e., a single $\mathbf{45_H}$, $\mathbf{16_H}$, $\mathbf{\overline{16}_H}$ and $\mathbf{10_H}$. Utilizing the SO(10)-invariant direct Yukawa couplings of the three chiral families to each other (e.g. couplings of the type $h_{ij}\mathbf{16_i 16_j 10_H}$) it leads to eight predictions – including $m_b^0 \approx m_\tau^0$, $m(\nu_L^\tau) \sim 1/20$ eV, together with $\sin^2 2\Theta^{osc}_{\nu_\mu \nu_\tau} \approx$ 0.87-0.96, $V_{ub} \approx 0.003$, $V_{us} \approx 0.22$ and $m_d \approx 8$ MeV. Remarkably enough, all of these are in agreement with observation to within 10 %. It is interesting, as we show below, that the same form of effective mass matrix of the three chiral (especially μ and τ) families can also be obtained for the case of ESSM simply by imposing an SO(10) group structure analogous to that of Ref. [12] on the off-diagonal Yukawa couplings of ESSM [shown in Eq. (1)], and by performing a see-saw block diagonalization that integrates out the heavy vectorlike families. Thus, the successes of Ref. [12] can essentially be retained for the case of ESSM, embedded in SO(10), as well. To see this briefly (details will be given in a separate note), let us go to the basis, denoted by a hat as above, in which the first family is entirely (or almost) decoupled from the two vectorlike families, so that $x_1 = x'_1 = y_1 = y'_1 = y_2 = y'_2 = 0$ [22]. For the convenience of writing, we drop the hat on the Yukawa couplings. Using only ($\mathbf{10_H}$, $\mathbf{16_H}$, $\mathbf{\overline{16}_H}$ and $\mathbf{45_H}$) the relevant leading terms of the effective superpotential involving the two chiral ($\mathbf{16_2}$ and $\mathbf{16_3}$) and the two vectorlike families ($\mathbf{16_V} + \mathbf{\overline{16}_V}$) that would conform with the Yukawa coupling matrix of Eq. (1) and also would yield (after integrating out Q and Q') a mass matrix analogous to that of Ref. [12] is given by:

$$\begin{aligned}
\hat{W}_{Yuk} &= h_V \mathbf{16_V \overline{16}_V} H_V + f_V \mathbf{16_V \overline{16}_V}(\mathbf{45_H}/M)\mathbf{1'_V} + h_{3V}\mathbf{16_3 16_V 10_H} \\
&+ \tilde{h}_{3V}\mathbf{16_3 16_V 10_H 45_H}/M + h_{3\bar{V}}\mathbf{16_3 \overline{16}_V} H_s + h_{2V}\mathbf{16_2 16_V 10_H}\left(\frac{X}{M}\right) \\
&+ a_{2V}\mathbf{16_2 16_V 10_H 45_H}/M + g_{2V}\mathbf{16_2 16_V 16^d_H 16_H}/M
\end{aligned} \tag{2}$$

where $v_0 \equiv \langle H_V\rangle \sim \mathbf{1'_V} \sim 1$ TeV $\sim \langle H_s\rangle > \langle H_u\rangle \sim 200\ GeV \gg \langle H_d\rangle$; $\langle \mathbf{16_H}\rangle \sim \langle \mathbf{45_H}\rangle \sim \langle X\rangle \sim M_{GUT}$ and $M \sim M_{string}$. It is presumed that owing to flavor symmetries [23] , f_V and h_{2V} terms require the presence of $\mathbf{45_H}$ and X, respectively, so that they are suppressed by one power of $\langle \mathbf{45_H}\rangle/M$ or $\langle X/M\rangle \sim$ (1/3-1/10) compared to the h_{3V} term [24]. The $\tilde{h}_{3V}$, a_{2V} and g_{2V} terms are also naturally suppressed (by SO(10) group theory) by a similar factor relative to the h_{3V} and h_V terms. Note, the VEV $\langle \mathbf{45_H}\rangle \propto B - L$ introduces a $B - L$

dependence, while $\langle \mathbf{16_H^d} \rangle$ introduces up-down distinction [here $\langle \mathbf{16_H^d} \rangle$ denotes the electroweak VEV of $\mathbf{16_H}$, which arises through a mixing between $\mathbf{16_H}$ and $\mathbf{10_H^d}$; see Ref. [12]]. Taking these into account, the parameters (or corresponding VEVs of certain entries) of the Yukawa matrix (1) in the rotated hat basis (discussed above) are given by (hat is suppressed):

$$\begin{aligned}
M_Q &= z_c \langle H_V \rangle = h_V^Q v_0 (1+\kappa_{B-L}) \\
M_{Q'} &= z'_f \langle H_V \rangle = h_V^{Q'} v_0 (1-\kappa_{B-L}) \\
x_f &= h_{3V}^Q (1+\delta_{B-L}) \\
x'_f &= h_{3V}^{Q'} (1-\delta_{B-L}) \\
p_f x_f &\equiv x_{2f} = h_{2V}^Q \left\langle \frac{X}{M} \right\rangle (1+\xi_{B-L}) + [g_{2V}^Q \sin\gamma \langle \mathbf{16_H} \rangle / M]_d \\
p'_f x'_f &\equiv x'_{2f} = h_{2V}^{Q'} \left\langle \frac{X}{M} \right\rangle (1-\xi_{B-L}) + [g_{2V}^{Q'} \sin\gamma \langle \mathbf{16_H} \rangle / M]_d \\
y^Q &= y'^{Q'} = h_{3\bar{V}}
\end{aligned} \tag{3}$$

These entries correspond to GUT scale values. The superscripts Q and Q' on h_V (and likewise on the other couplings) signify that even if $h_V^Q = h_V^{Q'}$ at the GUT-scale (owing to SO(10)), renormalization effects would introduce differences between the two couplings at the electroweak scale [see Eq. (10), Ref. [2]]. Here, $\kappa_{B-L} = \kappa$ for the heavy quarks (U, D, U' and D'), while $\kappa_{B-L} = -3\kappa$ for the heavy leptons (E and E'); likewise $\delta_{B-L} \equiv (\delta, -3\delta)$ for (q,l), and $\xi_{B-L} \equiv (\xi, -3\xi)$ for (q,l). The second term in x_{2f} and x'_{2f} contributes only to the down quarks and charged leptons. The parameter $\sin\gamma$ denotes the mixing between $\langle \mathbf{16_H} \rangle$ and $\langle \mathbf{10_d} \rangle$, where $\cos\gamma \approx (m_b/m_t)\tan\beta$ (see Ref. [12]). Since $\langle \mathbf{45_H} \rangle / M$ is expected to be small compared to unity, we expect the $(B-L)$-dependent parameters κ and δ to be typically $\leq 1/10$; however, with $a_{2V} \sim g_{2V}$ and $\langle X/M \rangle \sim \langle \mathbf{45_H} \rangle / M$, ξ_{B-L} (if present) is expected to be of the order of unity. It turns out that with the Yukawa couplings presented in Eqs. (1) and (3), together with the much suppressed direct Yukawa couplings of the electron with the muon and the tau families, all the successes of Ref. [12] are essentially preserved. This can be seen by integrating out the vectorlike families and examining the resulting 3×3 matrix for the light chiral families, which will have the same form as the mass matrices of Ref. [12] (in the leading see-saw approximation). There is one difference however in the prediction for m_b. Owing to renormalization effects corresponding to the running of the scale from M_{GUT} to $M_S \sim 1$ TeV, which distinguish between M_D, $M_{D'}$, M_E and $M_{E'}$ (see Ref. [2]), the ratio m_b/m_τ evaluated at M_S for ESSM (with $\kappa = \delta = 0$) turns out to be typically larger than that for MSSM [25] by nearly 20-25%. The $(B-L)$ dependent entries κ and δ exhibited in

Eq. (3), which are expected to be of order 1/10, would however have the right magnitude and the right sign (if $\kappa\delta$ is negative) to compensate adequately for this difference. In short, the pattern of Yukawa couplings given by Eq. (1), (2) and (3) does correspond to a *realistic mass matrix* for fermion masses and mixings in the case of ESSM, which preserves the major successes of Ref. [12] including especially the predictions of $m(\nu_\tau)$, ν_μ-ν_τ oscillation angle, V_{cb} and V_{ub}.

Details of the analysis of the fermion masses and mixings for the case of ESSM embedded in SO(10), including the κ and δ-terms, will be presented in a separate paper. Here, our main focus will be to study the new contributions to radiative transitions in the charged lepton sector that arise for the case of ESSM.

As we shall see shortly the new contribution to the amplitude for $\tau \to \mu\gamma$ arising from H_d-H_s mixing would vanish if $m_E = m_{E'}$. We note, however, that even if E and E' were exactly degenerate at the GUT-scale [i.e., with $f_V = 0$ and thus $\kappa = 0$, see Eqs. (2) and (3)], renormalization effects would split them near the electroweak scale, because E couples with W_L, but E' does not. For instance, with $M_E = M_{E'}$ and h_V large ($\approx$ 1-2, say), at GUT scale, one finds $(M_E/M_{E'})_{1\ TeV} = (z_l/z'_l)_{1\ TeV} \approx 0.273/0.185 \approx 1.47$ [see Eq. (10), Ref. [2]]. In the presence of the κ-term, which seems to be needed to account for the observed value of (m_b/m_τ) (see remarks above), it thus seems quite plausible (with $\kappa > 0$) that the degree of non-degeneracy of E and E' at the electroweak scale could lie typically in the range of (10-50)% (say) [26]. Thus, for concreteness in our analysys, that would be relevant especially for consideration of $\tau \to \mu\gamma$, we would allow:

$$(M_E/M_{E'})_{1\ TeV} \approx 1 + (10 \text{ to } 50)\ \% \ . \tag{4}$$

Now see-saw diagonalization of the 5×5 mass-matrix for charged leptons, following from Eqs. (1) and (3), leads to a μ-τ mass-matrix given by:

$$M_{\mu\tau} = \begin{array}{c} \\ \bar{\mu}_R \\ \bar{\tau}_R \end{array} \begin{array}{c} \begin{array}{cc} \quad\mu_L\quad & \quad\quad\tau_L\quad\quad \end{array} \\ \begin{pmatrix} 0 & xy'p/M_E \\ x'yp'/M_{E'} & xy'/M_E + x'y/M_{E'} \end{pmatrix} \end{array} v_d v_s \quad . \tag{5}$$

Here, all the entries (x, x', y, y', p and p') refer to the charged lepton sector (so the subscript l is suppressed). Using the parameters appearing in x_2, x'_2, x and x' in Eq. (3), and anticipating a correspondence with Ref. [12], one can express p_l and p'_l for charged leptons in terms of two effective parameters – i.e.,

$$p \equiv 2(\eta + 3\epsilon); \qquad\qquad p' \equiv 2(\eta - 3\epsilon) \ . \tag{6}$$

Owing to the SO(10)-constraint, the corresponding parameters p and p' for the b-s sector are $p_d = 2(\eta - \epsilon)$ and $p'_d = 2(\eta + \epsilon)$, respectively (compare with Ref. [12]). Note, if we drop the relatively small $(B-L)$-dependent δ and κ terms $[\mathcal{O}(1/10)]$ in x ,x', z and z'; see Eq. (3), we would have $x = x'$, $y = y'$ and $z = z'$ (therefore $M_E = M_{E'}$) at the GUT-scale due to SO(10) [27]. In this case, using $M_E = zv_0$ and $M_{E'} = z'v_0$, one would have the equality of the ratios:

$$xy'/M_E = x'y/M_{E'} \tag{7}$$

at the GUT scale. This would lead to *a very simple form* for the μ-τ mass-matrix [see Eq. (5)], with (xy'/M_E) being a common factor in all three elements of the matrix.

It is worth noting that in the context of the renormalization effects studied in Ref. [2] [where it was assumed that all the Yukawa couplings of vectorlike and the third family of fermions – x_f, x'_f, y_c, y'_c, z_f and z'_c – are large (~ 1 to 2) at the GUT-scale so that they acquire their respective quasi-fixed point values at the electroweak scale], the equality (7) and thus the simple form of the mass-matrix referred to above holds even at the electroweak scale. This is because, the ratio of the renormalized couplings at the electroweak scale – for instance for the leptons [see Eq. (10), Ref. [2]] – given by $xy'/z \approx 0.396 \times 0.251/0.273 \approx 0.364$ equals the ratio $x'y/z' \approx 0.368 \times 0.184/0.185 \approx 0.364$. Analogous equalities for the renormalized couplings are found to hold (see Ref. [2]) for the quark sector as well.

As a further remark, as long as $x = x'$, $y = y'$ and $z = z'$ at the GUT-scale (i.e., in the limit $\kappa = \delta = 0$), we would in fact expect the equality (7) to hold at the EW scale to a fairly good approximation (better than 10%), even if not all the Yukawa couplings are so large at the GUT-scale as to approach their quasi-fixed point values at the electroweak scale.

In the interest of simplicity in writing analytic expressions for the mixing angles, which would be relevant to radiative transitions, we would ignore the $(B-L)$-dependent δ and κ terms which are $\mathcal{O}(1/10)$ and assume (for reasons explained above) that the equality (7) and thus the simple form of the μ-τ mass-matrix holds to a good approximation at the electroweak scale. This would amount to making an error typically of 10-25 % in the radiative amplitudes [28], which would, however, not affect our conclusion. A more refined analysis will be presented elsewhere.

With the equality (7) holding (approximately) at the electroweak scale and the corresponding simple form of the mass-matrix, that results from Eq. (5), one can identify the parameters η and ϵ appearing in Eq. (6) precisely with those in Ref. [12]. From the fitting of fermion masses carried out in Ref. [12], one then has: $\eta \approx -0.15$ and $\epsilon \approx 0.095$, and thus [see Eq. (6)]:

$$p_l \approx 0.27; \qquad p_l' \approx -0.87\ . \tag{8}$$

The μ-τ mass-matrix (5), subject to Eq. (7), gets diagonalized by the simple 2×2 matrices:

$$U_L \approx \begin{bmatrix} 1 & p_l'/2 \\ -p_l'/2 & 1 \end{bmatrix}; \qquad U_R \approx \begin{bmatrix} 1 & p_l/2 \\ -p_l/2 & 1 \end{bmatrix} \tag{9}$$

and one gets:

$$m_\mu \approx -\left(\frac{p_l p_l'}{2}\right)\left(\frac{xy'}{M_E}\right)_l v_d v_s; \qquad m_\tau \approx \left(2+\frac{p_l p_l'}{2}\right)\left(\frac{xy'}{M_E}\right)_l v_d v_s\ . \tag{10}$$

Thus, $m_\mu/m_\tau \approx -p_l p_l'/4 \approx 1/17$, in good accord with observation. Analogous discussion will apply to the quark and neutrino sectors, which are not relevant here.

4 Radiative Transitions in ESSM

We will be interested in radiative transitions of charged leptons, in particular $\tau \to \mu\gamma$, $\mu \to \mu\gamma$ and $\mu \to e\gamma$. The corresponding amplitudes are defined by:

$$A(\Psi^i_L \to \Psi^j_R\gamma) \equiv A_{ij}(\bar{\Psi}_{jR}\sigma_{\mu\nu}q^\nu\Psi_{iL})\epsilon^\mu\ . \tag{11}$$

New contributions to these amplitudes would arise from (a) scalar loops involving (H_d, H_s) mixing (see Figs 1a,b), and also (b) (W and Z)-loops (see later). These new contributions arise from the Yukawa couplings of the chiral with the vectorlike families (i.e., x_i, y_i, x_i' and y_i') [see Eq. (1)]. We will use the hat basis discussed above, although not exhibit the hats (thus $x_1 = x_1' = y_1 = y_1' = y_2 = y_2' = 0$). We drop the subscripts f and c, both of which now correspond to charged leptons. The scalar loops are evaluated by exchanging the mass eigenstates $H_1 = H_d\cos\theta + H_s\sin\theta$, and $H_2 = -H_d\sin\theta + H_s\cos\theta$, with eigenvalues M_1 and M_2, respectively. The amplitude is found to be:

$$\begin{aligned} A_{ij}^{H_dH_s} &= (y_i'x_j/m_E)\mathcal{K}_E + (x_i'y_j/m_{E'})\mathcal{K}_{E'} \\ &= \left(\frac{y_i'x_j}{m_E}+\frac{x_i'y_j}{m_{E'}}\right)\frac{\mathcal{K}_E+\mathcal{K}_{E'}}{2} + \left(\frac{y_i'x_j}{m_E}-\frac{x_i'y_j}{m_{E'}}\right)\frac{\mathcal{K}_E-\mathcal{K}_{E'}}{2} \\ &\equiv A_{ij}^{(+)} + A_{ij}^{(-)} \end{aligned} \tag{12}$$

where $\mathcal{K}_F$ ($F = E$ or E') is given by [29]:

$$\mathcal{K}_F = -\frac{e\sin\theta\cos\theta\ (M_F)}{32\pi^2}[\mathcal{B}(M_F, M_1) - \mathcal{B}(M_F, M_2)] \tag{13}$$

with,

$$M_F\mathcal{B}(M_F, M_i) = \frac{r_i}{(1-r_i)^2}\left[3 - r_i + \frac{2\ln r_i}{1-r_i}\right] \tag{14}$$

where

$$r_i = M_F^2/M_i^2 \ . \tag{15}$$

Now H_d-H_s mixing denoted by the angle θ can arise through (i) a term in the superpotential $W \supset \lambda H_u H_d H_s$ [30], and (ii) a soft SUSY-breaking term $AH_uH_dH_s$ (involving only scalar fields). Using $\langle H_d^\dagger\rangle = v_d$ and $\langle H_s^\dagger\rangle = v_s$, these two terms together would induce a mass-mixing term $[(\hat{\lambda} v_d v_s)(H_d H_s) + h.c.]$ where

$$\hat{\lambda} = \lambda^2 + (A/v_s)\tan\beta. \tag{16}$$

Correspondingly, one obtains, for the $H_d - H_s$ mixing angle:

$$\sin\theta\cos\theta = (\hat{\lambda} v_d v_s)/(M_2^2 - M_1^2) \tag{17}$$

We should expect $\hat{\lambda}$ to be complex in general, owing to the phases in the A–term and/or v_s, but for now we shall assume $\hat{\lambda}$ to be real. We will comment in subsection 4.2 on the implications of a complex $\hat{\lambda}$ on the EDM of the muon, which turns out to be in the observable range in proposed experiments. At this stage, it is worth noting that to leading order in see-saw diagonalization, which serves to integrate out the heavy vectorlike families (Q, Q'), the mass matrix of the charged leptons in the three chiral families (baring small corrections $\lesssim$ few MeV that arise from direct entries in the 3×3 block of Eq. (1), see discussions above) are given by:

$$M_{ij} = \left(\frac{y_i' x_j}{m_E} + \frac{x_i' y_j}{m_{E'}}\right) v_d v_s \ . \tag{18}$$

Now, for discussions of $(g-2)_\mu$ and $\tau \to \mu\gamma$, we may ignore the electron family; thus M_{ij} is effectively a 2×2 matrix, and so is A_{ij}. Note that $A_{ij}^{(+)}$ of Eq. (12) is directly proportional to the mass-matrix M_{ij}. As a result, as we go to the physical basis by diagonalizing M_{ij}, $A_{ij}^{(+)}$ gets diagonalized as well. *Thus, to a very good approximation, $A_{ij}^{(+)}$ does not contribute to off-diagonal transitions like $\tau \to \mu\gamma$* (likewise, the analogous term in the quark sector does not contribute to $b \to s\gamma$), *but $A_{ij}^{(-)}$ does.* On the other hand, $A_{ij}^{(+)}$ makes bigger contribution, compared to $A_{ij}^{(-)}$, to diagonal transitions – that is to $(g-2)$ of the muon and the tau. We see from Eqs. (12) and (13) that $\mathcal{K}_F$ and therefore the new contributions to $\tau \to \mu\gamma$ arising from Fig. 1 would tend to vanish if $M_E \to M_{E'}$ (because in this case, $\mathcal{K}_E \to \mathcal{K}_{E'}$ and thus $A^{(-)}_{ij} \to 0$). While we expect $M_E \sim M_{E'}$, we do not of course have any reason to expect exact degeneracy of E and E'. For numerical purposes, we would take M_1 and M_2 to be comparable to within a factor of two (M_1 by choice being lighter) and $(M_E/M_{E'})$ to be away from unity as in Eq. (4).

To evaluate the new contributions to radiative transitions, we first go to the physical basis by diagonalizing the μ-τ mass-matrix M_{ij} with the transformation $M \to \hat{M} = U_R^\dagger M U_L$ [see Eqs (5), (7) and (9)], and then impose the same transformation on the matrix A_{ij} [Eq. (12)]; so that $A \to \hat{A} = U_R^\dagger A U_L$. The matrices $U_{L,R}$ are given approximately by Eq. (9). Noting that the diagonal elements of $\hat{M}$ are just m_μ and m_τ, which are proportional to those of $\hat{A}^{(+)}$, one then straightforwardly obtains:

$$a_\mu^H \equiv a_\mu^{H_d H_s} \approx \frac{m_\mu^2}{e}(\mathcal{K}_E + \mathcal{K}_{E'})/(v_d v_s) \tag{19}$$

where m_μ stands for $(-p_l p_l'/4)m_\tau$ [see Eq. (10)]. Likewise, using contribution from $\hat{A}^{(-)}$ [see Eq. (12)], one obtains:

$$\begin{aligned} A_L^H \equiv A(\tau_L \to \mu_R)^{H_d H_s} &\approx p(1+p'^2/4)(x_3 y_3'/M_E)(\mathcal{K}_E - \mathcal{K}_{E'})/2 \\ &\approx \left\{ p(1+p'^2/4)\frac{m_\tau}{2+pp'/2}\right\}\frac{1}{v_d v_s}(\mathcal{K}_E - \mathcal{K}_{E'})/2 \end{aligned} \tag{20}$$

$$\begin{aligned} A_R^H \equiv A(\tau_R \to \mu_L)^{H_d H_s} &\approx -p'(1+p^2/4)(x_3' y_3/M_{E'})(\mathcal{K}_E - \mathcal{K}_{E'})/2 \\ &\approx \left\{ -p'(1+p^2/4)\frac{m_\tau}{2+pp'/2}\right\}\frac{1}{v_d v_s}(\mathcal{K}_E - \mathcal{K}_{E'})/2 \end{aligned} \tag{21}$$

where we have used Eq. (10) for m_τ. All the parameters p, p', etc., correspond to the charged lepton sector. It thus follows that for a given choice of the spectrum (M_E, $M_{E'}$, M_1, M_2) and $\hat{\lambda}$ [see Eq (16)], we can calculate a_μ and $A(\tau_{L,R} \to \mu_{L,R} + \gamma)$ arising from H_d-H_s mixing. Note that $v_d v_s$ appearing in the denominator in Eqs. (20) and (21) cancels out because $\mathcal{K}_F \propto v_d v_s$ [see Eqs (13) and (17)]. We now proceed with the numerical evaluation of these radiative amplitudes for a few sample choices of the spectrum. They are listed in Table 1. We will return to a discussion of these contributions after presenting the contributions from the W-loop.

It should be mentioned that the supersymmetric partners of $(E_{L,R}, E'_{L,R})$ will also contribute to radiative transitions in the lepton sector. Such contributions will arise through diagrams analogous to Fig. 1, obtained by replacing (H_s, H_d) fields by their fermionic partners ($\tilde{H}_s$, $\tilde{H}_d$), $E_{L,R}$ by the scalar heavy leptons $\tilde{E}_{L,R}$ and $E'_{L,R}$ by $\tilde{E}'_{L,R}$. These diagrams are however suppressed somewhat relative to those shown in Fig. 1, mainly because the masses of the vector sleptons ($\tilde{E},\ \tilde{E}'$) are expected to be much larger than those of the fermions ($E,\ E'$). The scalar heavy leptons receive masses from the superpotential as well as from the soft SUSY breaking terms. For example, if $M_E = 300$ GeV and $m_0 = 500$ GeV (the soft SUSY breaking scalar mass parameter), then $M_{\tilde{E}} \simeq (M_E^2 + m_0^2)^{1/2} \simeq 580$ GeV, to be compared with the masses $M_{1,2}$ of the Higgs fields H_d and H_s of Fig. 1 which are in the range $100-250$ GeV. (The $\tilde{H}_d - \tilde{H}_s$ Higgsino mass term is comparable to M_E.) In any case, the flavor structure of these supersymmetric diagrams are identical to those in Fig. 1, so even if the new diagrams have comparable magnitudes, their effects can be mimicked by a redefinition of $\hat{\lambda}$. Thus we shall focus on the diagrams of Fig. 1 in our numerical evaluation of the radiative transitions.

4.1 New Contributions from the W-Loop

In ESSM, both (N_L, E_L) and (N_R, E_R) are doublets of $\mathrm{SU}(2)_L$; thus they both couple to W_L, while (N_L', E_L') and (N_R', E_R') do not. We will argue that the new (non-standard) contributions from the W-loop are strongly suppressed compared to those from the H_d-H_s loop. Allowing for the mass-mixing of the light and the heavy leptons [see Eq. (1)], the weak interaction Lagrangian contains terms given by:

$$\mathcal{L}_W^{(N)} = \left(g_W/\sqrt{2}\right) \sum_{i,a} \left[\bar{N}_L^a \gamma_\mu V_{iN_L^a} \Psi_L^i + \bar{N}_R^a \gamma_\mu V_{iN_R^a} \Psi_R^i\right] W_L^\mu + h.c. \tag{22}$$

Here $\Psi_{iL,R}$ denote *physical* charged leptons $(\mu, \tau, E)_{L,R}$, and $(N_{L,R}^a)_{a=1,2}$ denote the physical neutral heavy leptons given by

$$\begin{aligned} N_{L,R}^1 &= \cos\Theta_{L,R}^N N_{L,R} + \sin\Theta_{L,R}^N N_{L,R}' \\ N_{L,R}^2 &= -\sin\Theta_{L,R}^N N_{L,R} + \cos\Theta_{L,R}^N N_{L,R}' \end{aligned} \tag{23}$$

Note that these include N-N' mixing which is induced by the mass-matrix of Eq. (1). We refer the reader to Ref. [31] for diagonalization of the Q-Q' mass matrices in all four sectors and for expressions of the mixing angles. It is argued there that, including renormaliztion group effects, the mixing angles Θ_L^N and Θ_R^N are nearly equal (to better than 5%). The coefficients $V_{iN_L^a}$ and $V_{iN_R^a}$ are obtained by diagonalizing the mass-matrices in the leptonic up and down sectors (analogous to the CKM-matrix). The new contributions to radiative transitions due to the W-loop are shown in Fig. 2. Using Ref. [32], the contribution of the

W-loop to the radiative amplitude A_{ij} [defined in Eq. (11)] is given by:

$$A_{ij}^{(W)} = \frac{eM_N}{32\pi^2}\left(g_W^2/m_W^2\right)\sum_{a=1,2}\left(V_{iN_L^a}V^*_{jN_R^a}\right)F(x) \tag{24}$$

$$where F(x) = (2 - \frac{15}{2}x + 6x^2 - 3x^2\ln x - \frac{x^3}{2})/(1-x)^3 \tag{25}$$

Here $x \equiv M_N^2/m_W^2$ and the quantities $m_{\mu,\tau}^2/M_N^2$ are dropped. Diagonalizing the mass-matrix for the charged and neutral leptons, we get [31]:

$$\begin{aligned} V_{\tau_L N_L^1} &\approx -\left(\frac{\kappa_u^2}{\kappa_\lambda^2}\right)\left(\frac{\kappa_s}{\kappa_\lambda}\right)\left(\frac{1}{\eta_L^2-1}\right) \\ V_{\tau_L N_L^2} &\approx \left(\frac{\kappa_u}{\kappa_\lambda}\right)\left[1-\frac{\kappa_r^2}{\kappa_\lambda^2}+\frac{3}{8}\frac{\kappa_r^4}{\kappa_\lambda^4}\right] \\ V_{\tau_R N_R^1} &\approx \frac{\kappa_d}{\kappa_\lambda} \\ V_{\tau_R N_R^2} &\approx -\left(\frac{\kappa_u}{\kappa_\lambda}\right)\left(\frac{\kappa_s}{\kappa_\lambda}\right)\left(\frac{\kappa_d}{\kappa_\lambda}\right)\left(\frac{\eta_L}{\eta_L^2-1}\right)\ . \end{aligned} \tag{26}$$

Following the notations of Ref. [31] and [2], $\kappa_u \equiv x_3\langle H_u\rangle = x_3 v_u$, $\kappa_d \equiv x_3\langle H_d\rangle = x_3 v_d$, $\kappa_\lambda \equiv z\langle H_v\rangle = z v_0 \approx M_N \approx M_E$ (putting $z_f = z_c = z$ at the GUT-scale), and $\kappa_s \equiv y\langle H_s\rangle = y v_s$. The entity η_L denotes the renormalization of the Yukawa couplings due to SU$(2)_L$ gauge interactions for the effective momentum running from $M_{\rm GUT}$ to the electroweak scale ($\eta_L \approx 1.5$, see Ref. [31]). In writing Eq. (24), we have made the approximation that $\kappa_u \ll \kappa_\lambda$ and $\kappa_s < \kappa_\lambda$, and neglected the relevant small terms. For $M_N \sim 500$ GeV, we expect $\eta_u \equiv \kappa_u/\kappa_\lambda \approx 1/5$-$1/20$ (see footnote [27] in Ref. [3]); in particular, an explanation of the possible NuTeV-anomaly (if it is real) suggests $\eta_u \approx 1/10$-$1/15$. For the estimate presented below, we would use: $\eta_u \equiv \kappa_u/\kappa_\lambda \approx 1/10$.

The vertices given above for $W_L^+ \to \tau_L N_L^a$ and $W_L^+ \to \tau_R N_R^a$ would give the corresponding vertices for $W_L^+ \to \mu_L N_L^a$ and $W_L^+ \to \mu_R N_R^a$, with the insertion of an additional factor of $(p_l'/2)$ for the substitution $\tau_L \to \mu_L$ and of $(p_l/2)$ for $\tau_R \to \mu_R$. Using these substitutions and Eq. (26), the sum of the contributions from the N_1 and N_2-lines in the loop (Fig. 2) is given by :

$$A^W_{N_1+N_2}(\tau_L \to \mu_R\gamma) \approx e\left(\frac{\alpha_2}{8\pi}\right)\left(\frac{M_N F(x)}{m_W^2}\right)\left(\frac{\eta_u^2}{\eta_L+1}\right)\left(\frac{m_\tau}{2M_N}\right)\left(\frac{p_l}{2}\right) \tag{27}$$

$$A^W_{N_1+N_2}(\tau_R \to \mu_L\gamma) \approx e\left(\frac{\alpha_2}{8\pi}\right)\left(\frac{M_N F(x)}{m_W^2}\right)\left(\frac{\eta_u^2}{\eta_L+1}\right)\left(\frac{-m_\tau}{2M_N}\right)\left(\frac{p_l'}{2}\right)\ . \tag{28}$$

In above, we have used $(\kappa_u\kappa_s/\kappa_\lambda) \approx m_\tau/2$ – [see Eq. (10)]. Evaluating the functions $F(x)$, κ_E and $\kappa_{E'}$ numerically, we find that *because of the suppression factor* $\eta_u^2 \approx 10^{-2}$ in

Eqs. (27) and (28), the W-contributions to a_μ and to the $\tau \to \mu\gamma$ amplitude are strongly suppressed compared to those of the H_d-H_s loop, as long as $\hat{\lambda} \geq 5$. To be specific, we obtain: $(a_\mu^W/a_\mu^{H_d-H_s} \leq 1/50$, and $\left|A^W(\tau \to \mu\gamma)/A^{H_d-H_s}(\tau \to \mu\gamma)\right| \leq 1/10$ for $\hat{\lambda} \geq 5$. Of course, if $\hat{\lambda}$ is substantially less than 5, both the W and the H_d-H_s loop contributions to a_μ as well as to $A(\tau \to \mu\gamma)$ would be comparable, but rather small. Henceforth, we will use $\hat{\lambda} \geq 4$ (which is quite plausible for $\tan\beta \geq$ 3-5 (say)), and drop the W-loop contribution to $(g-2)_\mu$ and to the $\tau \to \mu\gamma$-amplitude. One can verify that the non-standard Z^0-loop contributions involving E and E' in the loop are extremely small ($\lesssim 1$ %) compared to those from the W-loop [33]. They are therefore dropped as well in subsequent discussions. The contributions from the H_d-H_s loop are listed in Table 1. The rate for $\tau \to \mu\gamma$ is calculated by using:

$$\Gamma(\tau \to \mu\gamma) = \left[|A(\tau_L \to \mu_R)|^2 + |A(\tau_R \to \mu_L)|^2\right] m_\tau^3/(16\pi) \tag{29}$$

where the amplitudes defined by Eq. (11) include contributions from only the H_d-H_s loop.

$(M_1, M_2, M_E, M_{E'})$	$\hat{\lambda}$	$(g-2)_\mu \times 10^{10}$ a_μ^H	$A(\tau \to \mu\gamma)$ $(A_L^H, A_R^H) \times 10^9$ GeV	BR$(\tau \to \mu\gamma)$
(1) (120, 200, 320, 280)	10	29.6	(0.79, 2.18)	2.7×10^{-7}
(2) (120, 200, 320, 280)	4	11.8	(0.32, 0.87)	4.3×10^{-8}
(3) (120, 200, 320, 220)	10	33	(2.2, 6.06)	2.1×10^{-6}
(4) (120, 250, 420, 300)	12.5	26.8	(1.85, 5.06)	1.5×10^{-6}
(5) (120, 250, 480, 380)	10	17.6	(0.97, 2.6)	3.8×10^{-7}
(6) (120, 250, 600, 450)	10	14.0	(1.03, 2.84)	4.6×10^{-7}
(7) (120,250,700,550)	10	11.4	(0.72, 2.06)	2.6×10^{-7}

Table 1. New contributions to a_μ and to $A(\tau \to \mu\gamma)$ due to H_d-H_s loop in ESSM. The masses $(M_1, M_2, M_E, M_{E'})$ are given in units of GeV. E and E' are expected to be degenerate to within (10-50)% at the electroweak scale [see Eq. (4)].

A glance at the table reveals the following features:

(1) For a decent range of the spectrum, with heavy leptons (E, E') having masses $\approx$ 300-600 GeV (say) and thus the heavy quarks having masses $\approx$ 700-1500 GeV, and for reasonable positive values of $\hat{\lambda} \approx$ 4-10 [which would arise plausibly for $\tan\beta \approx$ 3-10 (say), see Eq. (16)], ESSM provides a sizeable positive contribution to $a_\mu^{\rm ESSM} \approx (30\text{-}10) \times 10^{-10}$ (say).

(2) For the same range of the spectrum and the value of $\hat{\lambda}$ as above, with $a_\mu^{\rm ESSM} \approx$ (30-

$10) \times 10^{-10}$ (say), *ESSM makes a correlated contribution to the branching ration for $\tau \to \mu\gamma$,* which typically lies in the range of (0.4-15) $\times$ 10^{-7}. Given that the present experimental upper limit for $B(\tau \to \mu\gamma)$ is around 10^{-6} [19], *ESSM quite reasonably predicts that $\tau \to \mu\gamma$ decay should be discovered with a modest improvement of the current limit by a factor of 3-20.* Studies at B–factories can be sensitive to the level of few times 10^{-8} in this branching ratios, while LHC can probe even further.

It should be remarked that the ESSM-contribution to a_μ noted above is, of course, above and beyond the familiar SUSY-contribution to a_μ [10], which necessarily exists for ESSM as well. However, in the presence of $a_\mu^{\rm ESSM}$, even if the net new contribution to a_μ eventually needs to be in the range of (15-30) $\times$ 10^{-10} (say), bulk of this contribution can in principle come from $a_\mu^{\rm ESSM}$. That is if sleptons are not too light ($m_{\tilde{l}} \sim 400$ GeV, say) and if $\tan\beta$ is not too large ($\lesssim 10$, say), $a_\mu^{\rm SUSY}$ can be less than or of the order (5-10) $\times$ 10^{-10}, while $a_\mu^{\rm ESSM}$ can be of the order (10-20) $\times$ 10^{-10} (say). [Depending upon the sign of the μ parameter, with $\hat{\lambda} > 0$, the two contributions add or subtract.] However, in this case (i.e., with $a_\mu^{\rm ESSM} \sim (10-20) \times 10^{-10}$, $\tau \to \mu\gamma$ should be discovered with the improvement in its current limit by a factor of 3-20.

4.2 Electric Dipole Moment of the Muon

As noted in Sec. 5, the parameter $\hat{\lambda}$ in Eq. (16) is in general complex, with its imaginary part being proportional to the phases of the A–term and/or to the VEV v_s. A complex $\hat{\lambda}$ will lead to a nonzero electric dipole moment (EDM) of the muon (d_μ), arising from the same type of diagrams as in Fig. 1. d_μ can be estimated to be $d_\mu \simeq a_\mu^{ESSM}/(2m_\mu)\arg(\hat{\lambda})$. This is in the range $(1-3) \times 10^{-22}\arg(\hat{\lambda})$ e-cm for $a_\mu^{ESSM} = (10-30) \times 10^{-10}$. The present experimental limit on d_μ is $d_\mu \leq 10^{-18}$ e-cm. There is a proposal [36] to improve this limit down to the level of 10^{-24} e-cm or even 10^{-26} e-cm. The ESSM framework presented here will predict an observable signal in such experiments. It should be emphasized that the $H_d - H_s$ mixing contribution to the electron EDM (d_e) is extremely small in our framework since the electron has highly suppressed couplings to (E, E') fields. Thus the naive scaling $d_e/d_\mu \sim m_e/m_\mu$ will not hold in our case (unlike in the MSSM). A linear scaling of the EDMs with the lepton mass would have implied $d_\mu \leq 10^{-25}$ e-cm from the current limit on d_e [37].

4.3 $\mathbf{b \to s\gamma}$

Just like for $\tau \to \mu\gamma$, there would be new contributions to the amplitude for $b \to s\gamma$ decay through H_d-H_s loop involving $D_{L,R}$ and $D'_{L,R}$ heavy quark exchanges (compare Fig. 1), as well as through W loop involving (U, U')-exchanges (compare Fig. 2). We can simply obtain the new contributions to $b \to s\gamma$ amplitudes in ESSM by making the following substitutions in the corresponding amplitudes for $\tau \to \mu\gamma$, listed in Eqs. (20), (21), (27), and (28):

$$\begin{aligned} A(\tau_{L,R} \to \mu_{R,L}\gamma) &\longrightarrow A(b_{L,R} \to s_{R,L}\gamma) \\ (p_l, p'_l) &\longrightarrow (p_d, p'_d) \\ (M_E, M_{E'}) &\longrightarrow (M_D, M_{D'}) \\ m_\tau &\longrightarrow m_b \\ Q^{\rm em}_E = Q^{\rm em}_{E'} = e &\longrightarrow Q^{\rm em}_D = Q^{\rm em}_{D'} = e/3 \ . \end{aligned} \tag{30}$$

As noted in Sec. 3, we have:

$$\begin{aligned} p_l &= 2(\eta + 3\epsilon) \approx 0.27, & p_d &= 2(\eta - \epsilon) \approx -0.49 \\ p'_l &= 2(\eta - 3\epsilon) \approx -0.87, & p'_d &= 2(\eta + \epsilon) \approx -0.11 \end{aligned} \tag{31}$$

Using QCD renormalization factors for the effective momentum running from the GUT to the electroweak scale, we get [31]:

$$M_{D,D'} = \eta_c M_{E,E'} \tag{32}$$

where $\eta_c \approx 2.8$. For an estimate, consider a relatively light heavy lepton spectrum – i.e., $M_{E,E'} \approx (420, 300)$ GeV with $\hat{\lambda} = 10$ [case (4) in Table 1], and thus $M_{D,D'} \approx (1176, 840)$ GeV. Using the substitutions above, we get $A^{H_dH_s}_{R,L}(b_{R,L} \to s_{L,R}\gamma) \approx (8.5, 35.8) \times 10^{-11}$ GeV^{-1} $\times$ (0.689), where the factor (0.689) denotes the QCD renormalization of the effective operator (see e.g., [34]). Comparing with the Standard Model contribution (see e.g., [34,35]) $A_R(b_R \to s_L\gamma)^{\rm SM} \approx -\{(4G_F/\sqrt{2})(e/16\pi^2)V_{tb}V^*_{ts}\}\{2m_b\, c^{\rm eff}_7(\mu)\} \approx -7.5 \times 10^{-9}$ GeV^{-1}, where $c^{\rm eff}_7(\mu) \approx -0.312$, we see that the new contributions due to H_d-H_s loop involving $(A^{H_dH_s}_R, A^{H_dH_s}_L)$ are only about (0.6, 2.6)% of the Standard Model contribution for A_R, which are thus too small. As in the case of $\tau \to \mu\gamma$, the new contributions involving W-loop are even smaller. The unimportance of the new contributions to $b \to s\gamma$ amplitudes (in contrast to the case of $\tau \to \mu\gamma$) arises primarily because of (i) the difference in electric charges $Q_D/Q_E = 1/3$, (ii) the heaviness of the quark members (D, D') compared to the leptonic members (E, E') due to QCD renormalization, and (iii) the difference in the p-factors [see Eq. (31)].

4.4 New Contributions to $\mu \to e\gamma$

As explained in Sec. 3, if the entries in the upper 3×3 block of Eq. (1) are set strictly to zero, one can always go to the hat-basis in which the electron family would be completely decoupled from the other four families (μ, τ, Q and Q') and would remain massless. In this limit, the amplitude for $\mu \to e\gamma$ would of course vanish. The electron family does, however, get masses and mixings with the other families owing to small entries m_{ij} ($\lesssim$ a few MeV) in the upper 3×3 block of the mass-matrix, which can arise through VEVs inserted into higher dimensional operators. Given that there are new contributions to a_μ (i.e., to $\mu_L \to \mu_R\gamma$) in ESSM especially from the H_d-H_s loop, which was evaluated in a basis where the muon is almost physical, except for its small mixing with the electron, the amplitude for $\mu \to e\gamma$-transition can be obtained simply by inserting the e-μ mixing angles into $A(\mu_L \to \mu_R\gamma)$ [38]. Thus we get [following the definition in Eq. (11)]:

$$A(\mu_{L,R} \to e_{R,L}\gamma) \approx (e/2m_\mu)a_\mu^{\mathrm{ESSM}}\Theta^{e\mu}_{R,L} \tag{33}$$

where $\Theta^{e\mu}_{R,L} \approx m^l_{12,21}/m_\mu$. Here m^l_{12} and m^l_{21} are the $\bar{e}_R\mu_L$ and the $\bar{\mu}_R e_L$ mixing masses, while m^l_{11} (not shown) is the $\bar{e}_R e_L$ diagonal mass (all in the hat basis), and a_μ^{ESSM} is the contribution to a_μ from the H_d-H_s loop, listed in Table 1 [39]. [The amplitude $A(\mu \to e\gamma)$ would also get contributions by inserting e-τ mixing angles – i.e., $\Theta^{e\tau}_{R,L} \approx m^l_{13,31}/m_\tau$ – into $A(\tau \to \mu\gamma)$ [given by Eqs (20) and (21)]. One can estimate (using obvious notation) that $A(\mu_L \to e_R)_{\tau\to\mu}/A(\mu_L \to e_R)_{\text{Eq. (33)}} \sim (2/p)(1/2.5)(m_\mu/m_\tau)(m_{13}/m_{12}) \approx (1/5.6)(m_{13}/m_{12})$. The analogous ratio for $A(\mu_R \to e_L)$ is $\approx -(1/30)(m_{31}/m_{21})$. Thus, barring accidental cancellation in both channels, which is unlikely, it should suffice as an estimate to include only the contribution shown in Eq. (33), which should yield the right magnitude within a factor 2-3 (say).] Using Table 1 as a guide and setting $a_\mu^{ESSM} \equiv x_\mu(30 \times 10^{-10})$, and furthermore assuming for simplicity $\Theta^{e\mu}_L \approx \Theta^{e\mu}_R \equiv \Theta^{e\mu}$, we get [using Eq. (33)]:

$$\Gamma(\mu \to e\gamma)^{Th} = K(8 \times 10^{-22}\ GeV)(x_\mu\Theta^{e\mu})^2\ . \tag{34}$$

Here K denotes a correction factor of order one ($K \approx 1/4$ to 4, say), which can arise by allowing for contribution from $\tau \to \mu\gamma$ transition and for $\Theta^{e\mu}_L \neq \Theta^{e\mu}_R$, etc. The experimental limit $B(\mu \to e\gamma) < 1.6 \times 10^{-11}$ [19] thus provides an upper limit on $m^l_{12} \sim m^l_{21}$ (with $K \geqslant 1/4$, say) given by:

$$m_{e\mu} \lesssim (1/65,\ 1/42,\ 1/32,\ 1/22,\ 1/11)\ MeV \tag{35}$$

for a_μ^{ESSM} =(30, 20, 15, 10, 5)$\times 10^{-10}$. Here $m_{e\mu}$ may be viewed roughly as the average of m_{12}^l and m_{21}^l.

Apriori, one might have expected $m_{e\mu}$ to be at least of order $m_e \approx 1/2\ MeV$ (if not of order $m_\mu\sqrt{m_e/m_\mu}$) which is, however, a factor (30 to 10) higher than the values shown in Eq. (35). In this sense, if a sizeable contribution to a_μ ($\gtrsim 15 \times 10^{-10}$, say) should come from the H_d-H_s and W-loops in ESSM, a natural explanation for the large suppression of $m_{e\mu}$, as required by the limit $\Gamma(\mu \to e\gamma)$, would clearly be warranted. While this is a burden on ESSM, we should remark that given the smallness of the elements of the mass-matrix involving the first family, it is difficult to pin-down the SO(10)-structure of the corresponding Yukawa couplings, because these may arise from a variety of SO(10)-invariant higher dimensional operators. In fact, consistent with SO(10)-invariance, there can exist a mechanism [40] (analogous to that of the doublet-triplet splitting in SO(10) [41]), which would contribute to mixings of the first family with the other two, only in the quark-sector, but not in the lepton-sector. This could retain the successes of Ref. [12] as regards the predictions of the Cabibbo angle, V_{ub} and m_d, while yielding vanishing e-μ and e-τ mixings, and simultaneously $m_e \neq 0$. Details of this discussion will be presented in a separate paper.

In spite of this specific mechanism, however, it is hard to see why $m_{e\mu}$ and thus $\Theta_{e\mu}$ should be strictly zero. Even if $m_{e\mu}$ is suppressed by a factor of 50 to 100, say, compared to m_e (and that seems to be rather extreme), with $a_\mu^{ESSM} \gtrsim 10 \times 10^{-10}$ and $K \gtrsim 1/4$ [see Eq. (34)], we would expect:

$$B(\mu \to e\gamma) \gtrsim 1.6 \times 10^{-11}(1/14 \text{ to } 1/56)\ . \tag{36}$$

In short, within ESSM, the decay $\mu \to e\gamma$ is generically expected to occur at a decent level so that it should have been seen already. Even with a rather pessimistic scenario for $m_{e\mu}$ as mentioned above, the decay should be seen with an improvement in the current limit by a factor of 5 to 50 (say), especially if $a_\mu^{ESSM} \gtrsim 10 \times 10^{-10}$.

5 Concluding Remarks

The ESSM framework we have adopted here has been motivated on several grounds, as noted in our earlier papers [1,2] and summarized here in Sec. 2. ESSM has been embedded into an SO(10) unified theory which makes correlations among several observable quantities (such as those between $\tau \to \mu\gamma$, $b \to s\gamma$ and neutrino oscillations) possible. Such an embedding preserves the unification of gauge couplings and provides a quantitative understanding of the pattern of quark and lepton masses, including the smallness of V_{cb} and the largeness of the $\nu_\mu - \nu_\tau$ oscillation angle.

In this paper, we have studied the new contributions of ESSM to radiative processes including $\tau \to \mu\gamma$, $b \to s\gamma$, $\mu \to e\gamma$, $(g-2)_\mu$ and the muon EDM. We have shown that ESSM makes significant contributions especially to the decays $\tau \to \mu\gamma$ and $\mu \to e\gamma$ and simultaneously to $(g-2)_\mu$. For a large and plausible range of the relevant parameters (see Table 1), we obtain $a_\mu^{ESSM} \approx +(10-30) \times 10^{-10}$, and predict that $\tau \to \mu\gamma$ should be discovered with an improvement in the current limit by a factor of 3-20. The implication for the discovery of $\mu \to e\gamma$ is very similar. The EDM of the muon is expected to be in the range of 10^{-22} e-cm, which should be accessible to the next generation of experiments. Thus radiative processes can provide an effective probe of ESSM before a direct search for the heavy fermions is feasible at the LHC. The hallmark of ESSM is of course the existence of complete vectorlike families $(U,\ D,\ N,\ E)_{L,R}$ and $(U',\ D',\ E',\ N')_{L,R}$ with masses in the range of 200 GeV to 2 TeV)say), which will certainly be tested at the LHC and a future linear collider.

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[11] To obtain $a_\mu^{\rm SUSY} \approx 25 \times 10^{-10}$ (say), one would, however, need rather light sleptons ($m_{\tilde{l}} \approx$ 200-400 GeV) and $\tan\beta$ to be appropriately large ($\approx$ 6-24). Such light sleptons and correspondingly light squarks ($m_{\tilde{q}} \approx$ 400-800 GeV, say), together with moderately large $\tan\beta$, could, however, pose theoretical dilemmas pertaining to issues of (a) proton decay within SUSY grand unification, and possibly also (b) edm's of the electron and the neutron.

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[22] Masses and mixings of the first family fermions (in the hat basis) are attributed to higher dimensional operators consistent with flavor symmetries which directly provide entries of order 1 MeV in the 3×3 sector involving the chiral families.

[23] For example, to get a hierarchical pattern with the desired couplings for obtaining the Dirac masses of all fermions and Majorana masses of the right–handed neutrinos (and also to prevent undesirable couplings), one can assume a $U(1)_f \times (Z_3 \times Z_3') \times Z_2$ symmetry, with the introduction of two SO(10) singlets X and X', where the $Z_3 \times Z_3'$ transformation of the fields are essentially given in Ref. [2]. The singlets X and X' will be assumed to acquire GUT scale VEVs. To be specific, assign $U(1)_f$ charges of (2, 1, 0) to ($\mathbf{16}_1\ \mathbf{16}_2,\ \mathbf{16}_3$), -1 to X and zero to all other fields. Take the $Z_3 \times Z_3'$ transformations of the fields to be as follows: $\mathbf{16}_i \sim (\omega, 1)$, $\mathbf{16}_V \sim (1, \omega), \bar{\mathbf{16}}_V \sim (\omega, 1)$, $\mathbf{10}_H \sim (\omega^2, \omega^2)$, $H_V \sim (\omega^2, \omega^2)$, $H_s \sim (\omega, 1)$, $\mathbf{45}_H \sim (1, 1)$, $\mathbf{16}_H \sim (\omega, \omega)$, $\bar{\mathbf{16}}_H \sim (\omega^2, 1)$, $X \sim (1, 1)$ and $X' \sim (1, \omega^2)$, where $\omega^3 = 1$. Z_2 serves as a matter parity with all the Higgs fields ($\mathbf{45}_H$, $\mathbf{16}_H$, $\bar{\mathbf{16}}_H$, $\mathbf{10}_H$, X, X', H_V and H_s) being even and all matter fields

$(\mathbf{16}_i,\ \mathbf{16}_V, \bar{\mathbf{16}}_V)$ being odd. One can verify that the desired hierarchical couplings given in Eq. (2), as well as the ν_R Majorana masses (which would arise through couplings like $f_{33}\mathbf{16}_3\mathbf{16}_3\bar{\mathbf{16}}_H\bar{\mathbf{16}}_H/M$) would be allowed as needed, if $\langle X\rangle \sim \langle X'\rangle \sim 1/10$, and no undesirable term would be induced. With this choice, the first family masses should arise through yet unspecified higher order operators. Note that in the the context of string theory solutions, the apparent anomaly of $U(1)_f$ would typically be cancelled either by the presence of other massless fields or by the Green–Schwarz mechanism. Vanishing of the D–term of an anomalous $U(1)$ can generically induce GUT scale VEVs of certain fields (such as X and X') without a superpotential for such fields.

[24] Note, for ESSM, $M_{\rm GUT} \approx$ (1/2-2) $\times\ 10^{17}$ GeV [2], while $M_{\rm string} \sim$ (6-10) $\times\ 10^{17}$ GeV; thus $(M_{\rm GUT}/M_{\rm string})_{\rm ESSM} \sim$ (1/3-1/10).

[25] The case of MSSM embedded in SO(10), considered in Ref. [12], leads to a value for m_b, which is about 10 % higher than the observed value. Thus ESSM (with $\kappa = \delta = 0$) would lead to even larger values for m_b.

[26] With $\kappa \sim 1/10$, M_E can be lower than $M_{E'}$ by (say) 10-20 % at the electroweak scale, but the sign of the departure from unity in Eq. (4) will not alter our discussion of $\tau \to \mu\gamma$ (see next section).

[27] In addition, with $\kappa = \delta = 0$, there would be equalities between quark and lepton couplings, so that $z_u = z_d = z_l = z_\nu$; $x_u = x_l = x_d = x_\nu$, etc. and $M_U = M_D = M_{U'} = MD' = M_E = M_{E'}$ at the GUT-scale.

[28] Since $\tau \to \mu\gamma$ is rather sensitive to degree of non-degeneracy of E and E', we would still permit M_E and $M_{E'}$ to receive small contributions from the κ-term as suggested by m_b/m_τ. So, together with renormalization effects $(M_E/M_{E'})$ may vary in a range, such as that given by Eq. (4). Allowing for the κ-term would, however, not have a major effect on the relevant mixing angles.

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[30] This term is analogous to or could play the role of that in the Next to Minimal Supersymmetric Standard Model (NMSSM), which gives rise to the μ-term. Using $\langle H_s\rangle = v_s$, one would get $\mu = \lambda v_s$.

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[33] This is because, (a) the Z^0-current has a partial GIM-like invariant form in the subspaces of $(\mu_L, \tau_L, E_L, E_R)$ and $(\mu_R, \tau_R, E'_L, E'_R)$, and thus the new contributions involving physical (E, E') in the loop and physical (μ, τ) as external particles vanish except for $E - E'$ mixing which is small $(\simeq x_3 y'_3 v_d v_s / v_0^2 \lesssim (v_d/v_0)(v_s/v_0) < 1\%)$; and in addition (b) the effective coupling of Z^0 given by $(g_2/\cos\theta_W)(I_{3L} - Q_{\rm em}\sin^2\theta_W)$ is smaller than that of W^+.

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[38] One would of course get essentially the same answer if one calculates in the true physical basis for all fields, because the latter would induce non-diagonal couplings of the electron to the physical (E, E') and the N-fields, with the emission of (H_d-H_s) and W-fields respectively.

[39] $a_\mu^{\rm ESSM}$ in Eq. (33) does not, of course, include other contributions to a_μ such as those from supersymmetry which do not contribute to off-diagonal transitions.

[40] The mechanism in question introduces the effective vertices $\mathbf{16_1 16_H \hat{10}}$, $\mathbf{16_2 16_H 10'}$, $\mathbf{\hat{10} 10' 45_H}$, and GUT-scale mass term $\hat{M}\mathbf{\hat{10}^2}$ and $M'\mathbf{10'^2}$ in the superpotential, where the strengths of some of the vertices are presumed to be suppressed by powers of $\langle S\rangle/M$ owing to generalized flavor symmetries. Here, $\langle \mathbf{45_H}\rangle$ and $\langle \mathbf{16_H}\rangle$ acquire VEVs as stated in the text. Exchanging a $\mathbf{\hat{10}}$ that converts into $\mathbf{10'}$ (by utilizing the VEV of $\langle \mathbf{45_H}\rangle \propto B$-$L$) between the $(\mathbf{16_1 16_H})$ and $(\mathbf{16_2 16_H})$ pairs, one gets an effective operator of the form $\{\mathbf{16_1 16_2} \langle \mathbf{16_H^d}\rangle \langle \mathbf{16_H}\rangle \langle \mathbf{45_H}\rangle / M^2\}(\langle S\rangle/M)^n$, which would contribute

to mixing in the quark sector – i.e., to d-s mixing, but *not* to e-μ mixing. The electron can still get a suppressed diagonal mass of order 1/2 MeV through operators like $\{\mathbf{16_1 16_1 10_H} \langle \mathbf{45_H} \rangle^2 / M^2\} (\langle S \rangle / M)^m$, or simply through $\mathbf{16_1 16_1 10_H} (\langle S \rangle / M)^p$, which would make small corrections $\lesssim 1/2$ MeV to u and d-masses. Details of these considerations including the origin of ν_e-ν_μ mixing and of the large angle MSW solution for the solar neutrino puzzle will be taken up in a separate note.

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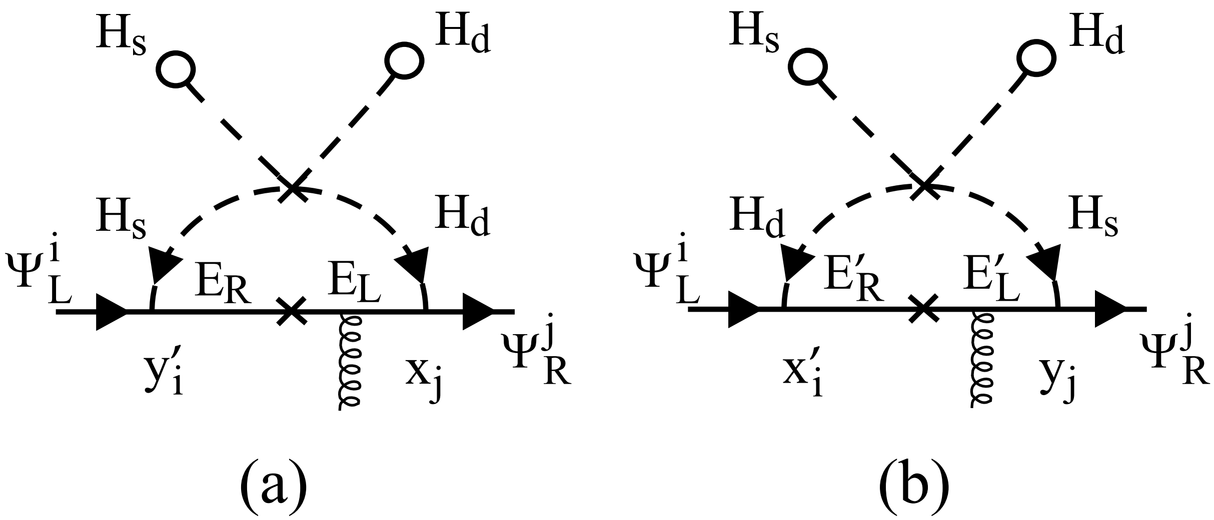

Figure 1: Contributions to $\ell_i \to \ell_j \gamma$ arising through H_d-H_s mixing

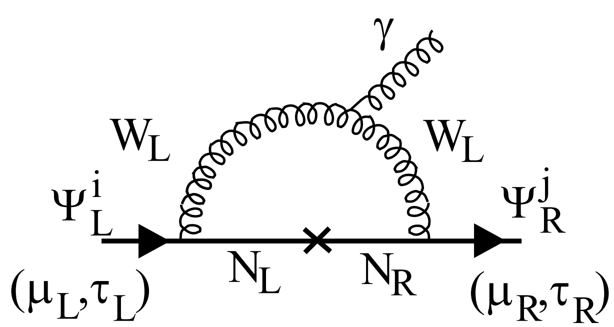

Figure 2: New contributions to radiative transitions from W-loop