

BARYON CHIRAL DYNAMICS*

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After contrasting the low energy effective theory for the baryon sector with one for the Goldstone sector, I use the example of pion nucleon scattering to discuss some of the progress and open issues in baryon chiral perturbation theory.

1 Higher, faster, swifter?

Many of the constraints of chiral symmetry on the interaction of pions and nucleons at low energies were worked out long before the advent of QCD (Current algebra, PCAC). Later it was realized that the corrections to these symmetry relations can be obtained by implementing the chiral symmetry and its breaking by the quark masses into an effective Lagrangian describing the interaction of mesons and baryons. This method is called chiral perturbation theory (CHPT).¹ It allows one to compute the expansion of QCD amplitudes and transition currents in powers of the external momenta and quark masses; it has become one of the standard tools to analyze the strong interactions at low energy.

Over the last few years, the progress in this field in the baryon sector has been twofold: on one hand, we have reached a new level of precision in many of the classical applications: by now, the full one loop result for the nucleon form factors and the pion-nucleon scattering amplitude in the isospin limit is known.^{2,3} Even the first two-loop result has been obtained: the chiral expansion of the nucleon mass has been worked out to fifth order.⁴ On the other hand, the framework has been extended and applied to a whole range of new processes: the effective Lagrangian has been extended to include electromagnetism, making it possible to disentangle strong and electromagnetic isospin violation.⁵ This effective theory of QCD+QED has been used to calculate next-to-leading order isospin violating effects in the pion-nucleon scattering amplitude⁶ and to study the properties of the π^-p bound state.⁷ Another extension of the framework incorporates the Δ -resonance as an ex-

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plicit degree of freedom into the effective Lagrangian, thereby summing up the potentially large higher-order terms in the chiral expansion associated with this resonance.⁸

Despite all of this impressive progress, we are still short of having the answers to some very old questions, like, for example, what is the value of the σ -term, and the agreement with data in many cases is not quite as satisfactory as in the Goldstone sector. In my talk, I will contrast the baryon with the meson sector and illustrate some of the peculiarities that arise, once the baryon field is included into the effective Lagrangian.

I will illustrate my discussion with the example of πN -scattering and conclude that the low energy theorems for this amplitude hold to high accuracy. Chiral symmetry governs the amplitude in a small region around the Cheng-Dashen point. However, the momentum dependence of the chiral representation for the amplitude is not accurate enough to make direct contact with experimental data. After discussing some of the difficulties associated with the extrapolation of the experimental results to the low energy region, I show how the simple structure of the result in the low energy theory can be implemented into a dispersive analysis of the data.

As I focus the discussion mostly on πN -scattering, I will fail to report on many important developments over the past few years. Fortunately, my sense of guilt for omitting electromagnetic probes of the nucleon was relieved by the plenary talks of Ed Brash, Helene Fonvieille, Frank Maas and Harald Merkel as well as a number of interesting talks on these matters in the parallel sessions. Unfortunately, there were no talks covering the few nucleon sector⁹, nor about the recent work on quenched¹⁰ and partially quenched¹¹ baryon CHPT.

2 Baryons versus Mesons

While this is not the place to give an introduction to CHPT¹², it is instructive to point out some of the differences between CHPT in the baryon and the Goldstone sector. All in all, the inclusion of the baryon field leads to three complications: i) in general, one has to deal with a larger number of low energy constants than in the vacuum sector, ii) from the viewpoint of the low energy theory, the physical region is at higher energies, iii) the singularity structure of the amplitudes is more complicated. On the upside, there are much more and more precise data available than in the meson sector.

2.1 Effective Lagrangian

For vanishing up- and down-quark masses, the pions are the Goldstone bosons associated with the spontaneous breaking of chiral symmetry. The interactions between Goldstone bosons tend to zero at low energies and they decouple from matter fields. Accordingly, the effective Lagrangian is organized in powers of derivatives on the Goldstone fields. At low energies, the terms with higher powers of derivatives on the meson field are suppressed by powers of the meson momenta. Because of decoupling, the interactions of the baryon and the meson involve at least one derivative on the meson field. The lowest-order, effective Lagrangian for the pion-nucleon interaction reads

$$\mathcal{L}_{\text{eff}}^N = -\frac{g_A}{2F_\pi} \bar{\psi} \gamma^\mu \gamma_5 \partial_\mu \pi \psi + \frac{1}{8F_\pi^2} \bar{\psi} \gamma^\mu i[\pi, \partial_\mu \pi] \psi + \dots \quad (1)$$

The ellipsis stands for terms which involve higher powers of the pion field. Their coefficients are fixed by chiral symmetry. At second order, the effective Lagrangian also contains terms proportional to the quark masses.

The fact that the lowest-order Lagrangian is fully determined by the nucleon mass and the matrix element of the axial charge shows how chiral symmetry constrains the interactions of mesons and baryons. However, the rapid increase in the number of parameters at higher orders makes it evident that it is not nearly as restrictive as in the meson sector. The number of parameters entering at each order is shown in brackets:

$$\begin{aligned} \mathcal{L}_{\pi\pi} &= \mathcal{L}_{\pi\pi}^{(2)} + \mathcal{L}_{\pi\pi}^{(4)} + \mathcal{L}_{\pi\pi}^{(6)} \\ &\quad (2) \qquad (7) \quad (53) \\ \mathcal{L}_{\pi N} &= \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)} + \mathcal{L}_{\pi N}^{(4)} \\ &\quad (2) \quad (7) \quad (23) \quad (118) \end{aligned}$$

The larger number of low energy constants arises from the spin- $\frac{1}{2}$ nature of the nucleon and because it stays massive in the chiral limit, so that the effective Lagrangian involves odd as well as even powers in the chiral expansion.

In a given process only a handful of the outrageous number of terms in the fourth-order Lagrangian will contribute. The fact that the effective Lagrangian contains 118 terms at fourth order,¹³ however, means that the chances that the same combination enters two different observables are rather dim: there will hardly be any symmetry relations valid to fourth order in the chiral expansion.

2.2 The low energy region and the role of resonances

The strongest constraints from chiral symmetry on the πN -scattering amplitude are obtained at unphysically small values of Mandelstam variables, at the Cheng-Dashen point $s = u = m_N^2$, $t = 2M_\pi^2$. In figure 1 the Mandelstam triangles for $\pi\pi$ - and πN -scattering are compared. The figure makes it evident that the physical threshold for πN -scattering is at higher energies: the increase in s from the Cheng-Dashen point to the threshold is of $O(M_\pi)$ for πN -scattering, while it is $O(M_\pi^2)$ for $\pi\pi$ scattering. At threshold, higher-order

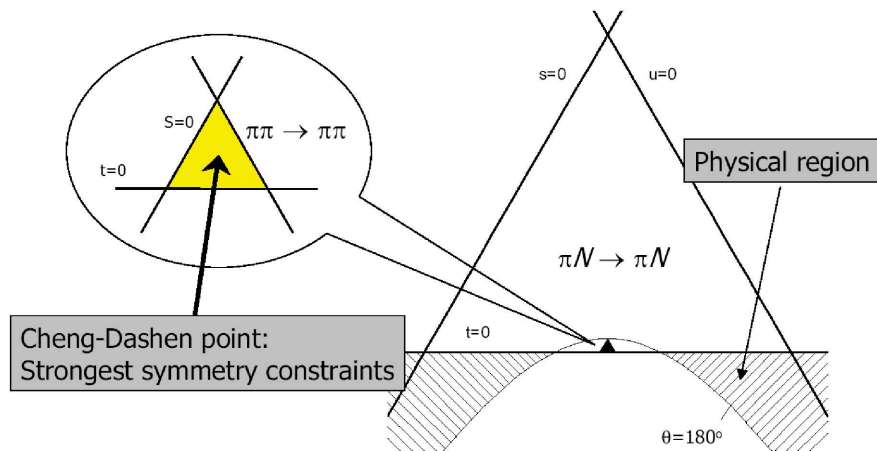


Figure 1. Comparison of the Mandelstam triangles for $\pi\pi$ - and πN -scattering.

terms in the chiral expansion will therefore be more important in πN - than in $\pi\pi$ -scattering.

This observation is confirmed by looking at the position of the first resonance. The increase in s from the Cheng-Dashen point to the first resonance is roughly the same in both cases: $m_\Delta^2 - m_N^2 \approx m_\rho^2$. The relevant expansion parameter for the resonance contributions at threshold is, however, much larger for πN -scattering: $2m_N M_\pi / (m_\Delta^2 - m_N^2) \approx 0.4 \gg 4M_\pi^2 / m_\rho^2 \approx 0.1$. While the effective theory for the meson sector will still yield meaningful results well above threshold, the Δ -resonance must be included into the Lagrangian if one wants to arrive at an accurate description of the meson nucleon amplitude above threshold.

This can be done in a systematic way by counting the mass difference $\delta = m_\Delta - m_N$ as a small quantity of the same order as M_π . This procedure

is referred to as “small scale expansion” and allows one to resum the potentially large corrections associated with the resonance.⁸ While it is certainly important to get a handle on the resonance contributions, a few words of caution are appropriate: we are still performing a low energy expansion and there are higher-order terms not associated with the resonance. In particular, the inclusion of the Δ has so far only be performed in the non-relativistic framework for baryon CHPT (to be discussed in the next subsection) and the higher-order kinematic corrections are important already in the threshold region. Furthermore, the effective theory which includes the Δ is not unique: recently it has been claimed that the effective Lagrangian is compatible with different counting schemes; it seems in general not possible to decide from first principles at which order a given operator enters.¹⁴ The πN -scattering amplitude has been calculated to third order in this combined expansion in δ and the meson momenta.¹⁵ The calculation confirms that the bulk of the Δ contribution stems from the resonance pole term. According to the authors, the energy range in which their results reproduces the existing data is only slightly larger than for the fourth-order calculation in pure CHPT.

2.3 Formulation of the effective theory

In the low energy expansion, the baryon four momentum P_μ has to be counted as a large quantity, since $P^2 = m_N^2$ is of the size of the typical QCD scale squared. If we choose a frame, where the baryon is initially at rest and let it interact with low energy pions, the nucleon will remain nearly static, its three momentum being of the order of the meson mass. The chiral expansion of the corresponding amplitudes in the momenta and masses of the mesons therefore leads to an expansion of the nucleon kinematics around the static limit. This expansion is implemented ab initio in the framework called heavy baryon chiral perturbation theory (HBCHPT),¹⁶ However, the expansion of the kinematics fails to converge in part of the low energy region. The breakdown is related to the fact that the expansion of the nucleon propagator in some cases ruins the singularity structure of the amplitudes. This makes it desirable to perform the calculations in a relativistic framework. In doing so, the correct analytic properties of the amplitudes are guaranteed, and one can address the question of their chiral expansion in a controlled way.

In the relativistic formulation of the effective theory a technical complication arises from the fact that in a standard regularization prescription, like dimensional regularization, the low energy expansion of the loop graphs starts in general at the same order as the corresponding tree diagrams.¹⁷ Since the contributions that upset the organization of the perturbation expansion

stem from the region of large loop momentum of the order of the nucleon mass, they are free of infrared singularities. In d -dimensions, the infrared singular part of the loop integrals can be unambiguously separated from the remainder, whose low energy expansion to any finite order is a polynomial in the momenta and quark masses. Moreover, the infrared singular and regular parts of the amplitudes separately obey the Ward identities of chiral symmetry. This ensures that a suitable renormalization of the effective coupling constants removes the infrared regular part altogether, so that we may drop the regular part of the loop integrals and *redefine* them as the infrared singular part of the integrals in dimensional regularization, a procedure referred to as *infrared regularization*.¹⁸ The representation of the various quantities of interest obtained in this way combines the virtues of HBCHPT and the relativistic formulation: both the chiral counting rules and Lorentz invariance are manifest at every stage of the calculation.

In the meantime, this relativistic framework has been used to calculate the scalar¹⁸, axial¹⁹ and electro-magnetic form factors²⁰ as well as the elastic pion-nucleon amplitude³ to fourth order in the chiral expansion. Recently, the Gerasimov-Drell-Hearn sum rule has been reanalyzed and it was found that the recoil corrections, which are summed up in the relativistic approach, are rather large.²¹

3 Pion-nucleon scattering

3.1 Low energy theorems

Chiral symmetry constrains the strength of the πN -interaction as well as the value of the scattering amplitudes at the Cheng-Dashen point. The fourth-order result for the scattering amplitude allows us to analyze the corrections to the low energy theorems that arise at leading order in the expansion and we find that the symmetry breaking corrections are rather small.

As a first example, let us consider the Goldberger-Treiman relation

$$g_{\pi N} = \frac{g_A m_N}{2 F_\pi} (1 + \Delta_{\text{GT}}).$$

If the masses of the up- and down-quarks are tuned to zero, the strength of the πN interaction is fully determined by g_A and F_π : $\Delta_{\text{GT}} = 0$. Up to and including terms of third order in M_π , the correction has the form

$$\Delta_{\text{GT}} = c M_\pi^2 + O(M_\pi^4).$$

It is remarkable that the correction neither involves a term of the form $M_\pi^2 \ln(\frac{M_\pi^2}{m_N^2})$ (a “chiral logarithm”) nor a correction of order M_π^3 . Such in-

frared singular terms *are* present in the chiral expansion of $g_{\pi N}$, g_A , F_π and m_N , but they cancel out in the above relation. To this order, the correction is thus analytic in the quark masses. If the low energy constant c is of typical size, $c \approx 1/\text{GeV}^2$, the correction to the Goldberger-Treiman relation is 2%. If one evaluates the above relation with value for the coupling constant given in Höhler's comprehensive review of πN -scattering²², one finds $\Delta_{\text{GT}} = 4\%$. The data accumulated since then seems to favor a smaller value of $g_{\pi N}$ reducing the correction to 2-3%.

Another well known low-energy theorem relates the value of the isosymmetric amplitude D^+ at the Cheng-Dashen point^a

$$\Sigma = F_\pi^2 \bar{D}^+(s = m_N^2, t = 2M_\pi^2)$$

to the scalar form factor

$$\langle N' | m_u \bar{u}u + m_d \bar{d}d | N \rangle = \sigma(t) \bar{u}'u.$$

The relation may be written in the form

$$\Sigma = \sigma(2M_\pi^2) + \Delta_{CD}.$$

The theorem states that the term Δ_{CD} vanishes up to and including contributions of order M_π^2 . The explicit expression obtained for Σ when evaluating the scattering amplitude to order q^4 again contains infrared singularities proportional to M_π^3 and $M_\pi^4 \ln M_\pi^2/m_N^2$. Precisely the same singularities, however, also show up in the scalar form factor at $t = 2M_\pi^2$, so that the result for Δ_{CD} is free of such singularities:

$$\Delta_{CD} = d M_\pi^4 + O(M_\pi^5).$$

A crude estimate like the one used in the case of the Goldberger-Treiman relation indicates that the term Δ_{CD} must be very small, of order 1 MeV. Unfortunately, the experimental situation concerning the magnitude of the amplitude at the Cheng-Dashen point leaves much to be desired. The inconsistencies between the results of the various partial wave analyses need to be clarified in order to arrive at a reliable value for $g_{\pi N}$. Only then it will be possible to extract a small quantity like the Σ -term from data.^b

^aThe bar indicates that the pseudo-vector Born term has been subtracted.

^bJugoslav Stahov has reported at the conference that discrepancies in the higher partial waves of different partial wave analyses can explain the inconsistencies between different determinations of the Σ -term.²³

3.2 Momentum dependence: analyticity and unitarity

To obtain the amplitudes in the region around the Cheng-Dashen point, the experimental results need to be extrapolated to the subthreshold region. The extrapolation can only be performed reliably, if the correct structure of the singularities of the amplitude is implemented into the data analysis. Having to deal with functions of two variables, this is not a simple task and while all modern partial wave analyses incorporate some of these constraints, subsequent analyses have not kept up with the high level of sophistication reached by the Karlsruhe-Helsinki collaboration in the eighties.

Because of the complexity of a dispersive analysis, it is tempting to use the representation obtained in chiral perturbation theory to perform the extrapolation to the unphysical region, since the use of a relativistic effective Lagrangian guarantees the correct analytic properties in the low energy region. The problem with this approach is that unitarity is not exact in the chiral representation, but only fulfilled to the order considered. At one loop level, the imaginary part will be given by the current algebra amplitudes squared. Since the corrections to the current algebra result become sizeable above threshold, the violation of unitarity will prevent an accurate extrapolation to the subthreshold region in this framework.

This is illustrated in figure 2, where we compare the result obtained in CHPT with the KA84 solution.²⁴ The parameters in the chiral representation have been adjusted to the KA84 solution at the threshold and we want to check the energy range in which we reproduce the KA84 solution. For the amplitude D^+ , the deviation in the region around the Cheng-Dashen point would translate into a 10 MeV uncertainty in the Σ -term. The accuracy is better in the case of the amplitude D^- , but also in this case the chiral representation starts to deviate soon after threshold.

There are various prescriptions²⁵ to fix the problem by hand: one can, e. g., use the K-matrix formalism to unitarize the amplitudes found in CHPT. Once some resonances are added in, these unitarized amplitudes usually fit the data very nicely, however, this “solution” has its price: the unitarizations usually ruin crossing symmetry and analyticity, by introducing unphysical singularities into the results, making their use for an extrapolation to lower energies doubtful.

We have set up a framework that combines the analytic structure found in CHPT with the constraints from unitarity:³ one starts by writing a dispersive representation for the result found in the low energy effective theory. This representation splits the amplitude into a polynomial part and nine functions of a *single* variable, which are given by integrals over the imaginary parts of the

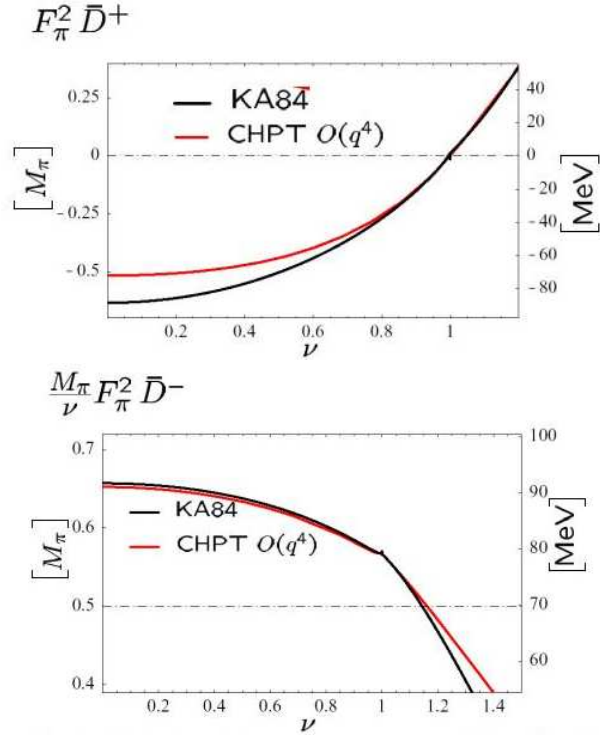


Figure 2. Real part of the pion-nucleon amplitude at zero momentum transfer. The variable ν denotes the lab. energy of the incoming pion. The reaction threshold is at $\nu = M_\pi$. The red line is the result obtained at the fourth order in the chiral expansion. The black curve corresponds to the KA84 solution.

amplitude. In the elastic region, unitarity then leads a set of coupled integral equations for these functions, similar to the Roy equations in $\pi\pi$ -scattering. Replacing the imaginary parts found in CHPT by the experimental imaginary parts in the inelastic region and solving the equations iteratively one arrives at a representation of the amplitude that fulfills both the constraints from unitarity and analyticity. In addition to the imaginary parts, this system of equations also needs four subtraction constants as an input. One of them can be expressed as an integral over the total cross section, while the other three need to be pinned down from the experimental information at low energies. The results from the study of pionic hydrogen, to be discussed below, should

subject these constants to stringent bounds.

3.3 Isospin violation, pionic hydrogen

To study strong isospin breaking, one needs to disentangle it from electromagnetic isospin violation. Since both are of similar magnitude, they need to be treated simultaneously, making it necessary to incorporate the photon field as an additional degree of freedom into the low energy effective Lagrangian. In the baryon sector, the corresponding Lagrangian has been worked out to third order⁵ in a simultaneous expansion in $m_q \sim q^2 \sim e^2$ and the result for the pion-nucleon scattering amplitude has been worked out to the same order.⁶

An important application of the low energy effective theory of QCD+QED is the extraction of the hadronic scattering length from the measurements of the strong interaction width and level shifts of hadronic atoms. The goal of the experiments with pionic hydrogen (the bound state of a π^- with a proton) at PSI²⁶ is to measure these quantities at the level of one per cent. In order to extract the pure QCD scattering lengths from the measurements, one needs to remove isospin breaking effects with high precision. The framework for the calculation has been set up and by now, the calculation of the strong energy shift has been carried out to next-to-leading order in isospin breaking.⁷ The results differ significantly from earlier potential model calculations which fail to consistently incorporate all of the interactions present even at the leading order. At present, the main uncertainty in the result of the effective theory is the value of the low energy constant f_1 , whose value is as yet unknown.

4 Conclusions

We have a good understanding of how chiral symmetry manifests itself in the baryon sector. Chiral symmetry breaking effects, on the other hand, are *small* and their determination from measurements is nontrivial. The reason being that, in many cases, we cannot directly confront the low energy theorems of the symmetry with the experimental data taken at higher energies. In this situation, the precise extrapolation of the data to lower energies becomes a central issue. While the representations obtained in CHPT are not suitable for this purpose, their analytic structure can be implemented into a dispersive analysis.

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