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## The Asymmetry between Matter and Antimatter<sup>\*</sup>

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## Abstract

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We live in a Universe dominated by matter, containing very little antimatter. The laws of physics, however, seem to include an almost exact symmetry between matter and antimatter. The near symmetry between matter and antimatter is CP, that is charge conjugation (C) times parity (P), rather than just charge conjugation.

Prior to the Dirac equation [1] there was no concept of such a symmetry, or indeed of the existence of antimatter. Instead there was simply a conservation law, the conservation of matter (or of atoms). Thus all the matter, all the stuff with mass, in the Universe, must always have been there. At the time, the prevailing view of western science, and indeed of western religions and philosophy, was that we live in a static unchanging Universe. The constancy of matter in such a Universe is no puzzle.

Hubble's law, the linear relationship of redshift of distant galaxies with the distance to them [2], provided the first key evidence for an expanding Universe. It appeared a year after Dirac's equation. With this discovery the science of cosmology began; questions about the physical basis of an evolving Universe were inevitable. Even in an evolving Universe, if matter is conserved, its presence can only be understood as an initial condition of the evolution. But Dirac's equation had started us down a path that has radically changed that situation.

To understand the initial struggle to interpret Dirac's work it helps to recognize that there were then only two known fundamental matter particles: electrons and protons. Understanding the nature of nuclei, particularly the concept of neutrons and their discovery, was a work in process. Neutron detection was only understood in 1932 [3]. So when Dirac's equation appeared physicists did not readily postulate new particle types to explain new phenomena, let alone to explain a peculiar result in a newly postulated equation.

The equation was the fruit of Dirac's effort to obtain a relativistic spin 1/2 equation of motion. The equation had one very successful feature, it gave the correct magnetic moment for an electron. This was, Dirac later said, "an unexpected bonus, completely unexpected" (Dirac reminiscing, 1977) [4]. But at the same time it contained an enigma, which manifests itself at first as the existence of negative energy states.

Such states are clearly unphysical. At best they suggest one is attempting to do perturbation theory around the wrong ground state or vacuum. At worst they say the theory is incurably sick, having no lowest energy state. For fermions, the exclusion principle offered a cure. A better vacuum or ground state can be found, in which all negative energy states are filled, but all positive energy states are empty. This transforms but does not remove the enigma. Dirac's equation then has excitations that must be interpreted as positively charged particles (holes in the negative-energy sea) in addition to the negatively charged electron states. What possible physical interpretation could be made of these objects? As Dirac later stated, "At that time ... everyone felt pretty sure that the electrons and the protons were the only elementary particles in nature" [5]. Thus, it seemed, the positively charged states must be protons. This interpretation had one obvious deficiency, and a second, more fatal, flaw that took a little longer to be noticed.

The obvious problem was that the equation required that the proton and the electron have the same mass. Indeed it gave an exact symmetry between the properties of the positive and negatively charged states. Dirac was not unaware of this, as his 1929 letter to Bohr makes clear: "as long as one neglects interactions one has complete symmetry between electrons and protons; ... However when the interaction between the electrons is taken into account, this symmetry is spoilt. I have not yet worked out mathematically the consequences of the interaction .... One can hope, however, that a proper theory of this will enable one to calculate the ratio of the masses of protons and electrons" [6]. I can only guess at what Dirac means by "the interactions", perhaps between the particles filling the negative energy states (though we now understand that there are no such effects). Whatever he was thinking, his wild hope that the symmetry can be removed by including a next-order correction is completely wrong!

Weyl, who knew a symmetry and its consequences when he saw one, stated, in November of 1930, "... indeed, ... the mass of the proton should be the same as the mass of the electron; furthermore ... this hypothesis leads to the essential equivalence of positive and negative electricity under all circumstances" "... the dissimilarity of the two kinds of electricity thus seems to hide a secret of nature which lies deeper than the dissimilarity of past and future ... a new crisis of quantum physics ..." [7].

The crisis deepened when Oppenheimer and, independently, Tamm noticed the second problem [8, 9]. They saw that the equation contained the possibility of particle-

hole annihilation. This was the kiss of death for the proton interpretation of the holes. Even if higher-order corrections could fix the mass, they could not remove this disaster. Such a process would destroy all possibility of stable matter. So in May 1931, Dirac made what he later called a "small step forward" [10]. He declared that "A hole, if there were one, would be an entirely new kind of particle, unknown to experimental physics, having the same mass, and opposite charge of the electron" [11]. The evasive phrase "if there were one" is interesting. It seems that the idea of predicting a new type of particle was overwhelmingly bold to Dirac.

Not much more than a year later, these particles were observed [12]. The observation of the positron raised a new question. Why is the world populated with electrons but not positrons? This cosmological issue was recognized by Pauli who said, in a remarkable June 1933 letter to Heisenberg: "... I do not believe in the hole theory, since I would like to have the asymmetry between positive and negative electricity in the laws of nature (it does not satisfy me to shift the empirically established asymmetry to one of the initial state)" [13]. As far as I know this is a first statement of the view, held today by many particle physicists and cosmologists, that it is unsatisfactory to appeal to initial conditions to explain the dominance of matter over antimatter in the Universe. Pauli also points out that to avoid having to appeal to initial conditions one must somehow remove the symmetry between matter and antimatter seen in Dirac's equation.

Experiments soon added more types of antimatter, antiprotons [14] and antineutrons [15] and eventually also additional particles, the mesons [16], which eluded classification as either matter or antimatter (today we see them as equal mixtures of both, with a basic substructure of a quark and an antiquark). These discoveries changed the basic law of conservation of matter. Annihilation and production of matter can occur, but only with annihilation or production of a matching amount of antimatter. The law of conservation of matter is replaced by conservation of baryon and lepton numbers, the number of particles minus antiparticles of a given fermion type. These two features, symmetry between matter and antimatter in the equations, and baryon and lepton number conservation, were considered exact laws. With them, the observed asymmetry of matter and antimatter in the Universe can only be accounted for by imposing an initial condition. The discovery of antimatter and of mesons was a complete revision of the concept of "matter." One can no longer define matter as that which has mass. Thus, by choosing the term "antimatter" physicists also changed the definition of the word matter. Unfortunately, at least in the US, middle school text books still give the out-dated definition "matter is that which has mass and occupies space." Today, seventy-some years after the discovery of positrons, most people in the US still think antimatter exists only science fiction. Publicity over recent results on the production of a small number of antihydrogen atoms may temporarily change this perception, but still leaves most people unaware that accelerators routinely produce large number of positron and of antiprotons. I find that many otherwise educated people are truly surprised to hear that antimatter is real, and commonly made in our laboratories.

Pauli's comment aside, most physicists for the next twenty-some years not only accepted the existence of antimatter, but also the exact symmetry, C or Charge Conjugation, between the laws of physics for matter and antimatter. All local field theories have a compound symmetry CPT, where P is parity and T is time-reversal. In QED, the direct descendent of Dirac's equation, all three of these symmetries are separately conserved. It seems most physicists expected this pattern to be a general one. In 1956 Lee and Yang pointed out that there was indeed no evidence for or against parity conservation in weak interactions [17]. Experiments soon showed that weak interactions are maximally parity violating, and also violate C symmetry [18].

However, while P and C symmetry were seen to be broken, a revised matterantimatter symmetry, CP, still appeared to be exact. Indeed, all known particle theories at the time, like Dirac's equation, automatically had CP symmetry once one imposed locality and hermiticity. Even the modern Standard Model would be in this class if there were only two generations of quarks and one Higgs boson [19]! One needs quite a number of different particle types and many independent couplings in the theory before one can have a field theory that has room for CP violation.

In 1957 such a theory would have been wildly speculative. Quarks were not yet invented. The four-Fermi theory of weak interactions fit the observations and preserved CP symmetry and T symmetry. It had the inconvenient feature that any higher order calculation gave non-renormalizable divergent answers! Since weak interactions are rare processes, the lowest order theory worked quite satisfactorily, so that embarrassment could be set aside for a while at least. As far as I know, no-one immediately made the leap that Pauli had earlier made, to the idea that a theory without CP symmetry would be preferable, until Sakharov's 1967 paper (discussed below), which comes after the empirical discovery of CP violation.

The assumption that CP is an exact symmetry of nature was destroyed by experiment in 1964. CP-symmetry violation was discovered in neutral K meson decays. Two states of similar mass but very different half life were assumed to be the CP-even and CP-odd combinations of the strange  $K^0$  meson and its CP-conjugate antiparticle of opposite strangeness  $\overline{K}^0$ . Only the CP-even state  $((K^0 + \overline{K}^0)/\sqrt{(2)}$  can decay to two pions, while the hadronic decays of the CP-odd state  $(K^0 - \overline{K}^0)/\sqrt{(2)}$  are suppressed by the much smaller phase space available for a three pion decay. This explained the large difference in half life for the two neutral K mesons. Christenson, Cronin, Fitch and Turlay, PRD July 1964 [20] observed the CP-forbidden two-pion decays of the long-lived neutral kaon. This proved that the definite half-life eigenstate is not an eigenstate of CP.

If the mass eigenstates are not CP eigenstates then the only possible conclusion is that CP symmetry is not exact. The experimental result was simple and irrefutable, and rapidly confirmed by others. At the time the result was a surprise and a puzzle. The modern theory of particle interactions, the three-generation Standard Model, allows such CP-violating effects, but that was not evident until almost ten years later.

Once CP is not a symmetry, the matter-antimatter asymmetry of the Universe could be a result of its evolution rather than simply the initial condition so disliked by Pauli. One of the first people to recognize this was Sakharov. In 1967 he proposed that baryons and antibaryons were present in equal quantities in the early Universe, and that the imbalance developed at some later time, a process nowadays referred to as "baryogenesis". Sakharov [21] showed that this requires

- baryon number changing processes in the early Universe
- CP-symmetry violation
- an out-of-equilibrium situation for the Universe at time of baryogenesis.

Sakharov's primary observation was that it follows from the equal masses of particles and their antiparticles that, if the two species are in thermal equilibrium, their populations are equal. If there is any baryon number-changing process then this thermal equilibrium will be achieved. This means that, whatever the initial condition, collisions and decays drive the baryon number density to zero. One must protect an initial condition with a conservation law for it to significantly affect the history of the Universe.

Sakharov further recognized that any transition from thermal equality of matter and antimatter to the present inequality must occur at an out-of-equilibrium stage in the history of the Universe. Furthermore, for any imbalance to develop, both baryon-number conservation and CP symmetry must be broken. CP symmetry would require a balance between any baryon-producing process and its CP inverse, antibaryon-production. Finally, baryon-number-changing processes must be rare, or "frozen out" at all later times. If not, these processes will gradually remove any baryon excess and return the system to thermal equilibrium equality between the baryons and antibaryons. For its time, this was a revolutionary paper, since the notion of baryon-number conservation was still firmly established in the theorist's canon.

To discuss whether, and if so when, Sakharov's three conditions are satisfied, requires a fully developed theory of particle interactions that includes both CPsymmetry breaking and baryon-number changing terms. This we now have, in the Standard Model of particle physics.

The early version of the theory was a two-generation theory, with four quarks and four leptons. Kobayashi and Maskawa addressed the issue of CP-symmetry breaking in this theory in 1973 [19]. They pointed out that the theory with two generations was automatically CP conserving, and further showed that a three-generation generalization of the Cabibbo structure of quark weak currents gave a theory that allowed CP violation, but with only a single CP-violating parameter. This at a time when most physicists were skeptical of the hypothesis of a fourth quark. Weinberg observed that another option to introduce CP violation, instead of adding additional quarks, was to add additional Higgs bosons [22].

A big change in attitude occurred in 1974–75. The first reason was the discovery

of particles containing charm quarks [23]. This discovery gave great credence to the Standard Model. By remarkable coincidence another new particle, the tau lepton, was pair-produced in the same energy region [24]. So by the time the second generation of particles was complete we had also found a third lepton. A third generation for the Standard Model was no longer a fringe suggestion. Kobayashi and Maskawa's three generation theory became the Standard Model, and with it, CP violation became an effect that could be included in theoretical discussions.

But is the single CP-violating parameter in the Kobayashi-Maskawa scheme the full story? Are other extensions of the theory, such as Weinberg's additional Higgs particle, needed? This is an active question in current work, both theory and experiment. To date, all quark weak decay results fit with a single set of Standard Model parameters, but both new measurements and new calculations continue to refine the issue.

Once we have a theory that includes CP violation we can address the question of the matter-antimatter asymmetry in the Universe. CP violation was only one of Sakharov's conditions for generation of this asymmetry. Equally striking was the requirement that baryon number not be a conserved quantity. The Standard Model indeed predicts that baryon number changing processes are possible. These processes are non-perturbative multiparticle effects, which are very rare at low temperature, and thus consistent with the observed long half-life of the proton (greater than  $10^{32}$  years). However at sufficiently high temperature in the early Universe these baryon-numberchanging processes would be frequent. So thermal equilibrium with equal amounts of matter and antimatter would prevail at high temperature in the Universe, no matter what the initial condition was. The existence of baryon-number-changing processes, which Sakharov showed is necessary to explain how an imbalance could develop, also makes that explanation essential.

The third condition for an imbalance to develop, that of an out-of-equilibrium situation, can be met in two basic ways. The first occurs whenever the Universe undergoes a phase transition, as will be explained below. The other possibility is that a massive particle type that interacts very weakly, produced in the hot early Universe, decays late enough that CP-inverse production processes are no longer probable. CP violation in the decay of such a particle can then produce a matterantimatter imbalance in the decay products.

Phase transitions occur when, due to expansion, the Universe cools to certain critical temperatures. The transition of interest here is the weak-scale phase transition, which occurs when the Universe cools enough that effective potential for the Higgs field develops a minimum at non-zero field. At this phase transition particle masses appear, and the large mass of the W and Z bosons suppress the baryon-numberchanging processes thereafter, so they become extremely rare. Thus any baryon number produced during this phase transition will persist. Further, this is the latest time in the Standard Model history of the Universe where it would be possible to change the overall baryon number.

Baryogenesis at this weak-scale phase transition [25] depends on it being first order phase transition. In such a transition, a bubble region forms with the correct lowtemperature vacuum inside it and a distinct bubble wall. The imbalance in matter over antimatter inside the bubble arises because CP violation gives different transmission probabilities through the bubble wall for matter and antimatter. However, for the Standard Model, the most recent limits on Higgs mass [26] destroy this scenario. The relevant phase transition is not first order! The picture of an expanding bubble with a well-defined bubble wall is lost for a second-order phase transition. Thus, in the minimal Standard Model, the scenario simply does not work.

For particle physicists this suggests a fascinating possibility—perhaps the failure is not in the cosmological scenario but in the underlying particle physics of the Standard Model. For example if one adds additional Higgs field then the conditions for a first-order phase transition can be recovered. Furthermore this can give additional CP-violation effects at the phase boundary. One can also investigate more complex extensions of the Standard Model, among them Grand Unified and supersymmetric theories. The list of ideas is long; only further particle physics experimentation can narrow it down. As experiments add further constraints the task of finding a theory that fits the data and gives a satisfactory picture for baryogenesis at a phase transition gets harder, but it can still be achieved.

An intriguing possibility of the second type has recently appeared. Three different types of evidence now all point to the fact that neutrinos have some tiny masses [27]. As with the quarks, the mass eigenstates and the weak-interaction-decay eigenstates

are misaligned. The observed phenomenon is neutrino-oscillation. Neutrinos are produced as particles with definite flavor but appear to oscillate to some other flavor (and back) as they travel through space (or through matter) due to the superposition of the different mass eigenstates in the definite flavor neutrino.

Neutrino masses are too small to be measured directly (so far), but observations can tell us something about the mass differences between two neutrino mass eigenstates that both contain some component of the original flavor eigenstate. The mass differences and mixing parameters are beginning to be explored. As for the case of quarks, there can be CP-violating parameters in the mixing matrix. Could some effect arising from the CP violation in the neutrino sector give the matter-antimatter asymmetry of the Universe?

One can indeed devise scenarios where this is the case. In any theory of neutrino mass there are some very massive neutrino species as well as the familiar light ones. Early in the evolution of the Universe the massive neutrino species are produced in collisions. These particles interact extremely little and decay long after their production via collisions has become improbable. This gives an out-of-equilibrium situation, satisfying Sakharov's second condition. CP violation in these decays can produce a net lepton number in the remaining light species. If, at this stage, all other interactions that can change (B-L) (baryon number minus lepton number) have been frozen out (have become improbable) this quantity is now fixed. The lepton asymmetry then thermalizes via processes that change both baryon and lepton number, but not their difference. This then produces a baryon excess. This possibility has been explored in a number of theoretical models, such as grand unified theory extensions of the Standard Model. The details depend on the particular extension. One can find models where this scenario can give the observed asymmetry [28]. Time will tell if these models are satisfactory in other aspects of their predictions as well.

So this brings us back to the fundamental particle physics question, what is the asymmetry in the underlying laws of physics between matter and antimatter? Is it correctly and fully described by the CP violation in the quark or the neutrino mixing matrices of the revised Standard Model (with neutrino masses) or not?

Some of the answers can be sought in the high energy laboratory. Indeed the large effort in B physics in high energy laboratories around the world has this focus. The

major goal is to make as many redundant measurements as possible of quantities that, in the Standard Model, determine the values of mixing-matrix elements, including the one CP-violating phase, (or the equivalent rephasing-invariant quantity known as the Jarlskog invariant) [29]. Do all these measurements give results consistent with a single set of values for the four independent parameters that define the CKM matrix? If so, we will have better-determined values for all these quantities. If not, then further study of the nature of the inconsistencies will point toward one or another possible addition to the Standard Model theory—and perhaps help us understand Baryogenesis too.

This work is well begun, the SLAC and KEK *B* factories have produced first results [30] and will yield additional interesting results over the next five to ten years. These will be complemented by other results from *B*-physics experiments at hadron colliders. *B* physics provides a wonderful laboratory to study CP violation because both the  $B_d$  and  $B_s$  mesons form a pair like the neutral *K* mesons where the flavor eigenstates are mixed to form mass eigenstates, thereby providing a sensitive probe for CP violation. Further, because of the large mass of the *b* quark, the *B* mesons have many possible decay channels, so they provide many different probes of the Standard Model CKM and CP-violation structure. This allows one to develop a redundant set of independent determinations of Standard Model parameters, and thus probe for inconsistencies that could serve as pointers to extensions of the theory.

The figure, taken from the Particle Data Group review of the quark mixing matrix [31] shows that consistent results are given by a variety of experimental routes to determine the parameters of the Standard Model quark mixing matrix, also known as the CKM matrix. The plot shows the favored region for the parameters  $(\overline{\rho}, \overline{\eta})$  which enter into the matrix element ratios  $V_{ub}^* V_{ud}/V_{cb}^* V_{cd} = \overline{\rho} + i\overline{\eta}$  and  $V_{tb}^* V_{td}/V_{cb}^* V_{cd} = 1 - \overline{\rho} - i\overline{\eta}$  in the Wolfenstein parametrization of this matrix [32]. The inputs for the value of  $V_{cb}$  and  $V_{ub}$  come from semileptonic decays of B mesons to states with and without charm respectively. Constraints for  $V_{td}$  are extracted from the mass difference for  $B_s$  mass eigenstates. The CP-violating quantities  $\epsilon_K$ , measured in decays of the long lived neutral K meson to two pions, and  $\sin 2\beta$ , measured by comparing the time dependence of  $B^0$  and  $\overline{B}^0$  decays to final states  $\psi K_s$  and  $\psi K_L$ ,

where  $\psi$  can be any charmonium resonance, give further constraints and show that the single Standard Model CP-violating parameter  $\overline{\eta}$  is non-zero. All these measurements, with the exception of the last, have significant theoretical uncertainties in relating the measurement to the Standard Model parameter [31], these dominate the widths of the allowed regions shown in the figure. The review discusses in detail how these issues have been handled in generating this plot, and also gives more detail on the experimental inputs. As long as all results are generally consistent the issue of how theoretical uncertainties are treated statistically is not critical; it will become very important at such time as any discrepancy appears between two measurements that fix related Standard Model parameters.

On the neutrino front as well there is much yet to be learned by further experiments. While not all the parameters of the very massive neutrino sector can be determined in experiments on the light neutrinos, some that are still unknown can be pinned down. The CP violation in the light neutrino sector will not be easy to detect, the outlook depends on the size of an as-yet-unmeasured mixing parameter. Measuring these quantities would provide some important constraints on model building and scenarios for leptogenesis.



Figure 1: Constraints on the parameters  $\overline{\rho}$  and  $\overline{\eta}$  of the Standard Model quark mixing matrix from a variety of experimental inputs [31].

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