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Complementary Observables for the Determination of $|V_{ub}|$ in Inclusive Semileptonic B Decays

Robert V. Kowalewski[a,1], Sven Menke[b,2]

[a] *Department of Physics and Astronomy, University of Victoria, P.O. Box 3055, Victoria, BC V8W3P6, Canada*

[b] *Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309, U.S.A.*

Abstract

The determination of $|V_{ub}|$ from inclusive semileptonic B decays is limited by uncertainties in modeling the decay distributions in $\mathrm{b} \to \mathrm{u}\ell\nu$ transitions. The largest uncertainties arise from the limited knowledge of the appropriate b quark mass and Fermi momentum to use in the parameterization of the shape function. This paper presents a new method in which these shape function parameters are constrained by the same data used to measure $|V_{ub}|$. The method requires measurements of the momenta of both the charged lepton and the neutrino in semileptonic B decays. From these quantities two complementary observables can be constructed, one for discriminating between $\mathrm{b} \to \mathrm{u}\ell\nu$ transitions and background and the other for constraining the shape function. Using this technique the uncertaintites in $|V_{ub}|$ from the shape function may be signficantly reduced.

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Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309

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1 Introduction

The precise determination of $|V_{ub}|$ is of comparable importance to the measurement of CP asymmetries in probing the CKM sector of the Standard Model. The most precise determination of $|V_{ub}|$ at present comes from a measurement of the endpoint of the charged lepton energy spectrum in inclusive semileptonic B decays[1], which has a total relative uncertainty of 16%. The major uncertainties in the endpoint measurement arise from the background from $\mathrm{b} \to \mathrm{c}\ell\nu$ decays and the theoretical prediction of the fraction of the $\mathrm{b} \to \mathrm{u}\ell\nu$ spectrum in the endpoint region. These two sources of error have opposite sensitivities to the lepton energy cut; raising the cut decreases the former and increases the latter. The total error is minimized with a lepton energy cut of $\sim 2.2\,\mathrm{GeV}$. Within the context of the lepton endpoint method, further progress requires an improved understanding of the shape function that describes the b quark mass and momentum distribution in the B meson.

In this paper we discuss a technique by which the theoretical uncertainty in extracting $|V_{ub}|$ from inclusive semileptonic decays at the $\Upsilon(4\mathrm{S})$ may be significantly reduced. The method requires the reconstruction of the neutrino momentum vector in addition to that of the charged lepton in semileptonic B decays.[1] Using this information two quantities are measured, one directly sensitive to the shape function parameters and another offering good discrimination between $\mathrm{b} \to \mathrm{u}\ell\nu$ and $\mathrm{b} \to \mathrm{c}\ell\nu$ decays. This approach using two complementary observables could be used in measurements of $|V_{ub}|$ at present e^+e^- B factories.

2 The $\mathrm{b} \to \mathrm{u}\ell\nu$ Generator

A Monte Carlo generator has been implemented to study $\mathrm{b} \to \mathrm{u}\ell\nu$ decays. The generator is based on the triple differential decay width of Ref. [3], which is calculated including $O(\alpha_\mathrm{s})$ corrections. The parton-level distributions are convolved with a shape function to obtain distributions in the experimental observables. The shape function can be written in terms of a single variable k_+. The b quark mass appearing in the parton-level distributions is replaced by $m_\mathrm{b} + k_+$. For the distribution of k_+ we take [3]

$$F(k_+) = N(1-\kappa)^a e^{(1+a)\kappa}; \quad \kappa = \frac{k_+}{m_\mathrm{B} - m_\mathrm{b}} \leq 1 \ . \tag{1}$$

The shape function can be paraterized using m_b and a or, equivalently, using $\overline{\Lambda} = m_\mathrm{B} - m_\mathrm{b}$ and $\lambda_1 = -3\overline{\Lambda}^2/(1+a)$.

3 Observables

Two types of observables are constructed. The first type allows discrimination between $\mathrm{b} \to \mathrm{u}\ell\nu$ decays and background. For this purpose, the invariant mass of the hadronic recoil, m_h, can be used, provided the other B meson in the event is fully reconstructed. Alternatively, a measurement of the missing momentum, which can be obtained without fully reconstructing the other B meson, can be combined with the charged lepton momentum to determine q^2, the invariant mass squared of the lepton pair. The measurement of q^2, in contrast to the neutrino energy itself, does not suffer from the unknown direction of the parent B meson in the $\Upsilon(4\mathrm{S})$ frame. Using the charged lepton

[1] Technical issues involving neutrino reconstruction will not be discussed here; such reconstruction has already been used in analyses of exclusive semileptonic decays [2].

energy and q^2 one can define a variable to discriminate between $\mathrm{b} \to \mathrm{u}\ell\nu$ and $\mathrm{b} \to \mathrm{c}\ell\nu$ decays that is essentially the maximum kinematically allowed invariant mass squared of the hadronic recoil system[2]:

$$\begin{aligned} s_{\mathrm{h}}^{\max} &= m_{\mathrm{B}}^2 + q^2 - 2m_{\mathrm{B}}E_\ell\sqrt{\frac{1\mp\beta}{1\pm\beta}} - 2m_{\mathrm{B}}\left(\frac{q^2}{4E_\ell}\right)\sqrt{\frac{1\pm\beta}{1\mp\beta}} \\ &\qquad \text{for } \pm E_\ell > \pm\frac{1}{2}\left(m_{\mathrm{B}} - m_{\mathrm{cut}}\right)\sqrt{\frac{1\pm\beta}{1\mp\beta}}\,, \qquad (2) \\ s_{\mathrm{h}}^{\max} &= m_{\mathrm{B}}^2 + q^2 - 2m_{\mathrm{B}}\sqrt{q^2} \quad \text{otherwise.} \end{aligned}$$

In the above $\beta = \sqrt{1-(2m_{\mathrm{B}}/m_{\Upsilon(4\mathrm{S})})^2} \simeq 0.06$ is the velocity of the B meson in the $\Upsilon(4\mathrm{S})$ frame and E_ℓ is the charged lepton energy in the $\Upsilon(4\mathrm{S})$ rest frame. Events containing $\mathrm{b} \to \mathrm{u}\ell\nu$ decays are selected by requiring $s_{\mathrm{h}}^{\max} < m_{\mathrm{cut}}^2$, with $m_{\mathrm{cut}} \simeq m_{\mathrm{D}}$; in this case no event with a true hadronic recoil mass above m_{cut} can enter the sample unless q^2 (or, less likely, E_ℓ) is misreconstructed. Constant values of $s_{\mathrm{h}}^{\max}$ map out contours in the q^2-E_ℓ plane as shown in Fig. 1. Decays from the dominant

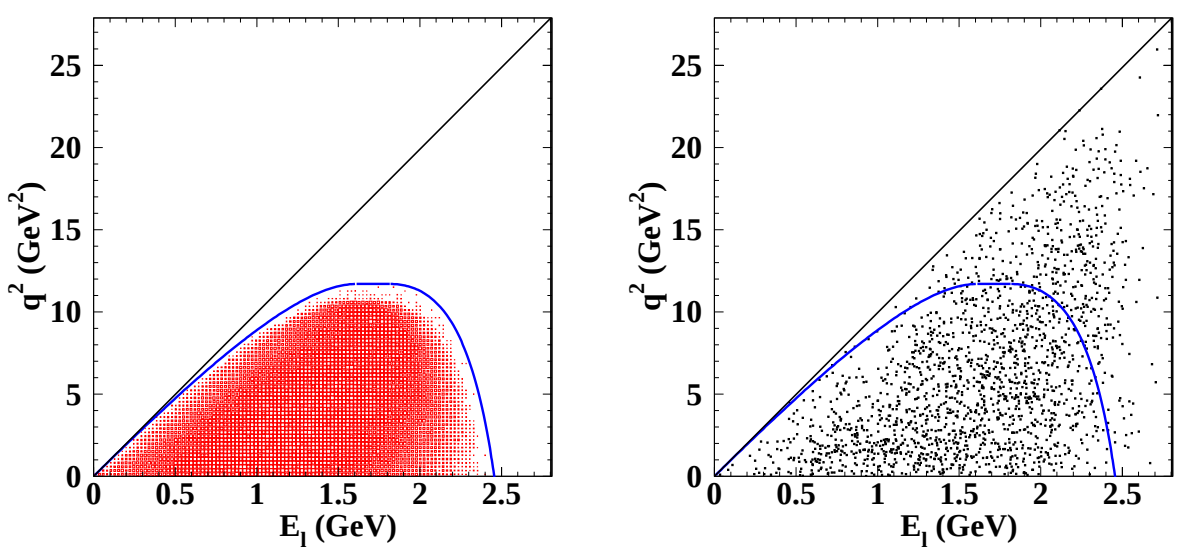

Figure 1: q^2 vs. E_ℓ in the $\Upsilon(4\mathrm{S})$ rest frame. The diagonal line and the curve are contours for $s_{\mathrm{h}}^{\max} = 0$ and m_{D}^2, respectively. The left plot shows $\mathrm{b} \to \mathrm{c}\ell\nu$ transitions and the right plot shows $\mathrm{b} \to \mathrm{u}\ell\nu$ transitions.

$\mathrm{b} \to \mathrm{c}\ell\nu$ process cannot contribute in the region $s_{\mathrm{h}}^{\max} < m_{\mathrm{D}}^2$ unless they are mismeasured.

As seen in Fig. 2, the shape of the $\mathrm{b} \to \mathrm{u}\ell\nu$ spectrum in m_{h}^2 is sensitive to both the assumed b quark mass and to the Fermi momentum, but the shape of the spectrum in $s_{\mathrm{h}}^{\max}$ is largely insensitive

[2]The expression is the maximum allowed recoil mass squared in the limit $\beta \to 0$; for non-zero β it remains a good variable for separating $\mathrm{b} \to \mathrm{u}\ell\nu$ from $\mathrm{b} \to \mathrm{c}\ell\nu$.

to the Fermi momentum. In these and subsequent plots the charged lepton energy is required to exceed 1 GeV in the $\Upsilon(4S)$ frame to ensure that it can be cleanly identified.[3]

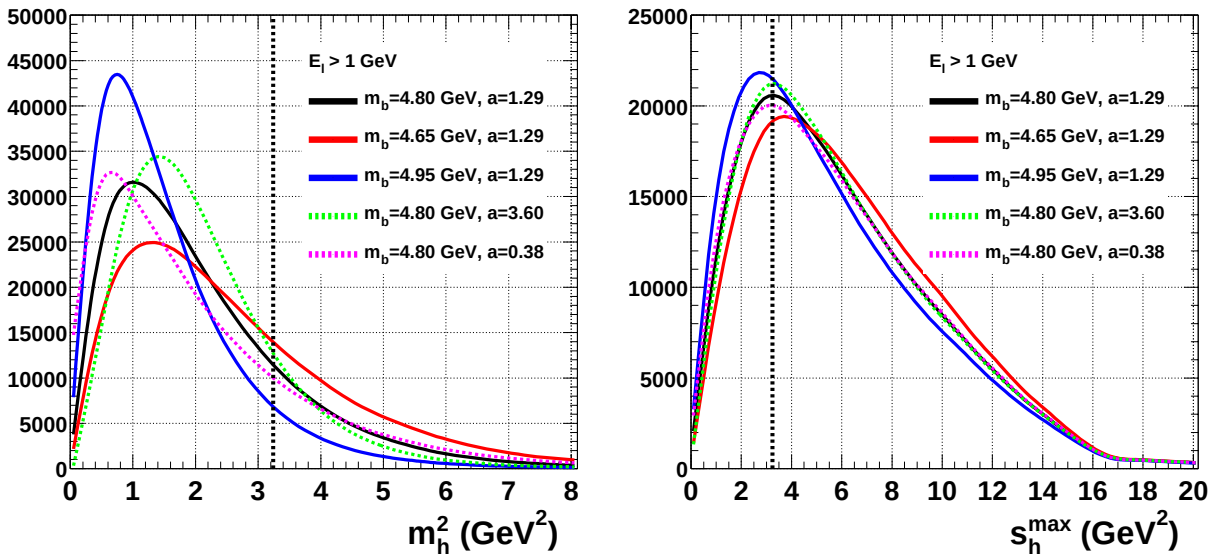

Figure 2: The effect of varying the shape function parameters on the distributions of $m_{\rm h}^2$ and $s_{\rm h}^{\rm max}$ for $\rm b \to u\ell\nu$ decays. The solid lines correspond to $a = 1.29$ and $m_{\rm b} = 4.65, 4.8$ and $4.95\,\rm GeV$; the dashed lines correspond to $a = 0.38$ and $a = 3.60$ with $m_{\rm b} = 4.8\,\rm GeV$. In each case the charged lepton exceeds 1 GeV in the $\Upsilon(4S)$ frame. The $\rm b \to u\ell\nu$ signal region is to the left of the dashed line.

The second type of observable is sensitive to the effective b quark mass. In the rest frame of the b quark, $m_{\rm b}$ is related to the W properties:

$$E_{\rm W}^{\rm (b)} = \frac{m_{\rm b}^2 + q^2 - m_{\rm u}^2}{2m_{\rm b}} \quad \Longrightarrow \quad m_{\rm b} \simeq E_{\rm W}^{\rm (b)} + |\vec{p}_{\rm W}^{\rm (b)}|$$

where $m_{\rm u}$ is the effective mass of the final state u quark and the equality in the last relation holds in the limit $m_{\rm u} \ll m_{\rm b}$. The quantity $E_{\rm W}^{\rm (b)} + |\vec{p}_{\rm W}^{\rm (b)}|$ is an estimator for the effective b quark mass. The boost from the b quark rest frame into the $\Upsilon(4S)$ rest frame smears the estimate but introduces little bias, since the boost direction is uncorrelated with the W direction and $\gamma \simeq 1$. The average value $\langle E_{\rm W} + |\vec{p}_{\rm W}|\rangle$, measured in the $\Upsilon(4S)$ rest frame from the lepton momenta, is an observable with substantial sensitivity to the effective b quark mass.

The strategy for determining $|V_{ub}|$ uses either $s_{\rm h}^{\rm max}$ or $m_{\rm h}$ to separate $\rm b \to u\ell\nu$ decays from background and $\langle E_{\rm W} + |\vec{p}_{\rm W}|\rangle$ to limit the variation in shape function parameters that must be considered.

How well can $\langle E_{\rm W} + |\vec{p}_{\rm W}|\rangle$ be measured? The r.m.s. of the generated $E_{\rm W} + |\vec{p}_{\rm W}|$ distribution is $\sim 0.25\,\rm GeV$ after cuts are applied on $s_{\rm h}^{\rm max}$ or $m_{\rm h}$ to select $\rm b \to u\ell\nu$ events. The experimental

[3]The efficiency of this requirement is 87% and is not very sensitive to the shape function.

resolution on missing energy will further broaden this; the r.m.s. missing energy resolution in the e^+e^- B factories is $O(0.2\,\mathrm{GeV})$. With B-factory data samples the statistical uncertainty in $\langle E_\mathrm{W} + |\vec{p}_\mathrm{W}|\rangle$ can be reduced below $\sim 0.02\,\mathrm{GeV}$. However, it will be an experimental challenge to understand systematic biases[4] at this level.

4 Extracting $|V_{ub}|$

The two parameters m_b and a of the shape function are varied to estimate the change in the fraction of $\mathrm{b} \to \mathrm{u}\ell\nu$ decays in the signal region. Fig. 3 shows the change in the fraction f_u of $\mathrm{b} \to \mathrm{u}\ell\nu$ transitions in the signal region versus $\langle E_\mathrm{W}+|\vec{p}_\mathrm{W}|\rangle$ for different choices of shape function parameters. The variations $m_\mathrm{b} = (4.8 \pm 0.15)\,\mathrm{GeV}$ and $a = 1.29^{+2.31}_{-0.91}$ correspond to $\overline{\Lambda} = (0.48 \pm 0.15)\,\mathrm{GeV}$ and $\lambda_1 = (-0.3^{+0.15}_{-0.20})\,\mathrm{GeV}^2$, respectively, and are a reasonable a priori choice at present. The observable $\langle E_\mathrm{W} + |\vec{p}_\mathrm{W}|\rangle$ has sensitivity to both m_b and a; the sensitivity to m_b is reduced (i.e. $\Delta\langle E_\mathrm{W} + |\vec{p}_\mathrm{W}|\rangle/\Delta m_\mathrm{b} < 1$) due to the cuts used to reject $\mathrm{b} \to \mathrm{c}\ell\nu$ decays.

With no measurement of $\langle E_\mathrm{W}+|\vec{p}_\mathrm{W}|\rangle$ the uncertainty on $|V_{ub}|$ due to the shape function (about $\sim 9\%$ for the requirement $s_\mathrm{h}^\mathrm{max} < 3.2\,\mathrm{GeV}^2$) is dominated by the uncertainty in m_b. A simultaneous measurement of $\langle E_\mathrm{W} + |\vec{p}_\mathrm{W}|\rangle$ could significantly reduce this uncertainty. For example, achieving a precision of $0.04\,\mathrm{GeV}$ on $\langle E_\mathrm{W} + |\vec{p}_\mathrm{W}|\rangle$ reduces the uncertainty on $|V_{ub}|$ to $\sim 5\%$ for a cut on $s_\mathrm{h}^\mathrm{max} < 3.2\,\mathrm{GeV}^2$ (see the shaded band in Fig. 3). Similar results are obtained if $m_\mathrm{h} < 1.7\,\mathrm{GeV}$ is used to select $\mathrm{b} \to \mathrm{u}\ell\nu$ decays. If a stiff cut on the lepton energy is required in addition to the cut on $s_\mathrm{h}^\mathrm{max}$, or if the cut on m_h is reduced to $1.5\,\mathrm{GeV}$, the theoretical error is increased and the ability to reduce it with a given precision on $\langle E_\mathrm{W} + |\vec{p}_\mathrm{W}|\rangle$ is decreased. However, a measurement of $\langle E_\mathrm{W} + |\vec{p}_\mathrm{W}|\rangle$ would provide a useful cross-check on the externally chosen parameter values in all cases.

5 Discussion

The use of $\langle E_\mathrm{W}+|\vec{p}_\mathrm{W}|\rangle$ to constrain the shape function is analogous to the use of the photon energy spectrum from $b \to s\gamma$ decays and amounts to using the virtual W to probe the b quark decay.

In order to use the $b \to s\gamma$ spectrum to constrain the uncertainty on $|V_{ub}|$, the γ spectrum must be measured to as low an energy as possible and deconvoluted to obtain an estimate of $\overline{\Lambda}$ (and, perhaps eventually, λ_1). Care must be taken to ensure that the theoretical expressions used to extract $\overline{\Lambda}$ from $b \to s\gamma$ and to determine the dependence of $|V_{ub}|$ on $\overline{\Lambda}$ are compatible.

There are several advantages to the approach described in this paper. Since the same event generator is used for extracting $|V_{ub}|$ and determining the relationship between $\langle E_\mathrm{W} + |\vec{p}_\mathrm{W}|\rangle$ and the shape function, no deconvolution is necessary. The need for a common theoretical framework for extracting and using the shape function parameters is trivially met; the parameters of the shape function need not even be extracted.

Quantifying the shape function parameters is of interest in its own right. In the absence of experimental cuts, the perturbative relationship between $\overline{\Lambda}$ and $\langle E_\mathrm{W} + |\vec{p}_\mathrm{W}|\rangle$ in the B meson rest frame can be determined (to $O(\alpha_s)$) from formulae in Ref. [3]:

$$\overline{\Lambda} = m_\mathrm{B} - \langle E_\mathrm{W} + |\vec{p}_\mathrm{W}|\rangle - \frac{91}{900}\frac{\alpha_\mathrm{s}}{\pi}\,\frac{2\,m_\mathrm{B}^2 - (2\,m_\mathrm{B} - \langle E_\mathrm{W} + |\vec{p}_\mathrm{W}|\rangle)^2}{\langle|\vec{p}_\mathrm{W}|\rangle}\ . \tag{3}$$

[4]Biases not properly accounted for in Monte Carlo simulations. Note, however, that uncertainties on the missing energy *resolution* do not affect $\langle E_\mathrm{W} + |\vec{p}_\mathrm{W}|\rangle$ directly.

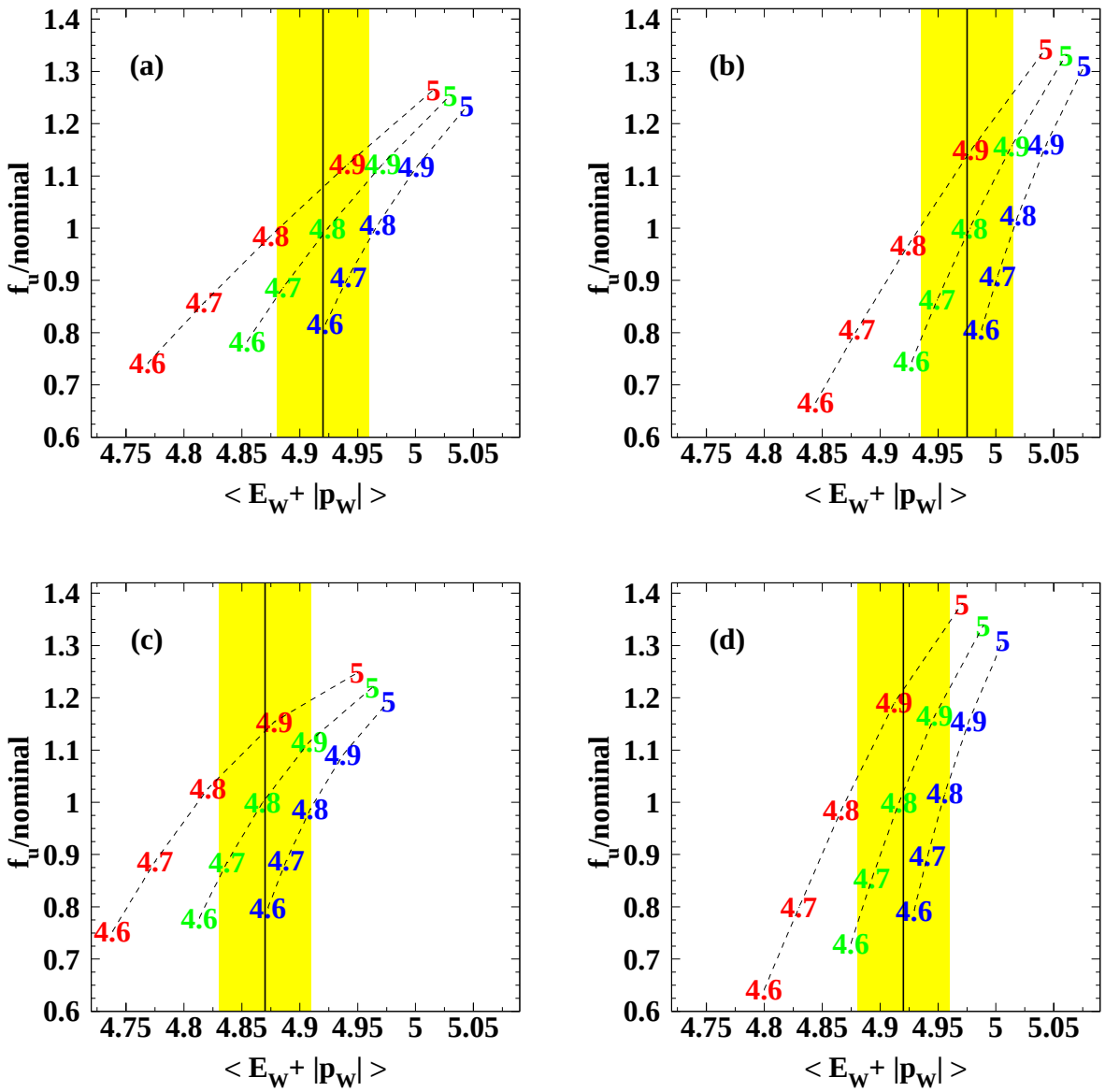

Figure 3: The ratio $f_{\rm u}(m_{\rm b}, a)/f_{\rm u}(4.8, 1.29)$ is shown versus $\langle E_{\rm W} + |\vec{p}_{\rm W}|\rangle$ for different choices of the parameters $m_{\rm b}$ and a, with the $m_{\rm b}$ values indicated by the numbers on the plot. The leftmost, center and rightmost curves correspond to choices $a = 0.38, 1.29$ and 3.60, respectively. The requirement $s_{\rm h}^{\rm max} < 3.2\,{\rm GeV}^2$ is used in (a); in (b) the additional requirement $E_\ell > 2.1\,{\rm GeV}$ is made. In (c) and (d) the requirements $m_{\rm h} < 1.7\,{\rm GeV}$ and $m_{\rm h} < 1.5\,{\rm GeV}$ are made, respectively. In each case the charged lepton energy is required to exceed $1\,{\rm GeV}$ in the $\Upsilon(4{\rm S})$ frame. The shaded bands show an example of the impact a measurement of $\langle E_{\rm W} + |\vec{p}_{\rm W}|\rangle$ with $0.04\,{\rm GeV}$ precision would have. The values of $f_{\rm u}(4.8, 1.29)$ corresponding to requirements (a)-(d) are, respectively, 0.22, 0.15, 0.67 and 0.57.

The relationship between $\overline{\Lambda}$ and the measured $\langle E_{\rm W} + |\vec{p}_{\rm W}|\rangle$ needs to be calculated in the presence of the experimental cuts required to reject $\mathrm{b} \to \mathrm{c}\ell\nu$ events. Extracting λ_1 is not feasible due to the large intrinsic width of the distribution and direct sensitivity to the missing energy resolution.

The method outlined in this paper holds promise for reducing the theoretical uncertainty in determining the fraction of $\mathrm{b} \to \mathrm{u}\ell\nu$ decays in the experimentally accessible region. The $s_{\rm h}^{\rm max}$ variable offers a way of extracting a clean signal for $\mathrm{b} \to \mathrm{u}\ell\nu$ decays with modest theoretical uncertainties. While it has a lower acceptance for $\mathrm{b} \to \mathrm{u}\ell\nu$ decays than does the recoil mass $m_{\rm h}$, it may be advantageous experimentally because it does not require full reconstruction of the second B meson in the event. The use of $\langle E_{\rm W} + |\vec{p}_{\rm W}|\rangle$ may reduce the range of parameter space that need be considered in evaluating the uncertainty on $|V_{ub}|$. The ultimate experimental precision on $\langle E_{\rm W} + |\vec{p}_{\rm W}|\rangle$ is hard to predict at present, but there is strong motivation to pursue such a measurement.

References

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