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Confronting the Conventional Ideas of Grand Unification with Fermion Masses, Neutrino Oscillations and Proton Decay

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CONFRONTING THE CONVENTIONAL IDEAS OF GRAND UNIFICATION WITH FERMION MASSES, NEUTRINO OSCILLATIONS AND PROTON DECAY*

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Abstract

It is noted that one is now in possession of a set of facts, which may be viewed as the *matching pieces of a puzzle*; in that all of them can be resolved by just one idea—that is grand unification. These include (i) the observed family-structure, (ii) quantization of electric charge, (iii) the meeting of the three gauge couplings, (iv) neutrino oscillations [in particular the value $\Delta m^2(\nu_\mu - \nu_\tau)$, suggested by SuperK], (v) the intricate pattern of the masses and mixings of the fermions, including the smallness of V_{cb} and the largeness of $\theta^{osc}_{\nu_\mu\nu_\tau}$, and (vi) the need for B–L as a generator to implement baryogenesis (via leptogenesis). All these pieces fit beautifully together within a single puzzle board framed by supersymmetric unification, based on either SO(10) or a string-unified G(224)-symmetry. The two notable pieces of the puzzle still missing, however, are proton decay and supersymmetry.

A concrete proposal is presented within a predictive SO(10)/G(224)-framework that successfully describes the masses and mixings of all fermions, including the neutrinos—with eight predictions, all in agreement with observation. Within this framework, a systematic study of proton decay is carried out, which (a) pays special attention to its dependence on the fermion masses, and (b) limits the threshold corrections so as to preserve natural coupling unification. The study updates prior work by Babu, Pati and Wilczek, in the context of both MSSM and its (interesting) variant, the so-called ESSM, by allowing for improved values of the matrix elements and of the short- and long-distance renormalization effects. It shows that a conservative upper limit on the proton lifetime is about (1/3 - 2)$\times 10^{34}$ years, with $\overline{\nu}K^+$ being the dominant decay mode, and quite possibly μ^+K^0 and $e^+\pi^0$ being prominent. This in turn strongly suggests that an improvement in the current sensitivity by a factor of five to ten (compared to SuperK) ought to reveal proton decay. Otherwise some promising and remarkably successful ideas on unification would suffer a major setback. For comparison, some alternatives to the conventional approach to unification pursued here are mentioned at the end.

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1 Introduction

The standard model of particle physics, based on the gauge symmetry $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{SU}(3)_C$ [1, 2] is in excellent agreement with observations, at least up to energies of order 100 GeV. Its success in turn constitutes a triumph of quantum field theory, especially of the notions of gauge invariance, spontaneous symmetry breaking, and renormalizability. The next step in the unification-ladder is associated with the concept of "grand unification", which proposes a unity of quarks and leptons, and simultaneously of their three basic forces: weak, electromagnetic and strong [3, 4, 5]. This concept was introduced on purely aesthetic grounds, in fact *before* any of the empirical successes of the standard model was in place. It was realized in 1972 that the standard model judged on aesthetic merits has some major shortcomings [3, 4]. For example, it puts members of a family into five scattered multiplets, assigning rather peculiar hypercharge quantum numbers to each of them, without however providing a compelling reason for doing so. It also does not provide a fundamental reason for the quantization of electric charge, and it does not explain why the electron and proton possess exactly equal but opposite charges. Nor does it explain the co-existence of quarks and leptons, and that of the three gauge forces—weak, electromagnetic and strong—with their differing strengths.

The idea of grand unification was postulated precisely to remove these shortcomings. It introduces the notion that quarks and leptons are members of one family, linked together by a symmetry group G, and that the weak, electromagnetic and strong interactions are aspects of one force, generated by gauging this symmetry G. The group G of course inevitably contains the standard model symmetry $G(213) = SU(2)_L \times U(1)_Y \times SU(3)_C$ as a subgroup. Within this picture, the observed differences between quarks and leptons and those between the three gauge forces are assumed to be low-energy phenomena that arise through a spontaneous breaking of the unification symmetry G to the standard model symmetry G(213), at a very high energy scale $M \gg 1$ TeV. As a *prediction* of the hypothesis, such differences must then disappear and the true unity of quarks and leptons and of the three gauge forces should manifest at energies exceeding the scale M.

The second and perhaps the most dramatic prediction of grand unification is proton decay. This important process, which would provide the window to view physics at truly short distances ($< 10^{-30}$ cm), is yet to be seen. Nevertheless, as I will stress in this talk, there has appeared over the years an impressive set of facts, favoring the hypothesis of grand unification which in turn suggest that the discovery of proton decay should be imminent. These include:

(a) The observed family structure: The five scattered multiplets of the standard model, belonging to a family, neatly become parts of a whole (*a single multiplet*), with their weak hypercharges predicted by grand unification, precisely as observed. It is hard to believe that this is just an accident. Realization of this feature calls for an extension of the standard model symmetry $\mathrm{G}(213) = \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{SU}(3)^C$ *minimally* to the symmetry group $\mathrm{G}(224) = \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{SU}(4)^C$ [3], which can be extended further into the simple group SO(10) [6], but not SU(5) [4]. The G(224) symmetry in turn introduces some additional attractive features (see Section 2), including especially the right-handed (RH) neutrinos (ν_R's) accompanying the left-handed ones (ν_L's), and B–L as a local symmetry. As we will see, both of these features now seem to be needed, on empirical grounds, to understand neutrino masses and to implement baryogenesis.

(b) Quantization of electric charge and the fact that $Q_{\mathrm{electron}} = -Q_{\mathrm{proton}}$: Grand Unification provides compelling reasons for both of these facts.

(c) Meeting of the gauge couplings: Such a meeting is found to occur at a scale $M_X \approx 2 \times 10^{16}$ GeV, when the three gauge couplings are extrapolated from their values measured at

LEP to higher energies, in the context of supersymmetry [7]. This dramatic phenomenon provides a strong support in favor of the ideas of both grand unification and supersymmetry [8]. Both of these features in turn may well emerge from a string theory [9] or M-theory [10] (see discussion in Section 3).

(d) $\Delta m^2(\nu_\mu - \nu_\tau) \sim (1/20eV)^2$: The recent discovery of atmospheric neutrino-oscillation at SuperKamiokande [11] suggests a value $\Delta m^2(\nu_\mu \nu_\tau) \sim (1/20 \text{ eV})^2$. It has been argued (see e.g. Ref. [12]) that precisely such a magnitude of $\Delta m^2(\nu_\mu \nu_\tau)$ can be understood very simply by utilizing the SU(4)-color relation $m(\nu_\tau)_{\text{Dirac}} \approx m_{\text{top}}$ and the SUSY unification scale M_X, noted above (See Section 4).

(e) Some intriguing features of fermion masses and mixings: These include: (i) the "observed" near equality of the masses of the b-quark and the τ-lepton at the unification-scale (i.e. $m_b^0 \approx m_\tau^0$) and (ii) the observed largeness of the ν_μ-ν_τ oscillation angle ($\sin^2 2\theta^{\text{osc}}_{\nu_\mu \nu_\tau} \geq 0.92$) [11], together with the smallness of the corresponding quark mixing parameter $V_{cb}(\approx 0.04)$ [13]. As shown in recent work by Babu, Wilczek and me [14], it turns out that these features and more can be understood remarkably well (see discussion in Section 5) within an economical and predictive SO(10)-framework based on a minimal Higgs system. The success of this framework is in large part due simply to the group-structure of SO(10). For most purposes, that of G(224) suffices.

(f) Baryogenesis: To implement baryogenesis [15] successfully, in the presence of electroweak sphaleron effects [16], which wipe out any baryon excess generated at high temperatures in the (B–L)-conserving mode, it has become apparent that one would need B–L as a generator of the underlying symmetry in four dimensions, whose spontaneous violation at high temperatures would yield, for example, lepton asymmetry (leptogenesis). The latter in turn is converted to baryon-excess at lower temperatures by electroweak sphalerons. This mechanism, it turns out, yields even quantitatively the right magnitude for baryon excess [17]. The need for B–L, which is a generator of SU(4)-color, again points to the need for G(224) or SO(10) as an effective symmetry near the unification-scale M_X.

The success of each of these six features (a)–(f) seems to be non-trivial. Together they make a strong case for both *the conventional ideas on supersymmetric grand unification* and simultaneously for the G(224)/SO(10)-route to such unification, as being relevant to nature at short distances $\leq (10^{16} \text{ GeV})^{-1}$, in four dimensions.[1] However, despite these successes, as long as proton decay remains undiscovered, the hallmark of grand unification—that is *quark-lepton transformability*—would remain unrevealed.

The relevant questions in this regard then are: What is the predicted range for the lifetime of the proton—in particular an upper limit—within the empirically favored route to unification mentioned above? What are the expected dominant decay modes within this route? Are these predictions compatible with current lower limits on proton lifetime mentioned above, and if so, can they still be tested at the existing or possible near-future detectors for proton decay?

Fortunately, we are in a much better position to answer these questions now, compared to a few years ago, because meanwhile we have learnt more about the nature of grand unification, and also there have been improved evaluations of the relevant matrix elements and short and long-distance renormalization effects. As noted above (see also Section 2 and Section 4), the neutrino masses and the meeting of the gauge couplings together seem to select out the supersymmetric G(224)/SO(10)-route to higher unification. The main purpose of my talk here will therefore be

[1] For comparison, some alternative attempts, including those based on the ideas of (a) large extra dimensions, and (b) unification occurring only in higher dimensions, are mentioned briefly in Section 6 G.

to address the questions raised above, in the context of this route. For the sake of comparison, however, I will state the corresponding results for the case of supersymmetric SU(5) as well.

My discussion will be based on a recent study of proton decay by Babu, Wilczek and me [14], an update presented in the Erice talk [18], and a subsequent update of the same as presented here. Relative to other analyses, this study has four distinctive features:

(i) It systematically takes into account the link that exists between proton decay and the masses and mixings of all fermions, including the neutrinos.

(ii) In particular, in addition to the contributions from the so-called "standard" $d = 5$ operators [19] (see Section 6), it includes those from a *new* set of $d = 5$ operators, related to the Majorana masses of the RH neutrinos [20]. These latter are found to be generally as important as the standard ones.

(iii) As discussed in the Appendix, the work also restricts GUT-scale threshold corrections, so as to preserve naturally coupling unification, in accord with the observed values of the three gauge couplings.

(iv) Finally, the present update incorporates recently improved values of the matrix elements, and the short and long-distance renormalization effects, which significantly enhance proton decay rate.

Each of these features turn out to be *crucial* to gaining a reliable insight into the nature of proton decay. Our study shows that the inverse decay rate for the $\overline{\nu}K^+$-mode, which is dominant, is less than about 1.2×10^{31} years for the case of MSSM embedded in minimal SUSY SU(5), and that it is less than about 10^{33} years for the case of MSSM embedded in SO(10). These upper bounds are obtained by making generous allowance for uncertainties in the matrix element and the SUSY-spectrum. Typically, the lifetime should of course be less than these bounds.

Proton decay is studied also for the case of the extended supersymmetric standard model (ESSM), that has been proposed a few years ago [21] on several grounds, based on the issues of (a) an understanding of the inter-family mass-hierarchy, (b) removing the mismatch between MSSM and string-unification scales, and (c) dilaton-stabilization (see Section 6 and the appendix). This case adds an extra pair of vector-like families at the TeV-scale, transforming as $\mathbf{16} + \overline{\mathbf{16}}$ of SO(10), to the MSSM spectrum. While the case of ESSM is fully compatible with both neutrino-counting at LEP and precision electroweak tests, it can of course be tested directly at the LHC through a search for the vectorlike fermions. Our study shows that, with the inclusion of only the "standard" $d = 5$ operators (defined in Section 6), ESSM, embedded in SO(10), can quite plausibly lead to proton lifetimes in the range of $10^{33} - 10^{34}$ years, for nearly central values of the parameters pertaining to the SUSY-spectrum and the matrix element. Allowing for a wide variation of the parameters, owing to the contributions from both the standard and the neutrino mass-related $d = 5$ operators (discussed in Section 6), proton lifetime still gets bounded above by about 2×10^{34} years, for the case of ESSM, embedded in SO(10) or a string-unified G(224).

For either MSSM or ESSM, embedded in G(224) or SO(10), due to contributions from the new operators, the μ^+K^0-mode is found to be prominent, with a branching ratio typically in the range of 10-50%. By contrast, minimal SUSY SU(5), for which the new operators are absent, would lead to branching ratios $\leq 10^{-3}$ for this mode. It is stressed that the $e^+\pi^0$-mode induced by gauge boson-exchange, in either SUSY SU(5) or SUSY SO(10), could have an inverse decay rate as short as about $(1 - 2) \times 10^{34}$ years.

Thus our study of proton decay, correlated with fermion masses, strongly suggests that discovery of proton decay should be imminent. Allowing for the possibility that the proton lifetime may well be closer to the upper bound stated above, a next-generation detector providing a net gain

in sensitivity in proton decay-searches by a factor of 5–10, compared to SuperK, would certainly be needed not just to produce proton-decay events, but also to clearly distinguish them from the background. It would of course also be essential to study the branching ratios of certain sub-dominant but crucial decay modes, such as the μ^+K^0 and $e^+\pi^0$. The importance of such improved sensitivity, in the light of the successes of supersymmetric grand unification, is emphasized at the end.

2 Advantages of the Symmetry G(224) as a Step to Higher Unification

As mentioned in the introduction, the hypothesis of grand unification was introduced to remove some of the conceptual shortcomings of the standard model (SM). To illustrate the advantages of an early suggestion in this regard, consider the five standard model multiplets belonging to the electron-family as shown:

$$\begin{pmatrix} u_r & u_y & u_b \\ d_r & d_y & d_b \end{pmatrix}_L^{\frac{1}{3}} ; \begin{pmatrix} u_r & u_y & u_b \end{pmatrix}_R^{\frac{4}{3}} ; \begin{pmatrix} d_r & d_y & d_b \end{pmatrix}_R^{-\frac{2}{3}} ; \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L^{-1} ; \left(e^-\right)_R^{-2} . \tag{1}$$

Here the superscripts denote the respective weak hypercharges Y_W (where $Q_{em} = I_{3L} + Y_W/2$) and the subscripts L and R denote the chiralities of the respective fields. If one asks: how one can put these five multiplets into just one multiplet, the answer turns out to be simple and unique. As mentioned in the introduction, the minimal extension of the SM symmetry G(213) needed, to achieve this goal, is given by the gauge symmetry [3]:

$$\mathrm{G}(224) \;=\; \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{SU}(4)^C . \tag{2}$$

Subject to left-right discrete symmetry ($L \leftrightarrow R$), which is natural to G(224), all members of the electron family become parts of a single left-right self-conjugate multiplet, consisting of:

$$F^e_{L,R} \;=\; \begin{bmatrix} u_r & u_y & u_b & \nu_e \\ d_r & d_y & d_b & e^- \end{bmatrix}_{L,R} . \tag{3}$$

The multiplets F^e_L and F^e_R are left-right conjugates of each other and transform respectively as (2,1,4) and (1,2,4) of G(224); likewise for the muon and the tau families. Note that the symmetries $\mathrm{SU}(2)_L$ and $\mathrm{SU}(2)_R$ are just like the familiar isospin symmetry, except that they operate on quarks and well as leptons, and distinguish between left and right chiralities. The left weak-isospin $\mathrm{SU}(2)_L$ treats each column of F^e_L as a doublet; likewise $\mathrm{SU}(2)_R$ for F^e_R. The symmetry SU(4)-color treats each row of F^e_L *and* F^e_R as a quartet; *thus lepton number is treated as the fourth color.* Note also that postulating either SU(4)-color or $\mathrm{SU}(2)_R$ forces one to introduce a right-handed neutrino (ν_R) for each family as a singlet of the SM symmetry. *This requires that there be sixteen two-component fermions in each family, as opposed to fifteen for the SM.* The symmetry G(224) introduces an elegant charge formula:

$$Q_{em} \;=\; I_{3L} \,+\, I_{3R} \,+\, \frac{B \,-\, L}{2} \tag{4}$$

expressed in terms of familiar quantum numbers I_{3L}, I_{3R} and -B–L, which applies to all forms of matter (including quarks and leptons of all six flavors, gauge and Higgs bosons). Note that the weak hypercharge given by $Y_W/2 = I_{3R} + \frac{B-L}{2}$ is now completely determined for all members of

the family. The values of Y_W thus obtained precisely match the assignments shown in Eq. (1). Quite clearly, the charges I_{3L}, I_{3R} and B–L, being generators respectively of $SU(2)_L$, $SU(2)_R$ and $SU(4)^c$, are quantized; so also then is the electric charge Q_{em}.

In brief, the symmetry G(224) brings some attractive features to particle physics. These include:
(i) Unification of all 16 members of a family within one left-right self-conjugate multiplet;
(ii) Quantization of electric charge, with a reason for the fact that $Q_{\rm electron} = -Q_{\rm proton}$
(iii) Quark-lepton unification (through SU(4) color);
(iv) Conservation of parity at a fundamental level [3, 22];
(v) Right-handed neutrinos ($\nu_R's$) as a compelling feature; and
(vi) B–L as a local symmetry.
As mentioned in the introduction, the two distinguishing features of G(224)—i.e. the existence of the RH neutrinos and B–L as a local symmetry—now seem to be needed on empirical grounds. Furthermore, SU(4)-color provides simple relations between the masses and mixings of quarks and leptons, while $SU(2)_L\times$ $SU(2)_R$ relates the mass-matrices in the up and down sectors. As we will see in Sections 4 and 5, these relations are in good accord with observations.

Believing in a complete unification, one is led to view the G(224) symmetry as part of a bigger symmetry, which itself may have its origin in an underlying theory, such as string theory. In this context, one may ask: Could the effective symmetry below the string scale in four dimensions (see Section 3) be as small as just the SM symmetry G(213), even though the latter may have its origin in a bigger symmetry, which lives only in higher dimensions? I will argue in Section 4 that the data on neutrino masses and the need for baryogenesis provide an answer to the contrary, suggesting that it is the *effective symmetry in four dimensions, below the string scale, which must minimally contain either* G(224) *or a close relative* $G(214) = SU(2)_L\times I_{3R}\times SU(4)^C$.

One may also ask: does the effective four dimensional symmetry have to be any bigger than G(224) near the string scale? In preparation for an answer to this question, let us recall that the smallest simple group that contains the SM symmetry G(213) is SU(5) [4]. It has the virtue of demonstrating how the main ideas of grand unification, including unification of the gauge couplings, can be realized. However, SU(5) does not contain G(224) as a subgroup. As such, it does not possess some of the advantages listed above. In particular, it does not contain the RH neutrinos as a compelling feature, and B–L as a local symmetry. Furthermore, it splits members of a family (not including ν_R) into two multiplets: $\overline{\mathbf{5}} + \mathbf{10}$.

By contrast, the symmetry SO(10) has the merit, relative to SU(5), that it contains G(224) as a subgroup, and thereby retains all the advantages of G(224) listed above. (As a historical note, it is worth mentioning that these advantages had been motivated on aesthetic grounds through the symmetry G(224) [3], and *all* the ideas of higher unification were in place [3, 4, 5], before it was noted that G(224) [isomorphic to SO(4)×SO(6)] embeds nicely into SO(10) [6]). Now, *SO(10) even preserves the 16-plet family-structure of G(224) without a need for any extension.* By contrast, if one extends G(224) to the still higher symmetry E_6 [23], the advantages (i)–(vi) are retained, but in this case, one must extend the family-structure from a 16 to a 27-plet, by postulating additional fermions. In this sense, there seems to be some advantage in having the effective symmetry below the string scale to be minimally G(224) [or G(214)] and maximally no more than SO(10). I will compare the relative advantage of having either a string-derived G(224) or a string-SO(10), in the next section. First, I discuss the implications of the data on coupling unification.

3 The Need for Supersymmetry: MSSM versus String Unifications

It has been known for some time that the precision measurements of the standard model coupling constants (in particular $\sin^2\theta_W$) at LEP put severe constraints on the idea of grand unification. Owing to these constraints, the non-supersymmetric minimal SU(5), and for similar reasons, the one-step breaking minimal non-supersymmetric SO(10)-model as well, are now excluded [24]. But the situation changes radically if one assumes that the standard model is replaced by the minimal supersymmetric standard model (MSSM), above a threshold of about 1 TeV. In this case, the three gauge couplings are found to meet [7], to a very good approximation, barring a few percent discrepancy which can be attributed to threshold corrections (see Appendix). Their scale of meeting is given by

$$M_X \approx 2 \times 10^{16}\,\mathrm{GeV} \quad \text{(MSSM or SUSY SU(5))}\ . \tag{5}$$

This dramatic meeting of the three gauge couplings, or equivalently the agreement of the MSSM-based prediction of $\sin^2\theta_W(m_Z)_{\rm th} = 0.2315 \pm 0.003$ [25] with the observed value of $\sin^2\theta_W(m_Z) = 0.23124 \pm 0.00017$ [13], provides a strong support for the ideas of both grand unification and supersymmetry, as being relevant to physics at short distances $\lesssim (10^{16}\,\mathrm{GeV})^{-1}$.

In addition to being needed for achieving coupling unification there is of course an independent motivation for low-energy supersymmetry—i.e. for the existence of SUSY partners of the standard model particles with masses of order 1 TeV. This is because it protects the Higgs boson mass from getting large quantum corrections, which would (otherwise) arise from grand unification and Planck scale physics. It thereby provides at least a technical resolution of the so-called gauge-hierarchy problem. *In this sense low-energy supersymmetry seems to be needed for the consistency of the hypothesis of grand unification.* Supersymmetry is of course also needed for the consistency of string theory. Last but not least, as a symmetry linking bosons and fermions, it is simply a beautiful idea. And it is fortunate that low-energy supersymmetry can be tested at the LHC, and possibly at the Tevatron, and the proposed NLC.

The most straightforward interpretation of the observed meeting of the three gauge couplings and of the scale M_X, is that a supersymmetric grand unification symmetry (often called GUT symmetry), like SU(5) or SO(10), breaks spontaneously at M_X into the standard model symmetry G(213), and that supersymmetry-breaking induces soft masses of order one TeV.

Even if supersymmetric grand unification may well be a good effective theory below a certain scale $M \gtrsim M_X$, it ought to have its origin within an underlying theory like the string/M theory. Such a theory is needed to unify all the forces of nature including gravity, and to provide a good quantum theory of gravity. It is also needed to provide a rationale for the existence of flavor symmetries (not available within grand unification), which distinguish between the three families and can resolve certain naturalness problems including those associated with inter-family mass hierarchy. In the context of string or M-theory, an alternative interpretation of the observed meeting of the gauge couplings is however possible. This is because, even if the effective symmetry in four dimensions emerging from a higher dimensional string theory is non-simple, like G(224) or even G(213), string theory can still ensure familiar unification of the gauge couplings at the string scale. In this case, however, one needs to account for the small mismatch between the MSSM unification scale M_X (given above), and the string unification scale, given by $M_{st} \approx g_{st} \times 5.2 \times 10^{17}$ GeV $\approx 3.6 \times 10^{17}$ GeV (Here we have put $\alpha_{st} = \alpha_{GUT}(\mathrm{MSSM}) \approx 0.04$) [26]. Possible resolutions of this mismatch have been proposed. These include: (i) utilizing the idea of *string-duality* [27] which allows a lowering of M_{st} compared to the value shown above, or alternatively (ii) the idea of the so-called "Extended Supersymmetric Standard Model" (ESSM) that assumes the existence of

two vector-like families, transforming as ($\mathbf{16}+\overline{\mathbf{16}}$) of SO(10), with masses of order one TeV [21], in addition to the three chiral families. The latter leads to a semi-perturbative unification by raising α_{GUT} to about 0.25-0.3. Simultaneously, it raises M_X, in two loop, to about $(1/2-2)\times 10^{17}$ GeV. (Other mechanisms resolving the mismatch are reviewed in Ref. [28]). In practice, a combination of the two mechanisms mentioned above may well be relevant. [2]

While the mismatch can thus quite plausibly be removed for a non-GUT string-derived symmetry like G(224) or G(213), a GUT symmetry like SU(5) or SO(10) would have an advantage in this regard because it would keep the gauge couplings together between M_{st} and M_X (even if $M_X \sim M_{st}/20$), and thus not even encounter the problem of a mismatch between the two scales. A supersymmetric four dimensional GUT-solution [like SU(5) or SO(10)], however, has a possible disadvantage as well, because it needs certain color triplets to become superheavy by the so-called doublet-triplet splitting mechanism (see Section 6 and Appendix), in order to avoid the problem of rapid proton decay. However, no such mechanism has emerged yet, in string theory, for the GUT-like solutions [29]. [3]

Non-GUT string solutions, based on symmetries like G(224) or G(2113) for example, have a distinct advantage in this regard, in that the dangerous color triplets, which would induce rapid proton decay, are often naturally projected out for such solutions [30, 31]. Furthermore, the non-GUT solutions invariably possess new "flavor" gauge symmetries, which distinguish between families. These symmetries are immensely helpful in explaining qualitatively the observed fermion mass-hierarchy (see e.g. Ref. [31]) and resolving the so-called naturalness problems of supersymmetry such as those pertaining to the issues of squark-degeneracy [32], CP violation [33] and quantum gravity-induced rapid proton decay [34].

Weighing the advantages and possible disadvantages of both, it seems hard at present to make a priori a clear choice between a GUT versus a non-GUT string-solution. As expressed elsewhere [35], it therefore seems prudent to keep both options open and pursue their phenomenological consequences. Given the advantages of G(224) or SO(10) in the light of the neutrino masses (see Sections 2 and 4), I will thus proceed by assuming that either a suitable four dimensional G(224)-solution [with the scale M_X being close to M_{st} (see footnote 2)], or a realistic four-dimensional SO(10)-solution (with the desired mechanism for doublet-triplet splitting) emerges effectively from an underlying string theory, at the "conventional" string-scale $M_{st} \sim 10^{17}$-10^{18} GeV, and that the G(224)/SO(10) symmetry in turn breaks spontaneously at the conventional GUT-scale of $M_X \sim 2\times 10^{16}$ GeV (or at $M_X \sim 5\times 10^{16}$ GeV for the case of ESSM, as discussed in footnote 2) to the standard model symmetry G(213). The extra dimensions of string/M-theory are assumed to be tiny with sizes $\leq M_X^{-1} \sim 10^{-30}$ cm, so as not to disturb the successes of GUT. In short, I assume that essentially *the conventional (good old) picture of grand unification, proposed and developed sometime ago [3, 4, 5, 6, 7], holds as a good effective theory above the unification*

[2] I have in mind the possibility of string-duality [27] lowering M_{st} for the case of semi-perturbative unification in ESSM (for which $\alpha_{st} \approx 0.25$, and thus, without the use of string-duality, M_{st} would have been about 10^{18} GeV) to a value of about (1-2)$\times 10^{17}$ GeV (say), and semi-perturbative unification [21] raising the MSSM value of M_X to about 5×10^{16} GeV$\approx M_{st}$(1/2 to 1/4) (say). In this case, an intermediate symmetry like G(224) emerging at M_{st} would be effective only within the short gap between M_{st} and M_X, where it would break into G(213). Despite this short gap, one would still have the benefits of SU(4)-color that are needed to understand neutrino masses (see Section 4), and to implement baryogenesis via leptogenesis. At the same time, since the gap is so small, the couplings of G(224), unified at M_{st} would remain essentially so at M_X, so as to match with the "observed" coupling unification, of the type suggested in Ref. [21].

[3] Some alternative mechanisms for doublet-triplet splitting, and for suppression of the $d = 5$ proton decay operators have been proposed in the context of higher dimensional theories. These will be mentioned briefly in Section 6 G.

scale M_X and up to some high scale $M \lesssim M_{st}$, with the added presumption that it may have its origin from the string/M-theory. Such a picture seems to be directly motivated on observational grounds such as those based on (a) coupling unification (discussed above), (b) neutrino masses including the (mass)^2-difference of the ν_μ-ν_τ system and the near maximal ν_μ-ν_τ oscillation angle (see discussions in the next sections), and (c) the fact that spontaneous violation of B–L local symmetry at high temperatures, seems to be needed to implement baryogenesis via leptogenesis.[4]

We will see that with the broad assumption mentioned above, an economical and predictive framework emerges, which successfully accounts for a host of observed phenomena pertaining to the masses and the mixings of all fermions, including neutrinos. It also makes some crucial testable predictions for proton decay. I next discuss the implications of the mass of ν_τ, or rather of $\Delta m^2(\nu_\mu \nu_\tau)$, as revealed by the SuperK data.

4 $\Delta m^2(\nu_\mu \nu_\tau)$: Evidence In Favor of the G(224) Route

One can obtain an estimate for the mass of ν_L^τ in the context of G(224) or SO(10) by using the following three steps (see e.g. Ref. [12]):

(i) Assume that B−L and I_{3R}, contained in a string-derived G(224) or SO(10), break near the unification-scale:

$$M_X \sim 2 \times 10^{16}\,\mathrm{GeV}\,, \tag{6}$$

through VEVs of Higgs multiplets of the type suggested by string-solutions—i.e. $\langle (1,2,4)_H \rangle$ for G(224) or $\langle \overline{\mathbf{16}}_H \rangle$ for SO(10), as opposed to $\mathbf{126}_H$ which seems to be unobtainable at least in weakly interacting string theory [39]. In the process, the RH neutrinos (ν_R^i), which are singlets of the standard model, can and generically will acquire superheavy Majorana masses of the type $M_R^{ij}\, \nu_R^{iT}\, C^{-1}\, \nu_R^j$, by utilizing the VEV of $\langle \overline{\mathbf{16}}_H \rangle$ and effective couplings of the form:

$$\mathcal{L}_M\,(SO(10)) \;=\; f_{ij}\; \mathbf{16}_i \cdot \mathbf{16}_j\; \overline{\mathbf{16}}_H\; \cdot \overline{\mathbf{16}}_H / M + h.c. \tag{7}$$

A similar expression holds for G(224). Here $i, j = 1, 2, 3$, correspond respectively to e, μ and τ families. Such gauge-invariant non-renormalizable couplings might be expected to be induced by Planck-scale physics, involving quantum gravity or stringy effects and/or tree-level exchange of superheavy states, such as those in the string tower. With f_{ij} (at least the largest among them) being of order unity, we would thus expect M to lie between $M_{\mathrm{Planck}} \approx 2 \times 10^{18}$ GeV and $M_{\mathrm{string}} \approx 4 \times 10^{17}$ GeV. Ignoring for the present off-diagonal mixings (for simplicity), one thus obtains [5]:

$$M_{3R} \;\approx\; \frac{f_{33} \langle \overline{\mathbf{16}}_H \rangle^2}{M} \;\approx\; f_{33}\,(2 \times 10^{14}\,\mathrm{GeV})\,\rho^2\,(M_{\mathrm{Planck}}/M) \tag{8}$$

This is the Majorana mass of the RH tau neutrino. Guided by the value of M_X, we have substituted $\langle \overline{\mathbf{16}}_H \rangle = (2 \times 10^{16}\,\mathrm{GeV})\,\rho$,where we expect $\rho \approx 1/2$ to 2 (say).

[4] Alternative scenarios such as those based on TeV-scale large extra dimensions [36] or string-scale being at a few TeV [37], or submillimeter-size even larger extra dimensions with the fundamental scale of quantum gravity being a few TeV [38], though intriguing, do not seem to provide simple explanations of these features: (a), (b) and (c). They will be mentioned briefly in Section 6 G.

[5] The effects of neutrino-mixing and of the more legitimate choice of $M = M_{string} \approx 4 \times 10^{17}$ GeV (instead of $M = M_{\mathrm{Planck}}$) on the values of $m(\nu_L^\tau)$ and of M_{3R} are considered in Ref. [14] and are reflected in our discussions in Section 5. The two effects together end up in yielding essentially the same mass for $m(\nu_L^\tau)$ as obtained within the simplified picture presented in this section, together with a value for $M_{3R} \approx$ (5-10) $\times 10^{14}$ GeV.

(ii) Now using SU(4)-color and the Higgs multiplet $(\mathbf{2},\mathbf{2},\mathbf{1})_H$ of G(224) or equivalently $\mathbf{10}_H$ of SO(10), one obtains the relation $m_\tau(M_X) = m_b(M_X)$, which is known to be successful. Thus, there is a good reason to believe that the third family gets its masses primarily from the $\mathbf{10}_H$ or equivalently $(2,2,1)_H$ (see Section 5). In turn, this implies:

$$m(\nu^\tau_{\rm Dirac}) \approx m_{\rm top}(M_X) \approx (100\text{-}120)\,{\rm GeV}\ . \tag{9}$$

Note that this relationship between the Dirac mass of the tau-neutrino and the top-mass is special to SU(4)-color. It does not emerge in SU(5).

(iii) Given the superheavy Majorana masses of the RH neutrinos as well as the Dirac masses as above, the see-saw mechanism [40] yields naturally light masses for the LH neutrinos. For ν^τ_L (ignoring flavor-mixing), one thus obtains, using Eqs.(8) and (9),

$$m(\nu^\tau_L) \approx \frac{m(\nu^\tau_{\rm Dirac})^2}{M_{3R}} \approx [(1/20)\,{\rm eV}\,(1\text{-}1.44)/f_{33}\,\rho^2]\,(M/M_{\rm Planck})\ . \tag{10}$$

In the next section, we discuss the masses and mixings of all three neutrinos. As we will see, given the hierarchical masses of quarks and charged leptons and the see-saw mechanism, we naturally obtain $m(\nu^\mu_L) \sim (1/10)m(\nu^\tau_L)$. We are thus led to predict that $\Delta m^2(\nu_\mu\nu_\tau)_{th} \equiv |m^2(\nu^\tau_L) - m^2(\nu^\mu_L)|_{th} \approx m^2(\nu^\tau_L)_{th} =$ square of the RHS of Eq. (10). Now SuperK result strongly suggests that it is observing ν^μ_L-ν^τ_L (rather than ν^μ_L-ν_X) oscillation, with a $\Delta m^2(\nu_\mu\nu_\tau)_{obs} \approx 3{\times}10^{-3}$ eV^2. It seems *truly remarkable* that the expected magnitude of $\Delta m^2(\nu_\mu\nu_\tau)$, given to a very good approximation by the square of the RHS of Eq. (10), is just about what is observed at SuperK, if $f_{33}\,\rho^2(M_{\rm Planck}/M) \approx 1.3$ to $1/2$. Such a range for $f_{33}\,\rho^2(M_{\rm Planck}/M)$ seems most plausible and natural (see discussion in Ref. [12]). Note that the estimate (10) crucially depends upon the supersymmetric unification scale, which provides a value for M_{3R}, as well as on SU(4)-color that yields $m(\nu^\tau_{\rm Dirac})$. *The agreement between the expected and the SuperK results thus clearly favors supersymmetric unification, and in the string theory context, it suggests that the effective symmetry below the string-scale should contain SU(4)-color.* Thus, minimally this effective symmetry should be either G(214) or G(224), and maximally as big as SO(10), if not E_6.

By contrast, if SU(5) is regarded as either a fundamental symmetry or as the effective symmetry below the string scale, there would be no compelling reason based on symmetry alone, to introduce a ν_R, because it is a singlet of SU(5). Second, even if one did introduce ν^i_R by hand, their Dirac masses, arising from the coupling $h^i\,\overline{\mathbf{5}}_i\langle\mathbf{5}_H\rangle\nu^i_R$, would be unrelated to the up-flavor masses and thus rather arbitrary [contrast with Eq. (9)]. So also would be the Majorana masses of the ν^i_R's, which are SU(5)-invariant, and thus can be even of order string scale . This would give extremely small values of $m(\nu^\tau_L)$ and $m(\nu^\mu_L)$ and thus of $\Delta m^2(\nu_\mu\nu_\tau)$, which would be in gross conflict with observation.

Before passing to the next section, it is worth noting that the mass of ν_τ or of $\Delta m^2(\nu_\mu\nu_\tau)$ suggested by SuperK, as well as the observed value of $\sin^2\theta_W$ (see Section 3), provide valuable insight into the nature of GUT symmetry breaking. They both favor the case of a *single-step breaking* (SSB) of SO(10) or a string-unified G(224) symmetry at a high scale of order M_X, into the standard model symmetry G(213), as opposed to that of a multi-step breaking (MSB). The latter would correspond, for example, to SO(10) [or G(224)] breaking at a scale M_1 into G(213), which in turn breaks at a scale $M_2 \ll M_1$ into G(213). One reason why the case of single-step breaking is favored over that of MSB is that the latter can accommodate but not really predict $\sin^2\theta_W$, whereas the former predicts the same successfully. Furthermore, since the Majorana mass of ν^τ_R arises arises only after B–L and I_{3R} break, it would be given, for the case of MSB, by

$M_{3R} \sim f_{33}(M_2^2/M)$, where $M \sim M_{st}$ (say). If $M_2 \ll M_X \sim 2 \times 10^{16}$ GeV, and M $> M_X$, one would obtain too low a value ($\ll 10^{14}$ GeV) for M_{3R} [compare with Eq. (8)], and thereby too large a value for $m(\nu_L^\tau)$, compared to that suggested by SuperK. By contrast, the case of single-step breaking (SSB) yields the right magnitude for $m(\nu_\tau)$ [see Eq. (10)].

Thus the success of the results on $m(\nu_\tau)$ and thereby on $\Delta m^2(\nu_\mu \nu_\tau)$ discussed above not only favors the symmetry SO(10) or G(224) beging effective in 4D at a high scale, but also clearly suggests that B–L and I_{3R} break near the conventional GUT scale $M_X \sim 2 \times 10^{16}$ GeV, rather than at an intermediate scale $\ll M_X$. In other words, the observed values of both $\sin^2\theta_W$ and $\Delta m^2(\nu_\mu \nu_\tau)$ favor only *the simplest pattern of symmetry-breaking*, for which SO(10) or a string-derived G(224) symmetry breaks in one step to the standard model symmetry, rather than in multiple steps. It is of course only this simple pattern of symmetry breaking that would be rather restrictive as regards its predictions for proton decay (to be discussed in Section 6). I next discuss the problem of understanding the masses and mixings of all fermions.

5 Understanding Aspects of Fermion Masses and Neutrino Oscillations in SO(10)

Understanding the masses and mixings of all quarks *in conjunction with* those of the charged leptons *and* neutrinos is a goal worth achieving by itself. It also turns out to be essential for the study of proton decay. I therefore present first a partial attempt in this direction, based on a quark-lepton unified G(224)/SO(10)-framework, which seems most promising [14]. A few guidelines would prove to be helpful in this regard. The first of these is motivated by the desire for economy [see (11)], and the rest (see below) by the data. In essence, we will be following (partly) a *bottom-up approach* by appealing to the data to provide certain clues as regards the pattern of the Yukawa couplings, and simultaneously a *top-down approach* by appealing to grand unification, based on the symmetry G(224)/SO(10), to restrict the couplings by the constraints of group theory. The latter helps to interrelate the masses and mixings of quarks with those of the charged leptons and the neutrinos. As we will see, it is these *interrelationships*, which permit predictivity, and are found to be remarkably successful. The guidelines which we adopt are as follows.

1) Hierarchy Through Off-diagonal Mixings: Recall earlier attempts [41] that attribute hierarchical masses of the first two families to mass matrices of the form:

$$M = \begin{pmatrix} 0 & \epsilon \\ \epsilon & 1 \end{pmatrix} m_s^{(0)}, \tag{11}$$

for the (d, s) quarks, and likewise for the (u, c) quarks. Here $\epsilon \sim 1/10$. The hierarchical patterns in Eq. (11) can be ensured by imposing a suitable flavor symmetry which distinguishes between the two families (that in turn may have its origin in string theory (see e.g. Ref [31]). Such a pattern has the virtues that (a) it yields a hierarchy that is much larger than the input parameter ϵ: $(m_d/m_s) \approx \epsilon^2 \ll \epsilon$, and (b) it leads to an expression for the Cabibbo angle:

$$\theta_c \approx \left| \sqrt{\frac{m_d}{m_s}} - e^{i\phi} \sqrt{\frac{m_u}{m_c}} \right|, \tag{12}$$

which is rather successful. Using $\sqrt{m_d/m_s} \approx 0.22$ and $\sqrt{m_u/m_c} \approx 0.06$, we see that Eq. (12) works to within about 25% for any value of the phase ϕ. Note that the square root formula (like

$\sqrt{m_d/m_s}$) for the relevant mixing angle arises because of the symmetric form of M in Eq. (11), which in turn is ensured if the contributing Higgs is a 10 of SO(10). A generalization of the pattern in Eq. (11) would suggest that the first two families (i.e. the e and the μ) receive masses primarily through their mixing with the third family (τ), with $(1,3)$ and $(1,2)$ elements being smaller than the $(2,3)$; while $(2,3)$ is smaller than the $(3,3)$. We will follow this guideline, except for the modification noted below.

2) The Need for an Antisymmetric Component: Although the symmetric hierarchical matrix in Eq. (11) works well for the first two families, a matrix of the same form fails altogether to reproduce V_{cb}, for which it yields:

$$V_{cb} \approx \left| \sqrt{\frac{m_s}{m_b}} - e^{i\chi} \sqrt{\frac{m_c}{m_t}} \right| . \tag{13}$$

Given that $\sqrt{m_s/m_b} \approx 0.17$ and $\sqrt{m_c/m_t} \approx 0.0.06$, we see that Eq. (13) would yield V_{cb} varying between 0.11 and 0.23, depending upon the phase χ. This is too big, compared to the observed value of $V_{cb} \approx 0.04 \pm 0.003$, by at least a factor of 3. We interpret this failure as a *clue* to the presence of an antisymmetric component in M, together with symmetrical ones (so that $m_{ij} \neq m_{ji}$), which would modify the relevant mixing angle to $\sqrt{m_i/m_j}\,\sqrt{m_{ij}/m_{ji}}$, where m_i and m_j denote the respective eigenvalues.

3) The Need for a Contribution Proportional to B–L: The success of the relations $m_b^0 \approx m_\tau^0$, and $m_t^0 \approx m(\nu_\tau)^0_{\rm Dirac}$ (see Section 4), suggests that the members of the third family get their masses primarily from the VEV of a SU(4)-color singlet Higgs field that is independent of B–L. This is in fact ensured if the Higgs is a 10 of SO(10). However, the empirical observations of $m_s^0 \sim m_\mu^0/3$ and $m_d^0 \sim 3m_e^0$ [42] call for a contribution proportional to B–L as well. Further, one can in fact argue that understanding naturally the suppression of V_{cb} (in the quark-sector) together with an enhancement of $\theta^{\rm osc}_{\nu_\mu\,\nu_\tau}$ (in the lepton sector) calls for a contribution that is not only proportional to B–L, but also antisymmetric in the family space (this later feature is suggested already in item (2)). We show below how both of these requirements can be met in SO(10), even for a minimal Higgs system.

4) Up-Down Asymmetry: Finally, the up and the down-sector mass matrices must not be proportional to each other, as otherwise the CKM angles would all vanish. Note that the cubic couplings of a single 10_H with the fermions in the 16's will not serve the purpose in this regard.

Following Ref. [14], I now present a simple and predictive mass-matrix, based on SO(10), that satisfies *all four* requirements (1), (2), (3) and (4). The interesting point is that one can obtain such a mass-matrix for the fermions by utilizing only the minimal Higgs system, that is needed anyway to break the gauge symmetry SO(10). It consists of the set:

$$H_{\rm minimal} = \{\mathbf{45}_H,\, \mathbf{16}_H,\, \overline{\mathbf{16}}_H,\, \mathbf{10}_H\} . \tag{14}$$

Of these, the VEV of $\langle \mathbf{45}_H \rangle \sim M_X$ breaks SO(10) into G(2213), and those of $\langle \mathbf{16}_H \rangle = \langle \overline{\mathbf{16}}_H \rangle \sim M_X$ break G(2213) to G(213), at the unification-scale M_X. Now G(213) breaks at the electroweak scale by the VEV of $\langle 10_H \rangle$ to $U(1)_{em} \times SU(3)^c$.

One might have introduced large-dimensional tensorial multiplets of SO(10) like $\overline{\mathbf{126}}_H$ and $\mathbf{120}_H$, both of which possess cubic level Yukawa couplings with the fermions. In particular, the coupling $\mathbf{16}_i\mathbf{16}_j(\mathbf{120}_H)$ would give the desired family-antisymmetric as well as (B–L)-dependent contribution. We do not however introduce these multiplets in part because there is a general argument suggesting that they do not arise at least in weakly interacting heterotic string solutions

[39], and in part also because mass-splittings within such large-dimensional multiplets could give excessive threshold corrections to $\alpha_3(m_z)$ (typically exceeding 20%), rendering observed coupling unification fortuitous. By contrast, the multiplets in the minimal set (shown above) can arise in string solutions. Furthermore, the threshold corrections for the minimal set are found to be naturally small, and even to have the right sign, to go with the observed coupling unification [14] (see Appendix).

The question is: can the minimal set of Higgs multiplets [see Eq. (14)] meet all the requirements listed above? Now $\mathbf{10}_H$ (even several $\mathbf{10}$'s) cannot meet the requirements of antisymmetry and (B-L)-dependence. Furthermore, a single $\mathbf{10}_H$ cannot generate CKM-mixings. This impasse disappears, however, as soon as one allows for not only cubic, but also effective non-renormalizable quartic couplings of the minimal set of Higgs fields with the fermions. These latter couplings could of course well arise through exchanges of superheavy states (e.g. those in the string tower) involving renormalizable couplings, and/or through quantum gravity.

Allowing for such cubic and quartic couplings and adopting the guideline (1) of hierarchical Yukawa couplings, as well as that of economy, we are led to suggest the following effective lagrangian for generating Dirac masses and mixings of the three families [14] (for a related but different pattern, involving a non-minimal Higgs system, see Ref. [43]).

$$\begin{aligned}\mathcal{L}_{\mathbf{Yuk}} \;=\; & h_{33}\,\mathbf{16}_3\,\mathbf{16}_3\,\mathbf{10}_H \;+\; [\,h_{23}\,\mathbf{16}_2\,\mathbf{16}_3\,\mathbf{10}_H \;+\; a_{23}\,\mathbf{16}_2\,\mathbf{16}_3\,\mathbf{10}_H\,\mathbf{45}_H/M \\ & +\; g_{23}\,\mathbf{16}_2\,\mathbf{16}_3\,\mathbf{16}_H\,\mathbf{16}_H/M] \;+\; \{a_{12}\,\mathbf{16}_1\,\mathbf{16}_2\,\mathbf{10}_H\,\mathbf{45}_H/M \\ & +\; g_{12}\,\mathbf{16}_1\,\mathbf{16}_2\,\mathbf{16}_H\,\mathbf{16}_H/M\}\,. \end{aligned} \tag{15}$$

Here, M could plausibly be of order string scale. Note that a mass matrix having essentially the form of Eq. (11) results if the first term $h_{33}\langle\mathbf{10}_H\rangle$ is dominant. This ensures $m_b^0 \approx m_\tau^0$ and $m_t^0 \approx m^0(\nu_{\rm Dirac})$. Following the assumption of progressive hierarchy (equivalently appropriate flavor symmetries [6]), we presume that $h_{23} \sim h_{33}/10$, while h_{22} and h_{11}, which are not shown, are assumed to be progressively much smaller than h_{23}. Since $\langle\mathbf{45}_H\rangle \sim \langle\mathbf{16}_H\rangle \sim M_X$, while $M \sim M_{st} \sim 10M_X$, the terms $a_{23}\langle\mathbf{45}_H\rangle/M$ and $g_{23}\langle\mathbf{16}_H\rangle/M$ can quite plausibly be of order $h_{33}/10$, if $a_{23} \sim g_{23} \sim h_{33}$. By the assumption of hierarchy, we presume that $a_{12} \ll a_{23}$, and $g_{12} \ll g_{23}$

It is interesting to observe the symmetry properties of the a_{23} and g_{23}-terms. Although $\mathbf{10}_H \times \mathbf{45}_H = \mathbf{10} + \mathbf{120} + \mathbf{320}$, given that $\langle\mathbf{45}_H\rangle$ is along B–L, which is used to implement doublet-triplet splitting (see Appendix), only $\mathbf{120}$ in the decomposition contributes to the mass-matrices. This contribution is, however, antisymmetric in the family-index and, at the same time, proportional to B–L. *Thus the a_{23} term fulfills the requirements of both antisymmetry and (B–L)-dependence, simultaneously* [7] . With only h_{ij} and a_{ij}-terms, however, the up and down quark mass-matrices will be proportional to each other, which would yield $V_{CKM} = 1$. This is remedied by the g_{ij} coupling,

[6] Although no explicit string solution with the hierarchy in all the Yukawa couplings in Eq. (15)—i.e. in h_{ij}, a_{ij} and g_{ij}—exists as yet, one can postulate flavor symmetries of the type alluded to (e.g. two abelian U(1) symmetries), which assign flavor charges not only to the fermion families and the Higgs multiplets, but also to a few (postulated) SM singlets that acquire VEVs of order M_X. The flavor symmetry-allowed effective couplings such as $\mathbf{16}_2\mathbf{16}_3\mathbf{10}_H\,\langle S\rangle/M$ would lead to $h_{23} \sim \langle S\rangle/M \sim 1/10$. One can verify that the full set of hierarchical couplings shown in Eq. (15) can in fact arise in the presence of two such U(1) symmetries. String theory (at least) offers the scope (as indicated by the solutions of Refs. [31] and [30]) for providing a rationale for the existence of such flavor symmetries, together with that of the SM singlets. For example, there exist solutions with the top Yukawa coupling being leading and others being hierarchical (as in Ref. [31]).

[7] The analog of $\mathbf{10}_H \cdot \mathbf{45}_H$ for the case of G(224) would be $\chi_H \equiv (2,2,1)_H \cdot (1,1,15)_H$. Although in general, the coupling of χ_H to the fermions need not be antisymmetric, for a string-derived G(224), the multiplet $(1,1,15)_H$ is

because, the $\mathbf{16}_H$ can have a VEV not only along its SM singlet component (transforming as $\tilde{\bar{\nu}}_R$) which is of GUT-scale, but also along its electroweak doublet component—call it $\mathbf{16}_d$—of the electroweak scale. The latter can arise by the the mixing of $\mathbf{16}_d$ with the corresponding doublet (call it $\mathbf{10}_d$) in the $\mathbf{10}_H$. The MSSM doublet H_d, which is light, is then a mixture of $\mathbf{10}_d$ and $\mathbf{16}_d$, while the orthogonal combination is superheavy (see Appendix). Since $\langle \mathbf{16}_d \rangle$ contributes only to the down-flavor mass matrices, but not to the up-flavor, the g_{23} and g_{12} couplings generate non-trivial CKM-mixings. *We thus see that the minimal Higgs system (as shown in Eq. (14)) satisfies* a priori *all the qualitative requirements (1)–(4), including the condition of* $V_{CKM} \neq 1$. I now discuss that this system works well even quantitatively.

With the six effective Yukawa couplings shown in Eq. (15), the Dirac mass matrices of quarks and leptons of the three families at the unification scale take the form:

$$
\begin{aligned}
U &= \begin{pmatrix} 0 & \epsilon' & 0 \\ -\epsilon' & 0 & \epsilon + \sigma \\ 0 & -\epsilon + \sigma & 1 \end{pmatrix} m_U, \\
D &= \begin{pmatrix} 0 & \epsilon' + \eta' & 0 \\ -\epsilon' + \eta' & 0 & \epsilon + \eta \\ 0 & -\epsilon + \eta & 1 \end{pmatrix} m_D, \\
N &= \begin{pmatrix} 0 & -3\epsilon' & 0 \\ 3\epsilon' & 0 & -3\epsilon + \sigma \\ 0 & 3\epsilon + \sigma & 1 \end{pmatrix} m_U, \\
L &= \begin{pmatrix} 0 & -3\epsilon' + \eta' & 0 \\ 3\epsilon' + \eta' & 0 & -3\epsilon + \eta \\ 0 & 3\epsilon + \eta & 1 \end{pmatrix} m_D. \qquad (16)
\end{aligned}
$$

Here the matrices are multiplied by left-handed fermion fields from the left and by anti–fermion fields from the right. (U, D) stand for the mass matrices of up and down quarks, while (N, L) are the Dirac mass matrices of the neutrinos and the charged leptons. The entries $1, \epsilon$,and σ arise respectively from the h_{33}, a_{23} and h_{23} terms in Eq. (15), while η entering into D and L receives contributions from both g_{23} and h_{23}; thus $\eta \neq \sigma$. Similarly η' and ϵ' arise from g_{12} and a_{12} terms respectively. Note the quark-lepton correlations between U and N as well as D and L arise because of $\mathrm{SU}(4)^C$, while the up-down correlations between U and D as well as N and L arise because of $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$. Thus, these correlations emerge just because of the symmetry property of G(224). The relative factor of -3 between quarks and leptons involving the ϵ entry reflects the fact that $\langle \mathbf{45_H} \rangle$ is proportional to (B–L), while the antisymmetry in this entry arises from the group structure of SO(10), as explained above[7]. As we will see, this ϵ-entry helps to account for (a) the differences between m_s and m_μ, (b) that between m_d and m_e, and most important, (c) the suppression of V_{cb} *together with* the enhancement of the ν_μ-ν_τ oscillation angle.

The mass matrices in Eq. (16) contain 7 parameters [8]: $\epsilon, \sigma, \eta, m_D = h_{33} \langle 10_d \rangle, m_U = h_{33} \langle 10_U \rangle$,

most likely to arise from an underlying 45 of SO(10) (rather than 210); in this case, the couplings of χ_H must be antisymmetric like that of $\mathbf{10}_H \cdot \mathbf{45}_H$.

[8]Of these, $m_U^0 \approx m_t^0$ can in fact be estimated to within 20% accuracy by either using the argument of radiative electroweak symmetry breaking, or some promising string solutions (see e.g. Ref. [31]).

η' and ϵ'. These may be determined by using, for example, the following input values: $m_t^{\rm phys} = 174$ GeV, $m_c(m_c) = 1.37$ GeV, $m_s(1\,{\rm GeV}) = 110$–116 MeV [44], $m_u(1\,{\rm GeV}) \approx 6$ MeV and the observed masses of e, μ and τ, which lead to (see Ref. [14], for details):

$$\sigma \simeq 0.110\,,\ \eta \simeq 0.151\,,\ \epsilon \simeq -0.095\,,\ |\eta'| \approx 4.4 \times 10^{-3} \ \text{and}\ \epsilon' \approx 2 \times 10^{-4}$$

$$m_U \simeq m_t(M_U) \simeq (100\text{-}120)\,{\rm GeV}\,,\ m_D \simeq m_b(M_U) \simeq 1.5\,{\rm GeV}\,. \tag{17}$$

Here, I will assume, only for the sake of simplicity, as in Ref. [14], that the parameters are real.[9] Note that in accord with our general expectations discussed above, each of the parameters σ, η and ϵ are found to be of order 1/10, as opposed to being [10] $O(1)$ or $O(10^{-2})$, compared to the leading (3,3)-element in Eq. (16). Having determined these parameters, we are led to a total of five predictions involving only the quarks (those for the leptons are listed separately):

$$m_b^0 \approx m_\tau^0(1 - 8\epsilon^2)\,;\ \ \text{thus}\ \ m_b(m_b) \simeq (4.6\text{-}4.9)\,{\rm GeV} \tag{18}$$

$$|V_{cb}| \simeq |\sigma - \eta| \approx \left|\sqrt{m_s/m_b}\left|\frac{\eta + \epsilon}{\eta - \epsilon}\right|^{1/2} - \sqrt{m_c/m_t}\left|\frac{\sigma + \epsilon}{\sigma - \epsilon}\right|^{1/2}\right| \simeq 0.045 \tag{19}$$

$$m_d\,(1{\rm GeV}) \simeq 8\,{\rm MeV} \tag{20}$$

$$\theta_C \simeq \left|\sqrt{m_d/m_s} - e^{i\phi}\sqrt{m_u/m_c}\right| \tag{21}$$

$$|V_{ub}/V_{cb}| \simeq \sqrt{m_u/m_c} \simeq 0.07\,. \tag{22}$$

In making these predictions, we have extrapolated the GUT-scale values down to low energies using $\alpha_3(m_Z) = 0.118$, a SUSY threshold of 500 GeV and $\tan\beta = 5$. The results depend weakly on these choices, assuming $\tan\beta \approx 2$-30. Further, the Dirac masses and mixings of the neutrinos and the mixings of the charged leptons also get determined. We obtain:

$$m_{\nu_\tau}^D(M_U) \approx 100\text{-}120\,{\rm GeV};\ m_{\nu_\mu}^D(M_U) \simeq 8\,{\rm GeV}, \tag{23}$$

$$\theta_{\mu\tau}^\ell \approx -3\epsilon + \eta \approx \sqrt{m_\mu/m_\tau}\left|\frac{-3\epsilon + \eta}{3\epsilon + \eta}\right|^{1/2} \simeq 0.437 \tag{24}$$

$$m_{\nu_e}^D \simeq [\,9\epsilon'^2/(9\epsilon^2 - \sigma^2)]\,m_U \simeq 0.4\,{\rm MeV} \tag{25}$$

$$\theta_{e\mu}^\ell \simeq \left|\frac{\eta' - 3\epsilon'}{\eta' + 3\epsilon'}\right|^{1/2}\sqrt{m_e/m_\mu} \simeq 0.85\sqrt{m_e/m_\mu} \simeq 0.06 \tag{26}$$

[9] Babu and I have recently studied supersymmetric CP violation within the G(224)/SO(10) framework, by using precisely the fermion mass-matrices as in Eq. (16). We have observed [33] that complexification of the parameters can lead to observed CP violation, without upsetting in the least the success of Ref. [14] (i.e. of the fermion mass-matrices of Eq. (16)) in describing the masses and mixings of all fermions, including neutrinos. Even with complexification the relative signs and the approximate magnitudes of the real parts of the parameters must be the same as in Eq. (17), to retain the success.

[10] This is one characteristic difference between our work and that of Ref. [43], where the (2,3)-element is even bigger than the (3,3).

$$\theta^{\ell}_{e\tau} \simeq \frac{1}{0.85}\sqrt{m_e/m_\tau}\,(m_\mu/m_\tau) \simeq 0.0012\,. \tag{27}$$

In evaluating $\theta^{\ell}_{e\mu}$, we have assumed ϵ' and η' to be relatively positive.

Given the bizarre pattern of quark and lepton masses and mixings, it seems remarkable that the simple and economical pattern of fermion mass-matrices, motivated in part by the assumption of flavor symmetries[6] which distinguish between the three families and in large part by the group theory of G(224)/SO(10), gives an overall fit to all of them [Eqs. (18) through (22)] which is good to within 10%. This includes the two successful predictions on m_b and V_{cb} [Eqs.(18) and (19)]. Note that in supersymmetric unified theories, the "observed" value of $m_b(m_b)$ and renormalization-group studies suggest that, for a wide range of the parameter $\tan\beta$, m^0_b should in fact be about 10-20% *lower* than m^0_τ [45]. This is neatly explained by the relation: $m^0_b \approx m^0_\tau(1-8\epsilon^2)$ [Eq. (18)], where exact equality holds in the limit $\epsilon \to 0$ (due to SU(4)-color), while the decrease of m^0_b compared to m^0_τ by $8\epsilon^2 \sim 10\%$ is precisely because the off-diagonal ϵ-entry is proportional to B–L [see Eq. (16)].

Specially intriguing is the result on $V_{cb} \approx 0.045$ which compares well with the observed value of $\simeq 0.04$. The suppression of V_{cb}, compared to the value of 0.17 ± 0.06 obtained from Eq. (13), is now possible because the mass matrices [Eq. (16)] contain an antisymmetric component $\propto \epsilon$. That corrects the square-root formula $\theta_{sb} = \sqrt{m_s/m_b}$ [appropriate for symmetric matrices, see Eq. (11)] by the asymmetry factor $|(\eta+\epsilon)/(\eta-\epsilon)|^{1/2}$ [see Eq. (19)], and similarly for the angle θ_{ct}. This factor suppresses V_{cb} if η and ϵ have opposite signs. The interesting point is that, *the same feature necessarily enhances the corresponding mixing angle $\theta^{\ell}_{\mu\tau}$ in the leptonic sector*, since the asymmetry factor in this case is given by $[(-3\epsilon+\eta)/(3\epsilon+\eta)]^{1/2}$ [see Eq. (24)]. This enhancement of $\theta^{\ell}_{\mu\tau}$ helps to account for the nearly maximal oscillation angle observed at SuperK (as discussed below). This intriguing correlation between the mixing angles in the quark versus leptonic sectors—*that is suppression of one implying enhancement of the other*—has become possible only because of the ϵ-contribution, which is simultaneously antisymmetric and is proportional to B–L. That in turn becomes possible because of the group-property of SO(10) or a string-derived G(224)[7].

Taking stock, we see an impressive set of facts in favor of having B–L as a gauge symmetry and in fact for the full SU(4)-color-symmetry. These include: (i) the suppression of V_{cb}, together with the enhancement of $\theta^{\ell}_{\mu\tau}$, mentioned above; (ii) the successful relation $m^0_b \approx m^0_\tau(1-8\epsilon^2)$; (iii) the usefulness again of the SU(4)-color-relation $m(\nu^\tau_{\rm Dirac})^0 \approx m^0_t$ in accounting for $m(\nu^\tau_L)$ (see Section 4); (iv) the agreement of the relation $|m^0_s/m^0_\mu| = |(\epsilon^2-\eta^2)/(9\epsilon^2-\eta^2)|$ with the data, in that the ratio is naturally *less than* 1, if $\eta \sim \epsilon$ [The presence of $9\epsilon^2$ in the denominator is because the off-diagonal entry is proportional to B–L.]; and finally (v), the need for (B–L)—as a local symmetry, to implement baryogenesis via leptogenesis, as noted in Section 1.

Turning to neutrino masses, while all the entries in the Dirac mass matrix N are now fixed, to obtain the parameters for the light neutrinos, one needs to specify those of the Majorana mass matrix of the RH neutrinos ($\nu^{e,\mu,\tau}_R$). Guided by economy and the assumption of hierarchy, we consider the following pattern [14]:

$$M^R_\nu = \begin{pmatrix} x & 0 & z \\ 0 & 0 & y \\ z & y & 1 \end{pmatrix} M_R\,. \tag{28}$$

As discussed in Section 4, the magnitude of $M_R \approx$ (5-10) $\times 10^{14}$ GeV can quite plausibly be justified in the context of supersymmetric unification[5] [e.g. by using $M \approx M_{st} \approx 4\times 10^{17}$ GeV in

Eq. (8)]. To the same extent, the magnitude of $m(\nu_\tau) \approx$ (1/10-1/30) eV, which is consistent with the SuperK value, can also be anticipated by allowing for $\nu_\mu - \nu_\tau$ mixing [see Ref. [14]]. Thus there are effectively three new parameters: x, y, and z. Since there are six observables for the three light neutrinos, one can expect three predictions. These may be taken to be $\theta^{\rm osc}_{\nu_\mu\nu_\tau}$, m_{ν_τ} [see Eq. (10)], and for example $\theta^{\rm osc}_{\nu_e\nu_\mu}$.

Assuming successively hierarchical entries as for the Dirac mass matrices, we presume that $|y| \sim 1/10, |z| \leq |y|/10$ and $|x| \leq z^2$. Now given that $m(\nu_\tau) \sim 1/20$ eV [as estimated in Eq. (10)], the MSW solution for the solar neutrino puzzle [46] suggests that $m(\nu_\mu)/m(\nu_\tau) \approx$ 1/8-1/20. With hierarchical neutrino masses, the higher value of the mass-ratio (like 1/8) holds only for the large angle MSW solution (see below). With the mass-ratio being in the range of 1/8-1/20, one obtains: $|y| \approx$ (1/17 to 1/21), with y having the same sign as ϵ [see Eq. (17)]. This solution for y obtains only by assuming that y has a hierarchical value $O(1/10)$ rather than $O(1)$. Combining now with the mixing in the μ-τ sector determined above [see Eq. (24)], one can then determine the ν_μ-ν_τ oscillation angle. The two predictions of the model for the neutrino-system are then:

$$m(\nu_\tau) \approx (1/10 \text{ - } 1/30)\,\text{eV} \tag{29}$$

$$\theta^{\rm osc}_{\nu_\mu\nu_\tau} \simeq \theta^\ell_{\mu\tau} - \theta^\nu_{\mu\tau} \simeq \left(0.437 + \sqrt{\frac{m_{\nu_2}}{m_{\nu_3}}}\right). \tag{30}$$

Thus,

$$\sin^2 2\theta^{\rm osc}_{\nu_\mu\nu_\tau} = (0.99, 0.975, 0.92, 0.87) \tag{31}$$

for

$$m_{\nu_2}/m_{\nu_3} = (1/8, 1/10, 1/15, 1/20)\,. \tag{32}$$

Both of these predictions are extremely successful.[11]

Note the interesting point that the MSW solution, and the requirement that $|y|$ should have a natural hierarchical value (as mentioned above), lead to y having the same sign as ϵ. Now, that (it turns out) implies that the two contributions in Eq. (30) must *add* rather than subtract, leading to an *almost maximal oscillation angle* [14]. The other factor contributing to the enhancement of $\theta^{\rm osc}_{\nu_\mu\nu_\tau}$ is, of course, also the asymmetry-ratio which increases $|\theta^\ell_{\mu\tau}|$ from 0.25 to 0.437 [see Eq. (24)]. We see that one can derive rather plausibly a large ν_μ-ν_τ oscillation angle $\sin^2 2\theta^{\rm osc}_{\nu_\mu\nu_\tau} \geq 0.92$, together with an understanding of hierarchical masses and mixings of the quarks and the charged leptons, while maintaining a large hierarchy in the seesaw derived neutrino masses ($m_{\nu_2}/m_{\nu_3} =$ 1/8-1/15), all within a unified framework including both quarks and leptons. In the example exhibited here, the mixing angles for the mass eigenstates of neither the neutrinos nor the charged leptons are really large, in that $\theta^\ell_{\mu\tau} \simeq 0.437 \simeq 23^\circ$ and $\theta^\nu_{\mu\tau} \simeq$ (0.22-0.35) $\approx$ (13-20.5)$^\circ$, *yet the oscillation angle obtained by combining the two is near-maximal.* This contrasts with most works in the literature in which a large oscillation angle is obtained either entirely from the neutrino sector (with nearly degenerate neutrinos) or almost entirely from the charged lepton sector.

Small Versus Large Angle MSW Solutions

In considerations of ν_e-ν_μ and ν_e-ν_τ oscillation angles, tiny *intrinsic* non-diagonal Majorana masses $\sim 10^{-3}$ eV of the LH neutrinos leading to $\nu^e_L\nu^\mu_L$ and $\nu^e_L\nu^\tau_L$-mixings, which can far exceed those

[11] In writing Eq. (31), the small angle approximation exhibited in Eq. (30) is replaced by the more precise expression, given in Eq. (12) of Ref. [14], with the further understanding that $\sqrt{m_\mu/m_\tau}$ appearing in Eq. (12) (of Ref. [14]) is replaced by the μ-τ mixing angle ≈ 0.437.

induced by the standard see-saw mechanism, can be rather important, especially for ν_e-ν_μ mixing. As explained below, such intrinsic masses can arise quite naturally through higher dimensional operators and can lead to the large angle MSW solution of the solar neutrino puzzle.

Let us first ignore the intrinsic Majorana masses of the LH neutrinos and include only those that arise through the standard see-saw mechanism, involving the superheavy Majorana masses of the RH neutrinos, with a pattern given, for example, by Eq. (28). Note that, while $M_R \approx$ (5-15) $\times 10^{14}$ GeV and $y \approx -1/20$ are better determined, the parameters x and z can not be obtained reliably at present because very little is known about observables involving ν_e. Taking, for concreteness, $m_{\nu_e} \approx (10^{-5}\text{-}10^{-4})$ (1 to few)) eV and $\theta^{\rm osc}_{e\tau} \approx \theta^{\ell}_{e\tau} - \theta^{\nu}_{e\tau} \approx 10^{-3} \pm 0.03$ as inputs, we obtain: $z \sim$ (1-5) $\times 10^{-3}$ and $x \sim$ (1 to few)(10^{-6}-10^{-5}), in accord with the guidelines of $|z| \sim |y|/10$ and $|x| \sim z^2$. This in turn yields: $\theta^{\rm osc}_{e\mu} \approx \theta^{\ell}_{e\mu} - \theta^{\nu}_{e\mu} \approx 0.06 \pm 0.015$. Note that the mass of $m_{\nu_\mu} \sim 3 \times 10^{-3}$ eV, that follows from a natural hierarchical value for $y \sim -(1/20)$, and $\theta_{e\mu}$ as above, go well with the small angle MSW explanation of the solar neutrino puzzle. In short the framework presented so far, that neglects intrinsic Majorana masses of the LH neutrinos altogether, generically tends to yield the small angle MSW solution.

As alluded to above, we now observe that small intrinsic non-seesaw masses of the LH neutrinos $\sim 10^{-3}$ eV, which could mix ν_{eL} and $\nu_{\mu L}$, can, however, arise quite naturally through higher dimensional operators in the superpotential of the form [12]: $W \supset \kappa_{12}\mathbf{16}_1\mathbf{16}_2\mathbf{16}_H\mathbf{16}_H\mathbf{10}_H\mathbf{10}_H/M^3_{\rm GUT}$. One can verify that such a term would lead to an *intrinsic* Majorana mixing mass term of the form $m^{(0)}_{12}\nu^e_L\nu^\mu_L$, with a strength given by $m^{(0)}_{12} \approx \kappa_{12}(\langle \mathbf{16}_H\rangle/M_{\rm GUT})^2(175\ {\rm GeV})^2/M_{\rm GUT} \approx$ (1.5-6) $\times 10^{-3} eV$, where we have put $\langle \mathbf{16}_H\rangle \approx$ (1-2)$M_{\rm GUT}$ and $M_{\rm GUT} \approx 2 \times 10^{16}$ GeV. Such an intrinsic Majorana mixing mass $\sim 10^{-3}$ eV, though small, is still much larger than what one would get for the corresponding term from the standard see-saw mechanism. Now, as discussed above, the diagonal $(\nu^\mu_L\nu^\mu_L)$ mass-term, arising from the standard see-saw mechanism can naturally be of order (3-8)$\times 10^{-3}$ eV (for $|y| \approx 1/20$ to $1/15$, say). In addition, the intrinsic contribution of the type mentioned above may in general also contribute to the diagonal $(\nu^\mu_L\nu^\mu_L)$ mass (depending upon flavor symmetries) which can be (few)$\times 10^{-3}$ eV. Thus, taking the net values of $m_{22} \approx$ (6-7)$\times 10^{-3}$ eV (say), $m^{(0)}_{12} \approx$ (3-4) $\times 10^{-3}$ eV, and $m^{(0)}_{11} \lesssim$ (1-2) $\times 10^{-3}$ eV, which are all very plausible, we obtain $m_{\nu_\mu} \approx$ (6-7) $\times 10^{-3}$ eV, $m_{\nu_e} \sim 1 \times 10^{-3}$ eV, so that $\Delta m^2_{12} \approx$ (3.6-5) $\times 10^{-5}$ eV^2, and $\sin^2 2\theta^{\rm osc}_{12} \approx$ 0.6-0.7. This goes well with the large angle MSW solution of the solar neutrino puzzle, which is now favored over the small angle solution by the SuperK data [47].

In summary, the intrinsic non-seesaw contribution to the Majorana masses of the LH neutrinos quite plausibly has the right magnitude for ν_e-ν_μ mixing, so as to lead to the rather large oscillation angle as mentioned above, in accord with the data. In contrast to the case of the ν_μ-ν_τ oscillation angle, however, given the smallness of the entries involving the first two families, the relatively large angle solution for $\nu_e - \nu^\mu$ oscillation may not be regarded as a firm prediction of the SO(10)/G(224)-framework presented here. It is nevertheless a very reasonable possibility.

It is worth noting that although the superheavy Majorana masses of the RH neutrinos cannot be observed directly, they can be of cosmological significance. The pattern given above and the arguments given in Section 3 and in this section suggests that $M(\nu^\tau_R) \approx$ (5-15) $\times 10^{14}$ GeV, $M(\nu^\mu_R) \approx$ (1-4) $\times 10^{12}$ GeV (for $|y| \approx 1/20$); and $M(\nu^e_R) \sim$ (1/2-10) $\times 10^9$ GeV (for $x \sim$ (1/2-10)$10^{-6} > z^2$). A mass of $\nu^e_R \sim 10^9$ GeV is of the right magnitude for producing ν^e_R following reheating and inducing lepton asymmetry in ν^e_R decay into $H^0 + \nu^i_L$, that is subsequently

[12] Such a term can be induced in the presence of, for example, a singlet S and a ten-plet (denoted by $\hat{\mathbf{10}}$), both having GUT-scale masses, and possessing renormalizable couplings of the form $a_i\mathbf{16}_i\mathbf{16}_H\widehat{\mathbf{10}}$, $b\widehat{\mathbf{10}}\,\mathbf{10}_H S$, $M_S SS$ and $\widehat{M}\widehat{\mathbf{10}}^2$. In this case, $\kappa_{12}/M^3_{\rm GUT} = a_1a_2b/(\widehat{M}^2M_S)$.

converted into baryon asymmetry by the electroweak sphalerons [16, 17].

In summary, we have proposed an economical and predictive pattern for the Dirac mass matrices, within the SO(10)/G(224)-framework, which is remarkably successful in describing the observed masses and mixings of *all* the quarks and charged leptons. It leads to five predictions for just the quark- system, all of which agree with observation to within 10%. The same pattern, supplemented with a similar structure for the Majorana mass matrix, accounts for both the nearly-maximal ν_μ-ν_τ oscillation angle and a (mass)^2-difference $\Delta m^2(\nu_\mu \nu_\tau) \sim (1/20 \text{ eV})^2$, suggested by the SuperK data. Given this degree of success, it makes good sense to study proton decay concretely within this SO(10)/G(224)-framework. The results of this study [14, 18] are presented in the next section, together with an update.

Before turning to proton decay, it is worth noting that much of our discussion of fermion masses and mixings, including those of the neutrinos, is essentially unaltered if we go to the limit $\epsilon' \to 0$ of Eq. (28). This limit clearly involves:

$$m_u = 0\,, \quad \theta_C \simeq \sqrt{m_d/m_s}\,, \quad m_{\nu_e} = 0\,, \quad \theta^\nu_{e\mu} = \theta^\nu_{e\tau} = 0$$

$$|V_{ub}| \simeq \sqrt{\frac{\eta - \epsilon}{\eta + \epsilon}}\sqrt{m_d/m_b}\,(m_s/m_b) \simeq (2.1)(0.039)(0.023) \simeq 0.0019\,. \tag{33}$$

All other predictions remain unaltered. Now, among the observed quantities in the list above, $\theta_C \simeq \sqrt{m_d/m_s}$ is a good result. Considering that $m_u/m_t \approx 10^{-5}$, $m_u = 0$ is also a pretty good result. There are of course plausible small corrections which could arise through Planck scale physics; these could induce a small value for m_u through the (1,1)-entry $\delta \approx 10^{-5}$. For considerations of proton decay, it is worth distinguishing between these two *extreme* variants which we will refer to as cases I and II respectively.

$$\text{Case I}: \quad \epsilon' \approx 2 \times 10^{-4}\,, \ \delta = 0$$

$$\text{Case II}: \quad \delta \approx 10^{-5}\,, \ \epsilon' = 0\,. \tag{34}$$

It is worth noting that the observed value of $|V_{ub}| \approx 0.003$ favors a non-zero value of ϵ' ($\approx$ (1-2) $\times 10^{-4}$). Thus, in reality, ϵ' may not be zero, but it may lie in between the two extreme values listed above. In this case, the predicted proton lifetime for the standard $d = 5$ operators would be intermediate between those for the two cases, presented in Section 6.

6 Expectations for Proton Decay in Supersymmetric Unified Theories

6.1 Preliminaries

Turning to the main purpose of this talk, I present now the reason why the unification framework based on SUSY SO(10) or G(224), together with the understanding of fermion masses and mixings discussed above, strongly suggest that proton decay should be imminent.

Recall that supersymmetric unified theories (GUTs) introduce two new features to proton decay: (i) First, by raising M_X to a higher value of about 2×10^{16} GeV (contrast with the non-supersymmetric case of nearly 3×10^{14} GeV), they strongly suppress the gauge-boson-mediated $d = 6$ proton decay operators, for which $e^+\pi^0$ would have been the dominant mode (for this

case, one typically obtains: $\Gamma^{-1}(p \to e^+\pi^0)|_{d=6} \approx 10^{35\pm1}$ years). (ii) Second, they generate $d = 5$ proton decay operators [19] of the form $Q_iQ_jQ_kQ_l/M$ in the superpotential, through the exchange of color triplet Higginos, which are the GUT partners of the standard Higgs(ino) doublets, such as those in the $\mathbf{5}+\overline{\mathbf{5}}$ of SU(5) or the 10 of SO(10). Assuming that a suitable doublet-triplet splitting mechanism provides heavy GUT-scale masses to these color triplets and at the same time light masses to the doublets (see e.g, the Appendix), these "standard" $d = 5$ operators, suppressed by just one power of the heavy mass and the small Yukawa couplings, are found to provide the dominant mechanism for proton decay in supersymmetric GUT [48, 49, 50, 51, 52].

Now, owing to (a) Bose symmetry of the superfields in $QQQL/M$, (b) color antisymmetry, and especially (c) the hierarchical Yukawa couplings of the Higgs doublets, it turns out that these standard $d = 5$ operators lead to dominant $\overline{\nu}K^+$ and comparable $\overline{\nu}\pi^+$ modes, but in all cases to highly suppressed $e^+\pi^0$, e^+K^0 and even μ^+K^0 modes. For instance, for minimal SUSY SU(5), one obtains (with $\tan\beta \leq 20$, say):

$$[\,\Gamma(\mu^+K^0)/\Gamma(\overline{\nu}K^+)\,]^{SU(5)}_{std} \sim [m_u/(m_c\,\sin^2\theta_c)]^2\,R \approx 10^{-3}\,, \tag{35}$$

where $R \approx 0.1$ is the ratio of the relevant $|$matrix element$|^2\times$(phase space), for the two modes.

It was recently pointed out that in SUSY unified theories based on SO(10) or G(224), which assign heavy Majorana masses to the RH neutrinos, there exists a new set of color triplets and thereby very likely a *new source* of $d = 5$ proton decay operators [20]. For instance, in the context of the minimal set of Higgs multiplets [13] $\{\mathbf{45}_H, \mathbf{16}_H, \overline{\mathbf{16}}_H$ and $\mathbf{10}_H\}$ (see Section 5), these new $d = 5$ operators arise by combining three effective couplings introduced before:—i.e., (a) the couplings $f_{ij}\mathbf{16}_i\mathbf{16}_j\overline{\mathbf{16}}_H\overline{\mathbf{16}}_H/M$ [see Eq. (7)] that are required to assign Majorana masses to the RH neutrinos, (b) the couplings $g_{ij}\mathbf{16}_i\mathbf{16}_j\mathbf{16}_H\mathbf{16}_H/M$, which are needed to generate non-trivial CKM mixings [see Eq. (15)], and (c) the mass term $M_{16}\mathbf{16}_H\overline{\mathbf{16}}_H$. For the f_{ij} couplings, there are two possible SO(10)-contractions (leading to a 45 or a 1) for the pair $\mathbf{16}_i\overline{\mathbf{16}}_H$, both of which contribute to the Majorana masses of the RH neutrinos, but only the non-singlet contraction (leading to $\mathbf{45}$), would contribute to $d = 5$ proton decay operator. In the presence of non-perturbative quantum gravity, one would in general expect the two contractions to have comparable strength. Furthermore, the couplings of $\mathbf{45}'$**s** lying in the string-tower or possibly below the string-scale, and likewise of singlets, to the $\mathbf{16}_i\cdot\overline{\mathbf{16}}_H$-pair, would respectively generate the two contractions. It thus seems most likely that both contractions would be present, having comparable strength. Allowing for a difference between the relevant projection factors for ν_R masses versus proton decay, and also for the fact that both contractions contribute to the former, but only the non-singlet one (i.e. 45) to the latter, we would set the relevant f_{ij} coupling for proton decay to be $(f_{ij})_p \equiv (f_{ij})_\nu \cdot K$, where $(f_{ij})_\nu$ defined in Section 4 directly yields ν_R - masses [see Eq. (8)]; and K is a relative factor, which generically is expected to be of order unity. [14] As a plausible range, we will take $K \approx 1/5$ to 2 (say). In the presence of the non-singlet contraction, the color-triplet Higginos in $\overline{\mathbf{16}}_H$ and $\mathbf{16}_H$ of mass M_{16} can be exchanged between $\tilde{q}_iq_j$ and $\tilde{q}_kq_l$-pairs (correspondingly, for G(224), the color triplets would arise from $(1,2,4)_H$ and $(1,2,\overline{4})_H$). This exchange generates a new set of $d = 5$ operators in the superpotential of the form

$$W_{\rm new} \propto (f_{ij})_\nu\, g_{kl} K\,(\mathbf{16}_i\,\mathbf{16}_j)\,(\mathbf{16}_k\,\mathbf{16}_l)\,\langle\overline{\mathbf{16}}_H\rangle\,\langle\mathbf{16}_H\rangle/M^2 \times (1/M_{16}), \tag{36}$$

[13] The origin of the new $d = 5$ operators in the context of other Higgs multiplets, in particular in the cases where $\mathbf{126}_H$ and $\overline{\mathbf{126}}_H$ are used to break B–L, has been discussed in Ref. [20].

[14] For the special case of $K = 0$ (which would arise if only the singlet-contraction of $(\mathbf{16}_i\cdot\overline{\mathbf{16}}_H)$ contributes), the new $d = 5$ operators shown in Eq. (36) would not, of course, contribute to proton decay.

which induce proton decay. Note that these operators depend, through the couplings f_{ij} and g_{kl}, both on the Majorana and on the Dirac masses of the respective fermions. *This is why within SUSY SO(10) or G(224), if the generic case of $K \neq 0$ holds, proton decay gets intimately linked to the masses and mixings of all fermions, including neutrinos.*

6.2 Framework for Calculating Proton Decay Rate

To establish notations, consider the case of minimal SUSY SU(5) and, as an example, the process $\tilde{c}\tilde{d} \to \bar{s}\bar{\nu}_\mu$, which induces $p \to \overline{\nu}_\mu K^+$. Let the strength of the corresponding $d = 5$ operator, multiplied by the product of the CKM mixing elements entering into wino-exchange vertices, (which in this case is $\sin\theta_C \cos\theta_C$) be denoted by $\widehat{A}$. Thus (putting $\cos\theta_C = 1$), one obtains:

$$\begin{aligned} \widehat{A}_{\tilde{c}\tilde{d}}(SU(5)) &= (h^u_{22}\, h^d_{12}/M_{H_C}) \sin\theta_c \\ &\simeq (m_c m_s \sin^2\theta_C/v_u^2)\,(\tan\beta/M_{H_C}) \\ &\simeq (1.9\times 10^{-8})\,(\tan\beta/M_{H_C}) \\ &\approx (2\times 10^{-24}\,\mathrm{GeV}^{-1})\,(\tan\beta/2)\,(2\times 10^{16}\,\mathrm{GeV}/M_{H_C}), \end{aligned} \tag{37}$$

where $\tan\beta \equiv v_u/v_d$, and we have put $v_u = 174$ GeV and the fermion masses extrapolated to the unification-scale—i.e. $m_c \simeq 300$ MeV and $m_s \simeq 40$ MeV. The amplitude for the associated four-fermion process $dus \to \overline{\nu}_\mu$ is given by:

$$A_5(dus \to \overline{\nu}_\mu) = \widehat{A}_{\tilde{c}\tilde{d}} \times (2f) \tag{38}$$

where f is the loop-factor associated with wino-dressing. Assuming $m_{\tilde{w}} \ll m_{\tilde{q}} \sim m_{\tilde{l}}$, one gets: $f \simeq (m_{\tilde{w}}/m^2_{\tilde{q}})(\alpha_2/4\pi)$. Using the amplitude for $(du)(s\nu_\ell)$, as in Eq. (38), ($\ell = \mu$ or τ), and the recently obtained matrix element and renormalization effects (see below), one then obtains [50, 51, 52, 14, 18]:

$$\begin{aligned} \Gamma^{-1}(p \to \overline{\nu}_\tau K^+) &\approx (0.15\times 10^{31})\,\mathrm{years} \times (0.32/A_L)^2 \\ &\times \left(\frac{0.93}{A_S}\right)^2 \left[\frac{0.014\,\mathrm{GeV}^3}{\beta_H}\right]^2 \left[\frac{(1/6)}{(m_{\tilde{W}}/m_{\tilde{q}})}\right]^2 \left[\frac{m_{\tilde{q}}}{1.2\,\mathrm{TeV}}\right]^2 \left[\frac{2\times 10^{-24}\,\mathrm{GeV}^{-1}}{\widehat{A}(\overline{\nu})}\right]^2 . \end{aligned} \tag{39}$$

Here β_H denotes the hadronic matrix element defined by $\beta_H u_L(\vec{k}) \equiv \epsilon_{\alpha\beta\gamma} \left\langle 0|(d^\alpha_L u^\beta_L) u^\gamma_L|p,\vec{k}\right\rangle$. While the range $\beta_H = (0.003\text{-}0.03)$ GeV^3 has been used in the past [51], given that one lattice calculation yields $\beta_H = (5.6 \pm 0.5)\times 10^{-3}$ GeV^3 [53], and a recent improved calculation yields $\beta_H \approx 0.014$ GeV^3 [54] (whose systematic errors that may arise from scaling violations and quenching are hard to estimate [54]), we will take as a conservative, but plausible, range for β_H to be given by (0.014 GeV^3)$(1/2 - 2)$. (Compare this with the range for $\beta_H = (0.006$ GeV$^3)(1/2 - 2)$ as used in Ref. [14]). A_S denotes the short-distance renormalization effect for the $d = 5$ operator which arises owing to extrapolation between the GUT and the SUSY-breaking scales [49, 51, 55]. The average value of $A_S = 0.67$, given in Ref. [51] for $m_t = 100$ GeV, has been used in most early estimates. For $m_t = 175$ GeV, one would, however, have $A_S \approx 0.93$ to 1.2 [55]. Conservatively, I would use $A_S = 0.93$; this would enhance the rate by a factor of two compared with previous estimates. A_L denotes the long-distance renormalization effect of the $d = 6$ operator due to QCD interaction that arises due to extrapolation between the SUSY breaking scale and 1 GeV [49]. Using the two-loop expression for A_L [56], together with the two-loop value for α_3, Babu and I find: $A_L \approx 0.32$, in

contrast to $A_L \approx 0.22$, used in previous works[15]. In what follows, I would use $A_L \approx 0.32$. This by itself would also increase the rate by a factor of $(0.32/0.22)^2 \approx 2$, compared to the previous estimates [49, 50, 51, 52, 14, 18]. Including the enhancements in both A_S and A_L, we thus see that the net increase in the proton decay rate solely due to new evaluation of renormalization effects is nearly a factor of four, compared to the previous estimates (including that in Ref. [14]).

Note that the familiar factors that appear in the expression for proton lifetime—i.e., M_{H_C}, $(1 + y_{tc})$ representing the interference between the $\tilde{t}$ and $\tilde{c}$ contributions, and $\tan\beta$ (see e.g. Ref. [51] and discussion in the Appendix of Ref. [14])—are all effectively contained in $\widehat{A}(\overline{\nu})$. In Ref. [14], guided by the demand of naturalness (i.e. absence of excessive fine tuning) in obtaining the Higgs boson mass, squark masses were assumed to lie in the range of 1 TeV$(1/\sqrt{2} - \sqrt{2})$, so that $m_{\tilde{q}} \lesssim 1.4$TeV. Recent work, based on the notion of focus point supersymmetry however suggests that squarks may be considerably heavier without conflicting with the demands of naturalness [58]. In the interest of obtaining a conservative upper limit on proton lifetime, we will therefore allow squark masses to be as heavy as about 2.4 TeV and as light as perhaps 600 GeV. [16]

Allowing for plausible and rather generous uncertainties in the matrix element and the spectrum we take:

$$\beta_H \;=\; (0.014\,\mathrm{GeV}^3)\,(1/2\text{-}2)$$

$$m_{\tilde{w}}/m_{\tilde{q}} \;=\; 1/6\,(1/2\text{-}2)\,, \quad \text{and} \quad m_{\tilde{q}} \;\approx\; m_{\tilde{\ell}} \;\approx\; 1.2\,\mathrm{TeV}\,(1/2\text{-}2)\,. \tag{40}$$

Using Eqs. (39–40), we get:

$$\Gamma^{-1}(p \to \overline{\nu}_\tau K^+) \approx (0.15 \times 10^{31}\,\text{years})\,[\,2 \times 10^{-24}\,\mathrm{GeV}^{-1}/\widehat{A}(\overline{\nu}_\ell)\,]^2 \times \{64\text{-}1/64\}\,. \tag{41}$$

Note that the curly bracket would acquire its upper-end value of 64, which would serve towards maximizing proton lifetime, only provided all the uncertainties in Eq. (41) are stretched to the extreme so that $\beta_H = 0.007$ GeV^3, $m_{\tilde{W}}/m_{\tilde{q}} \approx 1/12$ and $m_{\tilde{q}} \approx 2.4$ TeV. This relation, as well as Eq. (39) are general, depending only on $\widehat{A}(\overline{\nu}_\ell)$ and on the range of parameters given in Eq. (40). They can thus be used for both SU(5) and SO(10).

[15] In most previous works starting with Ref. [49] through [52], as well as in Refs. [14] and [18], the one-loop value of A_L was taken to be 0.22. It was, however, noted in Refs. [56] and [57] that there is a numerical error in the evaluation of the one-loop expression for A_L [49], and that the correct value for $A_L(\text{one}-\text{loop}) \approx 0.43$ (this remained unnoticed by most authors). The two-loop value for A_L (as stated above) is nearly 0.32, which is lower than 0.43 but higher that the previously used value of 0.22.

[16] We remark that if the recently reported (g-2)-anomaly for the muon [59], together with reevaluation of the contribution from light by light-scattering [60], is attributed to supersymmetry [61], one would need to have extremely light s-fermions [i.e. $m_{\tilde{l}} \approx 200$ - 400 GeV (say) and correspondingly, for promising mechanisms of SUSY-breaking, $m_{\tilde{q}} \lesssim 300 - 600$ GeV (say)], and simultaneously relatively large $\tan\beta(\approx$ 6-24). However, not worrying about grand unification, such light s-fermions, together with large or very large $\tan\beta$ would typically be in gross conflict with the limits on the edm's of the neutron and the electron, unless one can explain naturally the occurrence of minuscule phases ($\lesssim 1/200$ to $1/500$) and/or large cancellation. Thus, if the $(g-2)_\mu$-anomaly turns out to be real, it may well find a non-supersymmetric explanation, in accord with the edm-constraints which ordinarily seem to suggest that squarks are (at least) moderately heavy ($m_{\tilde{q}} \gtrsim 0.6 - 1$ TeV, say), and $\tan\beta$ is not too large ($\lesssim 3$ to 10, say). We mention in passing that the extra vector—like matter—specially a $16 + \overline{16}$ of SO(10)—as proposed in the so-called extended supersymmetric standard model (ESSM) [21, 62], with the heavy lepton mass being of order 200 GeV, can provide such an explanation [63]. Motivations for the case of ESSM, based on the need for (a) removing the mismatch between MSSM and string unification scales, and (b) dilaton-stabilization, have been noted in Ref. [21]. Since ESSM is an interesting and viable variant of MSSM, and would have important implications for proton decay, we will present the results for expected proton decay rates for the cases of both MSSM and ESSM in the discussion to follow.

The experimental lower limit on the inverse rate for the $\bar{\nu}K^+$ modes is given by Ref. [64],

$$\left[\sum_{\ell} \Gamma(p \to \overline{\nu}_\ell K^+)\right]^{-1}_{\rm expt} \geq 1.9 \times 10^{33}\,{\rm years}\,. \tag{42}$$

Allowing for all the uncertainties to stretch in the same direction (in this case, the curly bracket = 64), and assuming that just one neutrino flavor (e.g. ν_μ for SU(5)) dominates, the observed limit (Eq. (42)) provides an upper bound on the amplitude[17]:

$$\widehat{A}(\overline{\nu}_\ell) \;\leq\; 0.46 \times 10^{-24}\,{\rm GeV}^{-1} \tag{43}$$

which holds for both SU(5) and SO(10). Recent theoretical analyses based on LEP-limit on Higgs mass ($\gtrsim$ 114 GeV), together with certain assumptions about MSSM parameters (as in CMSSM) and/or constraint from muon g-2 anomaly [59] suggest that $\tan\beta \gtrsim 3$ to 5 [65]. In the interest of getting a conservative upper limit on proton lifetime, we will therefore use, as a conservative lower limit, $\tan\beta \geq 3$. We will however exhibit relevant results often as a function of $\tan\beta$ and exhibit proton lifetimes corresponding to higher values of $\tan\beta$ as well. For minimal SU(5), using Eqs. (37) and (43) and, conservatively $\tan\beta \geq 3$, one obtains a lower limit on M_{HC} given by:

$$M_{HC} \;\geq\; 13 \times 10^{16}\,{\rm GeV}\;\;({\rm SUSY\ SU(5)})\;. \tag{44}$$

At the same time, gauge coupling unification in SUSY SU(5) strongly suggests $M_{HC} \leq (1/2\text{-}1) \times 10^{16}$ GeV. (See Ref. [66] where an even more stringent upper bound on M_{HC} is suggested.) Thus we already see a conflict, in the case of minimal SUSY SU(5), between the experimental limit on proton lifetime on the one hand, and coupling unification and constraint on $\tan\beta$ on the other hand. To see this conflict another way, if we keep $M_{HC} \leq 10^{16}$ GeV (for the sake of coupling unification) we obtain from Eq. (37): $\widehat{A}({\rm SU(5)}) \geq 5.7 \times 10^{-24}\,{\rm GeV}^{-1}(\tan\beta/3)$. Using Eq. (41), this in turn implies that

$$\Gamma^{-1}(p \to \overline{\nu}K^+) \;\leq\; 1.2 \times 10^{31}\,{\rm years} \times (3/\tan\beta)^2 \quad ({\rm SUSY\ SU(5)})\;. \tag{45}$$

For $\tan\beta \geq 3$, a lifetime of 1.2×10^{31} years is thus a most conservative upper limit. In practice, it is unlikely that all the uncertainties, including these in M_{HC} and $\tan\beta$, would stretch in the same direction to nearly extreme values so as to prolong proton lifetime. Given the experimental lower limit [Eq. (42)], we see that minimal SUSY SU(5) is already excluded by a large margin by proton decay-searches. This is in full accord with the conclusion reached by other authors (see especially Ref. [66]). We have of course noted in Section 4 that SUSY SU(5) does not go well with neutrino oscillations observed at SuperK.

Now, to discuss proton decay in the context of supersymmetric SO(10), it is necessary to discuss first the mechanism for doublet-triplet splitting. Details of this discussion may be found in Ref. [14]. A synopsis is presented in the Appendix.

6.3 Proton Decay in Supersymmetric SO(10)

The calculation of the amplitudes $\widehat{A}_{\rm std}$ and $\widehat{A}_{\rm new}$ for the standard and the new operators for the SO(10) model, are given in detail in Ref. [14]. Here, I will present only the results. It is

[17] If there are sub-dominant $\overline{\nu}_i K^+$ modes with branching ratio R, the right side of Eq. (43) should be divided by $\sqrt{1+R}$.

found that the four amplitudes $\widehat{A}_{\rm std}(\overline{\nu}_\tau K^+)$, $\widehat{A}_{\rm std}(\overline{\nu}_\mu K^+)$, $\widehat{A}_{\rm new}(\overline{\nu}_\tau K^+)$ and $\widehat{A}_{\rm new}(\overline{\nu}_\mu K^+)$ are in fact very comparable to each other, within about a factor of two to five, either way. Since there is no reason to expect a near cancellation between the standard and the new operators, especially for both $\overline{\nu}_\tau K^+$ and $\overline{\nu}_\mu K^+$ modes, we expect the net amplitude (standard + new) to be in the range exhibited by either one. Following Ref. [14], I therefore present the contributions from the standard and the new operators separately.

One important consequence of the doublet-triplet splitting mechanism for SO(10) outlined briefly in the appendix and in more detail in Ref. [14] is that the standard $d = 5$ proton decay operators become inversely proportional to $M_{\rm eff} \equiv [\lambda\langle \mathbf{45}_H\rangle]^2/\ M_{10'} \sim M_X^2/M_{10'}$, rather than to M_{H_C}. Here, $M_{10'}$ represents the mass of $\mathbf{10}'_H$, that enters into the D-T splitting mechanism through effective coupling $\lambda \mathbf{10}_H \mathbf{45}_H \mathbf{10}'_H$ in the superpotential [see Appendix, Eq. (A1)]. As noted in Ref. [14], $M_{10'}$ can be naturally suppressed (due to flavor symmetries) compared to M_X, and thus $M_{\rm eff}$ correspondingly larger than M_X by even one to three orders of magnitude. It should be stressed that $M_{\rm eff}$ does not represent the physical masses of the color triplets or of the other particles in the theory. It is simply a parameter of order $M_X^2/M_{10'}$. *Thus values of $M_{\rm eff}$, close to or even exceeding the Planck scale, do not in any way imply large corrections from quantum gravity.* Now accompanying the suppression due to $M_{\rm eff}$, the standard proton decay amplitudes for SO(10) possess an intrinsic enhancement as well, compared to those for SU(5), owing primarily due to differences in their Yukawa couplings for the up sector (see Appendix C of Ref. [14]). As a result of this enhancement, combined with the suppression due to higher values of $M_{\rm eff}$, a typical standard $d = 5$ amplitude for SO(10) is given by (see Appendix C of Ref. [14])

$$\widehat{A}(\bar{\nu}_\mu K^+)^{SO(10)}_{std} \approx (h_{33}^2/M_{\rm eff})(2\times 10^{-5}),$$

which should be compared with $\widehat{A}(\bar{\nu}_\mu K^+)^{SU(5)}_{std} \approx (1.9\times 10^{-8})(\tan\beta/M_{H_C})$ [see Eq. (37)]. Note, taking $h_{33}^2 \approx 1/4$, the ratio of a typical SO(10) over SU(5) amplitude is given by $(M_{H_c}/M_{\rm eff})(88)(3/\tan\beta)$. Thus the enhancement by a factor of about 88 (for $\tan\beta = 3$), of the SO(10) compared to the SU(5) amplitude, is compensated in part by the suppression that arises from $M_{\rm eff}$ being larger than M_{H_c}.

In addition, note that in contrast to the case of SU(5), the SO(10) amplitude does not depend *explicitly* on $\tan\beta$. The reason is this: if the fermions acquire masses only through the $\mathbf{10}_H$ in SO(10), as is well known, the up and down quark Yukawa couplings will be equal. By itself, it would lead to a large value of $\tan\beta = m_t/m_b \approx 60$ and thereby to a large enhancement in proton decay amplitude. Furthermore, it would also lead to the bad relations: $m_c/m_s = m_t/m_b$ and $V_{CKM} = 1$. However, in the presence of additional Higgs multiplets, in particular with the mixing of $(\mathbf{16}_H)_d$ with $\mathbf{10}_H$ (see Appendix and Section 5), (a) $\tan\beta$ can get lowered to values like 3-20, (b) fermion masses get contributions from both $\langle \mathbf{16}_H\rangle_d$ and $\langle \mathbf{10}_H\rangle$, which correct all the bad relations stated above, and simultaneously (c) the explicit dependence of $\widehat{A}$ on $\tan\beta$ disappears. It reappears, however, through restriction on threshold corrections, discussed below.

Although $M_{\rm eff}$ can far exceed M_X, it still gets bounded from above by demanding that coupling unification, as observed [18], should emerge as a natural prediction of the theory as opposed to

[18] For instance, in the absence of GUT-scale threshold corrections, the MSSM value of $\alpha_3(m_Z)_{MSSM}$, assuming coupling unification, is given by $\alpha_3(m_Z)^\circ_{MSSM} = 0.125 \pm 0.13$ [7], which is about 5-8% higher than the observed value: $\alpha_3(m_Z)^\circ_{MSSM} = 0.118 \pm 0.003$ [13]. We demand that this discrepancy should be accounted for accurately by a net *negative* contribution from D-T splitting and from "other" threshold corrections [see Appendix, Eq. (A4)], without involving large cancellations. That in fact does happen for the minimal Higgs system $(45, 16, \overline{16})$ (see Ref. [14]).

being fortuitous. That in turn requires that there be no large (unpredicted) cancellation between GUT-scale threshold corrections to the gauge couplings that arise from splittings within different multiplets as well as from Planck scale physics. Following this point of view, we have argued (see Appendix) that the net "other" threshold corrections to $\alpha_3(m_Z)$ arising from the Higgs (in our case $\mathbf{45}_H$, $\mathbf{16}_H$ and $\overline{\mathbf{16}}_H$) and the gauge multiplets should be negative, but conservatively and quite plausibly no more than about 10%, at the electroweak scale. This in turn restricts how big can be the threshold corrections to $\alpha_3(m_Z)$ that arise from (D-T) splitting (which is positive). Since the latter is proportional to $\ln(M_{\rm eff}\cos\gamma/M_X)$ (see Appendix), we thus obtain an upper limit on $M_{\rm eff}\cos\gamma$. For the simplest model of D-T splitting presented in Ref. [14] and in the Appendix [Eq. (A1)], one obtains: $\cos\gamma \approx (\tan\beta)/(m_t/m_b)$. An upper limit on $M_{\rm eff}\cos\gamma$ thus provides an upper limit on $M_{\rm eff}$ which is inversely proportional to $\tan\beta$. In short, our demand of natural coupling unification, together with the simplest model of D-T splitting, introduces an implicit dependence on $\tan\beta$ into the lower limit of the SO(10)-amplitude—i.e. $\widehat{A}(SO(10)) \propto 1/M_{\rm eff} \geq$ [(a quantity) $\propto \tan\beta$]. These considerations are reflected in the results given below.

Assuming $\tan\beta \geq 3$ and accurate coupling unification (as described above), one obtains for the case of MSSM, a conservative upper limit on $M_{\rm eff} \leq 2.7\times 10^{18}$ GeV $(3/\tan\beta)$ (see Appendix and Ref. [14]). Using this upper limit, we obtain a lower limit for the standard proton decay amplitude given by

$$\widehat{A}(\overline{\nu}_\tau K^+)_{std} \geq \left[\begin{array}{cc} (7.8\times 10^{-24}\,{\rm GeV}^{-1})\,(1/6\text{-}1/4) & \text{case I} \\ (3.3\times 10^{-24}\,{\rm GeV}^{-1})\,(1/6\text{-}1/2) & \text{case II} \end{array}\right] \left(\begin{array}{c} \text{SO(10)/MSSM, with} \\ \tan\beta \geq 3 \end{array}\right). \quad (46)$$

Substituting into Eq. (41) and adding the contribution from the second competing mode $\overline{\nu}_\mu K^+$, with a typical branching ratio $R \approx 0.3$, we obtain

$$\Gamma^{-1}(\overline{\nu}K^+)_{std} \leq \left[\begin{array}{c} (0.18\times 10^{31}\,\text{years})\,(1.6\text{-}0.7) \\ (0.4\times 10^{31}\,\text{years})\,(4\text{-}0.44) \end{array}\right] \{64\text{-}1/64\} \left(\begin{array}{c} \text{SO(10)/MSSM, with} \\ \tan\beta \geq 3 \end{array}\right). \quad (47)$$

The upper and lower entries in Eqs. (46) and (47) correspond to the cases I and II of the fermion mass-matrix with the *extreme values* of ϵ'—i.e. $\epsilon' = 2\times 10^{-4}$ and $\epsilon' = 0$—respectively, (see Eq. (34)). The uncertainty shown inside the square brackets correspond to that in the relative phases of the different contributions. The uncertainty of {64 to 1/64} arises from that in β_H, $(m_{\tilde{W}}/m_{\tilde{q}})$ and $m_{\tilde{q}}$ [see Eq. (40)]. Thus we find that for MSSM embedded in SO(10), for the two extreme values of ϵ' (cases I and II) as mentioned above, the inverse partial proton decay rate should satisfy:

$$\begin{aligned} \Gamma^{-1}(p \to \overline{\nu}K^+)_{std} &\leq \left[\begin{array}{c} 0.20\times 10^{31^{+2.0}_{-1.7}}\,\text{years} \\ 0.32\times 10^{31^{+2.4}_{-1.86}}\,years \end{array}\right] \\ &\leq \left[\begin{array}{c} 0.2\times 10^{33}\,\text{years} \\ 1\times 10^{33}\,\text{years} \end{array}\right] \left(\begin{array}{c} \text{SO(10)/MSSM, with} \\ \tan\beta \geq 3 \end{array}\right). \end{aligned} \quad (48)$$

The central value of the upper limit in Eq. (48) corresponds to taking the upper limit on $M_{\rm eff} \leq 2.7\times 10^{18}$ GeV, which is obtained by restricting threshold corrections as described above (and in the Appendix) and by setting (conservatively) $\tan\beta \geq 3$. The uncertainties of matrix element, spectrum and choice of phases are reflected in the exponents. The uncertainty in the most sensitive entry of the fermion mass matrix—i.e. ϵ'—is incorporated (as regards obtaining an upper limit on the lifetime) by going from case I (with $\epsilon' = 2\times 10^{-4}$) to case II ($\epsilon' = 0$). Note that this increases the lifetime by almost a factor of six. Any non-vanishing intermediate value of ϵ' would

only shorten the lifetime compared to case II. In this sense, the larger of the two upper limits quoted above is rather conservative. We see that the predicted upper limit for case I of MSSM (with the extreme value of $\epsilon' = 2 \times 10^{-4}$) is lower than the empirical lower limit [Eq. (43)] by a factor of ten, while that for case II, i.e. $\epsilon' = 0$ (with all the uncertainties stretched as mentioned above) is about two times lower than the empirical lower limit.

Thus the case of MSSM embedded in SO(10) is already tightly constrained, to the point of being disfavored, by the limit on proton lifetime. The constraint is of course augmented especially by *our requirement of natural coupling unification* which prohibits accidental large cancellation between different threshold corrections[19] (see Appendix); and it will be even more severe, especially within the simplest mechanism of D-T splitting (as discussed in the Appendix), if $\tan\beta$ turns out to be larger than 5 (say). On the positive side, improvement in the current limit by a factor of even 2 to 3 ought to reveal proton decay, otherwise the case of MSSM embedded in SO(10), would be clearly excluded.

6.4 The case of ESSM

Before discussing the contribution of the new $d = 5$ operators to proton decay, an interesting possibility, mentioned in the introduction (and in footnote 16), that would be especially relevant in the context of proton decay, if $\tan\beta$ is large, is worth noting. This is the case of the extended supersymmetric standard model (ESSM), which introduces an extra pair of vector-like families [$\mathbf{16} + \overline{\mathbf{16}}$ of SO(10)], at the TeV scale [21, 62]. Adding such complete SO(10)-multiplets would of course preserve coupling unification. From the point of view of adding extra families, ESSM seems to be the minimal and also the maximal extension of the MSSM, that is allowed in that it is compatible with (a) LEP neutrino-counting, (b) precision electroweak tests, as well as (c) a semi-perturbative as opposed to non-perturbative gauge coupling unification [21, 62]. [20] *The existence of two extra vector-like families of quarks and leptons can of course be tested at the LHC.*

Theoretical motivations for the case of ESSM arise on several grounds: (a) it provides a better chance for stabilizing the dilaton by having a semi-perturbative value for $\alpha_{\rm unif} \approx 0.35$-$0.3$ [21], in contrast to a very weak value of 0.04 for MSSM; (b) owing to increased two-loop effects [21, 68], it raises the unification scale M_X to $(1/2 - 2) \times 10^{17}$ GeV and thereby considerably reduces the problem of a mismatch [28] between the MSSM and the string unification scales (see Section 3); (c) It lowers the GUT-prediction for $\alpha_3(m_Z)$ to (0.112–0.118) (in absence of unification-scale threshold corrections), which is in better agreement with the data than the corresponding value of (0.125–0.13) for MSSM; and (d) it provides a simple reason for inter-family mass-hierarchy [21, 62]. In this sense, ESSM, though less economical than MSSM, offers some distinct advantages.

In the present context, because of (b) and (c), ESSM naturally enhances the GUT-prediction for proton lifetime, in full accord with the data [64]. As explained in the appendix, the net result of these two effects—i.e. a raising of M_X and a lowering of $\alpha_3(m_Z)^{\circ}_{\rm ESSM}$—is that for ESSM embedded in SO(10), $\tan\beta$ can span a wide range from 3 to even 30, and simultaneously the value or the upper limit on $M_{\rm eff}$ can range from (60 to 6) $\times 10^{18}$ GeV, in full accord with our criterion for accurate coupling unification discussed above.

[19] Other authors (see e.g., Ref. [67]) have considered proton decay in SUSY SO(10) by allowing for rather large GUT-scale threshold corrections, which do not, however, go well with our requirement of "natural coupling unification".

[20] For instance, addition of *two* pairs of vector-like families at the TeV-scale, to the three chiral families, would cause gauge couplings to become non-perturbative below the unification scale.

As a result, in contrast to MSSM, ESSM allows for larger values of $\tan\beta$ (like 10 or 20), without needing large threshold corrections, and simultaneously without conflicting with the limit on proton lifetime.

To be specific, consider first the case of a moderately large $\tan\beta = 10$ (say), for which one obtains $M_{\rm eff} \approx 1.8 \times 10^{19}$ GeV, with the "other" threshold correction $-\delta_3'$ being about 5% (see Appendix for definition). In this case, one obtains:

$$\Gamma^{-1}(\overline{\nu}K^+)_{\rm std} \approx \left[\begin{array}{c}(1.6-0.7)\\(10-1)\end{array}\right]\{64-1/64\}\,(7\times 10^{31}\,\text{years})\left(\begin{array}{c}\text{SO(10)/ESSM, with}\\ \tan\beta = 10\end{array}\right). \qquad (49)$$

As before, the upper and lower entries correspond to cases I ($\epsilon' = 2\times 10^{-4}$) and II ($\epsilon' = 0$) of the fermion mass-matrix [see Eq. (34)]. The uncertainty in the upper and lower entries in the square bracket of Eq. (49) corresponds to that in the relative phases of the different contributions for the cases I and II respectively, while the factor {64-1/64} corresponds to uncertainties in the SUSY spectrum and the matrix element (see Eq. (40)).

We see that by allowing for an uncertainty of a factor of $(30-100)$ jointly from the two brackets proton lifetime arising from the standard operators would be expected to lie in the range of $(2.1-7)\times 10^{33}$ years, for the case of ESSM embedded in SO(10), even for a moderately large $\tan\beta = 10$. Such a range is compatible with present limits, but accessible to searches in the near future.

The other most important feature of ESSM is that, by allowing for larger values of $M_{\rm eff}$, especially for smaller values of $\tan\beta \approx 3$ to 5 (say), *the contribution of the standard operators by itself can be perfectly consistent with present limit on proton lifetime even for almost central or "median" values of the parameters pertaining to the SUSY spectrum, the relevant matrix element, ϵ' and the phase-dependent factor.*

For instance, for ESSM, one obtains $M_{\rm eff} \approx (4.5\times 10^{19}{\rm GeV})(4/\tan\beta)$, with the "other" threshold correction - δ_3' being about 5% [see Appendix and Eq. (A6)]. Now, *combining* cases I ($\epsilon' = 2\times 10^{-4}$) and II ($\epsilon' = 0$), we see that the square bracket in Eq. (49) which we will denote by [S], varies from 0.7 to 10, depending upon the relative phases of the different contributions and the values of ϵ'. Thus as a "median" value, we will take $[S]_{\rm med} \approx 2$ to 6. The curly bracket {64-1/64}, to be denoted by {C}, represents the uncertainty in the SUSY spectrum and the matrix element [see Eq. (40)]. Again as a "nearly central" or "median" value, we will take $\{C\}_{\rm med} \approx 1/6$ to 6. Setting $M_{\rm eff}$ as above we obtain

$$\Gamma^{-1}(\bar{\nu}K^+)_{\rm std}^{\text{"median"}} \approx [S]_{\rm med}\{C\}_{\rm med}(0.45\times 10^{33}{\rm years})(4/\tan\beta)^2({\rm SO(10)/ESSM}). \qquad (50)$$

Choosing a few sample values of the effective parameters [S] and {C}, with low values of $\tan\beta = 3$ to 5, the corresponding values of $\Gamma^{-1}(\bar{\nu}K^+)$, following from Eq. (50), are listed below in Table 1.

Note that ignoring contributions from the new $d = 5$ operators for a moment[21], the entries in Table 1 represent *a very plausible range of values* for the proton lifetime, for the case of ESSM embedded in SO(10), with $\tan\beta \approx 3$ to 5 (say), *rather than upper limits for the same.* This is because they are obtained for "nearly central" or "median" values of the parameters represented by the values of [S] and {C}, as discussed above. For instance, consider the cases {C}=1 and {C}=1/2 respectively, which (as may be inferred from the table) can quite plausibly yield proton

[21] As I will discuss in the next section, we of course expect the new $d = 5$ operators to be important and significantly influence proton lifetime (see e.g. Table 2). Entries in Table 1 could still represent the actual expected values of proton lifetimes, however, if the parameter K defined in 6.1 (also see 6.5) happens to be unexpectedly small ($\ll 1$).

lifetimes in the range of (2 to 5)$\times 10^{33}$ years Now {C}=1 corresponds, e.g., to $\beta_H = 0.014$ GeV^3 (the central value of Ref. [54]) $m_{\tilde{q}} = 1.2$ TeV and $m_{\tilde{W}}/m_{\tilde{q}} = 1/6$ [see Eq. (40)], while that of {C}=1/2 would correspond, for example, to $\beta_H = 0.014$ GeV^3, with $m_{\tilde{q}} \approx 710$ GeV and $m_{\tilde{W}}/m_{\tilde{q}} \approx 1/6$. *In short, for the case of ESSM, with low values of* $\tan\beta \approx$ *3 to 5 (say), squark masses can be well below 1 TeV, without conflicting with present limit on proton lifetime.* This feature is not permissible within MSSM embedded in SO(10).

Table 1: Proton lifetime, based on contributions from only the standard operators for the case of ESSM embedded in SO(10), with parameters being in the "median" range.

$\tan\beta = 3$ [S]=3 {C}=1/2 to 4	$\tan\beta = 3$ [S]=6 {C}=1/2 to 1	$\tan\beta = 5$ [S]=5.4 {C}=1 to 6	$\tan\beta = 5$ [S]=6 {C}=1 to 4
$\Gamma^{-1}(\bar{\nu}K^+)^{std}_{ESSM} \approx$ (1.2 to 10) $\times 10^{33}$ yrs	$\Gamma^{-1}(\bar{\nu}K^+)^{std}_{ESSM} \approx$ (2.5 to 5) $\times 10^{33}$ yrs	$\Gamma^{-1}(\bar{\nu}K^+)^{std}_{ESSM} \approx$ (1.6 to 10) $\times 10^{33}$ yrs	$\Gamma^{-1}(\bar{\nu}K^+)^{std}_{ESSM} \approx$ (1.8 to 7.3) $\times 10^{33}$ yrs

Thus, confining for a moment to the standard operators only, if ESSM represents low-energy physics, and if $\tan\beta$ is rather small (3 to 5, say), we do not have to stretch the uncertainties in the SUSY spectrum and the matrix elements to their extreme values (in contrast to the case of MSSM) in order to understand why proton decay has not been seen as yet, and still can be optimistic that it ought to be discovered in the near future, with a lifetime $\leq 10^{34}$ years. The results for a wider variation of the parameters are listed in Table 2, where contributions of the new $d = 5$ operators are also shown.

It should also be remarked that if in the unlikely event, all the parameters (i.e. β_H, $(m_{\tilde{W}}/m_{\tilde{q}})$, $m_{\tilde{q}}$ and the phase-dependent factor) happen to be closer to their extreme values so as to extend proton lifetime, and if $\tan\beta$ is small ($\approx$ 3 to 5, say) and at the same time the value of $M_{\rm eff}$ is close to its allowed upper limit (see Appendix), the standard $d = 5$ operators by themselves would tend to yield proton lifetimes exceeding even (0.8 to 2.5)$\times 10^{34}$ years for the case of ESSM, (see Eq. (49) and Table 2). In this case (with the parameters having nearly extreme values), however, as I will discuss shortly, the contribution of the new $d = 5$ operators related to neutrino masses [see Eq. (36)], are likely to dominate and quite naturally yield lifetimes bounded above in the range of $(1 - 10) \times 10^{33}$ years (see Section 6.5 and Table 2). *Thus in the presence of the new operators, the range of* $(10^{33} - 10^{34})$ *years for proton lifetime is not only very plausible but it also provides a reasonable upper limit, for the case of ESSM embedded in SO(10).*

6.5 Contribution from the new d=5 operators

As mentioned in Section 6.1, for supersymmetric G(224)/SO(10), there very likely exists a new set of $d = 5$ operators, related to neutrino masses, which can induce proton decay [see Eq. (42)]. The decay amplitude for these operators for the leading mode (which in this case is $\bar{\nu}_\mu K^+$) becomes proportional to the quantity $P \equiv \{(f_{33})_\nu \langle \overline{\mathbf{16}}_H \rangle / M\} h_{33} K/(M_{16} \tan\gamma)$, where $(f_{33})_\nu$ and h_{33} are the effective couplings defined in Eqs. (7) and (15) respectively, and M_{16} and $\tan\gamma$ are defined in the Appendix. The factor K, defined by $(f_{33})_p \equiv (f_{33})_\nu K$, is expected to be of order unity (see Section 6.1 for the origin of K). As a plausible range, we would take $K \approx 1/5$ to 2. Using $M_{16} \tan\gamma = \lambda' \langle \overline{\mathbf{16}}_H \rangle$ (see Appendix), and $h_{33} \approx 1/2$ (given by top mass), one gets:

$P \approx [(f_{33})_\nu/M](1/2\lambda')K$. Here M denotes the string or the Planck scale (see Section 4 and footnote 2); thus $M \approx (1/2-1)\times 10^{18}$ GeV; and λ' is a quartic coupling defined in the appendix. Validity of perturbative calculation suggests that λ' should not much exceed unity, while other considerations suggest that λ' should not be much less than unity either (see Ref. [14], Section 6 E). Thus, a plausible range for λ' is given by $\lambda' \approx (1/2-\sqrt{2})$. (Note it is only the upper limit on λ' that is relevant to obtaining an upper limit on proton lifetime). Finally, from consideration of ν_τ mass, we have $(f_{33})_\nu \approx 1$ (see Section 4). We thus obtain: $P \approx (5\times 10^{-19}\mathrm{GeV}^{-1})(1/\sqrt{2}$ to 4)K. Incorporating a further uncertainty by a factor of (1/2 to 2) that arises due to choice of the relative phases of the different contributions (see Ref. [14]), the effective amplitude for the new operator is given by

$$\widehat{A}(\bar{\nu}_\mu K^+)_{\rm new} \approx (1.5\times 10^{-24}\mathrm{GeV}^{-1})(1/2\sqrt{2} \text{ to } 8)K \tag{51}$$

Note that this new contribution is independent of $M_{\rm eff}$; *thus it is the same for ESSM as it is for MSSM, and it is independent of* $\tan\beta$. Furthermore, it turns out that the new contribution is also insensitive to ϵ'; thus it is nearly the same for cases I and II of the fermion mass-matrix. Comparing Eq. (51) with Eq. (46) we see that the new and the standard operators are typically quite comparable to one another. Since there is no reason to expect near cancellation between them (especially for both $\bar{\nu}_\mu K^+$ and $\bar{\nu}_\tau K^+$ modes), we expect the net amplitude (standard+new) to be in the range exhibited by either one. It is thus useful to obtain the inverse decay rate assuming as if the new operator dominates. Substituting Eq. (51) into Eq. (41) and allowing for the presence of the $\bar{\nu}_\tau K^+$ mode with an estimated branching ratio of nearly 0.4 (see Ref. [14]), one obtains

$$\Gamma^{-1}(\overline{\nu}K^+)_{\rm new} \approx (0.25\times 10^{31}\,\text{years})\,[8\text{-}1/64]\,\{64\text{-}1/64\}(K^{-2}\approx 25 \text{ to } 1/4)\,. \tag{52}$$

The square bracket represents the uncertainty reflected in Eq. (51), while the curly bracket corresponds to that in the SUSY spectrum and matrix element (Eq. (40)). Allowing for the net uncertainty factor at the upper end, arising jointly from the *three brackets* in Eq. (52) to be 1000 to 4000 (say), which can be realized for plausible range of values of the parameters (see below), the new operators related to neutrino masses, by themselves, lead to a proton decay lifetime given by:

$$\Gamma^{-1}(\overline{\nu}K^+)^{\rm upper}_{\rm new} \approx (2.5\text{-}10)\times 10^{33}\,\text{years (SO(10) or string G(224)) (Indep. of } \tan\beta)\,. \tag{53}$$

The superscript "upper" corresponds to estimated lifetimes near the upper end. For instance, taking the curly bracket in Eq. (52) to be ≈ 8 to 16 (say) [corresponding for example, to $\beta_H = 0.010$ GeV^3, $(m_{\tilde{W}}/m_{\tilde{q}}) \approx 1/12$ and $m_{\tilde{q}} \approx (1 \text{ to } 1.4)(1.2 \text{ TeV})$], instead of its extreme value of 64, and setting the square bracket in Eq. (52) to be ≈ 6, and $K^{-2} \approx 20$, which are quite plausible, we obtain: $\Gamma^{-1}(\bar{\nu}K^+)_{\rm new} \approx (2.5-5)\times 10^{33}$ years; independently of $\tan\beta$, for both MSSM and ESSM. Proton lifetime for other choices of parameters, which lead to similar conclusion, are listed in Table 2.

It should be stressed that the standard $d=5$ operators [mediated by the color-triplets in the 10_H of SO(10)] may naturally be absent for a string-derived G(224)-model (see e.g. Ref. [30] and [31]), but the new $d=5$ operators, related to the Majorana masses of the RH neutrinos and the CKM mixings, should very likely be present for such a model, as much as for SO(10). These would induce proton decay [22]. *Thus our expectations for the proton decay lifetime [as shown in Eq. (53)]*

[22] In addition, quantum gravity induced $d=5$ operators are also expected to be present at some level, depending upon the degree of suppression of these operators due to flavor symmetries (see e.g. Ref. [34]).

and the prominence of the μ^+K^0 mode (see below) hold for a string-derived G(224)-model, just as they do for SO(10). For a string - G(224) - model, however, *the new d=5 operators would be essentially the sole source of proton decay*[21].

Nearly the same situation emerges for the case of ESSM embedded in G(224) or SO(10), with low $\tan\beta(\approx 3$ to 10, say), especially if the parameters (including β_H, $m_{\tilde{W}}/m_{\tilde{q}}$, $m_{\tilde{q}}$, the phase-dependent factor as well as $M_{\rm eff}$) happen to be somewhat closer to their extreme values so as to extend proton lifetime. In this case, (that is for ESSM) as noted in the previous sub-section, the contribution of the standard $d = 5$ operators would be suppressed; and proton decay would proceed primarily via the new operators with a lifetime quite plausibly in the range of $10^{33} - 10^{34}$ years, as exhibited above.

6.6 The Charged Lepton Decay Modes ($p \to \mu^+K^0$ and $p \to e^+\pi^0$)

I now note a distinguishing feature of the SO(10) or the G(224) model presented here. Allowing for uncertainties in the way the standard and the new operators can combine with each other for the three leading modes i.e. $\overline{\nu}_\tau K^+$, $\overline{\nu}_\mu K^+$ and μ^+K^0, we obtain (see Ref. [14] for details):

$$B(\mu^+K^0)_{std+new} \approx [1\% \text{ to } 50\%]\ \kappa \ \ (\text{SO(10) or string G(224)}) \tag{54}$$

where κ denotes the ratio of the squares of relevant matrix elements for the μ^+K^0 and $\overline{\nu}K^+$ modes. In the absence of a reliable lattice calculation for the $\bar{\nu}K^+$ mode, one should remain open to the possibility of $\kappa \approx 1/2$ to 1 (say). We find that for a large range of parameters, the branching ratio $B(\mu^+K^0)$ can lie in the range of 20 to 40% (if $\kappa \approx 1$). This prominence of the μ^+K^0 mode for the SO(10)/G(224) model is primarily due to contributions from the new $d = 5$ operators. This contrasts sharply with the minimal SU(5) model, in which the μ^+K^0 mode is expected to have a branching ratio of only about 10^{-3}. In short, prominence of the μ^+K^0 mode, if seen, would clearly show the relevance of the new operators, and thereby reveal the proposed link between neutrino masses and proton decay [20].

The $d = 5$ operators as described here (standard and new) would lead to highly suppressed $e^+\pi^0$ mode, for MSSM or ESSM embedded in SO(10). The gauge boson-mediated $d = 6$ operators, however, still give (using the recently determined matrix element $\alpha_H = 0.015 \pm 0.001$ GeV^3 [54]) proton decaying into $e^+\pi^0$ with an inverse rate:

$$\Gamma^{-1}(p \to e^+\pi^0)^{\rm SO(10)/SU(5)}_{\rm MSSM} \approx 10^{35\pm1}\text{years} \ . \tag{55}$$

This can well be as short as about 10^{34} years. For the case of ESSM embedded into SO(10) [or for an analogous case embedded into SU(5)], there are two new features. Considering that in this case, both $\alpha_{\rm unif}$ and the unification scale M_X (thereby the mass M_V of the (X, Y) gauge bosons) are raised by nearly a factor of (6 to 7) and (2.5 to 5) respectively, compared to those for MSSM (see discussions in Section 6.4), and that the inverse decay rate is proportional to $(M_V^4/\alpha^2_{\rm unif})$, we expect

$$\Gamma^{-1}(p \to e^+\pi^0)^{\rm SO(10)/SU(5)}_{\rm ESSM} \approx (1 \text{ to } 17)\Gamma^{-1}(p \to e^+\pi^0)^{\rm SO(10)/SU(5)}_{\rm MSSM} \ . \tag{56}$$

The net upshot is that the gauge boson-mediated $d = 6$ operators can quite plausibly lead to observable $e^+\pi^0$ decay mode with an inverse decay rate in the range of 10^{34}-10^{35} years. For ESSM embedded in SO(10), there can be the interesting situation that both $\bar{\nu}K^+$ (arising from $d = 5$) and $e^+\pi^0$ (arising from $d = 6$) may have comparable rates, with proton having a lifetime $\sim$ (1/2-2) $\times 10^{34}$ years. It should be stressed that the $e^+\pi^0$-mode is the *common denominator* of all GUT

models (SU(5), SO(10), etc.) which unify quarks and leptons and the three gauge forces. Its rate as mentioned above is determined essentially by the SUSY unification-scale, without the uncertainty of the SUSY-spectrum. I should also mention that the $e^+\pi^0$-mode is predicted to be the dominant mode in the flipped SU(5) $\times$ U(1)-model [69]. For these reasons, intensifying the search for the $e^+\pi^0$-mode to the level of sensitivity of about 10^{35} years in the next generation proton decay detector should be well worth the effort.

Before summarizing the results of this section, I note below a few distinctive features of the conventional approach adopted here compared to those of some alternatives.

6.7 Conventional Versus Other Approaches

In these lectures, as elaborated in Section 3, I have pursued systematically the consequences for fermion masses, neutrino oscillations *and* proton decay of the assumption that essentially the conventional picture of SUSY grand unification [3, 4, 5, 6, 7] holds, providing a good effective theory in 4D between the conventional GUT-scale $M_X \sim 2 \times 10^{16}$ GeV (for ESSM, $M_X \sim (1/2\text{-}2) \times 10^{17}$ GeV) and the conventional string scale $M_{\rm st} \sim$ (few to 10) $\times\, 10^{17}$ GeV. Believing in an underlying string/M-theory, and yet knowing that a preferred ground state of this theory is not yet in hand, the attitude, based on a bottom-up approach, has been to subject the assumed effective theory of grand unification to as many low-energy tests as possible, and to assess its soundness on empirical grounds. With this in mind, I have assumed that either a realistic 4D SO(10)-solution (with the desired mechanism of doublet-triplet splitting operating in 4D), or a suitable string-derived G(224)-solution (with $M_X \sim (1/2)M_{\rm st}$, see footnote 2) emerges effectively from an underlying string theory at the conventional string scale as mentioned above, and that the G(224)/SO(10) symmetry breaks into G(213) at the conventional GUT-scale M_X. The extra dimensions of string/M-theory are assumed to be tiny lying between the GUT-scale size $\sim M_X^{-1}$ and the string-size $M_{\rm st}^{-1}$, so as not to disturb the successes of GUT (see below). As mentioned before, this conventional picture of grand unification described above seems to be directly motivated on observational grounds such as those based on (a) coupling unification or equivalently the agreement between the observed and the predicted values of $\sin^2\theta_W$ (see Section 3), (b) neutrino masses including $\Delta m^2(\nu_\mu\text{-}\nu_\tau)$ and (c) the fact that spontaneous violation of B–L local symmetry seems to be needed to implement baryogenesis via leptogenesis [16, 17]. The relevance of the group theory of G(224)/SO(10)-symmetry for the 4D theory is further suggested by the success of the predictions of the masses and the mixings of all fermions including neutrinos; these include $m_b^0 \approx m_\tau^0$, $m(\nu^\tau_{\rm Dirac}) \approx m_t(M_X)$, and the smallness of $V_{cb} \approx 0.04$ correlated with the largeness of $\sin^2 2\theta^{\rm osc}_{\nu_\mu\nu_\tau} \approx 1$ (see Section 5).

In contrast to this conventional approach based on a presumed string-unified G(224) or an SO(10)-symmetry, there are several alternative approaches (scenarios) which have been proposed in the literature in recent years. Of importance is the fact that in many of these alternatives an attempt is made to strongly suppress proton decay, in some cases exclusively the $d = 5$ operators (though not necessarily the $d = 6$), invariably utilizing a higher dimensional mechanism. Each of these alternatives is interesting in its own right. However, it seems to me that the collection of successes mentioned above is not (yet) realized within these alternatives. For comparison, I mention briefly only a few, leaving out many interesting variants.

One class of alternatives is based on the idea of large extra dimensions [36, 38], or a low string scale of order one TeV [37]. Though most intriguing, it does not seem to provide simple explanations for (a) coupling unification, (b) neutrino-masses (or their (mass)^2-differences) of the

observed magnitudes[23], (c) a large (or maximal) ν_μ-ν_τ oscillation angle, and (d) baryogenesis via leptogenesis that seems to require violation of B–L at high temperatures. Within this scenario, quantum-gravity induced proton decay would ordinarily be extra rapid. This is prevented, for example, by assuming that quarks and leptons live in different positions in the extra dimension. It appears to me that this idea (introduced just to prevent proton decay) however, sacrifices the simple reason for the co-existence quarks and leptons that is provided by a gauge unification of matter within a family as in G(224) or SO(10).

There is an alternative class of attempts, carried out again in the context of higher dimensional theories, which, in contrast to the case mentioned above, assume that the extra dimensions (d > 4) are all small, lying between (or around) the conventional GUT and string scales. The approach of this class of attempts is rather close in spirit to that of the conventional approach of grand unification pursued here (see Section 3). As may be seen from the discussions below, they could essentially coincide with the string-unified G(224)-picture presented here if the effective symmetry in 4D, below the string (or compactification) scale, contains at least the G(224) symmetry.

Motivated by the original attempts carried out in the context of string theory [71] most of the recent attempts in the class mentioned above are made in the spirit of a bottom-up approach[24] to physics near the GUT and the string scales. They assume, following the spirit of the results of Ref. [71], and of analogous results obtained for the free fermionic formulation of string theory [72] (for applications based on this formulation, see e.g., Ref. [73], [30], [31] and [74]), that grand unification occurs, through symmetries like E_6, SO(10) or SU(5), only in some higher dimension (d > 4), and that the breaking of the unification gauge symmetry to some lower symmetry containing the standard model gauge group as well as doublet-triplet splitting occurs in the process of compactification. More specifically the latter two phenomena take place through either (a) Wilson lines [71], or (b) orbifolds [75] (for an incomplete list of recent attempts based on orbifold compactification, see e.g., Refs. [76, 77, 78, 79, 80, 82, 83, 84, 85]), or (c) essentially equivalently by a set of boundary conditions together with the associated GSO projections for the free fermionic formulation (see e.g., [30, 31, 73, 74]), or (d) discrete symmetries operating in higher dimensions [86].

Most of these attempts end up not only in achieving (a) doublet-triplet splitting by projecting out the relevant color triplets from the zero mode-spectrum in 4D, and (b) gauge symmetry breaking, as mentioned above, but also (c) suppressing strongly or eliminating the $d = 5$ proton decay operators. It should be mentioned, however, that in some of these attempts (see e.g., [77]), the mass of the X gauge boson is suggested to be lower than the conventional GUT-scale of 2×10^{16} GeV by about a factor of 3 to 8; correspondingly they raise the prospect for observing the $d = 6$ gauge boson mediated $e^+\pi^0$ mode, which is allowed in [77].

One crucial distinction between the various cases is provided by the nature of the effective gauge symmetry that is realized in 4D, below the string (or compactification) scale. References [76, 77, 78, 80, 81, 82] assume a supersymmetric SU(5) gauge symmetry in 5D, which is broken down to the standard model gauge symmetry in 4D through compactification. References [83] and [84], on the other hand, assume a supersymmetric SO(10) gauge symmetry in 6D and show (interestingly enough) that there are two 5D subspaces containing G(224) and SU(5)×U(1) subgroups respectively, whose intersection leads to SU(3)×SU(2)×$U(1)_Y$×$U(1)_X$ in 4D, which

[23]By placing the singlet (right-handed) neutrino in the bulk, for example, one can get a light Dirac neutrino [70] with a mass $m_\nu \approx \kappa v_{EW} M^*/M_{\rm Pl} \approx \kappa(2\times10^{-5}$ eV), where $M^* \approx 1$ TeV, $M_{\rm Pl} \approx 10^{19}$ GeV (as in [70]), and κ is the effective Yukawa coupling. To get $m_\nu \sim 1/20$ eV (for SuperK), one would, however, need too large a $\kappa \sim 2\times10^3$ and/or too large a value for $M^* \gtrsim 100$ TeV; which would seem to face the gauge-hierarchy problem.

[24]This is of course also the case for the approach adopted here which is outlined in Section 3.

contains B–L. [An alternative construction using SO(10) in 5D leads just to the standard model gauge group in 4D [85].] While it is desirable to have B–L in 4D, consistent breaking of $U(1)_X$ (or B–L) and generating desired masses of the right handed neutrinos, not to mention the masses and the mixings of the other fermions, is not yet realized in these constructions.

For comparison, it seems to me that at the very least B–L should emerge as a generator in 4D to implement baryogenesis via leptogenesis, and also to protect RH neutrinos from acquiring a string-scale mass. This feature is not available in models which start with SU(5) in 5D. Furthermore, the full SU(4)-color symmetry, which of course contains B–L, plays a crucial role in yielding not only $m_b^0 \approx m_\tau^0$ but also (a) $m(\nu^\tau_{\rm Dirac}) \approx m_t(M_X)$ that is needed to account for $m(\nu_\tau)$ or rather $\Delta m^2(\nu_\mu\text{-}\nu_\tau)$, in accord with observation (see Section 4), and (b) the smallness of V_{cb} together with the near maximality of $\sin^2 2\theta^{\rm osc}_{\nu_\mu\nu_\tau}$ (see Section 5). The symmetry $SU(2)_L\times SU(2)_R$ is also most useful in that it relates the masses and mixings of the up and the down sectors. Without such relations, we will not have the predictivity of the framework presented in Section 5.

In short, as mentioned before, certain intriguing features of the masses and mixings of all fermions including neutrinos, of the type mentioned above, as well as the need for leptogenesis, seem to strongly suggest that the effective symmetry below the string-scale in 4D should contain minimally the symmetry G(224) [or a close relative G(214)] and maximally SO(10). The G(224)/SO(10)-framework developed here has turned out to be the most predictive, in large part by virtue of its group structure and the assumption of minimality of the Higgs system. Given that it is also most successful so far, as regards its predictions, derivation of such a picture from an underlying theory, especially at least that based on an effective G(224)-symmetry[25] in 4D leading to the pattern of Yukawa couplings presented here remains a challenge.[26] Pending such a derivation, however, given the empirical support it has received so far, it makes sense to test the supersymmetric G(224)/SO(10)-framework, and thereby the *conventional picture of grand unification* on which it rests, thoroughly. There are two notable missing pieces of this picture. One is supersymmetry which will be probed at the LHC and a future NLC. The other, that constitutes the hallmark of grand unification, is proton decay. The results of this section on proton decay are summarized below.

6.8 Section Summary

Given the importance of proton decay, a systematic study of this process has been carried out within the supersymmetric SO(10)/G(224)-framework[27], with special attention paid to its dependence on fermion masses and threshold effects. A representative set of results corresponding to different choices of parameters is presented in Tables 1 and 2. Allowing for the ESSM-variant, the

[25]For this case, following the examples of Refs. [30] and [31], the color triplets in the $\mathbf{10}_H$ of SO(10) would be projected out of the zero-mode spectrum, and thus the standard $d = 5$ operators which would have been induced by the exchange of such triplets would be absent, as in Refs. [76, 77, 78, 80, 81, 82, 83, 84, 85, 86]. But, as long as the Majorana masses of the RH neutrinos are generated as in Section 4, the new neutrino-mass related $d = 5$ proton decay operators would generically be present (see Section 6 E).

[26]In this regard, three-generation solutions containing the G(224)-symmetry in 4D have been obtained in the context of the fermionic formulation of string theory in Ref. [30], within type-I string vacua with or without supersymmetry in [87, 88, 89] in the context of D-brane inspired models in [90], within type-I string-construction or string-motivated models obtained from intersecting D-branes (with G(224) breaking into G(213) at $M_X \sim M_{\rm st}$) in [91, 92], in string model with unification at the string scale in [93], and in other contexts (see e.g. [94] and [95]).

[27]As described in Sections 3, 4 and 5.

study strongly suggests that an upper limit on proton lifetime is given by

$$\tau_{\rm proton} \leq (1/3\text{-}2) \times 10^{34} \text{ years}, \tag{57}$$

with $\overline{\nu}K^+$ being the dominant decay mode, and quite possibly μ^+K^0 and $e^+\pi^0$ being prominent. Although there are uncertainties in the matrix element, in the SUSY-spectrum, in the phase-dependent factor, $\tan\beta$ and in certain sensitive elements of the fermion mass matrix, notably ϵ' (see Eq. (48) for predictions in cases I versus II), this upper limit is obtained, for the case of MSSM embedded in SO(10), by allowing for a generous range in these parameters and stretching all of them in the same direction so as to extend proton lifetime. In this sense, while the predicted lifetime spans a wide range, the upper limit quoted above, in fact more like 10^{33} years, is most conservative, for the case of MSSM (see Eq. (48) and Table 1). It is thus tightly constrained already by the empirical lower limit on $\Gamma^{-1}(\overline{\nu}K^+)$ of 1.9×10^{33} years to the point of being disfavored. For the case of ESSM embedded in SO(10), the standard $d = 5$ operators are suppressed compared to the case of MSSM; as a result, by themselves they can naturally lead to lifetimes in the range of $(1 - 10) \times 10^{33}$ years, for nearly central values of the parameters pertaining to the SUSY-spectrum and the matrix element (see Eq. (50) and Table 1). Including the contribution of the new $d = 5$ operators, and allowing for a wide variation of the parameters mentioned above, one finds that the range of $(10^{33} - 2 \times 10^{34})$ years for proton lifetime is not only very plausible but it also provides a rather conservative upper limit, for the case of ESSM embedded in either SO(10) or G(224) (see Section 6.5 and Table 2). Thus our study provides a clear reason to expect that the discovery of proton decay should be imminent for the case of ESSM, and even more so for that of MSSM. The implication of this prediction for a next-generation detector is emphasized in the next section.

7 Concluding Remarks

The preceding sections show that, but for two missing pieces—supersymmetry and proton decay—the evidence in support of grand unification is now strong. It includes: (i) the observed family-structure, (ii) quantization of electric charge, (iii) the meeting of the gauge couplings, (iv) neutrino-oscillations as observed at SuperK, (v) the intricate pattern of the masses and mixings of all fermions, including the neutrinos, and (vi) the need for B–L as a generator, to implement baryogenesis. Taken together, these not only favor grand unification but in fact select out a particular route to such unification, based on the ideas of supersymmetry, SU(4)-color and left-right symmetry. Thus they point to the relevance of an effective string-unified G(224) or SO(10)-symmetry in four dimensions, as discussed in Sections 3 and 4.

Based on a systematic study of proton decay within the supersymmetric SO(10)/G(224)-framework, that (a) allows for the possibilities of both MSSM and ESSM, and (b) incorporates the improved values of the matrix element and renormalization effects, I have argued that a conservative upper limit on the proton lifetime is about $(1/3\text{-}2)\times10^{34}$ years.

So, unless the fitting of all the pieces listed above is a mere coincidence, it is hard to believe that that is the case, discovery of proton decay should be around the corner. In particular, as mentioned in the Introduction, one expects that candidate events should very likely be observed in the near future already at SuperK, if its operation is restored. However, allowing for the possibility that proton lifetime may well be near the upper limit stated above, a next-generation detector providing a net gain in sensitivity by a factor five to ten, compared to SuperK, would be needed to produce real events and distinguish them unambiguously from the background. Such

an improved detector would of course be essential to study the branching ratios of certain crucial though (possibly) sub-dominant decay modes such as the μ^+K^0 and $e^+\pi^0$ as mentioned in Section 6.6.

The reason for pleading for such improved searches is that proton decay would provide us with a wealth of knowledge about physics at truly short distances ($< 10^{-30}$ cm), which cannot be gained by any other means. Specifically, the observation of proton decay, at a rate suggested above, with $\overline{\nu}K^+$ mode being dominant, would not only reveal the underlying unity of quarks and leptons but also the relevance of supersymmetry. It would also confirm a unification of the fundamental forces at a scale of order 2×10^{16} GeV. Furthermore, prominence of the μ^+K^0 mode, if seen, would have even deeper significance, in that in addition to supporting the three features mentioned above, it would also reveal the link between neutrino masses and proton decay, as discussed in Section 6. *In this sense, the role of proton decay in probing into physics at the most fundamental level is unique.* In view of how valuable such a probe would be and the fact that the predicted upper limit on the proton lifetime is at most a factor of three to ten higher than the empirical lower limit, the argument in favor of building an improved detector seems compelling.

To conclude, the discovery of proton decay would undoubtedly constitute a landmark in the history of physics. It would provide the last, missing piece of gauge unification and would shed light on how such a unification may be extended to include gravity in the context of a deeper theory.

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APPENDIX: A Natural Doublet-Triplet Splitting Mechanism in SO(10)

In supersymmetric SO(10), a natural doublet–triplet splitting can be achieved by coupling the adjoint Higgs $\mathbf{45_H}$ to a $\mathbf{10_H}$ and a $\mathbf{10'_H}$, with $\mathbf{45_H}$ acquiring a unification–scale VEV in the B–L direction [96, 97]: $\langle \mathbf{45_H} \rangle = (a, a, a, 0, 0) \times \tau_2$ with $a \sim M_U$. As discussed in Section 5, to generate CKM mixing for fermions we require $(\mathbf{16_H})_d$ to acquire a VEV of the electroweak scale. To ensure accurate gauge coupling unification, the effective low energy theory should not contain split multiplets beyond those of MSSM. Thus the MSSM Higgs doublets must be linear combinations of the $\mathrm{SU}(2)_L$ doublets in $\mathbf{10_H}$ and $\mathbf{16_H}$. A simple set of superpotential terms that ensures this and incorporates doublet-triplet splitting is [14]:

$$W_H \,=\, \lambda\, \mathbf{10_H}\, \mathbf{45_H}\, \mathbf{10'_H} \,+\, M_{10}\, \mathbf{10'_H}^{\,2} \,+\, \lambda'\, \overline{\mathbf{16}}_H\, \overline{\mathbf{16}}_H\, \mathbf{10}_H \,+\, M_{16}\, \mathbf{16}_H \overline{\mathbf{16}}_H\,. \tag{A1}$$

A complete superpotential for $\mathbf{45_H}$, $\mathbf{16_H}$, $\overline{\mathbf{16}}_\mathbf{H}$, $\mathbf{10}_H$, $\mathbf{10'}_H$ and possibly other fields, which ensure that (a) $\mathbf{45_H}$, $\mathbf{16_H}$ and $\overline{\mathbf{16}}_\mathbf{H}$ acquire unification scale VEVs with $\langle \mathbf{45_H} \rangle$ being along the $(B\text{-}L)$ direction; (b) that exactly two Higgs doublets (H_u, H_d) remain light, with H_d being a linear combination of $(\mathbf{10_H})_d$ and $(\mathbf{16_H})_d$; and (c) there are no unwanted pseudoGoldstone bosons, can

be constructed. With $\langle \mathbf{45_H} \rangle$ in the B–L direction, it does not contribute to the Higgs doublet mass matrix, so one pair of Higgs doublet remains light, while all triplets acquire unification scale masses. The light MSSM Higgs doublets are [14]

$$H_u \,=\, \mathbf{10}_u \,, \quad H_d \,=\, \cos\gamma\, \mathbf{10}_d \,+\, \sin\gamma\, \mathbf{16}_d \,, \tag{A2}$$

with $\tan\gamma \equiv \lambda' \langle \overline{\mathbf{16}}_\mathbf{H} \rangle / M_{16}$. Consequently, $\langle \mathbf{10} \rangle_d = (\cos\gamma)\, v_d$, $\langle \mathbf{16}_d \rangle = (\sin\gamma)\, v_d$, with $\langle H_d \rangle = v_d$ and $\langle \mathbf{16}_d \rangle$ and $\langle \mathbf{10}_d \rangle$ denoting the electroweak VEVs of those multiplets. Note that H_u is purely in $\mathbf{10}_H$ and that $\langle \mathbf{10}_d \rangle^2 + \langle \mathbf{16}_d \rangle^2 = v_d^2$. This mechanism of doublet-triplet (DT) splitting is the simplest for the minimal Higgs systems. It has the advantage that it meets the requirements of both D-T splitting and CKM-mixing. In turn, it has three special consequences:

(i) It modifies the familiar SO(10)-relation $\tan\beta \equiv v_u/v_d = m_t/m_b \approx 60$ to [28]:

$$\tan\beta / \cos\gamma \approx m_t/m_b \approx 60 \;. \tag{A3}$$

As a result, even low to moderate values of $\tan\beta \approx 3$ to 10 (say) are perfectly allowed in SO(10) (corresponding to $\cos\gamma \approx 1/20$ to $1/6$).

(ii) The most important consequence of the DT-splitting mechanism outlined above is this: In contrast to SU(5), for which the strengths of the standard $d = 5$ operators are proportional to $(M_{H_c})^{-1}$ (where $M_{H_C} \sim few \times 10^{16}$ GeV (see Eq. (44)), for the SO(10)-model, they become proportional to $M_{\rm eff}^{-1}$, where $M_{\rm eff} = (\lambda a)^2/M_{10'} \sim M_X^2/M_{10'}$. As noted in Ref. [14], $M_{10'}$ can be naturally smaller (due to flavor symmetries) than M_X and thus $M_{\rm eff}$ correspondingly larger than M_X by even one to three orders of magnitude. Now the proton decay amplitudes for SO(10) in fact possess an intrinsic enhancement compared to those for SU(5), owing primarily due to differences in their Yukawa couplings for the up sector (see Appendix C in Ref. [14]). As a result, these larger values of $M_{\rm eff} \sim (10^{18} - 10^{19})$ GeV are in fact needed for the SO(10)-model to be compatible with the observed limit on the proton lifetime. At the same time, being bounded above by considerations of threshold effects (see below), they allow optimism as regards future observation of proton decay.

(iii) $M_{\rm eff}$ gets bounded above by considerations of coupling unification and GUT-scale threshold effects as follows. Let us recall that in the absence of unification-scale threshold and Planck-scale effects, the MSSM value of $\alpha_3(m_Z)$ in the $\overline{\rm MS}$ scheme, obtained by assuming gauge coupling unification, is given by $\alpha_3(m_Z)^\circ_{\rm MSSM} = 0.125 - 0.13$ [7]. This is about 5 to 8% *higher* than the observed value: $\alpha_3(m_Z) = 0.118 \pm 0.003$ [13]. Now, assuming coupling unification, the net (observed) value of α_3, for the case of MSSM embedded in SU(5) or SO(10), is given by:

$$\alpha_3(m_Z)_{\rm net} = \alpha_3(m_Z)^\circ_{\rm MSSM} + \Delta\alpha_3(m_Z)^{\rm MSSM}_{\rm DT} + \Delta'_3 \tag{A4}$$

where $\Delta\alpha_3(m_Z)_{\rm DT}$ and Δ'_3 represent GUT-scale threshold corrections respectively due to doublet-triplet splitting and the splittings in the other multiplets (like the gauge and the Higgs multiplets), all of which are evaluated at m_Z. Now, owing to mixing between $\mathbf{10}_d$ and $\mathbf{16}_d$ [see Eq. (A2)], one finds that $\Delta\alpha_3(m_Z)_{\rm DT}$ is given by $[\alpha_3(m_Z)^2/2\pi](9/7)\ln(M_{\rm eff}\cos\gamma/M_X)$ [14].

As mentioned above, constraint from proton lifetime sets a lower limit on $M_{\rm eff}$ given by $M_{\rm eff} > (1-6)\times 10^{18}$GeV. Thus, even for small $\tan\beta \approx 2$ (i.e. $\cos\gamma \approx \tan(\beta/60) \approx 1/30$), $\Delta\alpha_3(m_Z)_{\rm DT}$ is

[28] It is worth noting that the simple relationship between $\cos\gamma$ and $\tan\beta$—i.e. $\cos\gamma \approx \tan\beta/(m_t/m_b)$—would be modified if the superpotential contains an additional term like $\lambda'' \mathbf{16}_H \cdot \mathbf{16}_H \cdot \mathbf{10}'_H$, which would induce a mixing between the doublets in $\mathbf{10}'_d$, $\mathbf{16}_d$ and $\mathbf{10}_d$. That in turn will mean that the upper limit on $M_{\rm eff}\cos\gamma$ following from considerations of threshold corrections (see below) will not be strictly proportional to $\tan\beta$. I thank Kaladi Babu for making this observation.

positive; and it increases logarithmically with $M_{\rm eff}$. Since $\alpha_3(m_Z)^{\circ}_{\rm MSSM}$ is higher than $\alpha_3(m_Z)_{\rm obs}$, and as we saw, $\Delta\alpha_3(m_Z)_{\rm DT}$ is positive, it follows that the corrections due to *other* multiplets denoted by $\delta'_3 = \Delta'_3/\alpha_3(m_Z)$ should be appropriately negative so that $\alpha_3(m_Z)_{\rm net}$ would agree with the observed value.

In order that coupling unification may be regarded as a natural prediction of SUSY unification, as opposed to being a mere coincidence, it is important that the magnitude of the net other threshold corrections, denoted by δ'_3, be negative but not any more than about 8 to 10% in magnitude (i.e. $-\delta'_3 \leq (8-10)\%$). It was shown in Ref. [14] that the contributions from the gauge and the minimal set of Higgs multiplets (i.e. $\mathbf{45}_H, \mathbf{16}_H, \overline{\mathbf{16}}_H$ and $\mathbf{10}_H$) leads to threshold correction, denoted by δ'_3, which has in fact a negative sign and quite naturally a magnitude of 4 to 8%, as needed to account for the observed coupling unification. The correction to $\alpha_3(m_Z)$ due to Planck scale physics through the effective operator $F_{\mu\nu}F^{\mu\nu}\mathbf{45}_H/M$ does not alter the estimate of δ'_3 because it vanishes due to antisymmetry in the SO(10)- contraction.

Imposing that δ'_3 (evaluated at m_Z)be negative and not any more than about 10-11% in magnitude in turn provides a restriction on how big the correction due to doublet-triplet splitting—i.e. $\Delta\alpha_3(m_Z)_{{\rm D}\bar{T}}$—can be. That in turn sets an upper limit on $M_{\rm eff}\cos\gamma$, and thereby on $M_{\rm eff}$ for a given $\tan\beta$. For instance, for MSSM, with $\tan\beta = (2,3,8)$, one obtains (see Ref. [14]): $M_{\rm eff} \leq (4, 2.66, 1)\times 10^{18}$ GeV. Thus, conservatively, taking $\tan\beta \geq 3$, one obtains:

$$M_{\rm eff} \lesssim 2.7\times 10^{18}{\rm GeV}\ ({\rm MSSM}) \quad (\tan\beta \geq 3)\ . \tag{A5}$$

Limit on $M_{\rm eff}$ For The case of ESSM

Next consider the restriction on $M_{\rm eff}$ that would arise for the case of the extended supersymmetric standard model (ESSM), which introduces an extra pair of vector-like families ($16+\bar{16}$) of SO(10)) at the TeV scale [21](see also footnote 16). In this case, $\alpha_{\rm unif}$ is raised to 0.25 to 0.3, compared to 0.04 in MSSM. Owing to increased two-loop effects the scale of unification M_X is raised to $(1/2-2)\times 10^{17}$ GeV, while $\alpha_3(m_Z)^{\circ}_{\rm ESSM}$ is lowered to about 0.112-0.118 [21, 68].

With raised M_X, the product $M_{\rm eff}\cos\gamma \approx M_{\rm eff}(\tan\beta)/60$ can be higher by almost a factor of five compared to that for MSSM, without altering $\Delta\alpha_3(m_Z)_{\rm DT}$. Furthermore, since $\alpha_3(m_Z)^{\circ}_{\rm MSSM}$ is typically lower than the observed value of $\alpha_3(m_Z)$ (contrast this with the case of ESSM), for ESSM, $M_{\rm eff}$ can be higher than that for MSSM by as much as a factor of 2 to 3, without requiring an enhancement of δ'_3. The net result is that for ESSM embedded in SO(10), $\tan\beta$ can span a wide range from 3 to even 30 (say) and simultaneously the upper limit on $M_{\rm eff}$ can vary over the range (60 to 6)$\times 10^{18} GeV$, satisfying

$$M_{\rm eff} \lesssim (6\times 10^{18}{\rm GeV})(30/\tan\beta)\ ({\rm ESSM}), \tag{A6}$$

with the unification-scale threshold corrections from "other" sources denoted by $\delta'_3 = \Delta'_3/\alpha_3(m_Z)$ being negative, but no more than about 5% in magnitude. As noted above, such values of δ'_3 emerge quite naturally for the minimal Higgs system. Thus, one important consequence of ESSM is that by allowing for larger values of $M_{\rm eff}$ (compared to MSSM), without entailing larger values of δ'_3, it can be perfectly compatible with the limit on proton lifetime for almost *central values* of the parameters pertaining to the SUSY spectrum and the relevant matrix elements (see Eq. (40)). Further, larger values of $\tan\beta$ (10 to 30, say) can be compatible with proton lifetime only for the case of ESSM, but not for MSSM. These features are discussed in the text, and also exhibited in Table 2.

Table 2: Values of proton lifetime $\left(\Gamma^{-1}(p \to \bar{\nu} K^+)\right)$ for a wide range of parameters.

Parameters (spectrum/Matrix element)	MSSM → SO(10) **Std. d=5** Intermed. ϵ' & phase†		ESSM → SO(10) **Std. d=5** Intermed. ϵ' & phase†		$\left\{\begin{array}{c}\text{MSSM}\\ \text{or}\\ \text{ESSM}\end{array}\right\} \to$ G(224)/SO(10) **New d=5††**
	$\tan\beta$=3	$\tan\beta$=10	$\tan\beta$=5	$\tan\beta$=10	Independent of $\tan\beta$
Nearly "central" {C}=2	0.2×10^{32} yrs	1.6×10^{30} yrs	0.25×10^{34} yrs*	0.7×10^{33} yrs	0.50×10^{33} yrs††
Intermediate {C}=8	0.7×10^{32} yrs	0.6×10^{31} yrs	1×10^{34} yrs*	2.8×10^{33} yrs	2×10^{33} yrs††
Nearly Extreme {C}=32	0.3×10^{33} yrs	2.6×10^{31} yrs	4×10^{34} yrs*	1.1×10^{34} yrs	8×10^{33} yrs††

∗In this case, lifetime is given by the last column.

• Since we are interested in exhibiting expected proton lifetime near the upper end, we are not showing entries in Table 2 corresponding to values of the parameters for the SUSY spectrum and the matrix element [see Eq. (40), for which the curly bracket {C} appearing in Eqs. (47), (49), (52)] would be less than one (see however Table 1). In this context, we have chosen here "nearly central", "intermediate" and "nearly extreme" values of the parameters such that the said curly bracket is given by 2, 8 and 32 respectively, instead of its extreme upper-end value of 64. For instance, the curly bracket would be 2 if $\beta_H = (0.0117)$ GeV^3, $m_{\tilde{q}} \approx 1.2$ TeV and $m_{\tilde{W}}/m_{\tilde{q}} \approx (1/7.2)$, while it would be 8 if $\beta_H = 0.010$ GeV^3, $m_{\tilde{q}} \approx 1.44$ TeV and $m_{\tilde{W}}/m_{\tilde{q}} \approx 1/10$; and it would be 32 if, for example, $\beta_H = 0.007$ GeV^3, $m_{\tilde{q}} \approx \sqrt{2}(1.2$ TeV) and $m_{\tilde{W}}/m_{\tilde{q}} \approx 1/12$.

† All the entries for the standard $d = 5$ operators correspond to taking an intermediate value of $\epsilon' \approx (1 \text{ to } 1.4) \times 10^{-4}$ (as opposed to the extreme values of 2×10^{-4} and zero for cases I and II, see Eq. (34)) and an intermediate phase-dependent factor such that the uncertainty factor in the square bracket appearing in Eqs. (47) and (49) is given by 5, instead of its extreme values of $2 \times 4 = 8$ and $2.5 \times 4 = 10$, respectively.

†† For the new operators, the factor [8-1/64] appearing in Eq. (52) is taken to be 6, and K^{-2}, defined in Section 6.1, is taken to be 25, which are quite plausible, in so far as we wish to obtain reasonable values for proton lifetime at the upper end.

• The standard $d = 5$ operators for both MSSM and ESSM are evaluated by taking the upper limit on M_{eff} (defined in the text) that is allowed by the requirement of natural coupling unification. This requirement restricts threshold corrections and thereby sets an upper limit on M_{eff}, for a given $\tan\beta$ (see Section 6 and Appendix).

∗ For all cases, the standard and the new $d = 5$ operators must be combined to obtain the net amplitude. For the three cases of ESSM marked with an asterisk, and other similar cases which arise for low $\tan\beta \approx 3$ to 6 (say), the standard $d = 5$ operators by themselves would lead to proton lifetimes typically exceeding (0.25-4) $\times 10^{34}$ years. For these cases, however, the contribution from the new $d = 5$ operators would dominate, which quite naturally lead to lifetimes in the range of $(10^{33} - 10^{34})$ years (see last column).

• As shown above, the case of MSSM embedded in SO(10) is tightly constrained to the point of being disfavored by present empirical lower limit on proton lifetime Eq. (42) [see discussion following Eq. (48)].

• Including contributions from the standard and the new operators, the case of ESSM, embedded in either G(224) or SO(10), is, however, fully consistent with present limits on proton lifetime for a wide range of parameters; at the same time it provides optimism that proton decay will be discovered in the near future, with a lifetime $\leq 10^{34}$ years.

• The lower limits on proton lifetime are not exhibited. In the presence of the new operators, these can typically be as low as about 10^{29} years (even for the case of ESSM embedded in SO(10)). Such limits and even higher are of course long excluded by experiments.

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