S. Gardner *

Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309 and

Department of Physics and Astronomy, University of Kentucky, Lexington, Kentucky 40506-0055[†]

Abstract

Direct CP violation can exist in untagged, neutral *B*-meson decays to certain self-conjugate, hadronic final states. As a necessary condition, the resonances which appear therein must permit the identification of distinct, CP-conjugate states — in analogy to stereochemistry, we term such states "CP-enantiomers." These states permit the construction of CP-even and CP-odd amplitude combinations and of observables sensitive to their interference, which are non-zero if direct CP violation is present. The decay $B \to \pi^+ \pi^- \pi^0$, containing the distinct CP-conjugate states $\rho^+ \pi^$ and $\rho^- \pi^+$, provides one such example of a CP-enantiomeric pair. We illustrate the possibilities in various multi-particle final states.

PACS numbers:

[†] Permanent Address.

The measurement of a non-zero value of $\operatorname{Re}(\epsilon'/\epsilon)$ in $K \to \pi\pi$ decays establishes the existence of direct CP violation in nature [1], and provides an important first check of the mechanism of CP violation in the Standard Model (SM). Numerically, however, $\operatorname{Re}(\epsilon'/\epsilon)$ is very small. In the SM, this results, in part, from the weakness of inter-generational mixing [2]; the associated CP-violating parameter δ_{KM} in the Cabibbo-Kobayashi-Maskawa (CKM) matrix need not be small [3]. Indeed, the measurement of a large CP-asymmetry in $B^0(B^0) \to J/\psi K_s$ decay and related modes [4], induced through the interference of $B^0 - \bar{B}^0$ mixing and direct decay, suggests that $\delta_{KM} \sim \mathcal{O}(1)$ [5]. Nevertheless, the observation of direct CP violation in the *B*-meson system is needed to clarify the mechanism of CP violation, to confirm that the Kobayashi-Maskawa (KM) phase drives the CP-violating effects seen. In the SM, direct CP violation is anticipated to be much larger in B-meson decays than in K-meson decays [6]. The observation of direct CP violation in B-meson decays would falsify models in which the CP-violating interactions are "essentially" superweak [7, 8]. In this paper, we discuss how the presence of direct CP violation can be elucidated in untagged B-meson decays — the practical advantage of this strategy is the far larger statistical sample of events available.

The rich resonance structure of the multiparticle $(n \ge 2)$ final states accessible in heavy meson decays provides the possibility of observing direct CP violation without tagging the flavor of the decaying, neutral meson. The familar condition for the presence of direct CP violation, $|A_{\bar{f}}/A_f| \neq 1$, can be met by a non-zero value of the partial rate asymmetry, so that, seemingly, one would want to distinguish empirically a decay with amplitude A_f from that of its CP-conjugate mode with amplitude $\bar{A}_{\bar{f}}$. However, in neutral B, D-meson decays to self-conjugate final states [9–11], direct CP violation in untagged decays may nevertheless occur. Let us articulate the conditions. We must be able to separate the selfconjugate final state, via the resonances which appear, into distinct, CP-conjugate states. This condition finds it analogue in stereochemistry: we refer to molecules which are nonsuperimposable, mirror images of each other as enantiomers [12]. Accordingly, we refer to non-superimposable, CP-conjugate states as CP enantiomers. In $B \to \pi^+ \pi^- \pi^0$ decay, e.g., the intermediate states $\rho^+\pi^-$ and $\rho^-\pi^+$ form CP enantiomers, as they are distinct, CP-conjugate states. As a result, we can form combinations of amplitudes of either even or odd character under CP. The resulting interference between the CP-even and CP-odd amplitudes realized in the overlapping resonance bands of the Dalitz plot can generate an asymmetry reflective of direct CP violation.

We shall use $B \to \pi^+ \pi^- \pi^0$ decay as a paradigm of how direct CP violation can occur in untagged *B*-meson decays. In what follows, we shall largely follow the notation and conventions of Quinn and Silva [13]. Consider the amplitudes for $B^0(\bar{B}^0) \to \pi^+ \pi^- \pi^0$ decay:

$$A(B^{0}(p_{B}) \to \pi^{+}(p_{+})\pi^{-}(p_{-})\pi^{0}(p_{0})) = f_{+}(u) a_{+-} + f_{-}(s) a_{-+} + f_{0}(t) a_{00},$$

$$\bar{A}(\bar{B}^{0}(p_{B}) \to \pi^{+}(p_{+})\pi^{-}(p_{-})\pi^{0}(p_{0})) = f_{+}(u) \bar{a}_{+-} + f_{-}(s) \bar{a}_{-+} + f_{0}(t) \bar{a}_{00}, \qquad (1)$$

where the two-body decay amplitudes are given by $a_{+-} = A(B^0 \to \rho^+ \pi^-)$, $a_{-+} = A(B^0 \to \rho^- \pi^+)$, and $a_{00} = A(B^0 \to \rho^0 \pi^0)$ and f_i is the form factor describing $\rho^i \to \pi \pi$. We have used $s = (p_- + p_0)^2$, $t = (p_+ + p_-)^2$, and $u = (p_+ + p_0)^2$ [?]. For clarity, note that $\bar{a}_{+-} = \bar{A}(\bar{B}^0 \to \rho^+ \pi^-)$ and $\bar{a}_{-+} = \bar{A}(\bar{B}^0 \to \rho^- \pi^+)$. Since $\rho^+ \pi^-$ and $\rho^- \pi^+$ are distinct, CP-conjugate states, we can form amplitudes of definite CP-even and CP-odd character, namely $a_g = a_{+-} + a_{-+}$ and $a_u = a_{+-} - a_{-+}$, respectively. That is, if we define $\bar{a}_g = \bar{a}_{+-} + \bar{a}_{-+}$ and $\bar{a}_u = \bar{a}_{+-} - \bar{a}_{-+}$, we see that the CP conjugate of a_q is \bar{a}_q , whereas the CP conjugate of a_u

is $-\bar{a}_u$. With $a_n = 2a_{00}$ we have

$$A_{3\pi} \equiv A(B^0 \to \pi^+ \pi^- \pi^0) = f_g(u, s) a_g + f_u(u, s) a_u + f_n(t) a_n$$

$$\bar{A}_{3\pi} \equiv \bar{A}(\bar{B}^0 \to \pi^+ \pi^- \pi^0) = f_g(u, s) \bar{a}_g + f_u(u, s) \bar{a}_u + f_n(t) \bar{a}_n , \qquad (2)$$

where $f_g(u,s) = (f_+(u) + f_-(s))/2$, $f_u(u,s) = (f_+(u) - f_-(s))/2$, and $f_n(t) = f_0(t)/2$. Neglecting the width difference of the *B*-meson mass eigenstates, as $\Delta\Gamma \equiv \Gamma_H - \Gamma_L$ and $|\Delta\Gamma| \ll \Gamma \equiv (\Gamma_H + \Gamma_L)/2$, the decay rate into $\pi^+\pi^-\pi^0$ for a B^0 meson at time t = 0 is given by [15]

$$\Gamma(B^{0}(t) \to \pi^{+}\pi^{-}\pi^{0}) = |A_{3\pi}|^{2}e^{-\Gamma t} \left[\frac{1+|\lambda_{3\pi}|^{2}}{2} + \frac{1-|\lambda_{3\pi}|^{2}}{2}\cos(\Delta m t) - \mathrm{Im}\lambda_{3\pi}\sin(\Delta m t)\right],$$
(3)

whereas the analogous decay rate for a \bar{B}^0 meson at time t = 0 is given by

$$\Gamma(\bar{B}^{0}(t) \to \pi^{+}\pi^{-}\pi^{0}) = |A_{3\pi}|^{2}e^{-\Gamma t} \left[\frac{1+|\lambda_{3\pi}|^{2}}{2} - \frac{1-|\lambda_{3\pi}|^{2}}{2}\cos(\Delta m t) + \mathrm{Im}\lambda_{3\pi}\sin(\Delta m t)\right].$$
(4)

Note that $\lambda_{3\pi} \equiv q\bar{A}_{3\pi}/pA_{3\pi}$ and $\Delta m \equiv M_H - M_L$. We neglect $\Delta\Gamma$, so that |q/p| = 1. Untagged observables, for which the identity of the *B* meson at t = 0 is unimportant, correspond to $\Gamma(B^0(t) \to \pi^+\pi^-\pi^0) + \Gamma(\bar{B}^0(t) \to \pi^+\pi^-\pi^0) \propto |A_{3\pi}|^2 + |\bar{A}_{3\pi}|^2$. We have

$$|A_{3\pi}|^{2} + |\bar{A}_{3\pi}|^{2} = \sum_{i} (|a_{i}| + |\bar{a}_{i}|^{2})|f_{i}|^{2} + 2\sum_{i < j} \left[\operatorname{Re}(f_{i}f_{j}^{*}) \operatorname{Re}(a_{i}a_{j}^{*} + \bar{a}_{i}\bar{a}_{j}^{*}) - \operatorname{Im}(f_{i}f_{j}^{*}) \operatorname{Im}(a_{i}a_{j}^{*} + \bar{a}_{i}\bar{a}_{j}^{*}) \right] , \quad (5)$$

where $i, j \in g, u, n$, noting that i, j labels are not repeated in the sum labelled "i < j". The different products $f_i f_j^*$ are distinguishable through the Dalitz plot of this decay, so that the coefficients of these functions are empirically distinct [13]. For our purposes the crucial point is that these observables, as first noted by Quinn and Silva [13], can be of CP-odd character. In particular, the presence of

$$a_g a_u^* + \bar{a}_g \bar{a}_u^*$$
 and/or $a_n a_u^* + \bar{a}_n \bar{a}_u^*$ (6)

is reflective of direct CP violation. Physically these observables correspond to a population asymmetry under the exchange of u and s (or of p_+ and p_-) across the Dalitz plot in the regions where the ρ^i bands overlap. To make the geometric sense of this construction clear, consider a Dalitz plot in u versus s, that is, in the invariant masses of the $\pi^+\pi^0$ and $\pi^-\pi^0$ pairs, respectively — such a plot is shown in Fig. 1 of Ref. [16]. We wish to consider the population asymmetry about the u = s "mirror line" in the regions where the ρ^i bands overlap. In $B \to \rho \pi$ decay, the ρ^i bands overlap in the corners of the Dalitz plot, so that we wish to consider the asymmetry about the u = s line for i) the overlapping ρ^+ and ρ^- bands and ii) for the overlaps of the ρ^{\pm} bands with the ρ^0 band. The first asymmetry determines the first amplitude combination in Eq. (6), whereas the second determines the second amplitude combination. A population asymmetry about the u = s line is also a signature of direct CP violation. However, non-zero values of the amplitude combinations of Eq. (6) do not guarantee its existence as cancellations, though likely incomplete, can occur. The direct CP-violating observables of Eq. (6) can persist even if the strong phases of the a_i amplitudes were zero. To illustrate, we parametrize $a_j = T_j \exp(-i\alpha) + P_j$ and $P_j/T_j = r_j \exp(i\delta_j)$, where $r_j > 0$ and δ_i is the strong phase of interest. Thus

$$a_g a_u^* + \bar{a}_g \bar{a}_u^* = -2T^g T^{u*} \sin \alpha \left[r_g \sin \delta_g + r_u \sin \delta_u - i(r_g \cos \delta_g - r_u \cos \delta_u) \right] . \tag{7}$$

The real and imaginary parts of this relation are each observable, as they correspond to distinct f_i -dependent terms in Eq. (5). The combination $T^g T^{u*}$ can be complex, though we assume it to be real for crispness of discussion. In the imaginary part, we see that direct CP violation can exist if the strong phases vanish, i.e., if $\delta_u = \delta_g = 0$. Indeed, merely r_g or r_u must be non-zero to realize direct CP violation were $\sin \alpha \neq 0$. Theoretical estimates suggest that r_g and r_u are both non-zero and unequal [17]. In constrast, a partial rate asymmetry can be written as

$$|a_g|^2 - |\bar{a}_g|^2 = -4|T^g|^2 r_g \sin \delta_g \, \sin \alpha \,, \tag{8}$$

yielding the familiar result that both r_g and δ_g must be non-zero to yield direct CP violation were $\sin \alpha \neq 0$. Such conditions are realized in the real part of Eq. (7) as well, so that the direct CP-violating observables we propose can be manifest irrespective of the strong phases, as they can be non-zero were the strong phases either zero or 90 degrees. This greater flexibility arises as the combination $P^g/T^g - P^{u*}/T^{u*}$ appears in Eq. (7), whereas $P^g/T^g - P^{g*}/T^{g*}$, e.g., appears in the partial rate asymmetry.

Interestingly, similar considerations arise in the angular analysis of $B \to V_1 V_2$ decays: there, too, the interference of CP-even and CP-odd amplitudes can beget direct CP violation in untagged decays [18]. There are three helicity amplitudes, labelled by the helicity $\lambda \in$ $(0, \pm 1)$ of either vector meson in $B \to V_1 V_2$ decay. Working in a transversity basis [19], we can define the amplitudes $A_{\parallel} \equiv (A_{+1} + A_{-1})/\sqrt{2}$ and $A_{\perp} \equiv (A_{+1} - A_{-1})/\sqrt{2}$ [20] to realize definite CP-even and CP-odd combinations, respectively, of these amplitudes. The full angular distribution of the summed amplitudes for B^0 and \bar{B}^0 decay permits the extraction of the imaginary part of the amplitude combinations of Eq. (6), under the identification $a_g \to A_{\parallel}, a_u \to A_{\perp}$, and $a_n \to A_0$. Moreover, these untagged contributions are insensitive to the strong phase [21].

The conditions which permit the realization of direct CP violation in untagged modes are quite general. We need only consider self-conjugate final states whose resonances encode enantiomeric pair correlations. Self-conjugate final states can be realized through the $b \rightarrow dq\bar{q}$ decays of B_d mesons and $b \rightarrow sq\bar{q}$ decays of B_s mesons, where $q \in u, d, s, c$ quarks. The KM picture of CP violation suggests that direct CP-violating effects ought be enhanced by a factor of $\mathcal{O}(1/\lambda^2) \sim 20$ in B_d meson decays. Thus the goals of direct CP violation searches in B_d and B_s meson decays are quite distinct. The appearance of direct CP violation in B_d meson decays would substantiate the KM picture of CP violation, whereas its appearance in B_s decays would signal the presence of new physics. Physics with B_s mesons is important to the future B-physics programs at the Tevatron [22] and at the LHC [23]. The effective tagging efficiency ϵ_{eff} is significantly smaller in a hadronic environment, cf. $\epsilon_{\text{eff}} \sim 7\%$ [24] with $\epsilon_{\text{eff}} \sim 27\%$ [25, 26] at the B-factories, so that the untagged studies we propose significantly enable direct CP violation searches at these facilities [?].

Let us enumerate three-, four-, and five-particle final states in B_d decay which could yield direct CP violation in the KM picture. We thus focus on $b \to du\bar{u}$ and $b \to dc\bar{c}$ decays, and some possibilities are given in Table I — we do not attempt to be exhaustive. The CP-enantiomers are useful in the sense we have illustrated in $B \to \rho \pi$ decay: they permit the formation of manifestly CP-odd amplitude combinations which can be probed through

asymmetries in the population of events in the regions where the resonances of the CPenantiomeric pair overlap. We expect the CP-violating effects to be much larger for broad resonances such as the ρ and $K^*(892)$. Note that the final states $K^+K^-\pi^0$ and $K^+K^-\pi^+\pi^-$, with the CP enantiomers indicated, also lend themselves to direct CP violation searches in B_s decay. Multiparticle final states can support more than one CP-enantiomeric pair, as illustrated in $B_d \to \pi^+ \pi^- \pi^+ \pi^- \pi^0$ decay. In the case of CP enantiomers with more than one spin one particle, as in $(a_1(1260)^+ \rho^-, a_1(1260)^- \rho^+)$, a caution is in order. The presence of two spin-one particles in the final state implies that partial waves with L = 0, 1, or 2can occur; the factor $(-1)^{L}$ impacts the CP of the state. The sum and difference of the amplitudes associated with $B^0 \to a_1(1260)^+ \rho^-$ and $\bar{B}^0 \to a_1(1260)^- \rho^+$ decay still yield states of definite CP for any particular L, but for L = 0, 2 the sum of amplitudes is CPeven, whereas for L = 1 the sum of amplitudes is CP-odd. In either event for fixed L the amplitude combination of Eq. (7) appears in the interference of the resonance contributions and drives a population asymmetry under the exchange of the momentum of a π^+ emerging from the $a_1(1260)^+$ and of the π^- from the ρ^- in the region over which the resonances of the CP-enantiomeric pair overlap. States of fixed L can be realized through a helicity analysis; the formation of the A_{\perp} amplitude, e.g., selects the L = 1 state [19]. In the absence of a helicity analysis, both CP-even and CP-odd contributions are subsumed in " a_q " and " a_u ," so that a population asymmetry in this case can exist without direct CP violation. Thus for pairs with two spin one particles, a helicity analysis is required; otherwise, for pairs with multiple spinned particles, the construction is not useful.

The observation of direct CP violation in B-meson decays in itself is crucial to establishing the mechanism of CP violation. Nevertheless, we would also like to interpret such results in terms of the parameters of the CKM matrix. An assumption of isospin symmetry can codify and potentially determine the hadronic parameters needed to interpret the mixing-induced CP-asymmetry in $b \rightarrow dq\bar{q}$ transitions to charmless final states. Relevant to the modes we discuss are the isospin-based analyses which yield $\sin(2\alpha)$ in $B \to \rho \pi$ [13, 27, 28] and $B \to a_1 \pi$ [29] decays. These analyses, however, do not determine the parameters necessary to interpret direct CP violation; the terms containing $\sin \alpha$ and $\cos \alpha$ are multiplied by unknown hadronic parameters. Nevertheless, were $\sin(2\alpha)$ determined and direct CP violation observed, the SM value of $\sin \alpha$ could be inferred, modulo discrete ambiguities. Interpreting direct CP-violating observables directly in terms of the underlying weak parameters may not prove possible. Theoretical progress has been made, however, in the computation of partial-rate asymetries in some two-body decays, see, e.g., Refs. [30, 31]. Alternatively, more phenomenological treatments indicate that the presence of resonances in certain channels can enhance the associated partial rate asymmetry [32, 33] and aid in the extraction of weak phase information [34].

We have discussed the conditions under which the rich resonance structure of hadronic B decays can be exploited to search for direct CP violation in untagged decays. Our method is sufficiently general to enable direct CP violation searches in B_s and D meson decays as well. In some channels the untagged search we propose complements tagged, time-dependent analyses in $B \to \rho \pi$ and $B \to a_1 \pi$ decays. Nevertheless, the gain in statistical power realized in untagged versus tagged searches, i.e., roughly a factor of 2 at the B-factories and of 4 in a hadronic environment such as at CDF, argues for a more comprehensive program.

Acknowledgements S.G. thanks H.R. Quinn for key discussions and input, L. Dixon for useful comments, and J.D. Bjorken for a helpful conversation. S.G. acknowledges the SLAC Theory Group for gracious hospitality and is supported by the U.S. Department of Energy

3-particles	CP-enantiomers
$\pi^+\pi^-\pi^0$	$(ho^+\pi^-, ho^-\pi^+)$
$K^+K^-\pi^0$	$(K^*(892)^+K^-, K^*(892)^-K^+)$
$D^+D^-\pi^0$	$(D^*(2010)^+D^-, D^*(2010)^-D^+)$
$D^0 \bar{D}^0 \pi^0$	$(D^*(2007)^0 \bar{D}^0, \bar{D}^*(2007)^0 D^0)$
4-particles	CP-enantiomers
$\pi^+\pi^-\pi^0\pi^0$	$(\rho^+\pi^-\pi^0, \rho^-\pi^+\pi^0)$
$\pi^+\pi^-\pi^+\pi^-$	$(a_1(1260)^+\pi^-, a_1(1260)^-\pi^+)$
$K^+K^-\pi^+\pi^-$	$(K^*(892)^0 K^- \pi^+, \bar{K}^*(892)^0 K^+ \pi^-)$
$D^0 \bar{D}^0 \pi^+ \pi^-$	$(D^*(2010)^+ \bar{D}^0 \pi^-, D^*(2010)^- D^0 \pi^+)$
$D^+D^-\pi^0\pi^0$	$(D^*(2010)^+D^-\pi^0, D^*(2010)^-D^+\pi^0)$
5-particles	CP-enantiomers
$\pi^+\pi^-\pi^+\pi^-\pi^0$	$(\rho^+\pi^-\pi^+\pi^-, \rho^-\pi^-\pi^+\pi^+)$
	$(ho^+ ho^0\pi^-, ho^- ho^0\pi^+)^a$
	$(a_1(1260)^+\pi^-\pi^0, a_1(1260)^-\pi^+\pi^0)$
	$(a_1(1260)^+\rho^-, a_1(1260)^-\rho^+)^a$
	$(a_0(980)^+\pi^-, a_0(980)^-\pi^+)$
	$(b_1(1235)^+\pi^-, b_1(1235)^-\pi^+)$

TABLE I: B_d decays to certain three-, four-, and five-particle, self-conjugate final-states and some of the CP-enantiomers they contain.

^aA helicity analysis is required; see text.

under contracts DE-FG02-96ER40989 and DE-AC03-76SF00515.

* Electronic address: gardner@pa.uky.edu

- A. Alavi-Harati *et al.* [KTeV Collaboration], Phys. Rev. Lett. **83**, 22 (1999); V. Fanti *et al.* [NA48 Collaboration], Phys. Lett. B **465**, 335 (1999); G. D. Barr *et al.* [NA31 Collaboration], Phys. Lett. B **317**, 233 (1993).
- [2] S. Bosch *et al.*, Nucl. Phys. B **565**, 3 (2000).
- [3] N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973).
- [4] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. 87, 091801 (2001); K. Abe *et al.* [Belle Collaboration], Phys. Rev. Lett. 87, 091802 (2001).
- [5] See, e.g., Y. Nir, arXiv:hep-ph/0109090.
- [6] M. Bander, D. Silverman, and A. Soni, Phys. Rev. Lett. 43, 242 (1979).

- [7] L. Wolfenstein, Phys. Rev. Lett. **13**, 562 (1964).
- [8] L. Wolfenstein, "The superweak theory 35 years later," in *Kaon Physics*, J. L. Rosner and B. D. Winstein, eds., University of Chicago Press, Chicago, IL, 2001.
- [9] A. B. Carter and A. I. Sanda, Phys. Rev. Lett. 45, 952 (1980); Phys. Rev. D 23, 1567 (1981).
- [10] I. I. Bigi and A. I. Sanda, Nucl. Phys. B **193**, 85 (1981); Nucl. Phys. B **281**, 41 (1987).
- [11] I. Dunietz and J. L. Rosner, Phys. Rev. D 34, 1404 (1986).
- [12] John D. Roberts and Marjorie C. Caserio, Basic Principles in Organic Chemistry, 2nd ed., W. A. Benjamin, Inc., Menlo Park, CA, 1977.
- [13] H. R. Quinn and J. P. Silva, Phys. Rev. D 62, 054002 (2000).
- [14] S. Gardner and U.-G. Meißner, arXiv:hep-ph/0112281, to appear in Phys. Rev. D.
- [15] Y. I. Azimov, N. G. Uraltsev, and V. A. Khoze, Yad. Fiz. 45, 1412 (1987).
- [16] B. Aubert *et al.* [BABAR Collaboration], arXiv:hep-ex/0107058.
- [17] See, e.g., A. Ali, G. Kramer, and C. D. Lu, Phys. Rev. D 58, 094009 (1998).
- [18] A. E. Snyder, private communication.
- [19] I. Dunietz et al., Phys. Rev. D 43, 2193 (1991); A. S. Dighe et al., Phys. Lett. B 369, 144 (1996).
- [20] C. P. Jessop *et al.* [CLEO Collaboration], Phys. Rev. Lett. **79**, 4533 (1997).
- [21] G. Valencia, Phys. Rev. D **39**, 3339 (1989).
- [22] K. Anikeev et al., "B physics at the Tevatron: Run II and beyond," arXiv:hep-ph/0201071.
- [23] P. Ball *et al.*, arXiv:hep-ph/0003238.
- [24] T. Affolder et al. [CDF Collaboration], Phys. Rev. D 61, 072005 (2000).
- [25] B. Aubert *et al.* [BABAR Collaboration], arXiv:hep-ex/0201020.
- [26] K. Abe *et al.* [Belle Collaboration], arXiv:hep-ex/0202027.
- [27] H. J. Lipkin, Y. Nir, H. R. Quinn, and A. Snyder, Phys. Rev. D 44, 1454 (1991).
- [28] A. E. Snyder and H. R. Quinn, Phys. Rev. D 48, 2139 (1993).
- [29] P. F. Harrison and H. R. Quinn [BABAR Collaboration], eds., "The BaBar physics book: Physics at an asymmetric B factory," SLAC-R-0504.
- [30] M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, Phys. Rev. Lett. 83, 1914 (1999);
 Nucl. Phys. B 591, 313 (2000); 606, 245 (2001).
- [31] H.-n. Li and H. L. Yu, Phys. Rev. Lett. 74, 4388 (1995); Phys. Rev. D 53, 2480 (1996);
 Y. Y. Keum, H.-n. Li, and A. I. Sanda, Phys. Rev. D 63, 054008 (2001).
- [32] D. Atwood and A. Soni, Z. Phys. C 64, 241 (1994); Phys. Rev. Lett. 74, 220 (1995); D. Atwood, G. Eilam, M. Gronau, and A. Soni, Phys. Lett. B 341, 372 (1995); G. Eilam, M. Gronau, and R.R. Mendel, Phys. Rev. Lett. 74, 4984 (1995).
- [33] R. Enomoto and M. Tanabashi, Phys. Lett. B **386**, 413 (1996).
- [34] S. Gardner, H. B. O'Connell, and A. W. Thomas, Phys. Rev. Lett. 80, 1834 (1998).
- [] We have implicitly summed over the ρ^i polarization. Defining $\langle \pi^0(p_0)\pi^-(p_-)|\rho^-(p_\rho,\epsilon)\rangle \equiv g_\rho \epsilon \cdot (p_0 p_-)$ and $\langle \rho^i(\epsilon, p_\rho)\pi^-(p_\pi)|\mathcal{H}_{\text{eff}}|B^0(p_B)\rangle \equiv 2\epsilon^* \cdot p_\pi \bar{\eta}^i$, where \mathcal{H}_{eff} is the $|\Delta B| = 1$ effective Hamiltonian, we find $A(B^0(p_B) \to \pi^+(p_+)\pi^-(p_-)\pi^0(p_0)) = -\bar{\eta}^0(s-u)f_0(t) + \bar{\eta}^+(s-t)f_+(u) + \bar{\eta}^-(t-u)f_-(s)$, where the pions' masses are given by $M_{\pi^\pm} = M_{\pi^0} = M_{\pi}$. The form factor $f_i(x)$ can be described by a Breit-Wigner form $g_\rho/(x M_\rho^2 + i\Gamma_\rho M_\rho)$, or a more sophisticated function, consistent with the theoretical constraints of analyticity, time-reversal-invariance, and unitarity, see Ref. [14] for all details.
- [] Recall that ϵ_{eff} , a conflation of the tagging efficiency ϵ and the mistag fraction w given by $\epsilon_{\text{eff}} = \epsilon (1 2w)^2$, drives the statistical error in an asymmetry measurement as per $1/\sqrt{\epsilon_{\text{eff}}N}$, where N is the number of untagged events.