

Measuring the Higgs boson's parity using $\tau \rightarrow \rho\nu$ *

G. R. Bower^a, T. Pierzchała^b, Z. Wąs^{c,d} and M. Worek^b

^a *Stanford Linear Accelerator Center, Stanford University
Menlo Park, CA 94025, USA.*

^b *Institute of Physics, University of Silesia
Uniwersytecka 4, 40-007 Katowice, Poland.*

^c *Institute of Nuclear Physics
Kawiorzy 26a, 30-055 Cracow, Poland.*

^d *CERN, Theory Division, CH-1211 Geneva 23, Switzerland.*

Abstract

We present a very promising method for a measurement of the Higgs boson parity using the $H/A \rightarrow \tau^+\tau^- \rightarrow \rho^+\bar{\nu}_\tau\rho^-\nu_\tau \rightarrow \pi^+\pi^0\bar{\nu}_\tau\pi^-\pi^0\nu_\tau$ decay chain. The method is both model independent and independent of the Higgs production mechanism. Angular distributions of the τ decay products which are sensitive to the Higgs boson parity are defined and are found to be measurable using typical properties of a future detector for an e^+e^- linear collider. The prospects for the measurement of the parity of a Higgs boson with a mass of 120 GeV are quantified for the case of e^+e^- collisions of 500 GeV center of mass energy with an integrated luminosity of $500fb^{-1}$. The Standard Model Higgsstrahlung production process is used as an example.

* This work is partly supported by the Polish State Committee for Scientific Research (KBN) grants Nos 5P03B09320, 2P03B00122 and the European Community's Human Potential Programme under contract HPRN-CT-2000-00149 Physics at Colliders. This work is also supported in part by the United States Department of Energy contract DE-AC03-76SF00515.

1 Introduction

If the Higgs mechanism is realized in nature, a complete determination of its properties is among the central tasks for future colliders. In particular, determining the \mathcal{CP} properties of the Higgs boson is one of the important goals of a future e^+e^- linear collider operating at a center-of-mass energy between 350–1000 GeV [1–3]. Different models, such as the Standard Model (\mathcal{SM}), the general Two Higgs Doublet Models ($2\mathcal{HDM}$), the Minimal Supersymmetric Standard Model (\mathcal{MSSM}), and others predict different properties for the details of the Higgs boson signature. It is of great importance to verify that, whatever scenario is realized in nature, the scientific program of a linear collider will be able to distinguish among these models.

Several methods have been proposed to distinguish between a scalar ($\mathcal{J}^{PC} = 0^{++}$) and a pseudoscalar ($\mathcal{J}^{PC} = 0^{-+}$) spin zero Higgs particle [4–7]. In this study we investigate the case of a Higgs boson which is light enough that the W^+W^- decay channel remains closed. Then the most promising decay channel for the model independent parity determination is $H/A \rightarrow \tau^+\tau^-$. It was previously proposed [4] that the best τ decay channel for parity determination would be $\tau^\pm \rightarrow \pi^\pm\nu$. The weakness of this method is that the Higgs boson rest frame needs to be precisely reconstructed.

To address resolution issues it is necessary to perform Monte Carlo studies where the significant details of theoretical effects and detector conditions can be included. In Ref. [8] we have extended the algorithm of Refs. [9, 10] for generating Higgs decay (independently from its production mechanism) to the pair of τ -leptons including the complete spin correlation matrix used in the subsequent decay of the τ leptons. The reaction chain $e^+e^- \rightarrow Z(H/A)$, $H/A \rightarrow \tau^+\tau^-$; $\tau^\pm \rightarrow \pi^\pm\bar{\nu}_\tau(\nu_\tau)$ was studied. It was found that even small effects of smearing seriously deteriorate the measurement resolution. An independent study [11], using the PANDORA Monte Carlo generator of Ref. [12] confirms that result. This leaves no doubt that if we want to use the Higgs boson τ decay channel for the measurement of the Higgs parity, a better technique is required.

In this paper we continue to investigate Higgs decay into a $\tau^+\tau^-$ pair. We extend our study to the $H \rightarrow \tau^+\tau^-$ decay with $\tau^\pm \rightarrow \rho^\pm\bar{\nu}_\tau(\nu_\tau)$ (*i.e.* the channel with the largest branching ratio) and then $\rho^\pm \rightarrow \pi^\pm\pi^0$. Spin correlations are distributed over *three* levels of the decay chain. Complicated geometrical distributions need to be defined. We have found the Monte Carlo method particularly useful already at the level of defining the observables. All of the main results are cross-checked using two independent analyses based on an extension of universal interface (Ref. [8]) for TAUOLA, and with the PANDORA Monte Carlo generator [12] which is interfaced to PYTHIA 6.1 [13] for fragmentation and decay processes.

The rest of the paper is organized as follows. The theoretical considerations which are used to understand the decay chain of the Higgs particle are explained in section 2 where we also give some details of the TAUOLA Monte Carlo simulation. In section 3 we define the observable we use to distinguish between the scalar and pseudoscalar Higgs boson. In section 4 we list our assumptions on smearing and we discuss imposed cuts, detector effects and necessary adaptations introduced into our observables. Main numerical results

are also given in this section. Section 5, a summary, closes the paper.

2 Determination of the \mathcal{CP} quantum numbers of the Higgs boson

The H/A parity information must be extracted from the correlations between τ^+ and τ^- spin components which are further reflected in correlations between the τ decay products in the plane transverse to the $\tau^+\tau^-$ axes. This is because the decay probability, see [4],

$$\Gamma(H/A \rightarrow \tau^+\tau^-) \sim 1 - s_{\parallel}^{\tau^+} s_{\parallel}^{\tau^-} \pm s_{\perp}^{\tau^+} s_{\perp}^{\tau^-} \quad (1)$$

is sensitive to the τ^{\pm} polarization vectors s^{τ^-} and s^{τ^+} (defined in their respective rest frames). The symbols \parallel/\perp denote components parallel/transverse to the Higgs boson momentum as seen from the respective τ^{\pm} rest frames. This suggests that the experimentally clean $\tau^+\tau^-$ final state may be the proper instrument to study the parity of the Higgs boson.

The spin of the τ lepton is not directly observable but it manifests itself in the distributions of its decay products. Depending on the decay channel, the polarimetric strength is different. The first Monte Carlo program for e^+e^- colliders where the density matrix of the τ lepton pair was used was KORALB [14,15]. Let us recall some basic properties of that solution, which we have adopted to the case of the Higgs boson decay in Ref. [8]. The algorithm is organized in two steps. In the first step, the τ lepton pair is generated and the τ leptons are decayed in their respective rest frames as if there were no spin effects at all. In the second step, the spin weight is calculated and rejection is performed. If the event is rejected, only the generation of the τ lepton decays is repeated. The spin weight is given by the following formula

$$wt = \frac{1}{4} \left(1 + \sum_{i,j=1,3} R_{i,j} h^i h^j \right) \quad (2)$$

where, as a consequence of formula (1), the components $R_{3,3} = -1$, $R_{1,1} = \pm 1$, $R_{2,2} = \pm 1$ (respectively for scalar and pseudoscalar) and all other components are zero¹.

In the following, we focus on the $\tau^{\pm} \rightarrow \rho^{\pm} \nu$ decay channel. It is interesting because it has, by far, the largest branching ratio (25%). However, in comparison to $\tau^{\pm} \rightarrow \pi^{\pm} \nu$ decay, its polarimetric force is more than a factor of 2 smaller. It was found, see *e.g.* [16,17], that in many cases this can be improved if information on the ρ decay products, *i.e.* on details of the decay $\tau^{\pm} \rightarrow \pi^{\pm} \pi^0 \nu$, are used. This is of no surprise because the polarimetric vector is given by the formula

$$h^i = \mathcal{N} \left(2(q \cdot N) q^i - q^2 N^i \right) \quad (3)$$

¹ See Ref. [8] for detailed definition of the quantization frames used for the spins of the τ^+ and τ^- .

where \mathcal{N} is a normalization function, q is the difference of the π^\pm and π^0 four-momenta and N is the four-momentum of the τ neutrino (all defined in the τ rest frame) see, *e.g.* [18]. Obviously, any control on the vector q can be advantageous. It is of interest to note that in the τ lepton rest frame, when $m_{\pi^\pm} = m_{\pi^0}$ is assumed, the term

$$q \cdot N = (E_{\pi^\pm} - E_{\pi^0})m_\tau. \quad (4)$$

Thus, to exploit this part of the polarimetric vector, we need to have some handle on the difference of the π^\pm and π^0 energies in their respective τ leptons rest frames. Otherwise, the effect of this part of the polarimetric vector cancels out and one is left with the part proportional to the ρ (equivalently ν) momentum. Without using the energy difference we would arrive at a case nearly identical to the one where the τ decays to a single π except with the disadvantage that the coefficient diminishes the spin effects by more than a factor of 2.

Already from this preliminary discussion we realize that the appropriate observable must rely on a distribution constructed out of at least 4 four-vectors. The Monte Carlo method is already essential at the level of designing the observable. Our search for improvements of the results obtained in [4,8] began with a study of acollinearity distributions defined in the Higgs boson rest frame, when instead of the $\tau \rightarrow \pi\nu$ decay mode the mode $\tau \rightarrow \rho\nu$ was used. The difference between the scalar and pseudoscalar Higgs particle which was still visible for $\tau \rightarrow \pi\nu$ turned out to be practically invisible in the case of $\tau \rightarrow \rho\nu$ once detector smearings were introduced. We found these results rather discouraging, and we have turned our attention to another possibility. As inspiration we have used two methods, one used at LEP 1 for the measurement of the τ polarization and another one, proposed for low energy e^+e^- colliders, see *e.g.* [16,17].

2.1 The Monte Carlo

For the following discussion all the Monte Carlo samples have been generated with the TAUOLA library [18–20]. The PHOTOS [21, 22] Monte Carlo program could be used for generating radiative corrections in the decays of the Higgs boson and τ leptons, but it was switched off. For the production of the τ lepton pairs the Monte Carlo program PYTHIA 6.1 [13] is used.² The production process $e^+e^- \rightarrow ZH \rightarrow \mu^+\mu^-(q\bar{q})H$ has been chosen with a Higgs boson mass of 120 *GeV* and a center-of-mass energy of 500 *GeV*. The effects of initial state bremsstrahlung were included in the PYTHIA generation.

For the τ lepton pair decay with full spin effects included in the $H \rightarrow \tau^+\tau^-$, $\tau^\pm \rightarrow \rho^\pm \bar{\nu}_\tau(\nu_\tau)$, $\rho^\pm \rightarrow \pi^\pm \pi^0$ chain, the interface explained in Ref. [8] was used. It is an extended version of the standard universal interface of Refs. [9,10]. For the sake of confidence we have confirmed all numerical results presented in this paper with the second simulation using the PANDORA Monte Carlo generator [12].

² It was shown that the interface can work as well in the same manner with the HERWIG [23] generator.

3 The acoplanarity of the ρ^+ and ρ^- decay products

In this section we advocate a new observable where we ignore the part of the polarimetric vector proportional to the ρ (equivalently ν) momentum in the τ rest frame. We rely only on the part of the vector due to the differences of the π^\pm and π^0 momenta, which manifests the spin state of the ρ^\pm .

In the Higgs rest frame the ρ momentum represents a larger fraction of the Higgs's energy than the neutrino. Therefore, we abandon the reconstruction of the Higgs rest frame and instead we use the $\rho^+\rho^-$ rest frame which has the advantage that it is built only from directly visible decay products of the ρ^+ and ρ^- .³

We take for both ρ 's the $\pi^\pm\pi^0$ decay channel. In the rest frame of the $\rho^+\rho^-$ system we define the acoplanarity angle, φ^* , between the two planes spanned by the immediate decay products (the π^\pm and π^0) of the two ρ 's.

3.1 Defining an optimal variable

The variable φ^* alone does not distinguish the scalar and psuedoscalar Higgs. To do this we must go further. The $\tau \rightarrow \pi^\pm\pi^0\nu$ spin sensitivity is proportional to the energy difference of the charged and neutral pion (in the τ rest frame), see formula (4). We have to separate events into two zones, \mathcal{C} and \mathcal{D} ,

$$\mathcal{C} : \quad y_1 y_2 > 0$$

$$\mathcal{D} : \quad y_1 y_2 < 0$$

where,

$$y_1 = \frac{E_{\pi^+} - E_{\pi^0}}{E_{\pi^+} + E_{\pi^0}} ; \quad y_2 = \frac{E_{\pi^-} - E_{\pi^0}}{E_{\pi^-} + E_{\pi^0}}. \quad (5)$$

E_{π^\pm} and E_{π^0} are the π^\pm, π^0 energies in the respective τ^\pm rest frames. If the cuts are applied using the four-momenta from the generation level boosted to the τ rest frame without any smearing then we call them respectively \mathcal{C}_{bare} and \mathcal{D}_{bare} . If the cuts are applied using the smeared four-momenta boosted to the replacement τ^\pm rest frames (defined below), then they are called \mathcal{C}_{reco} and \mathcal{D}_{reco} .

In Fig. 1 we plot the distribution of φ^* , where the left hand plot contains the events where the energy difference between the π^+ and π^0 defined in the τ^+ rest frame is of the same sign as the energy difference of π^- and π^0 defined in τ^- rest frame (selection \mathcal{C}_{bare} and formula 5). The right hand plot contains the events with the opposite signs for the two energy differences (selection \mathcal{D}_{bare}). It can be seen that the differences between

³This may open the way to study the parity of the Higgs boson in hadron machines like the Tevatron and the LHC.

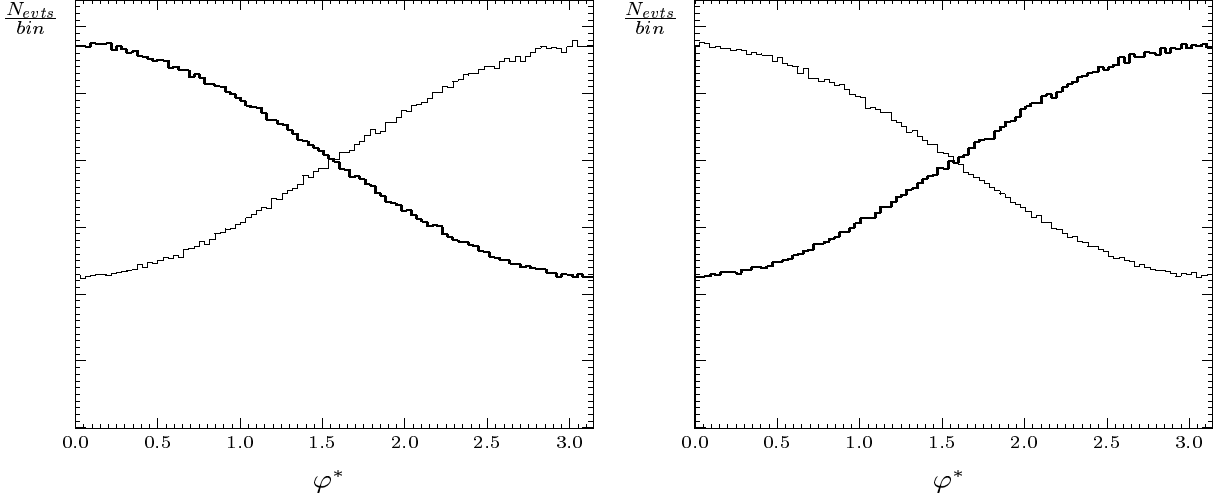


Figure 1: The $\rho^+\rho^-$ decay products' acoplanarity distribution angle, φ^* , in the rest frame of the $\rho^+\rho^-$ pair. A cut on the differences of the $\pi^\pm \pi^0$ energies defined in their respective τ^\pm rest frames to be of the same sign, selection \mathcal{C}_{bare} , is used in the left plot and the opposite sign, selection \mathcal{D}_{bare} , is used for the right plot. No smearing is done. Thick lines denote the case of the scalar Higgs boson and thin lines the pseudoscalar one.

the scalar and pseudoscalar Higgs boson are large. If the energy difference cut was not applied, we would have completely lost sensitivity to the Higgs boson parity.

Unfortunately, since the τ -lepton is not measurable, such a selection cut cannot be used directly. We now go on to define our choice for detection parameters. Then we will define realizable methods for making the energy cuts, \mathcal{C}_{reco} and \mathcal{D}_{reco} , and we will discuss phenomenologically sound results.

4 Detector Effects

To test the feasibility of the measurement, some assumptions about the detector effects have to be made. We include, as the most critical for our discussion, effects due to inaccuracies in the measurements of the π momenta. We assume Gaussian spreads of the 'measured' quantities with respect to the generated ones, and we use the following algorithm to reconstruct the energies of π 's in a measurable substitute for their respective τ^\pm rest frames.

1. **Charged pion momentum:** We assume a 0.1% spread on its energy and direction.
2. **Neutral pion momentum:** We assume an energy spread of $\frac{5\%}{\sqrt{E[\text{GeV}]}}$. For the θ and ϕ angular spread we assume $\frac{1}{3} \frac{2\pi}{1800}$. (These resolutions can be achieved with a

15% energy error and a $2\pi/1800$ direction error in the gammas resulting from the π^0 decays.)

3. **The replacement τ lepton rest frames and the π^\pm and π^0 energy differences:** We define replacements for the τ^\pm four-momenta in the $\rho^+\rho^-$ rest frame (which is a replacement for the Higgs rest frame). For the energy of each replacement τ four-momenta we use half of the Higgs boson mass. For the three-momenta we use the three-momenta of the ρ^\pm in the $\rho^+\rho^-$ rest frame and then scale them so the mass of the resulting four-vector is the τ mass. Finally, we boost the $\pi^+, \pi^0, \pi^-, \pi^0$ momenta to the respective rest frames of their replacement τ^+ and τ^- . The π energies defined this way are used in the \mathcal{C}_{reco} and \mathcal{D}_{reco} energy difference cuts.

In principle, when we compare predictions for scalar and pseudoscalar, we should take care of the possible differences in the production mechanisms. However, we have ignored this point in our study and use the same Higgsstrahlung production mechanism for both the scalar and pseudoscalar Higgs boson. The choice of a production mechanism is necessary to estimate basic properties only, in particular the size of the cross section. Except for that, our analysis and its conclusions are nearly independent of the production mechanism. We have checked that it is indeed the case by varying beam energies and choosing other production mechanisms as well.

4.1 The results of detector smearing

If we use the true, generator level, τ rest frame to define the energy differences between the ρ decay products but smear the momenta of the pions when used in the calculation of the acoplanarity angle, φ^\bullet , the resulting distributions (not shown) are very similar to the unsmeared case of Fig. 1. When we use the selection cuts \mathcal{C}_{reco} and \mathcal{D}_{reco} (and thereby use the replacement τ rest frames as well as smearing the π momenta) we obtain the results shown in Fig. 2. We see that the effects to be measured diminish but remain clearly visible⁴.

4.2 The potential measurement resolution

To determine the Higgs parity at an operating linear collider, a set of events will be collected over a period of time. An event selection will be made from the data set to isolate Higgsstrahlung events where the process $H/A \rightarrow \tau^+\tau^- \rightarrow \rho^+\bar{\nu}_\tau\rho^-\nu_\tau \rightarrow \pi^+\pi^0\bar{\nu}_\tau\pi^-\pi^0\nu_\tau$ occurred. This sample will be reconstructed and the distribution of the measured variable, φ^\bullet , defined in sections 3 and 5, will be compared with simulated reconstructed distributions for a scalar and a pseudoscalar Higgs such as those shown in Fig. 2. A goodness of fit test such as an unbinned maximum likelihood will be performed for both hypotheses.

⁴ We have studied several options for the definition of the separation cuts \mathcal{C}_{reco} and \mathcal{D}_{reco} . In one case we have directly used the smeared laboratory frame energies. In another, we have used all the information available from the reconstruction of the Higgs boson rest frame. All of these choices lead to practically identical versions of Fig. 2.

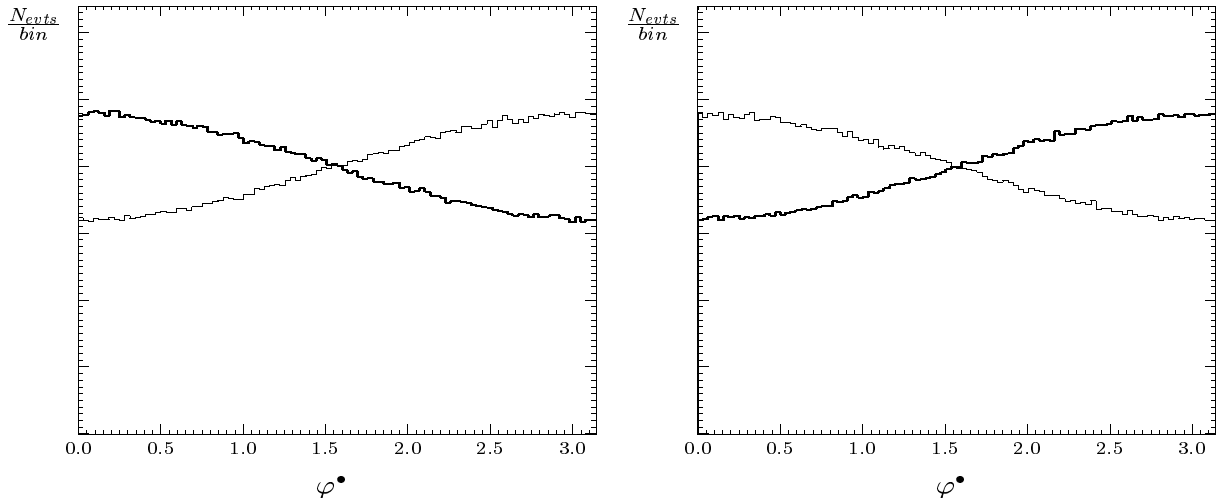


Figure 2: The $\rho^+\rho^-$ decay products' acoplanarity distribution angle, φ^\bullet , in the rest frame of the $\rho^+\rho^-$ pair. A cut on the differences of the $\pi^\pm \pi^0$ energies defined in their respective replacement τ^\pm rest frames to be of the same sign, selection \mathcal{C}_{reco} , is used in the left plot and the opposite sign, selection \mathcal{D}_{reco} , is used for the right plot. All smearing is included. Thick lines denote the case of the scalar Higgs boson and thin lines the pseudoscalar one.

One hypothesis will be favored over the other and statistical techniques will be applied to estimate the confidence level of the conclusion.

For present purposes, it is important to note that the level of certainty obtained will depend upon the specific sample collected. Another run under the same conditions with the same integrated luminosity would, in general, result in a sample of a different size and with different fit results due to the random sampling effects inherent in all measurements. Thus, we cannot a priori predict the level of certainty that will be achieved when the measurement is eventually made. However, we can simulate a large set of possible data samples that might be collected and gather some insight into the range of certainties possible.

We have applied a binned maximum likelihood technique to a set of approximately 1300 simulated data samples where half the samples were derived from a \mathcal{CP} even Higgs and half from a \mathcal{CP} odd Higgs. The Higgs mass was taken to be 120 GeV, the beam energy was 250 GeV per beam. The ρ^\pm were reconstructed from their gammas and π^\pm decay products. The π^\pm were smeared 0.1% in energy and direction, the gamma energy was smeared 15% and their direction by $2\pi/1800$, a typical calorimeter cell size. We assumed an integrated luminosity of 500 fb^{-1} . Beamsstrahlung and ISR effects are included. To account for detector acceptance effects, event selection efficiency and impurity we assumed an overall efficiency of 60%. Studies [24, 25] have shown these are realistic estimates of event selection efficiencies and purities.

Based on these assumptions, using a binned likelihood fit and comparing each data sample with the distributions for a \mathcal{CP} even and a \mathcal{CP} odd Higgs and comparing the resulting fits, we find that every data sample will identify the correct parent distribution with a confidence level of at least 95% and 86% of all samples will make the correct identification above the 3σ confidence level. Thus, we see that we have an excellent chance of correctly determining the \mathcal{CP} of the Higgs with this technique.

5 Summary

We have studied the possibility of distinguishing a scalar from a pseudoscalar light Higgs (*i.e.*, below the threshold of WW decay) using its decay to a pair of τ leptons and their subsequent decays to $\tau^\pm \rightarrow \nu_\tau \rho^\pm$ and $\rho^\pm \rightarrow \pi^\pm \pi^0$.

We have discussed an observable which is very promising. Using reasonable assumptions of \mathcal{SM} production cross section, and about measurement resolutions we find that with 500 fb^{-1} of luminosity at a 500 GeV e^+e^- linear collider the \mathcal{CP} of a 120 GeV Higgs can be measured to a confidence level greater than 95%. To confirm the method we have used two distinct Monte Carlo programs [12] and [8, 10] and the observables were coded independently. We emphasize that the technique is both model independent and independent of the Higgs production mechanism and depends only on good measurements of the Higgs decay products. Thus, this method may be applicable to other production modes including those available at proton colliders as well as at electron colliders.

Finally we note that several improvements are still possible. If, instead of unweighted events in our distributions as given in Fig. 2 we use events weighted with $weight = |y_1 y_2|$ (see formula 5) the statistical significance will increase⁵ by a factor of about 1.5. Other multi-meson final states such as $\tau \rightarrow \pi^+ \pi^+ \pi^- \nu$ can be used to increase the statistical samples. As argued in [26] they should lead to additional data of spin significance comparable to our $\tau \rightarrow \rho \nu$ channel (if the appropriate observables are found). Finally, we may expect that a measurement of the τ flight direction may turn out to be helpful, similarly as it was the case for the τ polarization measurement at LEP 1 [27, 28]. For each new production mode that can be analyzed using this technique the sample size increases accordingly. We may expect the final statistical significance for the parity measurement to be at least factor of 2-3 times better than in our conclusions. Such improvements will lead to more complex observables with many special cases and require good control of the systematic errors. We leave them to future studies but we think that, because of such possibilities, our estimation of the parity resolution will turn out to be largely conservative.

Acknowledgements

Discussions over the years with the experimental/phenomenological community analyzing τ lepton data were essential for creating the necessary background to start this work

⁵Instead of the weighted events we can fit 3 dimensional distributions spanned by the variables φ^* , y_1 , y_2 .

and we acknowledge the work of people involved in the TAUOLA project, in particular P. Golonka, S. Jadach, J. Kühn and E. Richter-Wąs. M. Peskin has played an essential role in this project and we also thank D. Muller for helpful discussions.

References

- [1] ACFA Linear Collider Working Group Collaboration, "*Particle physics experiments at JLC*", [hep-ph/0109166](#).
- [2] NLC Collaboration, "*2001 Report on the Next Linear Collider: A Report submitted to Snowmass 2001*", [SLAC-R-571](#).
- [3] F. Richard, J. Schneider, D. Trines, and A. Wagner, "*TESLA Technical Design Report Part I: Executive Summary*", [hep-ph/0106314](#).
- [4] M. Kramer, J. H. Kühn, M. L. Stong, and P. M. Zerwas, *Z. Phys.* **C64** (1994) 21, [hep-ph/9404280](#).
- [5] V. Barger, K. Cheung, A. Djouadi, B. A. Kniehl, and P. M. Zerwas, *Phys. Rev.* **D49** (1994) 79–90.
- [6] K. Hagiwara and M. Stong, *Z. Phys.* **C62** (1994) 99–108.
- [7] K. Hagiwara, S. Ishihara, J. Kamoshita, and B. Kniehl, *Eur. Phys. J.* **C14** (2000) 457–468.
- [8] Z. Wąs and M. Worek, "*Transverse spin effects in $H/A \rightarrow \tau^+\tau^-$; $\tau^\pm \rightarrow \nu X^\pm$, Monte Carlo approach*", [hep-ph/0202007](#).
- [9] T. Pierzchała, E. Richter-Wąs, Z. Wąs, and M. Worek, *Acta Phys. Polon.* **B32** (2001) 1277–1296, [hep-ph/0101311](#).
- [10] P. Golonka, T. Pierzchała, E. Richter-Wąs, Z. Wąs, and M. Worek, enlarged version of the document [hep-ph/0009302](#), in preparation, to be published in *Comput. Phys. Commun.*, see also <http://wasm.home.cern.ch/wasm/>
- [11] G. Bower, to be published in the proceedings of the Chicago LC Workshop, 7-9 January '02, transparencies available at: http://needmore.physics.indiana.edu/~rickv/nlc/chicago_agenda.html.
- [12] M. Iwasaki and M. E. Peskin, in preparation, see: http://www-sldnt.slac.stanford.edu/nld/new/Docs/Generators/PANDORA_PYTHIA.html.
- [13] T. Sjostrand *et al.*, *Comput. Phys. Commun.* **135** (2001) 238.
- [14] S. Jadach and Z. Wąs, *Acta Phys. Polon.* **B15** (1984) 1151, Erratum: **B16** (1985) 483.

- [15] S. Jadach and Z. Wąs, *Comput. Phys. Commun.* **36** (1985) 191.
- [16] J. L. Harton, *Nucl. Phys. Proc. Suppl.* **40** (1995) 463–473.
- [17] C. A. Nelson, *Nucl. Phys. Proc. Suppl.* **40** (1995) 525–540, [hep-ph/9411235](#).
- [18] S. Jadach, J. H. Kühn, and Z. Wąs, *Comput. Phys. Commun.* **64** (1990) 275.
- [19] M. Jeżabek, Z. Wąs, S. Jadach, and J. H. Kühn, *Comput. Phys. Commun.* **70** (1992) 69.
- [20] S. Jadach, Z. Wąs, R. Decker, and J. H. Kühn, *Comput. Phys. Commun.* **76** (1993) 361.
- [21] E. Barberio, B. van Eijk, and Z. Wąs, *Comput. Phys. Commun.* **66** (1991) 115.
- [22] E. Barberio and Z. Wąs, *Comput. Phys. Commun.* **79** (1994) 291–308.
- [23] G. Corcella, I. G. Knowles, G. Marchesini, S. Moretti, K. Odagiri, P. Richardson, M. H. Seymour, and B. R. Webber, *JHEP* **0101** (2001) 010, [hep-ph/0011363](#).
- [24] G. R. Bower, “*Proceedings of the Worldwide Study on Physics and Experiments with Future Linear e^+e^- Colliders*”, (1999) 1049-1057.
- [25] G. R. Bower, “*Physics and Experiments with Future Linear e^+e^- Colliders LCWS 2000*”, (2000) 920-923.
- [26] J. H. Kühn, *Phys. Rev.* **D52** (1995) 3128–3129, [hep-ph/9505303](#).
- [27] ALEPH Collaboration, A. Heister *et al.*, *Eur. Phys. J.* **C20** (2001) 401–430, [hep-ex/0104038](#).
- [28] I. Nikolic, CERN-THESIS-99-031.