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# A Novel Circular $TE_{01}$-Mode Bend for Ultra-High-Power Applications

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**Abstract— Future Linear Colliders and Accelerators require rf systems and components that are capable of handling hundreds of megawatts power levels at x-band frequencies and higher. Standard rf components that have been in use for a long time such as waveguide bends, directional couplers and hybrids, can not be used because of peak field considerations. Indeed, one has to reinvent most of these components taking into account the constraints imposed by ultra-high-power operation. Here, we present a new design for circular waveguides bends propagating the low-loss $TE_{01}$ mode. The bend has smooth walls and low field levels. We present a simple synthesis process for designing such device. The general philosophy of this technique can be applied to other components as well. We describe the detailed design of the bend and compare our design with finite element simulations and experimental data. The bend has very low ohmic losses, and the $TE_{01}$ mode is transmitted with virtually perfect mode purity.**

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## I. Introduction

Recently, ultra-high-power rf systems at X-band and above have received a lot of attention in different laboratories around the world because of the desire to design and construct a future linear collier. For a review of these activities the reader is referred to [1-2]. These systems are required to generate and manipulate hundreds of megawatts. Standard rf components that have been in use for a long time such as waveguide bends, directional couplers and hybrids, can not be used directly because of peak field considerations. Indeed, one has to reinvent most of these components while taking into account the constraints imposed by ultra-high-power operation. Usually most of these comports are made with oxygen-free high-conductivity copper and the operation takes place under ultra-high vacuum conditions. Experimental work at x-band showed that peak electric fields should not exceed 500 kV/cm[3]. Peak magnetic field should be limited so that the pulsed surface heating does not exceed 30 $C^o$ [4]

For these systems the $TE_{01}$ mode in circular waveguide is very attractive because it has low loss properties, and there are no electrical field lines that terminate on the walls of the waveguide. The low-loss $TE_{01}$ mode in circular waveguide has been utilized for several decades. It was used in several applications including communication systems, antenna feeds and rf systems for high energy accelerators.

Waveguide bends are important components for waveguide rf circuits and transport lines. The first analysis for bends in circular guides carrying the $TE_{01}$ mode is found in [5]. Since then several design concepts have been introduced to for these bends [6,7,8]. However, none of these designs are really suitable for ultra-high-power operations.

*Here, we present a new idea for perturbing the cross section of the waveguide smoothly to allow ultra high power operation*. There are several numerical techniques for the analysis of waveguide bends, see for example [9]. However, since there are several constraints on the design including mode purity, these are not convenient for the design process. We present a *simple synthesis* process for designing such bends that guarantees mode purity. We later verify our results with finite element simulations and experimental data.

## II. DESIGN METHODOLOGY AND SYNTHESIS PROCESS

The bend design is shown in Fig. 1. The bend comprises two circular to square tapers, two adiabatic tapers from square to rectangular cross section, and a connecting rectangular waveguide bend.

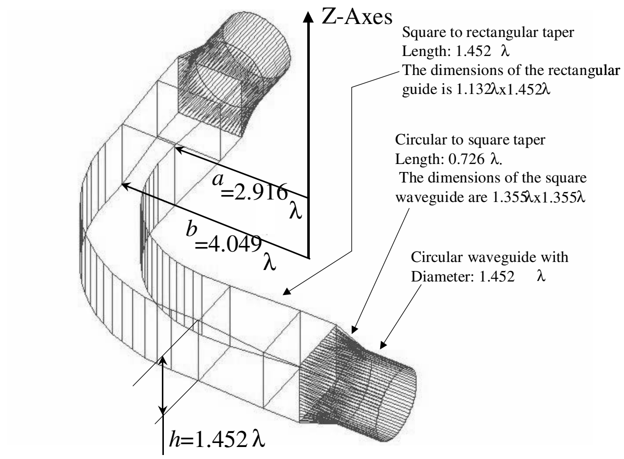


Figure 1. Bend Geometry. All dimensions are given in terms of free space wavelength.

### A. *The Circular to Square Taper*

In [10] it was noted that smooth transitions from rectangular to circular waveguide preserve their common reflection symmetries. The S-matrix of the transition connects modes of the same symmetry class, and for a sufficiently adiabatic transition preserves their TE (or TM) character. It is then also nonreflecting and, in the absence of degeneracy, its modal connections are one to one and order preserving. These properties enable us to carry out all of the RF manipulations in the more easily handled over-moded rectangular waveguide.

For the circular to square taper presented here the circular waveguide diameter is chosen such that all modes with cut-off frequency above that of the $TE_{01}$ mode do not propagate. The square waveguide is just large enough to allow both $TE_{20}$ and $TE_{02}$ modes to propagate. However, it does not allow the propagation of $TE_{22}$ and $TM_{22}$ modes. Because of reflection symmetries, only the two degenerate modes, $TE_{20}$ and $TE_{02}$, are exited in the rectangular guide when the $TE_{01}$ mode is incident in the circular waveguide. Using a finite element code, the design process for this taper is done by simply increasing the length until the reflection coefficient for the $TE_{01}$ mode in the circular guide is small enough. Because of degeneracy the combination between the two modes in the square waveguide could be regarded as one single mode. When exciting the circular guide with the $TE_{21}$ mode, again, it couples to the same two rectangular modes $TE_{02}$ and $TE_{20}$. However, the phase between them is a 180-degree different from the previous case, i.e., when they are excited with the $TE_{01}$ mode in the circular guide.

### *B. The Square To Rectangular Taper*

For reasons that will become clear in the next subsection we needed to perform the bend in a rectangular waveguide rather than a square one. To get to the cross section of the rectangular guide we used an adiabatic taper. The rectangular cross section of the guide still propagates both $TE_{02}$ and $TE_{20}$ modes. However, they are no longer degenerate. A straight rectangular cross section is inserted after the taper to adjust the phase difference between the $TE_{20}$ and the $TE_{02}$ modes

### *C. The Rectangular Bend*

The modes in a rectangular waveguide bend are well known, see for example [11]. The modes are referenced as TE or TM to the Z direction; where the Z direction is normal to the bend plane; see Fig. 1. For TE modes one can write the $z$ component of the magnetic field explicitly as

$$H_z(r,\phi,z) = A\left[Y_\nu'(k_t a)J_\nu(k_t r) - J_\nu'(k_t a)Y_\nu(k_t r)\right]\sin\left(\frac{n\pi}{h}z\right)e^{j\nu\varphi}; \qquad (1)$$

where $J_\nu(.)$ is the Bessel function of first kind and order $\nu$, $Y_\nu(.)$ is Neumann's function of order $\nu$, $a$ is the bend's inner radius, $h$ is the bend height, $n$ is an integer, $Y_\nu'$ and $J_\nu'$ means derivatives of these function with respect to the argument, and $k_t$ is the transverse wave number given by,

$$k_t^2 + \left(\frac{n\pi}{h}\right)^2 = \left(\frac{\omega}{c}\right)^2 \quad ; \qquad (2)$$

where $\omega$ is the rf angular frequency and $c$ is the speed of light in free space.

The incident field of the $TE_{20}$ mode in the rectangular guide has two variations along the Z-direction. Because the bend is uniform along the Z-direction the excited fields in the bend must respect the symmetries of the incident field, hence one must choose $n$ =2.

Eq. (1) indeed satisfies the boundary condition of $\left.\frac{\partial H_z}{\partial r}\right|_{r=a} = 0$. To satisfy the boundary conditions at the outer radius of the bend $b$, one must choose the order $\nu$ such that $\left.\frac{\partial H_z}{\partial r}\right|_{r=b} = 0$ ; i.e,

$$Y'_\nu(k_t a)J'_\nu(k_t b) - J'_\nu(k_t a)Y'_\nu(k_t b) = 0. \qquad (3)$$

Similarly for TM to Z modes one can write

$$H_z(r,\phi,z) = A[Y_\nu(k_t a)J_\nu(k_t r) - J_\nu(k_t a)Y_\nu(k_t r)]\cos\left(\frac{n\pi}{h}z\right)e^{j\nu\phi}. \qquad (4)$$

Applying similar arguments to the case of the TE modes, because the incident $TE_{20}$ mode has no variation along the Z-direction one must choose $n$=0. The boundary condition of $E_z|_{r=a} = 0$ is satisfied by Eq. (3). To satisfy the boundary conditions at the outer radius of the bend $b$, one must choose the order $\nu$ such that $E_z|_{r=b} = 0$ ; i.e,

$$Y_\nu(k_t a)J_\nu(k_t b) - J_\nu(k_t a)Y_\upsilon(k_t b) = 0 \qquad (5)$$

One can choose the dimensions of the bend such that only *two* TE modes with $n$=2 can propagate. At the same time, one can also choose these dimensions so that only *two* TM modes with $n$=0 can propagate. This gives us a total of *four* modes that can *propagate and are excited* because of an incident $TE_{01}$ mode in the circular guide.

Because of reciprocity, if the relative phases and amplitudes of these four modes are the same at the input and output of the rectangular bend, then one has to excite a pure $TE_{01}$ mode at the output circular guide if a pure $TE_{01}$ mode is exited at the input circular guide. The rectangular bend is relatively compact and the four modes will propagate through it with negligible loss, hence, the amplitudes of these modes are the same at the beginning and end. However, one must adjust the bend dimensions to phase them for proper recombination at the output

Let the four modes have azimuthal propagation constants $\nu_1$ through $\nu_4$. The total phase shift for each mode $i$ around the bend is given by $\nu_i\phi_0$; where $\phi_0$ is the bend angle. In this paper we give a design and experimental results for a $90^\circ$ bend, i.e., $\phi_0 = \pi/2$. The necessary and sufficient conditions for the relative phases to be equal at both ends of the bend are that the difference between the total phase shifts for all the modes be a multiples of $2\pi$. For a compact bend, with totally different propagation constants for all four modes this implies,

$$\nu_1\phi_0 = \nu_2\phi_0 + 2\pi = \nu_3\phi_0 + 4\pi = \nu_4\phi_0 + 6\pi \qquad (6)$$

Eq.(6) represents three conditions to be satisfied by $\nu_1$ through $\nu_4$. Since we have three degrees of freedom, namely the inner radius *a*, the outer radius *b* and the bend height *h*, one can hope, using Eqs. (3) and (4), to find a set of dimensions that satisfy Eq. (6). This is possible, and the dimensions are shown in Fig. 1. An HFSS[12] simulation of the taper is shown in Fig. 2 where the colors represents the relative electrical field strength.

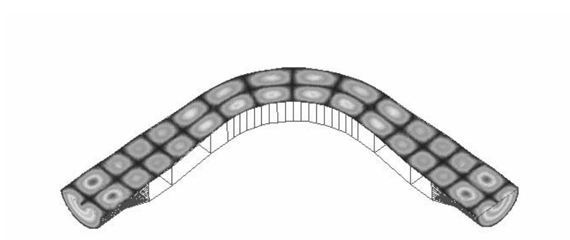

Figure 2. HFSS simulation shows a time snap shot of the relative electrical field strength at both circular ports and at the middle plane of the bend. Because of symmetry only one half of the bend is simulated

## III. EXPERIMENTAL RESULTS

We built four copies of the bend design shown in Fig.1. The dimensions were chosen so that the center frequency is 11.424 GHz; the Next Linear Collider (NLC) frequency [1]. The $TE_{01}$ mode was excited using a wrap-around mode converter [3]. We used an HP8510C network analyzer for these measurements. The results of the measured transmission through the bends are also shown in Fig. 3. This figure shows a near perfect transmission through this bend. For comparison, the results of the HFSS simulation are shown in this figure. We attribute the small differences to manufacturing tolerances and the measurement accuracy. The tapers were made with manufacturing tolerances of 100 microns. The measurement accuracy is mainly affected by the accuracy of the algorithm used to remove the mode converters response from the measurements. However, the level of losses in these bends is remarkably small, and the measurements agree well with the simulation.

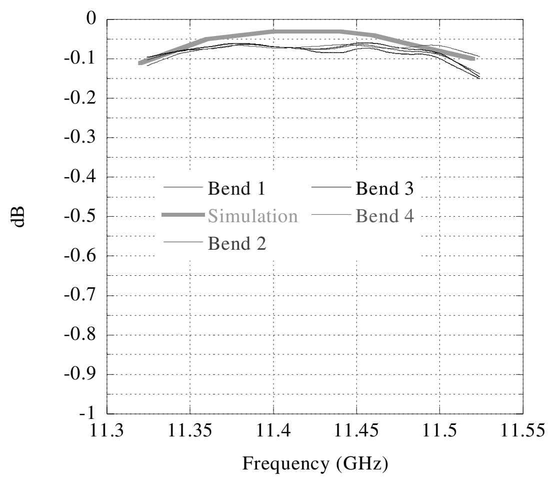


Fig. 3 Measured Transmission through two $TE_{01}$ mode converters and the 90 –degree bend. Center Frequency is 11.424 GHz, and the measurements are performed over a 1% bandwidth.

## IV. Conclusion

We presented a novel idea for a waveguide bend that supports the $TE_{01}$ mode in circular waveguides. This bend is suitable of ultar-high-power operations. Both simulations and experimental results showed very good results. The design methodology could be used to produce bends with deferent angles.

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