A UNITARY AND RENORMALIZABLE THEORY OF THE STANDARD MODEL IN GHOST-FREE LIGHT-CONE GAUGE [1]

Prem P. Srivastava [a,b,c] [2]

and

Stanley J. Brodsky [c] [3]

[a] *Instituto de Física, Universidade do Estado de Rio de Janeiro, RJ 20550*
[b] *Theoretical Physics Department, Fermilab, Batavia, IL 60510*
[c] *Stanford Linear Accelerator Center*
Stanford University, Stanford, California 94309

Abstract

Light-front (LF) quantization in light-cone (LC) gauge is used to construct a unitary and simultaneously renormalizable theory of the Standard Model. The framework derived earlier for QCD is extended to the Glashow, Weinberg, and Salam (GWS) model of electroweak interaction theory. The Lorentz condition is automatically satisfied in LF-quantized QCD in the LC gauge for the free massless gauge field. In the GWS model, with the spontaneous symmetry breaking present, we find that the 't Hooft condition accompanies the LC gauge condition corresponding to the massive vector boson. The two transverse polarization vectors for the massive vector boson may be chosen to be the same as found in QCD. The non-transverse and linearly independent third polarization vector is found to be parallel to the gauge direction. The corresponding sum over polarizations in the Standard model, indicated by $K_{\mu\nu}(k)$, has several simplifying properties similar to the polarization sum $D_{\mu\nu}(k)$ in QCD. The framework is ghost-free, and the interaction Hamiltonian of electroweak theory can be expressed in a form resembling that of covariant theory, except for few additional instantaneous interactions which can be treated systematically. The LF formulation also provides a transparent discussion of the Goldstone Boson (or Electroweak) Equivalence Theorem, as the illustrations show.

1   Introduction

The quantization of relativistic field theory at fixed light-front time $\tau = (t - z/c)/\sqrt{2}$, which was proposed by Dirac [1], has found important applications [2, 3, 4, 5] in gauge field theory, string theory [6], and M-theory [7], and it has become a useful alternative tool for the analysis of nonperturbative problems in quantum chromodynamics [8]. Light-front quantization has been employed in the nonabelian bosonization [9] of the field theory of N free Majorana fermions. The (nonperturbative) degenerate vacuum structures, the θ-vacua in the Schwinger model and their absence in the Chiral Schwinger model, were shown [10, 11] to follow transparently in the *front form* theory, along with the natural emergence in the former case of their continuum normalization. Also the requirement of the microcausality [12] implies that the LF framework is more appropriate for quantizing [13] the self-dual (chiral boson) scalar field.

LF quantization is especially useful for quantum chromodynamics, since it provides a rigorous extension of many-body quantum mechanics to relativistic bound states: the quark, and gluon momenta and spin correlations of a hadron become encoded in the form of universal process-independent, Lorentz-invariant wavefunctions [2]. The LF quantization of QCD in its Hamiltonian form thus provides an alternative to lattice gauge theory for the computation of nonperturbative quantities such as the spectrum as well as the LF Fock state wavefunctions of relativistic bound states [3].

We have recently presented a systematic study [14] of light-cone (LC) gauge LF-quantized QCD theory following the Dirac method [15, 16] and constructed the Dyson-Wick S-matrix expansion based on LF-time-ordered products. In our analysis [14] one imposes the light-cone gauge condition as a linear constraint using a Lagrange multiplier, rather than a quadratic form. We then find that the LF-quantized free gauge theory simultaneously satisfies the covariant gauge condition $\partial \cdot A = 0$ as an operator condition as well as the LC gauge condition. The resulting Feynman rule for the gauge field propagator in the LC gauge is doubly transverse

$$\left\langle 0 \right| T(A^a_{\ \mu}(x) A^b_{\ \nu}(0)) \left| 0 \right\rangle = \frac{i\delta^{ab}}{(2\pi)^4} \int d^4k \; e^{-ik\cdot x} \; \frac{D_{\mu\nu}(k)}{k^2 + i\epsilon} \tag{1}$$

where

$$D_{\mu\nu}(k) = -g_{\mu\nu} + \frac{n_\mu k_\nu + n_\nu k_\mu}{(n \cdot k)} - \frac{k^2}{(n \cdot k)^2} \, n_\mu n_\nu, \qquad n^\mu D_{\mu\nu} = k^\mu D_{\mu\nu} = 0,$$

and n_μ is the null four-vector, gauge direction. Thus only physical degrees of freedom propagate. The remarkable properties of (the projector) $D_{\nu\mu}$ provide much simplification in the computations of loop amplitudes. In the case of tree graphs, the term proportional to $n_\mu n_\nu$ cancels against the instantaneous gluon exchange term.

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The renormalization constants in the non-Abelian theory can be shown to satisfy the identity $Z_1 = Z_3$ at one-loop order, as expected in a theory with only physical gauge degrees of freedom. The QCD β function computed in the noncovariant LC gauge [14] agrees with the conventional theory result [17, 18]. Dimensional regularization and the Mandelstam-Leibbrandt prescription [19, 20, 21] for LC gauge were used to define the Feynman loop integrations [22]. Ghosts only appear in association with the Mandelstam-Liebbrandt prescription. There are no Faddeev-Popov or Gupta-Bleuler ghost terms.

In this paper we shall extend our LC gauge – LF quantization analysis to the Glashow, Weinberg and Salam (GWS) model of electroweak Interactions based on the nonabelian gauge group $SU(2)_W \times U(1)_Y$ [23]. It contains a nonabelian Higgs sector which triggers spontaneous symmetry breaking (SSB). A convenient way of implementing SSB and the (tree level) Higgs mechanism in the *front form* theory is known [24, 25, 26]. One separates the quantum fluctuation fields from the corresponding *dynamical bosonic condensate* (or zero-longitudinal-momentum-mode) variables, *before* applying the Dirac procedure in order to construct the Hamiltonian formulation[4]. This procedure by itself should determine in a *front form* theory if the condensate variable is a c- or a q- number (operator). In the description of SSB they are shown to be background constants. In the Schwinger model, in contrast, it is shown [10] to be an operator. Its occurrence in the model is crucial for showing, also in the LF framework, the degenerate vacuum structure (θ-vacua), known in the conventional theory since long time ago.

The tree level Lagrangian of the GWS model written in terms of the set of (tree level) parameters, for example, $(e, m_W, m_Z, m_h, m_u, m_d)$ is constructed and quantized on the LF. The model has the underlying initial gauge symmetry even after we rewrite it such that it bestows quadratic mass terms to some of the vector bosons. One is thus required to fix the gauge even when quantizing the theory in its spontaneously broken symmetry phase. For example, in the unitary gauge the Goldstone fields are gauged away, leaving behind only physical degrees of freedom. The resulting massive gauge field then carries the Proca propagator for which $D_{\mu\nu}(k) \rightarrow \left[-g_{\mu\nu} + \frac{k^\mu k^\nu}{M^2} \right]$ in (1). Because of the growing momentum dependence of the gauge propagator, the power counting renormalizability of the theory becomes very difficult to verify in this gauge. 't Hooft, however, demonstrated it by inventing the renormalizable R_ξ gauges [27, 28] and employing gauge-symmetry-preserving dimensional regularization. This framework, however, requires one to include in the theory Faddeev-Popov ghost fields, even in abelian theory, where the ghost fields couple to physical Higgs field as well. Several additional parameters ξ^γ, ξ^Z, ξ^W are introduced in the theory. Their renormalization must be taken also into account and the physical S-matrix elements should be shown not to depend on them.

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[4]See Appendix A and Ref. [3] for references to other alternative discussions on SSB.

In contrast, in the LC gauge LF-quantized theory framework for GWS model, there are no ghosts to consider, neither in the abelian nor in the nonabelian case. The massive gauge field propagator has good asymptotic behavior, the massive would-be Goldstone fields can be taken as physical degrees of freedom, and the proof of renormalizability becomes straightforward. Together with the previous work on QCD we thus obtain the simultaneous realization of unitary and renormalizable gauge, in our framework, for the Standard Model theory of strong and electro-weak interactions.

We start by considering in Section 2 the simpler case of the abelian Higgs model. The ingredients introduced here will be used later in the quantization of the non-abelian GWS model. The tree level Higgs Lagrangian when re-written in terms of the chosen tree level parameters e, M, and m_h still has the underlying gauge symmetry. We construct the LF Hamiltonian framework in the LC gauge, $A_- = 0$, following closely the procedure adopted in our paper on QCD. In the present case, where the gauge field mass M is generated by the Higgs mechanism, we find that the operator 't Hooft condition, $\partial \cdot A = M\,\eta$, where η is the would-be Goldstone field which also acquires the same mass, accompanies simultaneously the LC gauge condition. This is in contrast to the case of massless QCD where we have correspondingly the Lorentz condition.

The polarization vectors of the gauge field, which are all physical, are constructed, and their simplifying properties are discussed in detail. The interaction Hamiltonian which carries also an instantaneous term (derived in Appendix B) in the LF-quantized theory is constructed. The Fourier transform of the free gauge field, the propagators of the massive vector boson, the would-be Goldstone field, and the Higgs boson are derived. The LF quantization of the GWS model, which contains a nonabelian Higgs sector, is considered in Section 3.

Appendix B discusses a systematic procedure for constructing the instantaneous interaction terms required in the LF quantized field theory. It is illustrated by considering the Yukawa theory, abelian Higgs model, and QCD. In our LF framework A_+ and ψ_- are nondynamical and dependent field components. While taking care of the dependency, but without removing away these variables, we are able to recast the ghost free interaction Hamiltonian in a form close to that of covariant gauge theory. Despite a few additional instantaneous terms, it is straightforward to handle them in the Dyson-Wick expansion constructed in the LF-quantized theory. The nice properties of the gauge propagator turn it into a practical computational framework.

The Goldstone Boson (or Electroweak) equivalence theorem [29] becomes transparent in our framework. Its content is illustrated by the computation of Higgs bosons and top quark decays in Section 4. The computation of muon decay shows the relevance of the instantaneous interactions for recovering the manifest Lorentz invariance in the physical gauge [30] theory framework.

A new aspect of LF quantization, is that the third polarization of the quantized massive vector field A^μ with four momentum k^μ has the form $E_\mu^{(3)} = n_\mu M/n \cdot k$.

Since $n^2 = 0$, this non-transverse polarization vector has zero norm. However, when one includes the constrained interactions of the Goldstone particle, the effective longitudinal polarization vector of the vector particle is $E^{(3)}_{\text{eff}\,\mu} = E^{(3)}_\mu - k_\mu\, k \cdot E^{(3)}/k^2$ which is identical to the usual polarization vector of a massive vector with norm $E^{(3)}_{\text{eff}} \cdot E^{(3)}_{\text{eff}} = -1$. Thus, unlike the conventional quantization of the Standard Model, the Goldstone particle only provides part of the physical longitudinal mode of the electroweak particles.

2 The Quantization of the Abelian Higgs Model in LC Gauge

The implementation of spontaneous symmetry breaking (SSB) and the tree level Higgs mechanism on the LF have been understood for some time. A convenient description of SSB, which is useful for constructing [25] the tree-level Lagrangian in the Higgs model, is reviewed in Appendix A. The relevant differences in the LF quantized theory in the presence of SSB, when compared with the conventional theory treatment, may already be seen in the abelian Higgs model discussed below. The results obtained here will be utilized later in the quantization of the GWS model which carries in it a non-abelian Higgs sector (Section 3).

The abelian theory is described by

$$\mathcal{L} = -\frac{1}{4}\, F_{\mu\nu} F^{\mu\nu} + |\mathcal{D}_\mu \phi|^2 - V(\phi^\dagger \phi) \tag{2}$$

where ϕ is a complex scalar field, $\mathcal{D}_\mu = (\partial_\mu + ieA_\mu)$, and $V(\phi) = \mu^2\, \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2$ with $\lambda > 0$ and $\mu^2 < 0$. We choose the bosonic condensate, $< 0|\phi|0 >= v/\sqrt{2}$, to be real and separate it from the fluctuation field φ

$$\phi(x) = \frac{1}{\sqrt{2}}\, v + \varphi = \frac{1}{\sqrt{2}}\, ([\, v +\, h(x)\,] + i\eta(x)\,) \tag{3}$$

such that the real fields $h(x)$ and $\eta(x)$ carry vanishing vacuum expectation values.

The tree level Lagrangian, when the SSB is present, may be rewritten as

$$
\begin{aligned}
\mathcal{L} =\ & -\frac{1}{4}\, F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} M^2 A_\mu A^\mu + \frac{1}{2}(\partial_\mu \eta)^2 + M\, A^\mu \partial_\mu \eta + \frac{1}{2}(\partial_\mu h)^2 - \frac{1}{2}\, m_h^2\, h^2 \\
& +\ e\,(h\partial_\mu \eta - \eta\partial_\mu h)\, A^\mu + e\,M\, A_\mu A^\mu\, h + \frac{e^2}{2}(h^2 + \eta^2)\, A_\mu A^\mu \\
& -\ \frac{e\, m_h^2}{2\,M}\,(\eta^2 + h^2)\, h - \frac{\lambda}{4}\,(\eta^2 + h^2)^2 + const.
\end{aligned}
\tag{4}
$$

where e, M, and m_h indicate the tree level parameters defined by $M = e\,v$, $m_h^2 = 2\lambda v^2 = -2\mu^2$ indicating the physical squared mass of the Higgs field $h(x)$, $2\,\lambda\,v =$

$e\, m_h^2/M$, and $2\,\lambda = e^2\, m_h^2/M^2$. We note the presence of the mixed bi-linear term involving the Goldstone field η and the gauge field.

In view of the underlying local $U(1)$ gauge symmetry, one possible choice of the gauge may be taken to be such that the Goldstone mode η is eliminated, the so called "unitary (or unitarity) gauge", where only the physical fields appear in the Lagrangian. The gauge field is massive and its (Proca) propagator falls off more slowly than $1/k^2$ for large k. The perturbation theory renormalizability in this gauge is then not simple to demonstrate. The alternative of "renormalizability" or R_ξ gauges were introduced by 't Hooft [27]. The gauge-fixing term is here assumed to be $\mathcal{L}_{GF} = -(\partial \cdot A - \xi M\eta)^2/(2\xi)$. The bi-linear mixing of η and A_μ is then eliminated, and for any finite value of ξ, all of the propagators in this class of gauges fall off as $1/k^2$. The theory may also be shown [31] to be perturbatively renormalizable. We note, however, that in the Faddeev-Popov quantization procedure we are required to introduce also the auxiliary ghost fields in the theory with the corresponding piece in the Lagrangian $\mathcal{L}_{\text{Ghost}} = \bar{c}[-\partial\cdot\partial - \xi M^2(1+h/v)]c$, which contains the coupling of ghost fields with the physical Higgs field. In the nonabelian theory there are, in addition, the coupling of ghosts with the gauge field resulting from the term $\bar{c}^a(-\partial \cdot \mathcal{D})_{ab}c^b$.

In what follows we will quantize the *front form* theory described by the Lagrangian (3) in the LC gauge where the ghost fields are seen to decouple in both the nonabelian and abelian theories. The LF coordinates are defined as $x^\mu = (x^+ = x_- = (x^0 + x^3)/\sqrt{2}$, $x^- = x_+ = (x^0 - x^3)/\sqrt{2}$, $x^\perp)$, where $x^\perp = (x^1, x^2) = (-x_1, -x_2)$ are the transverse coordinates and $\mu = -, +, 1, 2$. The coordinate $x^+ \equiv \tau$ will be taken as the LF time, while x^- is regarded as the longitudinal spatial coordinate. The LF components of any tensor, for example, the gauge field, are similarly defined, and the metric tensor $g_{\mu\nu}$ may be read from $A^\mu B_\mu = A^+ B^- + A^- B^+ - A^\perp B^\perp$. Also k^+ indicates the longitudinal momentum, while k^- is the corresponding LF energy. Note that the LF Minkowski space coordinates are not related to the conventional ones, (x^0, x^1, x^2, x^3), by a finite Lorentz transformation.

We follow the arguments given in Ref. [14] and introduce auxiliary Lagrange multiplier field $B(x)$ carrying the canonical dimension three. The *linear* gauge-fixing term (BA_-) along with the ghost term $\bar{c}(-\partial \cdot \mathcal{D}_-)c$ are added to the Lagrangian (4) such as to ensure the Becchi-Rouet-Stora [32] symmetry of the action. The relevant free field propagators are thus determined from the following bi-linear terms in the action

$$
\int d^2x^\perp dx^- \quad \left\{ \frac{1}{2}\left[(F_{+-})^2 - (F_{12})^2 + 2F_{+\perp}F_{-\perp}\right] + BA_- \right.
$$
$$
+\frac{1}{2}M^2(2A_+A_- - A_\perp A_\perp) + M(A_+\partial_-\eta + A_-\partial_+\eta - A_\perp\partial_\perp\eta)
$$
$$
\left. +(\partial_+\eta)(\partial_-\eta) - \frac{1}{2}\partial_\perp\eta\partial_\perp\eta \right.
$$

$$+(\partial_+ h)(\partial_- h) - \frac{1}{2}\partial_\perp h \partial_\perp h - \frac{1}{2}m_h^2 h^2 + \cdots \Big\} \tag{5}$$

where we note that the fields A_+ as well as B have no kinetic terms, and they enter in the action as auxiliary Lagrange multiplier fields.

The canonical momenta following from (5) are $\pi^+ = 0$, $\pi_B = 0$, $\pi^\perp = F_{-\perp}$, $\pi^- = F_{+-} = (\partial_+ A_- - \partial_- A_+)$, $\pi_\eta = (\partial_- \eta + MA_-)$, and $\pi_h = \partial_- h$, which indicate that we are dealing with a constrained dynamical system. The Dirac procedure [1] will be followed in order to construct a self-consistent Hamiltonian theory framework, which is useful for the canonical quantization and in the study of the relativistic invariance. The canonical Hamiltonian density is

$$\begin{aligned}
\mathcal{H}_c &= \frac{1}{2}(\pi^-)^2 + \frac{1}{2}(F_{12})^2 - A_+(\partial_- \pi^- + \partial_\perp \pi^\perp + M^2 A_- + M\partial_- \eta) + \frac{1}{2}M^2 A_\perp A_\perp \\
&\quad + MA_\perp \partial_\perp \eta + \frac{1}{2}\partial_\perp h \partial_\perp h + \frac{1}{2}m_h^2 h^2 + \frac{1}{2}\partial_\perp \eta \partial_\perp \eta - BA_- + \cdots
\end{aligned} \tag{6}$$

The primary constraints are $\pi^+ \approx 0$, $\pi_B \approx 0$ and $\chi^\perp \equiv \pi^\perp - \partial_- A_\perp + \partial_\perp A_- \approx 0$, $\chi_\eta \equiv \pi_\eta - \partial_- \eta - MA_- \approx 0$, and $\chi_h \equiv \pi_h - \partial_- h \approx 0$, where $\approx$ stands for the *weak* equality relation. We now require the persistency in τ of these constraints employing the preliminary Hamiltonian, which is obtained by adding to the canonical Hamiltonian the primary constraints multiplied by undetermined Lagrange multiplier fields. In order to obtain the Hamilton's equations of motion, we assume initially the standard Poisson brackets for all the dynamical variables present in the theory.

We are then led to the following secondary constraints

$$\begin{aligned}
\Phi \equiv \partial_- \pi^- + \partial_\perp \pi^\perp + M\partial_- \eta &\approx 0, \\
A_- &\approx 0
\end{aligned} \tag{7}$$

which are already present in (5) multiplied by Lagrange multiplier fields. Requiring also the persistency of Φ and A_- leads to another secondary constraint

$$\Psi \equiv \pi^- + \partial_- A_+ \approx 0. \tag{8}$$

The procedure stops at this stage, and no more constraints are seen to arise, since further repetition leads to equations which would merely determine the multiplier fields.

We analyze now the nature of the LF phase space constraints derived above. In spite of the introduction of the gauge-fixing term, there still survives a first class constraint $\pi_B \approx 0$, while the other ones are second class. An inspection of the equations of motion shows that we may add [15] to the set found above an additional external constraint $B \approx 0$. This would make the whole set of constraints in the theory second class. Dirac brackets satisfy the property such that we can set the constraints as

strong equality relations inside them. The equal-τ Dirac bracket $\{f(x), g(y)\}_D$ which carries this property is straightforward to construct [15, 16]. Hamilton's equations now employ the Dirac brackets rather than the Poisson ones. The phase space constraints on the light front: $\pi^+ = 0$, $A_- = 0$, $\chi^\perp = 0$, $\chi_\eta = 0$, $\chi^h = 0$, $\Phi = 0$, $\Psi = 0$, $\pi_B = 0$, and $B = 0$ thus effectively eliminate B and all the canonical momenta from the theory. The surviving dynamical variables in LC gauge are found to be h, η and $A_\perp$, while A_+ is a dependent variable which satisfies $\partial_-(\partial_- A_+ - \partial_\perp A_\perp - M\eta) = 0$.

The canonical quantization of the theory at equal-τ is performed *via* the correspondence $i\{f(x), g(y)\}_D \to [f(x), g(y)]$, where the latter indicates the commutator (or ant-commutator) among the corresponding field operators. The equal-LF-time commutators of the transverse components of the gauge field are found to be

$$\left[A_\perp(\tau, x^-, x^\perp), A_{\perp'}(\tau, y^-, y^\perp) \right] = i\delta_{\perp\perp'} K(x, y)$$

where $K(x, y) = -(1/4)\epsilon(x^- - y^-)\delta^2(x^\perp - y^\perp)$. The commutators are nonlocal in the longitudinal coordinate, but there is no violation [13] of the microcausality principle on the LF. At equal LF-time, $(x - y)^2 = -(x^\perp - y^\perp)^2 < 0$, is nonvanishing for $x^\perp \neq y^\perp$, but $\delta^2(x^\perp - y^\perp)$ vanishes for such spacelike separation. The commutators of the transverse components of the gauge fields are physical, having the same form as the commutators of scalar fields in the *front form* theory. We find also

$$\begin{aligned}
\left[\eta(\tau, x^-, x^\perp), \eta(\tau, y^-, y^\perp) \right] &= iK(x, y) \\
\left[\eta(\tau, x^-, x^\perp), A_\perp(\tau, y^-, y^\perp) \right] &= 0
\end{aligned} \tag{9}$$

and some other nonvanishing ones

$$\begin{aligned}
\left[\partial_- A_+(\tau, x^-, x^\perp), \eta(\tau, y^-, y^\perp) \right] &= i\, M K(x, y) \\
\left[\partial_- A_+(\tau, x^-, x^\perp), A_\perp(\tau, y^-, y^\perp) \right] &= i\, \partial_\perp K(x, y) \\
\left[h(\tau, x^-, x^\perp), h(\tau, y^-, y^\perp) \right] = iK(x, y) \, . &
\end{aligned} \tag{10}$$

The structure of the commutators found in the LC gauge quantized theory on the LF indicates that in our framework the 't Hooft (gauge) condition, $\partial \cdot A - M\eta = 0$, is simultaneously incorporated as an operator equation, along with the LC gauge condition $A_- = 0$. This is in parallel to the result shown [14] in the earlier work on (massless) QCD where the Lorentz condition was found to be automatically incorporated. It gave rise there to the doubly transverse gauge field propagator which simplified greatly the computations of loop corrections and allowed for a transparent discussion of the renormalization theory and unitarity relations in the physical LC gauge.

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The *reduced* free LF Hamiltonian density in LC gauge, on making use of the constraints above, is shown to be

$$H_0{}^{LF} = \frac{1}{2}(\partial_\perp A_{\perp'})(\partial_\perp A_{\perp'}) + \frac{1}{2}M^2 A_\perp A_\perp + \frac{1}{2}(\partial_\perp \eta)(\partial_\perp \eta) + \frac{1}{2}M^2\eta^2 + \frac{1}{2}(\partial_\perp h)(\partial_\perp h) + \frac{1}{2}m_h^2 h^2$$
(11)

where the bi-linear cross terms are eliminated due to the presence of the 't Hooft condition in the framework.

The Hamilton's equations are found to lead to $(\partial \cdot \partial + M^2)A_\mu = 0$, $(\partial \cdot \partial + M^2)\eta = 0$ and $(\partial \cdot \partial + m_h^2)h = 0$. Taking into consideration the commutators among the field operators as derived above, we may write the momentum space expansions of the free (or interaction representation) field operators. Following the procedure parallel to that employed in Ref. [14] we may write

$$A^\mu(x) = \frac{1}{\sqrt{(2\pi)^3}} \int d^2k^\perp dk^+ \frac{\theta(k^+)}{\sqrt{2k^+}} \sum_{(\alpha)} E_{(\alpha)}{}^\mu(k) \left[a_{(\alpha)}(k^+, k^\perp)e^{-ik\cdot x} + a^\dagger_{(\alpha)}(k^+, k^\perp)e^{ik\cdot x} \right]$$
(12)

and

$$\eta(x) = \frac{1}{\sqrt{(2\pi)^3}} \int d^2k^\perp dk^+ \frac{\theta(k^+)}{\sqrt{2k^+}} \left[b(k^+, k^\perp)e^{-ik\cdot x} + b^\dagger(k^+, k^\perp)e^{ik\cdot x} \right] \ .$$
(13)

Here $k^2 = M^2$, $(\perp) = (1),(2)$, $(\alpha) = (\perp),(3)$, $a_{(\alpha)}(k) = a^{(\alpha)}(k)$, $a_{(3)}(k) = -ib(k)$, and the nonvanishing commutator $[a_{(\alpha)}(k), a^\dagger_{(\beta)}(l)] = \delta_{\alpha\beta}\, \delta^2(k_\perp - l_\perp)\, \delta(k^+ - l^+)$.

The three physical polarization vectors $E^\mu_{(\alpha)}(k) = E^{(\alpha)\mu}(k)$ of the massive gauge field (the mass arising through Higgs mechanism), satisfying $E^{(\alpha)}_-(k) = 0$, are constructed as follows. The two which are transverse to k_μ may be taken to be the same as defined in the earlier work on QCD, viz,

$$E^\mu_{(\perp)}(k) = E^{(\perp)\mu}(k) = -D^\mu_\perp(k)$$
(14)

with

$$D_{\mu\nu}(k) = D_{\nu\mu}(k) = -g_{\mu\nu} + \frac{n_\mu k_\nu + n_\nu k_\mu}{(n \cdot k)} - \frac{k^2}{(n \cdot k)^2} n_\mu n_\nu,$$
(15)

where the null four-vector n_μ indicates the gauge direction, whose components have been chosen conveniently to be $n_\mu = \delta_\mu{}^+$, $n^\mu = \delta^\mu{}_-$. We note that $E^{(\perp)}_+ = k^\perp/k^+$, $E^{(\perp)}_{\perp'} = g_{\perp\perp'} = -\delta_{\perp\perp'}$. They are also transverse to the gauge direction n_μ. The doubly transverse property [14] was very useful in the loop computations in QCD. We have

$$\sum_{(\perp)=1,2} E^{(\perp)}_\mu(k)E^{(\perp)}_\nu(k) = D_{\mu\nu}(k), \qquad\qquad g^{\mu\nu}E^{(\perp)}_\mu(k)E^{(\perp')}_\nu(k) = g^{\perp\perp'}$$
(16)

$$k^\mu E^{(\perp)}_\mu(k) = 0, \qquad\qquad n^\mu E^{(\perp)}_\mu \equiv E^{(\perp)}_- = 0$$
(17)

such that they are *spacelike* 4-vectors. The linearly independent non-transverse third polarization vector for the massive vector boson, in our LC gauge framework, is a *null* 4-vector being parallel to the gauge direction

$$E^{(3)}_\mu(k) = E_{(3)\mu}(k) = -\frac{M}{k^+}\,n_\mu, \qquad q\cdot E^{(3)}(k) = -M\frac{q^+}{k^+},$$
$$k\cdot E^{(\alpha)}(k) = -M\,\delta_{(\alpha)(3)}, \qquad E^{(3)}(k)\cdot E^{(\alpha)}(q) = 0 \tag{18}$$

such that

$$E^{(3)}_{\text{eff}\,\mu}(k) \equiv E^{(3)T} = E^{(3)}_\mu(k) - (k\cdot E^{(3)}(k))\,(k_\mu/k^2) = E^{(3)}_\mu(k) + M(k_\mu/k^2) \tag{19}$$

is *spacelike* and transverse to k_μ with $E^{(3)}_{\text{eff}}(k)\cdot E^{(3)}_{\text{eff}}(k) = -M^2/k^2 = -1$.

The sum over the three physical polarizations is given by $K_{\mu\nu}$

$$K_{\mu\nu}(k) = \sum_{(\alpha)} E^{(\alpha)}_\mu E^{(\alpha)}_\nu = D_{\mu\nu}(k) + \frac{M^2}{(k^+)^2}\,n_\mu n_\nu$$
$$= -g_{\mu\nu} + \frac{n_\mu k_\nu + n_\nu k_\mu}{(n\cdot k)} - \frac{(k^2 - M^2)}{(n\cdot k)^2}\,n_\mu n_\nu, \tag{20}$$

which satisfies: $k^\mu\,K_{\mu\nu}(k) = (M^2/k^+)\,n_\nu$ and $k^\mu\,k^\nu\,K_{\mu\nu}(k) = M^2$. We recall also [14]

$$D_{\mu\lambda}(k)D^\lambda{}_\nu(k) = D_{\mu\perp}(k)D^\perp{}_\nu(k) = -D_{\mu\nu}(k),$$
$$k^\mu D_{\mu\nu}(k) = 0, \qquad\qquad n^\mu D_{\mu\nu}(k) \equiv D_{-\nu}(k) = 0,$$
$$D_{\lambda\mu}(q)\,D^{\mu\nu}(k)\,D_{\nu\rho}(q') = -D_{\lambda\mu}(q)D^\mu_\rho(q'). \tag{21}$$

The expansion of the transverse components of the gauge field is then rewritten as

$$A_\perp(x) = -A^\perp = -\frac{1}{\sqrt{(2\pi)^3}}\int d^2k^\perp dk^+\,\frac{\theta(k^+)}{\sqrt{2k^+}}\left[a_{(\perp)}(k)e^{-ik\cdot x} + a^\dagger_{(\perp)}(k)e^{ik\cdot x}\right] \tag{22}$$

which, together with the independent (would be Goldstone) field η, describe the massive gauge field. It is convenient to also define the dependent gauge field component, A_+, by using the 't Hooft condition, $\partial\cdot A|_{A_-=0} = M\eta$ incorporated in our LC gauge framework. We find

$$A_+(x) = -\frac{1}{\sqrt{(2\pi)^3}}\int d^2k^\perp dk^+\,\frac{\theta(k^+)}{\sqrt{2k^+}}\left[a_{(+)}(k)e^{-ik\cdot x} + a^\dagger_{(+)}(k)e^{ik\cdot x}\right] \tag{23}$$

if $a_{(+)} = a^{(+)}$ is defined such that

$$k^+ a_{(+)}(k) = \left[k_\perp\,a_{(\perp)}(k) - iM\,b(k)\right] = \left[k_\perp a_{(\perp)}(k) + Ma_{(3)}\right]. \tag{24}$$

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while we set $a_{(-)}(k) = a^{(-)}(k) = 0$ in view of $A_- = 0$. The following nonvanishing commutator is straightforward to derive

$$\left[a_{(\mu)}(k),\, a^\dagger_{(\nu)}(l)\right] = K_{\mu\nu}(k)\delta^2(k_\perp - l_\perp)\delta(k^+ - l^+) \qquad (25)$$

where $\mu, \nu = -, +, \perp$. Following the standard procedure, the free propagator of the massive gauge field A_μ is found to be

$$\langle 0|T\left(A_\mu(x)A_\nu(y)\right)|0\rangle = \frac{i}{(2\pi)^4}\int d^4k \frac{K_{\mu\nu}(k)}{(k^2 - M^2 + i\epsilon)}\, e^{-i\,k\cdot(x-y)}. \qquad (26)$$

It does not have the bad high energy behavior found in the (Proca) propagator in the unitary gauge formulation, where the would-be Nambu-Goldstone boson is gauged away. For $M \to 0$ it reduces to the doubly transverse propagator found [14] in connection with the LF quantized QCD in the LC gauge.

The Higgs field $h(x)$ commutes with other field operators, and its propagator is $i/(k^2 - m_h{}^2 + i\epsilon)$. The commutation relations in (8) imply that the field η has an off-diagonal nonvanishing propagator with the component A_+, viz., $\langle 0|T\left(\eta(x)A_+(y)\right)|0\rangle \neq 0$. The $\eta\,\eta$ propagator is given by $i/(k^2 - M^2 + i\epsilon)$. If we use the ML prescription to handle the $1/k^+$ singularity along with the dimensional regularization, the general power-counting analysis becomes available [14]. The propagators in the framework have good asymptotic behavior; the divergences encountered are no worse than in QED. The proof of perturbative renormalizability in the LC gauge in the *front form* quantized theory presented here may be given straightforwardly along the lines performed earlier in the conventional [31] equal-time theory. In view of the simplifying properties of $K_{\mu\nu}$ (and $D_{\mu\nu}$), the absence of ghost fields, and the availability of the power counting rules, when we employ the dimensional regularization along with ML prescription, the effort required in our framework is comparable, as in the case of the previous work on QCD, to that in the conventional theory computations.

Some comments on the polarization vectors in LC gauge are in order. With the restriction $E^{(\alpha)}_- = 0$ there are only three linearly independent polarization vectors[5] as discussed above. $E^{(\perp)}_\mu(k)$ are transverse with respect to both n_μ and k_μ while the non-transverse $E^{(3)}_\mu(k)$ is parallel to the gauge direction n_μ, being equal to the sum of a transverse piece (T) ($\equiv E_{\text{eff}}$) and a longitudinal one (L), when referred to the 4-vector k_μ:

$$E^{(\alpha)T}_\mu(k) = (g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2})\, E^{(\alpha)\nu}(k), \qquad k \cdot E^{(\alpha)T}(k) = 0$$

$$E^{(\alpha)L}_\mu(k) = \frac{k_\mu}{k^2}\,(k \cdot E^{(\alpha)}(k)) = -M\frac{k_\mu}{k^2}\,\delta_{(\alpha)(3)},$$

$$k \cdot E^{(\alpha)L}(k) = k \cdot E^{(\alpha)}(k) = -M\delta_{(\alpha)(3)}, \qquad E^{(\alpha)T}(k) \cdot E^{(\beta)L}(k) = 0 \qquad (27)$$

---

[5]It is easily shown that n_μ, n^*_μ, $E^{(\perp)}_\mu(k)$, where $n^*_\mu = \delta^-_\mu$ is the null vector dual to $n_\mu = \delta^+_\mu$ constitute a convenient basis for 4-vectors in the LF theory.

such that $E_\mu^{(\perp)T}(k) = E_\mu^{(\perp)}(k)$, $E_\mu^{(\perp)L}(k) = 0$, $E_\mu^{(3)L}(k) = -M\,(k_\mu/k^2)$, $E_\mu^{(3)T}(k) = M\,(k_\mu/k^2 - n_\mu/k^+)$, and $E_-^{(3)\,L,T}(k) \neq 0$, $E^{(3)\,L}(k) \cdot E^{(3)\,L}(k) = M^2/k^2 = +1$.

The following analogous decomposition of $K_{\mu\nu}$ is useful in computations

$$K_{\mu\nu}(k) = K_{\mu\nu}^T(k) + K_{\mu\nu}^L(k) \tag{28}$$

where[6]

$$
\begin{aligned}
K_{\mu\nu}^L(k) &= (\frac{M^2}{k^2})\, d_{\mu\nu}(k) \\[2mm]
K_{\mu\nu}^T(k) &= K_{\mu\nu}(k) - K_{\mu\nu}^L(k) = D_{\mu\nu}(k) + M^2 \left(\frac{n_\mu n_\nu}{(n \cdot k)^2} - \frac{d_{\mu\nu}(k)}{k^2} \right) \\[2mm]
&= (k^2 - M^2) \left[\frac{d_{\mu\nu}(k)}{k^2} - \frac{n_\mu n_\nu}{k^{+2}} \right]
\end{aligned} \tag{29}
$$

where

$$d_{\mu\nu}(k) = -g_{\mu\nu} + \frac{n_\mu k_\nu + n_\nu k_\mu}{(n \cdot k)}, \qquad k^\mu\, d_{\mu\nu}(k) = \frac{k^2}{k^+} n_\nu, \qquad k^\mu k^\nu\, d_{\mu\nu}(k) = k^2\ . \tag{30}$$

They are symmetric and some interesting properties are $K_{\mu-}^L(k) = K_{\mu-}^T(k) = d_{\mu-}(k) = 0$, $k^\mu\, K_{\mu\nu}^T(k) = 0$, $k^\nu\, K_{\mu\nu}^T(k) = 0$, $k^\mu\, K_{\mu\nu}(k) = k^\mu\, K_{\mu\nu}^L(k) = (M^2/k^+)n_\nu$, $k^\mu k^\nu\, K_{\mu\nu}(k) = k^\mu k^\nu\, K_{\mu\nu}^L(k) = M^2$. From the properties of $D_{\mu\nu}(k)$ we easily derive

$$K_{\mu\rho}(k)K_\nu^\rho(k) = d_{\mu\rho}(k)d_\nu^\rho(k) = -D_{\mu\nu}(k) \tag{31}$$

and

$$
\begin{aligned}
K_{\mu\rho}^L(k)\, K_\nu^{T\rho}(k) &= -\frac{M^2(k^2 - M^2)}{(k^2)^2}\, D_{\mu\nu}(k), \\[2mm]
K_{\mu\rho}^L(k)\, K_\nu^{L\rho}(k) &= -\frac{M^4}{(k^2)^2}\, D_{\mu\nu}(k), \\[2mm]
K_{\mu\rho}^T(k)\, K_\nu^{T\rho}(k) &= -\frac{(k^2 - M^2)^2}{(k^2)^2}\, D_{\mu\nu}(k)\ .
\end{aligned} \tag{32}
$$

For completeness we note that

$$\sum_{(\alpha)} \left[E_\mu^{(\alpha)L} E_\nu^{(\alpha)L} + E_\mu^{(\alpha)L} E_\nu^{(\alpha)T} + E_\mu^{(\alpha)T} E_\nu^{(\alpha)L} \right] = K_{\mu\nu}^L(k) + \frac{M^2}{k^2} \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \tag{33}$$

while

$$\sum_{(\alpha)} E_\mu^{(\alpha)T}(k)\, E_\nu^{(\alpha)T}(k) = K_{\mu\nu}^T(k) - \frac{M^2}{k^2} \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right)\ . \tag{34}$$

---

[6] $K_{\mu\nu}^L(k) \neq \sum_{(\alpha)} E_\mu^{(\alpha)L}(k)\, E_\nu^{(\alpha)L}(k)$.

2.1 The Interaction Hamiltonian

The interaction Hamiltonian, in LC gauge $A_- = 0$, is derived to be

$$
\begin{aligned}
- \quad \mathcal{H}_{int} &= \mathcal{L}_{int} \\
= \quad & e\, M\, A_\mu A^\mu\, h - \frac{e\, m_h^2}{2\, M}\, (\eta^2 + h^2)\, h + e\, (h\, \partial_\mu \eta - \eta\, \partial_\mu h)\, A^\mu + \frac{e^2}{2}(h^2 + \eta^2)\, A_\mu A^\mu \\
- \quad & \frac{\lambda}{4}\, (\eta^2 + h^2)^2 - \frac{e^2}{2}\left(\frac{1}{\partial_-} j^+\right)\left(\frac{1}{\partial_-} j^+\right)
\end{aligned}
\tag{35}
$$

where $j_\mu = (h\, \partial_\mu \eta - \eta \partial_\mu h)$. The last term here is the additional quartic instantaneous interaction in the LF theory quantized in the LC gauge (Appendix B). No new instantaneous cubic interaction terms are introduced. The massive gauge field, when the mass is generated by the Higgs mechanism, is described in our LC gauge framework by the independent fields $A_\perp$ and η; the component A_+ is dependent one.

3 The GWS Model of Electroweak Interactions

3.1 The Quantization of the $SU(2) \otimes U(1)$ Non-Abelian Higgs Model in LC Gauge

A condensed review of the GWS model will be given below to define our notation. The model constructs a unified description of the electromagnetic and weak interactions by employing the spontaneously broken gauge theory based on the nonabelian gauge group $SU_W(2) \otimes U_Y(1)$, the direct product of the *weak* isospin and the abelian *hypercharge* groups. The corresponding hermitian generators are ($\vec{t}$ and t_Y) respectively with $\vec{t} = (t_1, t_2, t_3)$, and $t_Y = Y\, I$. Here $\vec{t}$ are isospin generators, I is the identity matrix, and Y indicates the *hypercharge*. For the spontaneous breaking a complex scalar field, Higgs doublet Φ, in the iso-spinor representation, with $t = 1/2$, $\vec{t} = \vec{\sigma}/2$, is introduced

$$
\Phi = \begin{pmatrix} G^+ \\ \chi^o \end{pmatrix} .
\tag{36}
$$

The value $Y(\Phi) = 1/2$ is assigned to it by convention such that the upper component G^+ corresponds to the unit eigenvalue of the $(U(1)_{em}$ or Charge) generator $Q = (t_3 + Y)$ and the lower one to the value zero. Under $SU_W(2) \otimes U_Y(1)$ it transforms as

$$
\Phi(x) \rightarrow e^{ig\, \vec{t}\cdot\vec{\alpha}(x)}\, e^{ig'\, t_Y\, \alpha_Y(x)}\ \Phi(x)
\tag{37}
$$

where g and g' indicate the two gauge coupling constants while $\alpha_a(x)$ are the gauge transformation parameters. The gauge covariant derivative may be defined as

$$
\mathcal{D}_\mu = (I\, \partial_\mu - ig\, \vec{A}_\mu \cdot \vec{t} - ig'\, Y\, I\, B_\mu)
\tag{38}
$$

where $\vec{A}_\mu$ and B_μ are real valued gauge fields.

The nonabelian gauge theory Lagrangian is written as

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} - \frac{1}{4} F_{\mu\nu}^Y F_Y^{\mu\nu} + (\mathcal{D}_\mu \Phi)^\dagger \mathcal{D}_\mu \Phi - V(\Phi^\dagger \Phi) \tag{39}$$

where the gauge invariant scalar potential contains, at most, quartic terms in Φ, so that the theory is renormalizable

$$V(\Phi) = \mu^2 \, \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2 \tag{40}$$

where $\lambda > 0$ and $\mu^2 < 0$. The gauge field strengths are $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g \, f_{abc} A_\mu^b A_\nu^c$ where $a, b, c = 1, 2, 3$ are the $SU(2)$ gauge group indices, $f_{abc} \equiv \epsilon_{abc}$, while $F_{\mu\nu}^Y = \partial_\mu B_\nu - \partial_\nu B_\mu$.

The description [24] of SSB in the abelian case (Appendix A) can be extended to the nonabelian one straightfordwardly. It may be shown [25] here too that none of the symmetry generators break the LF vacuum symmetry, but the expression which counts the number of Goldstone bosons is found to be identical to the one in the conventional theory [28]. On the LF the tree level theory of the non-abelian Higgs mechanism is straightforward to construct [25]. Its quantization in the LC gauge parallels closely to that of the abelian Higgs theory.

It is convenient again to introduce real fields h, ϕ_1, ϕ_2, $\phi_3 \equiv G^o$ which have vanishing vacuum expectation values and write

$$G^+ \equiv -i \, \phi^- = -\frac{i}{\sqrt{2}} \, (\, \phi_1(x) - i \, \phi_2(x))$$

$$\chi^o = \frac{v}{\sqrt{2}} + \frac{1}{\sqrt{2}} \, (\, h(x) + i \, G^o(x) \,) \tag{41}$$

where $v = \sqrt{-\mu^2/\lambda}$. In other words $\Phi = \Phi_{cl} + \varphi$ such that

$$\Phi_{cl} \equiv \langle 0|\Phi|0\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \tag{42}$$

which is taken to be the classical vacuum configuration[7] in the SSB case when $\mu^2 < 0$. This parameterization of Φ_{cl} can always be assumed if we make use of the (global) symmetry of the action under $SU_W(2)$ and $U_Y(1)$. We verify that $t_a \, \Phi_{cl} \neq 0$ but $Q\Phi_{cl} \equiv (t_3 + Y) = 0$ where the linear combination Q is the generator of the unbroken residual $U(1)_{em}$ symmetry. We note also that $\Phi^\dagger \Phi = (\phi_1^2 + \phi_2^2 + \phi_3^2 + \sigma^2)/2$ where, $\sigma = (\, v + h(x) \,)$. The potential V defined above is invariant under the larger $O(4) \approx$

---

[7]The stability of the asymmetric solution while the instability of the symmetric one may be inferred from the study of the dynamical (partial differential) equations of motion as usual.

$SU(2) \times SU(2)$ symmetry, which is broken by the field σ when it acquires a non zero vacuum expectation value.

The gauge field combinations $(W^{\pm}_\mu, Z)$ and photon A_μ (see below) are useful

$$
\begin{aligned}
W^{\pm}_\mu &= \frac{1}{\sqrt{2}}(A^1_\mu \mp i A^2_\mu) \\
Z_\mu &= (A^3_\mu \cos\theta_W - B_\mu \sin\theta_W) \\
A_\mu &= (B_\mu \cos\theta_W + A^3_\mu \sin\theta_W) \, .
\end{aligned}
\tag{43}
$$

Here θ_W is the Weinberg angle such that $g \sin\theta_W = g' \cos\theta_W = e$ and e is the electronic charge. The gauge covariant derivative may be conveniently re-expressed as

$$
\mathcal{D}_\mu = \partial_\mu - i\frac{g}{\sqrt{2}}\left(W^+_\mu\, t_+ + W^-_\mu\, t_-\right) - i\frac{g}{\cos\theta_W} Z_\mu\,(t_3 - Q\sin^2\theta_W) - i\,e\,Q\,A_\mu \tag{44}
$$

where $Q = (t_3 + Y)$ indicates the electric charge and $t_\pm = (t_1 \pm i\, t_2) = (\sigma_1 \pm i\,\sigma_2)/2$. We find

$$
\mathcal{D}_\mu\Phi = \begin{pmatrix} \partial_\mu G^+ - im_W W^+_\mu - i[\frac{g\cos(2\theta_W)}{2\cos\theta_W} Z_\mu + eA_\mu]G^+ - \frac{ig}{2}W^+_\mu(h + iG^o) \\[2mm] \frac{1}{\sqrt{2}}\partial_\mu(h + iG^o) + \frac{ig}{\sqrt{2}}m_Z Z_\mu - \frac{ig}{\sqrt{2}}W^-_\mu G^+ + \frac{ig}{\sqrt{2}}\frac{1}{2\cos\theta_W}Z_\mu(h + iG^o) \end{pmatrix} \tag{45}
$$

while $(\mathcal{D}^\mu\Phi)^\dagger \mathcal{D}_\mu\Phi =$

$$
|\partial_\mu G^+ - im_W W^+_\mu - i[\frac{g\cos(2\theta_W)}{2\cos\theta_W} Z_\mu + eA_\mu]G^+ - \frac{ig}{2}W^+_\mu(h + iG^o)|^2
$$

$$
+ \frac{1}{2}|\partial_\mu(h + iG^o) + ig\,m_Z Z_\mu - ig\,W^-_\mu G^+ + ig\,\frac{1}{2\cos\theta_W}Z_\mu(h + iG^o)|^2 \, . \tag{46}
$$

Also

$$
\begin{aligned}
V &= \frac{1}{2}m_h^2 h^2 + 2\lambda v\left[G^+G^- + \frac{1}{2}(G^{o2} + h^2)\right]h + \lambda\left[G^+G^- + \frac{1}{2}(G^{o2} + h^2)\right]^2 \\
&= \lambda\left[G^+G^- + \frac{1}{2}(G^{o2} + h^2) + v\,h + \frac{v^2}{2} + \frac{\mu^2}{2\lambda}\right]^2
\end{aligned}
\tag{47}
$$

where we set $m_W = gv/2$, $m_Z = m_W/\cos\theta_W$ indicating the vector boson masses. Interaction vertices are the cubic and quartic terms in these expressions. For example, the cubic Higgs boson interaction with charged vector bosons is

$$
\left[g\,m_W\, W^-_\mu W^{+\mu} - i\frac{g}{2}[(\partial_\mu G^-)W^{+\mu} - (\partial_\mu G^+)W^{-\mu}] + 2\lambda v\, G^+G^-\right]h. \tag{48}
$$

The quadratic terms in the bosonic Lagrangian which define the free theory are

$$
-\frac{1}{4}\,(\partial_\mu A_\nu - \partial_\nu A_\mu)^2
$$
$$
-\frac{1}{4}\,(\partial_\mu Z_\nu - \partial_\nu Z_\mu)^2 \;\; + \;\; \frac{1}{2}\,m_Z^2\,Z_\mu Z^\mu + \frac{1}{2}(\partial^\mu G^o)\,\partial_\mu G^o + m_Z\,Z_\mu\,\partial^\mu G^o
$$
$$
-\frac{1}{2}\,(\;\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+\;)(\;\partial^\mu W^{-\nu} - \partial^\nu W^{-\mu}\;) + m_W^2\,W_\mu^- W^{+\mu}
$$
$$
+ \;\; (\partial_\mu G^-)\,\partial^\mu G^+ - i\,m_W[(\partial_\mu G^-)\,W^{+\mu} - (\partial_\mu G^+)W^{-\mu}]
$$
$$
+ \;\; \frac{1}{2}(\partial^\mu h)\,\partial_\mu h - \frac{1}{2}\,m_h^2\,h^2 \; . \tag{49}
$$

No mass terms arise for the (Goldstone) fields $G^\pm$ and G^o or for the photon field A_μ. We note the *tree level* relations $(m_h/m_W)^2 = 8\lambda/g^2$ and $m_h^2/m_W = (4/g)\,\lambda\,v$, $m_W^2/(m_Z^2 \cos^2\theta_W) = 1$, $(v/\sqrt{2}) = (\sqrt{8}G_F)^{-1/2} \approx 174$ GeV, and $G_F/\sqrt{2} = g^2/(8m_W^2) = 1/(2\,v^2)$. The bi-linear terms corresponding to the charged fields may be rewritten in terms of the real field components as[8]

$$
-\frac{1}{4}\,\left(\partial_\mu A_\nu^1 - \partial_\nu A_\mu^1\right)^2 \;\; + \;\; \frac{1}{2}\,m_W^2\,A_\mu^1 A^{1\mu} + \frac{1}{2}(\partial^\mu \phi_1)\,\partial_\mu \phi_1 + m_W\,A_\mu^1\,\partial^\mu \phi_1
$$
$$
-\frac{1}{4}\,\left(\partial_\mu A_\nu^2 - \partial_\nu A_\mu^2\right)^2 \;\; + \;\; \frac{1}{2}\,m_W^2\,A_\mu^2 A^{2\mu} + \frac{1}{2}(\partial^\mu \phi_2)\,\partial_\mu \phi_2 + m_W\,A_\mu^2\,\partial^\mu \phi_2 \; . \tag{51}
$$

The quantization in the LC gauge, $A_- = Z_- = W_-^\pm = 0$, is now straightforward. We take over the discussion in Section 2 on the abelian Higgs theory and the one given in the earlier paper [14] on QCD for the massless gauge field. For comparison, we recall that the conventional R_ξ gauges in the equal-time framework requires us to include in the theory also the ghost fields, which interact with the Higgs and other physical fields. Moreover, W_μ, Z_μ, and A_μ may carry different parameters ξ^W, ξ^Z, and ξ^γ respectively in the gauge-fixing terms. The renormalization of these parameters also has to be taken into consideration, and it is required to show that the physical amplitudes do not depend on them. The LC gauge framework being discussed contains no ghost fields. The 't Hooft conditions corresponding to the massive vector bosons read as: $\partial \cdot W^\pm = \pm i\,m_W\,G^\pm$, $\partial \cdot Z = m_Z\,G^o$, while for the massless field we obtain [14] the Lorentz condition $\partial \cdot A = 0$. The momentum space

---

[8]

$$
\frac{1}{\sqrt{2}}\,(F_{\mu\nu}^1 \mp i\,F_{\mu\nu}^2) \;=\; \partial_\mu W_\nu^\pm - \partial_\nu W_\mu^\pm \pm i\,g\,(W_\mu^\pm A_\nu^3 - W_\nu^\pm A_\mu^3)
$$
$$
F_{\mu\nu}^Y \;=\; [(\partial_\mu A_\nu - \partial_\nu A_\mu)\cos\theta_W - (\partial_\mu Z_\nu - \partial_\nu Z_\mu)\sin\theta_W]
$$
$$
(\partial_\mu A_\nu^3 - \partial_\nu A_\mu^3) \;=\; [(\partial_\mu Z_\nu - \partial_\nu Z_\mu)\cos\theta_W + (\partial_\mu A_\nu - \partial_\nu A_\mu)\sin\theta_W] \tag{50}
$$

expansions of the quantized field operators are easily found to be

$$A^\mu(x) = \frac{1}{\sqrt{(2\pi)^3}} \int d^3k \frac{\theta(k^+)}{\sqrt{2k^+}} \sum_{(\perp)} E^\mu_{(\perp)}(k) \left[a_{(\perp)}(k)e^{-ik\cdot x} + a^\dagger_{(\perp)}(k)e^{ik\cdot x} \right]$$

$$W^+_\mu(x) = \frac{1}{\sqrt{(2\pi)^3}} \int d^3k \frac{\theta(k^+)}{\sqrt{2k^+}} \sum_{(\alpha)} E^\mu_{(\alpha)}(k) \left[a^W_{(\alpha)}(k)e^{-ik\cdot x} + b^{W\dagger}_{(\alpha)}(k)e^{ik\cdot x} \right]$$

$$Z^\mu(x) = \frac{1}{\sqrt{(2\pi)^3}} \int d^3k \frac{\theta(k^+)}{\sqrt{2k^+}} \sum_{(\alpha)} E^\mu_{(\alpha)}(k) \left[a^Z_{(\alpha)}(k)e^{-ik\cdot x} + a^{Z\dagger}_{(\alpha)}(k)e^{ik\cdot x} \right] \quad (52)$$

where $d^3k \equiv d^2k^\perp dk^+$, $(\perp) = (1), (2)$, and $(\alpha) = (\perp), (3)$.

For completeness, we collect here the cubic and quartic self interactions of the gauge fields arising from the $F^a_{\mu\nu} F^{a\mu\nu}$ term

$$ig \left[(\partial_\mu W^+_\nu - \partial_\nu W^+_\mu) W^{-\mu} - (\partial_\mu W^-_\nu - \partial_\nu W^-_\mu) W^{+\mu} \right] A^{3\nu}$$
$$+ig\, W^+_\mu W^-_\nu \, (\partial^\mu A^{3\nu} - \partial^\nu A^{3\mu})$$
$$+g^2 \left[\frac{1}{4}(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu)^2 - W^+_\mu W^-_\nu A^3_\rho A^3_\sigma (g^{\mu\nu} g^{\rho\sigma} - g^{\rho\mu} g^{\sigma\nu}) \right] \quad (53)$$

where $A^3_\mu = [A_\mu \sin\theta_W + Z_\mu \cos\theta_W]$. Note that the complete $W^+W^-\gamma$ coupling, for example, includes the interaction terms carrying $G^\pm$ fields arising from the $|\mathcal{D}_\mu \Phi|^2$ term.

3.2 Fermionic Fields

The LC gauge LF quantization when the fermionic fields are also present is done by following closely the discussion [14, 33] given in QCD. The fermionic matter content of GWS model has three generations with each one containing quarks and leptons. The left-handed components of the fermion fields are assigned to the iso-spinor representation while the right-handed to the singlet of $SU(2)_W$. For example, in the first generation with quarks (u, d) and leptons (ν_e, e^-) we make the following assignments

$$\psi_L: \quad \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \quad \begin{pmatrix} u \\ d \end{pmatrix}_L \quad \in t = \frac{1}{2}; \quad (u_R, d_R, e^-_R) \quad \in t = 0 \quad (54)$$

Here $\psi_L = [(1-\gamma_5)/2]\psi$, $\bar\psi_L = \bar\psi_L [(1+\gamma_5)/2]$, $\psi_R = [(1+\gamma_5)/2]\psi$, $\gamma_5 = \gamma_5^\dagger$, $\gamma_5^2 = I$ etc. Each left-handed doublet is assigned a value of the hypercharge Y similar to that of the Higgs doublet. For example, $Y(u_R) = Q(u_R) = Q(u_L) = Q(u) = (Y + 1/2)$ and $Q(d) = (Y - 1/2) = Y(d_R)$, where $Y = Y(u_L) = Y(d_L)$. We recall $Y(e^-_L) = -1/2$ and $Y(u_L) = 1/6$.

We base our discussion below on a single pair of generic fields $\psi \equiv (u, d)^T$ with its left-handed components carrying the hypercharge Y. It may stand for (ν_e, e^-), (t, b), (c, s), etc. The gauge invariant weak interaction Lagrangian for massless fermions may be written as

$$\bar{\psi}_L \, i \, \gamma^\mu \, \mathcal{D}_\mu \, \psi_L + \bar{u}_R \, i \, \gamma^\mu \, \mathcal{D}_\mu \, u_R + \bar{d}_R \, i \, \gamma^\mu \, \mathcal{D}_\mu \, d_R \; . \tag{55}$$

The assignments of the chiral components to distinct representations of $SU_W(2)$ and the requirement of the gauge invariance do not allow one to introduce directly the fermionic mass terms in the Lagrangian. Such terms may, however, be generated through SSB if the following gauge invariant Yukawa interaction is added to the theory

$$-\lambda_d \, (\, \bar{\psi}_L \, \Phi \,) d_R - \lambda_u \, (\, \bar{\psi}_L \, i \, \sigma_2 \Phi^* \,) \, u_R + h.c. \tag{56}$$

Here λ_u, λ_d, are real couplings, without any connection with the weak interaction coupling constant, and we used $Y(\Phi^*) = -1/2$. We find the generation of the mass terms: $-(m_u \, \bar{u}u + m_d \, \bar{d}d)$, where we set $\lambda_d \, v = \sqrt{2} \, m_d$, $\lambda_u \, v = \sqrt{2} m_u$. The Yukawa interaction terms are

$$-\frac{g}{\sqrt{2}} (\frac{m_d}{m_W}) \left[\bar{u} \frac{(1+\gamma_5)}{2} d \, G^+ + \bar{d} \frac{(1-\gamma_5)}{2} u \, G^- + \frac{1}{\sqrt{2}} \bar{d} d \, h + \frac{i}{\sqrt{2}} \bar{d}\gamma_5 d \, G^o \right] \tag{57}$$

$$-\frac{g}{\sqrt{2}} (\frac{m_u}{m_W}) \left[-\bar{u} \frac{(1-\gamma_5)}{2} d \, G^+ - \bar{d} \frac{(1+\gamma_5)}{2} u \, G^- + \frac{1}{\sqrt{2}} \bar{u} u \, h - \frac{i}{\sqrt{2}} \bar{u}\gamma_5 u \, G^o \right]$$

adding thereby additional parameters in the model.

The full fermionic Lagrangian is obtained from (55) and (56). Besides the Yukawa interactions in (57) it contains also the following terms

$$\bar{u} \left[i \, \gamma^\mu (\partial_\mu - ieQ(u)A_\mu) - m_u \right] u + \bar{d} \left[i \, \gamma^\mu (\partial_\mu - ieQ(d)A_\mu) - m_d \right] d$$

$$+g \left(W_\mu^+ J_W^{\mu+} + W_\mu^- J_W^{\mu-} + Z_\mu J_Z^\mu \right) \tag{58}$$

where

$$\begin{aligned}
J_W^{\mu+} &= \frac{1}{\sqrt{2}} \left(\bar{\psi}_L \gamma^\mu \, t_+ \, \psi_L \right) = \frac{1}{2\sqrt{2}} \bar{u}\gamma^\mu (1 - \gamma_5) d \\
J_W^{\mu-} &= \frac{1}{\sqrt{2}} \left(\bar{\psi}_L \gamma^\mu \, t_- \, \psi_L \right) = \frac{1}{2\sqrt{2}} \bar{d}\gamma^\mu (1 - \gamma_5) u \\
J_{em}^\mu &= Q(u) \, \bar{u}\gamma^\mu \, u + Q(d) \, \bar{d}\gamma^\mu \, d \\
J_Z^\mu &= \frac{1}{\cos \theta_w} \left[\bar{\psi}_L \, \gamma^\mu \, t_3 \, \psi_L - \sin^2 \, \theta_W \, J_{em}^\mu \right] \\
&= \frac{1}{\cos \theta_w} \left[\frac{1}{4} \bar{u} \, \gamma^\mu \, (1 - \gamma_5) \, u - \frac{1}{4} \bar{d}\gamma^\mu \, (1 - \gamma_5) \, d - \sin^2 \, \theta_W \, J_{em}^\mu \right] \tag{59}
\end{aligned}$$

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such that at the tree level there are no flavor changing neutral currents. The surviving $U(1)_{em}$ gauge symmetry is also manifest.

The construction above gives the tree level description of the GWS model in terms of the set of tree level parameters $(e, m_W, m_Z, m_h, m_u, m_d)$ or alternatively $(e, \sin\theta_W, v, m_h, m_u, m_d)$. The KM matrix can be incorporated easily in our discussion. The LF quantization of the GWS model is performed following the discussions in Section 2, Ref. [14], and the discussion in Appendix B. The procedure closely follows the one adopted in connection with the discussion [14] in LC gauge LF quantized QCD. In the GWS model we also have to take care in addition of Yukawa interactions. Besides the tree level interactions written above, in the LF quantized theory we also have instantaneous interaction in $\mathcal{H}_{int}^{LF}$ (see Appendix B). They are responsible for the restoration of the Lorentz covariance in the computation of physical matrix elements etc. The LF propagators of the fields in LC gauge quantized GWS model are collected in Appendix C.

4   Illustrations

4.1   Decay $h \to W + W$

This decay is interesting also in connection with the Goldstone boson or electroweak equivalence theorem. It is clear from the expressions of the relevant interaction vertices in Section 2 and Section 3 that it suffices to consider the abelian theory. The $A\,A\,h$ interaction term gives the decay into two transverse vector bosons. The matrix element is

$$\mathcal{M}_1 = (ieM)\,2\,E^{(\alpha)}(k) \cdot E^{(\beta)}(k') = -2ieM\,E_\perp^{(\alpha)}(k)\,E_\perp^{(\beta)}(k')\ . \tag{60}$$

where $P_\mu = k_\mu + k'_\mu$ is the 4-momentum of the Higgs particle. The $\eta^2\,h$ interaction term produces longitudinal bosons in the Higgs decay. The corresponding matrix element is

$$\begin{aligned}
\mathcal{M}_2 &= -i\,\frac{\lambda v}{M^2}\,2\,(ik \cdot E^{(\alpha)}(k))\,(ik' \cdot E^{(\beta)}(k')) \\
&= i\,e\,\frac{m_h^2}{M}\,\delta_{(\alpha)(3)}\delta_{(\beta)(3)}\ .
\end{aligned} \tag{61}$$

Finally, the $\eta\,A\,h$ vertex gives

$$\mathcal{M}_3 = -i\,\frac{e}{M}\,2\,[k^\mu\,k^\nu + k'^\mu\,k'^\nu + k^\mu\,k'^\nu]\,E_\mu^{(\alpha)}(k)\,E_\nu^{(\beta)}(k')\ . \tag{62}$$

The total matrix element is

$$\mathcal{M}_{(\alpha)(\beta)} = 2\,i\,e\,M\left[g_{\mu\nu} + \frac{1}{2}\,\frac{m_h^2}{M^4}\,k_\mu\,k'_\nu - \frac{1}{M^2}\,(k^\mu\,k^\nu + k'^\mu\,k'^\nu + k^\mu\,k'^\nu)\right]E_\mu^{(\alpha)}(k)\,E_\nu^{(\beta)}(k')\ . \tag{63}$$

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Using mass-shell conditions we may rewrite

$$\mathcal{M}_{(\alpha)(\beta)} = 2\,i\,e\,M\left[g_{\mu\nu} + a\,k_\mu\,k'_\nu + b\,(k^\mu\,k^\nu + k'^\mu\,k'^\nu)\right]E_\mu^{(\alpha)}(k)\,E_\nu^{(\beta)}(k') \qquad (64)$$

where $a = (k \cdot k')/M^4$ and $b = -1/M^2$. It is straightforward to compute the sum over polarizations of the squared matrix element[9] We find

$$\sum_{(\alpha)}\sum_{(\beta)}|\mathcal{M}_{(\alpha)(\beta)}|^2 = (2\,e\,M)^2\left[2 + \frac{(k\cdot k')^2}{M^4}\right] \qquad (65)$$

which agrees, as it should, with the result found when we use the unitary (or Proca) gauge.

The discussion in the nonabelian theory of the Higgs decays into gauge boson pair $W^+ W^-$ is parallel to that of the abelian theory as can be seen from the expressions in (35) and (48) of the corresponding Higgs couplings. We need only to replace $e \to g/2$ and $M \to m_W$ in the discussion above. We find

$$\sum_{(\alpha)}\sum_{(\beta)}|\mathcal{M}_{(\alpha)(\beta)}|^2 = \frac{g^2\,m_h^4}{4\,m_W^2}\left[1 + 4\,\frac{m_W^2}{m_h^4}\,(3\,m_W^2 - m_h^2)\right] \quad . \qquad (66)$$

In the limit $m_h \gg m_W$ the leading term is the first one. It derives solely from $\mathcal{M}_2$, e.g., from the decay to the would-be Goldstone particle η, as if we set the gauge field as vanishing in the interaction Lagrangian. Similar discussions of other two body decays of the Higgs boson may be given.

The additional contributions to the matrix element coming from the would-be Goldstone bosons are found to be manifestly displayed. The matrix element $\mathcal{M}_2$, which derives solely from the would-be Goldstone field, receives, compared to the others, an $(m_h/m_W)^2$ enhancement factor. The result is general and has been given the name of the Goldstone boson or electroweak equivalence theorem [29]. In the LF quantized theory it is revealed transparently, and the physics of the longitudinal gauge bosons and Higgs field can be described, under certain conditions, very well in terms of the scalar self-interactions present in the initial Lagrangian while ignoring the gauge fields. This would not be true in the decay under discussion if the mass of the Higgs boson is found, as currently expected, to be around 115 GeV. In fact, $[...] \approx [1 + 0.91]$ for $m_W/m_h \approx 0.699$.

4.2 Muon Decay

The cancellation of the noncovariant terms in the previous illustration is seen easily also in muon decay, where the noncovariant gauge propagator is involved. However,

---

[9]We use the simplifying properties of $K_{\mu\nu}$, the relation $k_\mu\,k_\nu\,K^{\mu\nu}(k') = -M^2 + 2\,(k\cdot k')\,k^+/k'^+$, and the mass-shell conditions.

in this case we must also take into account a contribution from an instantaneous interaction.

The terms in the interaction Lagrangian density responsible for the process are read from (57), (58), and (59)

$$
\frac{g}{2\sqrt{2}}\left[\bar{\nu}_{\mu^-}\left(1+\gamma_5\right)\left(\gamma\cdot W^+ + \frac{i\,m_\mu}{m_W^2}\,\partial\cdot W^+\right)\mu^- +
$$

$$
\bar{\mu}^-\left(\gamma\cdot W^- - \frac{im_\mu}{m_W^2}\,\partial\cdot W^-\right)\left(1-\gamma_5\right)\nu_{\mu^-} + (\mu\to e) + \cdots\right]
$$

$$
+\text{ quartic instantaneous interaction.} \tag{67}
$$

Here we have made use of the 't Hooft conditions, $G^\pm = \mp i(\partial\cdot W^\pm)/m_W$ for convenience. The matrix element for the muon decay in momentum space, excluding the instantaneous interaction contribution, reads as

$$
\left(\frac{ig}{2\sqrt{2}}\right)^2\,\bar{u}(\nu_\mu)\left(1+\gamma_5\right)\left(\gamma^\mu - \frac{m_\mu}{m_W^2}\,k^\mu\right)u(\mu)\,\frac{K_{\mu\nu}(k)}{(k^2 - m_W^2 + i\epsilon)}\times
$$

$$
\bar{u}(e)\left(\gamma^\nu - \frac{m_e}{m_W^2}\,k^\nu\right)\left(1-\gamma_5\right)v(\bar{\nu}_e) \tag{68}
$$

where $K_{\mu\nu}(k)$ is given in (20). On using simplifying properties of $K_{\mu\nu}(k)$ (Section 2) it reduces to (suppressing the constant factor)

$$
\bar{u}(\nu_\mu)\left(1+\gamma_5\right)\gamma^\mu\,u(\mu)\,\frac{K_{\mu\nu}(k)}{(k^2 - m_W^2 + i\epsilon)}\,\bar{u}(e)\,\gamma^\nu(1-\gamma_5)\,v(\bar{\nu}_e)
$$

$$
-\,\frac{m_\mu}{(k^2 - m_W^2 + i\epsilon)\,k^+}\,\bar{u}(\nu_\mu)\left(1+\gamma_5\right)u(\mu)\,\bar{u}(e)\,\gamma^+(1-\gamma_5)\,v(\bar{\nu}_e)
$$

$$
-\,\frac{m_e}{(k^2 - m_W^2 + i\epsilon)\,k^+}\,\bar{u}(\nu_\mu)\left(1+\gamma_5\right)\gamma^+\,u(\mu)\,\bar{u}(e)\,(1-\gamma_5)\,v(\bar{\nu}_e)
$$

$$
+\,\frac{m_\mu m_e}{(k^2 - m_W^2 + i\epsilon)\,m_W^2}\,\bar{u}(\nu_\mu)\left(1+\gamma_5\right)u(\mu)\,\bar{u}(e)\,(1-\gamma_5)\,v(\bar{\nu}_e)\,. \tag{69}
$$

Consider the contributions from the first term. The noncovariant terms carrying the $1/k^+$ dependence in $K_{\mu\nu}$ cancel the second and the third terms. Also an instantaneous contribution comes from the last term in the expression of $K_{\mu\nu}$

$$
-\frac{1}{k^{+2}}\,\bar{u}(\nu_\mu)\left(1+\gamma_5\right)\gamma^+\,u(\mu)\,\bar{u}(e)\,\gamma^+(1-\gamma_5)\,v(\bar{\nu}_e)\,. \tag{70}
$$

It gets compensated by the additional quartic instantaneous interaction term in our LC gauge framework, which is easily derived by following the straightforward procedure given in Appendix B. The final result agrees with the covariant matrix element found in the unitary gauge.

4.3 Decay $t \to b + W^+$

The relevant interaction terms in the present case are

$$\frac{g}{2\sqrt{2}} \, \bar{b} \left[\gamma \cdot W^- \, (1 - \gamma_5) + \left(\frac{(m_t - m_b)}{m_W} + \frac{(m_t + m_b)}{m_W} \gamma_5 \right) G^- \right] t \, + \, h.c. \tag{71}$$

The matrix element may be written as

$$\frac{ig}{2\sqrt{2}} \, \bar{u}^{(r)}(b) \left[\gamma^\mu \, (1 - \gamma_5) - \frac{m_t}{m_W^2} \, k^\mu \, (1 + \gamma_5) \right] u^{(s)}(t) \, E_\mu^{(\alpha)}$$

$$= \frac{ig}{2\sqrt{2}} \, \bar{u}^{(r)}(b) \left[\gamma \cdot E^{(\alpha)}(k) \, (1 - \gamma_5) + \frac{m_t}{m_W} \, \delta_{(\alpha)(3)} \, (1 + \gamma_5) \right] u^{(s)}(t) \, . \tag{72}$$

Here we have set $m_b = 0$ for simplicity, and we recall that $(\alpha) = (\perp), (3)$ indicate the three polarization states of the massive vector boson as discussed in Section 2. For the spinor field we follow the notation of Ref. [14]. The m_t enhancement of the matrix element containing solely the would-be Goldstone bosons G^+ is similar to that in the Higgs decay described above. It is another illustration of the electroweak equivalence theorem. Since the Higgs boson couples to fermion mass, the heavy fermion contributions do not decouple. The sum over spins and polarizations of the squared invariant matrix element here is found to be proportional to

$$\left[q^\mu p^\nu + q^\nu p^\mu - (q \cdot p) \, g^{\mu\nu} + \left(\frac{m_t}{m_W} \right)^2 \left(q \cdot p \, \frac{k^\mu k^\nu}{m_W^2} - q^\mu k^\nu - q^\nu k^\mu \right) \right] K_{\mu\nu}(k)$$

$$= \, [q^\mu p^\nu + q^\nu p^\mu - (q \cdot p) \, g^{\mu\nu}] \, d_{\mu\nu}(k) + \left(\frac{m_t}{m_W} \right)^2 \left(q \cdot p - 2m_W^2 \, \frac{q^+}{k^+} \right) \tag{73}$$

where the mass-shell conditions such as $2k \cdot q = (m_t^2 - m_W^2)$, $2k \cdot p = (m_t^2 + m_W^2)$, $q^2 = 0$ have been used. Collecting together the noncovariant terms, we rewrite it as

$$= \, - [q^\mu p^\nu + q^\nu p^\mu - (q \cdot p) \, g^{\mu\nu}] \, g_{\mu\nu} + \frac{1}{k^+} \left(2 q \cdot k \, p^+ + 2k \cdot p \, q^+ - 2q \cdot p - 2m_t^2 \, q^+ \right)$$

$$+ \left(\frac{m_t}{m_W} \right)^2 q \cdot k \, = \, - [q^\mu p^\nu + q^\nu p^\mu - (q \cdot p) \, g^{\mu\nu}] \, g_{\mu\nu} + \left(\frac{m_t}{m_W} \right)^2 q \cdot k$$

$$= \, \left(-g_{\mu\nu} + \frac{k^\mu k^\nu}{m_W^2} \right) [q^\mu p^\nu + q^\nu p^\mu - (q \cdot p) \, g^{\mu\nu}] \, . \tag{74}$$

The noncovariant terms cancel out giving the covariant result of the unitary gauge[10].

---

[10] $\Gamma = \frac{G_F \, m_t^3}{8\sqrt{2}\pi} \, (1 - \frac{m_W^2}{m_t^2})^2 \, (1 + 2 \frac{m_W^2}{m_t^2})$.

Conclusions

The canonical quantization of LC gauge GWS electroweak theory in the *front form* has been derived by using the Dirac procedure to construct a self-consistent LF Hamiltonian theory. Combining this with our previous work on light-front QCD [14], we obtain a simultaneously unitary and renormalizable gauge formulation of the Standard Model of the strong and electroweak interactions.

The ghost-free interaction Hamiltonian of the Standard Model has been obtained in a compact form by retaining the dependent components A_+ and ψ_- in the formulation. Its form closely resembles the interaction Hamiltonian of covariant theory, except for the presence of a few additional instantaneous interactions. Their derivation is given in Appendix B. The resulting Dyson-Wick perturbation theory expansion based on equal-LF-time ordering is also constructed, allowing one to perform higher-order computations in a straightforward fashion. In contrast, in the conventional equal-time framework utilizing R_ξ gauges, one is required to retain ghost fields which interact with the physical fields. Moreover, W_μ, Z_μ, and A_μ can carry different parameters ξ^W, ξ^Z, and ξ^γ, respectively, in the gauge-fixing terms. The renormalization of these parameters then also has to be taken into consideration, and it is required to show that the physical amplitudes do not depend on them. In view of the additional simplifying properties of $K_{\mu\nu}$ and the (projector) $D_{\mu\nu}$, computations in our framework require an effort comparable to that of conventional covariant gauge theory.

In our LC gauge LF framework, the free massive gauge fields in the electroweak theory satisfy simultaneously the 't Hooft conditions as an operator equation. In the limit of vanishing mass of the vector boson, the gauge field propagator goes over to the doubly transverse gauge, $(n^\mu D_{\mu\nu}(k) = k^\mu D_{\mu\nu}(k) = 0)$, the propagator found [14] in QCD, in view of the Lorentz condition in the theory. As discussed in Section 2, the factor $K_{\mu\nu}(k)$ in the gauge propagator also carries important simplifying properties, similar to the ones associated with the projector $D_{\mu\nu}(k)$. The transverse polarization vectors for massive or massless vector boson may be taken to be $E^\mu_{(\perp)}(k) \equiv -D^\mu_\perp(k)$, whereas the non-transverse third one in the massive case is found to be parallel to the LC gauge direction $E^{(3)}_\mu(k) = -(M/k^+)\,n_\mu$. Its projection along the direction transverse to k_μ shares the spacelike vector property carried by $E^\mu_{(\perp)}(k)$.

The Goldstone boson or electroweak equivalence theorem [29] becomes transparent in our formulation. Its content is illustrated in Section 4 by considering Higgs and top decays. The computation of muon decay shows the relevance of the instantaneous interactions for recovering manifest Lorentz invariance in the physical gauge [30] theory framework. They also correspond [14] to the semi-classical (or nonrelativistic) limit frequently employed in the conventional equal-time quantized theory.

The singularities in the noncovariant pieces of the field propagators may be defined using the causal ML prescription for $1/k^+$ when we employ dimensional regularization, as was shown also in our earlier work on QCD. The power-counting rules in LC gauge

then become similar to those found in covariant gauge theory.

We recall the explicit demonstration [14] of the simplifying equality $Z_1 = Z_3$ in QCD in our LC gauge framework. Similar Ward identities are expected in the GWS model as well. These Ward identities simplify the task of computing higher-loop corrections to physical processes.

Our light-front formulation of the Standard Model also provides the basis for an "event amplitude generator" [34] for high energy physics reactions where each particle's final state is completely labelled in momentum, helicity, and phase. The application of the light-front time evolution operator P^- to an initial state will systematically generate the tree and virtual loop graphs of the T-matrix in light-front time-ordered perturbation theory. In our ghost-free light-cone gauge framework, the virtual loop integrals only involve integration over the momenta of particles with physical polarization and physical phase space $\prod d^2 k_{\perp i} dk_i^+$. Renormalized amplitudes can be explicitly constructed by subtracting from the divergent loops amplitudes with nearly identical integrands corresponding to the contribution of the relevant mass and coupling counter terms (the "alternating denominator method") [35].

Acknowledgments

Appendix A

Spontaneous Symmetry Breaking Description on the LF

We first consider, due to its relevance to the discussion in Section 2, the abelian case where the scalar theory Lagrangian with $U(1)$ symmetry is given by

$$\mathcal{L} = \partial_+ \phi^\dagger \partial_- \phi + \partial_- \phi^\dagger \partial_+ \phi - \partial_\perp \phi^\dagger \partial_\perp \phi - V(\phi^\dagger \phi) \tag{75}$$

where $V(\phi) = \mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2$ with $\lambda > 0$ and $\mu^2 < 0$. To canonically quantize the theory we must construct an Hamiltonian framework for the constrained dynamics described by the above Lagrangian. The Dirac procedure [15] is convenient to use. *Before* applying it, however, we make [24] the separation[11]

$$\phi(\tau, x^-, x^\perp) = \omega(\tau, x^\perp) + \varphi(\tau, x^-, x^\perp)$$

---

[11]Such a decomposition may also be shown to follow [5] as an external [15] gauge-fixing condition, corresponding to a first class constraint in the theory, when we apply the Dirac procedure. We note that $\int d^2 x^\perp dx^- \varphi = 0$ such that φ has vanishing zero-longitudinal momentum-mode.

The field φ indicates the quantum fluctuations above the *dynamical condensate* (or zero-longitudinal-momentum-mode) variable $\omega(\tau, x^{\perp})$. The LF Hamiltonian framework is found to contain in it also a (second class) constraint equations [24], which relates the condensate variables with the fluctuation fields. The variable ω is shown [24, 25, 26] to have vanishing Dirac brackets with itself and with φ. It is thus a c-number (background field) in the quantized theory[12] The constraint equations[13] in the present case are

$$\int d^2x^{\perp} dx^- [\partial_{\perp}\partial_{\perp}\phi - \frac{\delta V}{\delta\phi^{\dagger}}] = 0, \qquad \int d^2x^{\perp} dx^- [\partial_{\perp}\partial_{\perp}\phi^{\dagger} - \frac{\delta V}{\delta\phi}] = 0. \qquad (76)$$

In the following discussion we only consider the case where $\partial_{\perp}\omega = 0$. At the classical (tree) level, since the fluctuations φ are assumed bounded, it follows [24, 25] that $\delta V/\delta\phi|_{\phi=\omega} = \delta V/\delta\phi^{\dagger}|_{\phi=\omega} = 0$. This coincides with the result in the conventional equal-time framework. It is obtained there after imposing additional constraints, which are based on physical considerations (seemingly not available or evident on the LF). The possible values of ω are $\omega = 0$ or $\omega^{\dagger}\omega = -\mu^2/(2\lambda)$. The stability of these solutions may be studied as usual from the Lagrange equations; the nonvanishing ω gives rise to stable solutions in the Nambu-Goldstone phase under study. The (classical) vacuum state is degenerate and characterized by a fixed value of $\omega = \sqrt{-\mu^2/(2\lambda)}\, e^{i\delta}$ where δ is real and arbitrary. In view of the invariance of the action under the phase symmetry transformations: $\varphi \to e^{i\alpha}\varphi$, $\omega \to e^{i\alpha}\omega$, we may, without any loss of generality, conveniently assume $\omega \equiv v/\sqrt{2}$ where $v = \sqrt{-\mu^2/\lambda}$ is a fixed real constant. A phase transformation would not leave this classical vacuum state invariant, and the symmetry is said to be broken spontaneously (see also Section 3.1)

At the quantum level, on the other hand, the LF field theoretic generator of $U(1)$ symmetry annihilates the LF vacuum state, independent of the broken symmetry or not. The symmetry transformations always leave the LF vacuum invariant, while the SSB is manifested, for example, in the non-conservation of some of the symmetry currents [25, 26]. These features are true in general.

The Dirac procedure is straightforward to apply, and the quantized theory is obtained by invoking the correspondence of the Dirac brackets with the commutators of the corresponding quantized field operators. In the LF quantized theory we find the following non-vanishing equal-x^+ commutator

$$[\varphi(x^+, x^-, x^{\perp}), \varphi(y^+, y^-, y^{\perp})]\,|_{x^+=y^+} = -\frac{i}{4}\epsilon(x^- - y^-)\delta^2(x^{\perp} - y^{\perp}) \qquad (77)$$

---

[12]In the Schwinger model it is shown [10] to be a q-number or an operator and where its presence gives rise to the *chiral* and the θ or *condensate* vacua. In the case of the Chiral Schwinger model ω may be eliminated from the theory by a field re-definition resulting in a different degenerate vacuum structure.

[13]They may [24] also be obtained by integrating the Lagrange equations but we must construct LF Hamiltonian frame to canonically quantize the theory.

which does not violate the principle of microcausality on the LF, in spite of the non-locality present in it along the x^- direction. The hermitian symmetry field theoretic generator is constructed straightforwardly

$$G(x^+) = \int d^2x^\perp dx^- j_-, \qquad \text{where}$$

$$j_\mu = i\left[\varphi^\dagger\,\partial_\mu\varphi - \varphi\,\partial_\mu\varphi^\dagger\right] \qquad (78)$$

such that $[\varphi(x), G] = \varphi$, $[\varphi(x)^\dagger, G] = -\varphi^\dagger$. The on-shell conserved Noether symmetry current is given by

$$J_\mu = i\left[\phi^\dagger\,\partial_\mu\phi - \phi\,\partial_\mu\phi^\dagger\right], \qquad \partial_\mu J^\mu = 0 \qquad (79)$$

which shows that the symmetry current $(\phi = v/\sqrt{2} + \varphi)$

$$j_\mu = J_\mu - \frac{i\,v}{\sqrt{2}}\partial_\mu(\varphi - \varphi^\dagger)$$

$$\partial^\mu j_\mu = \frac{i\,v}{\sqrt{2}}\partial\cdot\partial\,(\varphi - \varphi^\dagger) \qquad (80)$$

is not conserved in the broken phase. In the LF quantized theory, the two currents j_μ and J_μ, however, give rise to the same charge or generator, if the surface terms may be ignored.

The LF commutator may be realized by the following momentum space expansion

$$\varphi(x) = \frac{1}{\sqrt{(2\pi)^3}}\int d^2k^\perp dk^+ \frac{\theta(k^+)}{\sqrt{2k^+}}\left[a(k)e^{-ik\cdot x} + b^\dagger(k)e^{ik\cdot x}\right] \qquad (81)$$

where the nonvanishing commutators are $[a(k), a^\dagger(l)] = [b(k), b^\dagger(l)] = \delta^2(k_\perp - l_\perp)$ $\delta(k^+ - l^+)$. The symmetry generator in momentum space is found to be

$$G = \int d^2k^\perp dk^+ \,\theta(k^+)\left[a^\dagger(k)a(k) - b^\dagger(k)b(k)\right]. \qquad (82)$$

In the LF quantized theory only this term is present. It is already normal ordered and annihilates the LF vacuum. This is in contrast to the case of equal-time quantized conventional theory, where there is an additional term[14] in the field theoretic symmetry generator which does not annihilate the corresponding conventional vacuum state. The LF vacuum thus remains invariant under the symmetry transformations

---

[14]In the equal-time quantized theory we have instead $\partial_t(\varphi - \varphi^\dagger)$ in the expression of j_0 in (80). It does not drop out upon coordinate space integration, and there is an additional term in the corresponding generator which may not annihilate the vacuum state. The description of SSB [26] is thus somewhat different in the two *forms* of the theory.

independent of the SSB in the theory. The broken symmetry manifests [25] itself in the non-conservation of (some) symmetry currents or in the operator LF Hamiltonian.

Higgs mechanism in LF quantized theory[25]

The description below is relevant to the *front form* theory of the GWS model in Section 3 which has a non-abelian Higgs sector.

The SSB of continuous symmetry in the non-abelian case is discussed in refs. [25, 26] by considering an isospin-multiplet ϕ_i, $i = 1, 2, \cdots$, of real scalar fields. We separate first the *dynamical zero modes* or *condensates* from the quantum fluctuations, $\phi_i(\tau, x^-, x^\perp) = \omega_i(\tau, x^\perp) + \varphi_i(\tau, x^-, x^\perp)$. Then the Hamiltonian framework is constructed following the Dirac method. We find in it, in addition to the commutators and the Hamiltonian, a set of coupled constraint equations. At the tree level they yield $V_i'(\omega) - \partial_\perp \partial_\perp \omega_i = 0$. For space independent ω we find the same expression as found in the conventional theory.

It was also shown that the presence, in the case of continuous SSB, of the transverse directions was crucial for showing that the (dynamical) zero modes have vanishing Dirac brackets with the non-zero ones. This furnishes us a new simple proof of the *Coleman theorem* on the absence of Goldstone bosons in two dimensions, when we discuss the SSB on the LF.

The field theoretic generators are now $G_a = -i \int dx^\perp dx^- (\partial_- \varphi_i) (t_a)_{ij} \varphi_j$. It is easily checked to be already normal ordered, as in the abelian case, and we need not impose it. The symmetry generators on the LF thus annihilate the LF vacuum independent of the form of the scalar potential and its symmetry is not broken. We find $[\varphi_i(x), G_a] = (t_a)_{ij} \varphi_j$, $[\omega_i, G_a] = 0$, and $[G_a, G_b] = i f_{abc} G_c$ which is consistent with the generators annihilating the LF vacuum. Not all the generators, however, commute with the Hamiltonian when SSB is present, say, when ω_i are determined from $(\lambda \omega_i \omega_i - m^2) = 0$. There may survive a residual unbroken symmetry if a set of linearly independent generators still commutes with the LF Hamiltonian. Such generators may be found by solving $(\tilde{t}_a)_{ij} \omega_j = 0$ where $\tilde{t}_a$ are appropriate linearly independent combinations, depending on the iso-vector $\omega = \{\omega_i\}$ chosen, of the matrix generators t_a of the initial symmetry group. The corresponding generators $\tilde{G}_a$ commute with the Hamiltonian written in terms of φ_i and fixed constants ω_i. The counting of the number of Goldstone bosons is thus done as in the conventional theory. The tree level Higgs Lagrangian is re-written by the same procedure as in the conventional theory discussions, as done also in Section 3. The quantized theories of Higgs model though are different in the two *forms* of the theory as seen in Sections 2 and 3.

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Appendix B

Instantaneous Interactions in LF Quantized Theory

The additional instantaneous interactions in our LC gauge LF theory framework in GWS model may be found straightforwardly by following the procedure indicated in Ref. [14]. Such nonlocal terms are also required, as shown there, in order to restore the Lorentz covariance of physical matrix elements. They seem to have been missed in the conventional theory discussions [30, 14] in noncovariant gauges. It is worth stressing that they are also present in *front form* Yukawa theory, which is not even a gauge theory, as is shown below. Some other illustrations related to the abelian Higgs model, QCD, and the Yukawa couplings in GWS model are also briefly described. The instantaneous interactions arise when we take into account the fact that the nondynamical field components ψ_- and A_+ are not independent fields. The *front form* theory framework, however, permits us to re-express the interaction Hamiltonian in terms of the full spinor and gauge fields, as previously shown in QCD. It results in an alternative ghost-free and practical framework, in view of the Dyson-Wick expansion, for the computations in the Standard model. Unitarity and renormalizability are also manifest.

LF quantized Yukawa theory

The LF quantization of the free spinor field was discussed in Ref. [33] and the LF propagator of its dynamical component derived; it was also shown not to contain any instantaneous term in it. We recall that in the *front form* theory the spinor field[15] is naturally decomposed into a dynamical field component $\psi_+ \equiv \Lambda^+\psi$ and a nondynamical auxiliary field $\psi_- \equiv \Lambda^-\psi$, $\psi = \psi_+ + \psi_-$, where $\Lambda^\pm$, with $\Lambda^+ + \Lambda^- = 1$, are hermitian projection operators. Written in the LF coordinates, the free Dirac Lagrangian may, in fact, be re-written as

$$
\begin{aligned}
\mathcal{L}^o &= \bar{\psi}\left(i\gamma \cdot \partial - m\right)\psi \\
&= \bar{\psi}\left(\Lambda^+ + \Lambda^-\right)\frac{\partial \mathcal{L}^o}{\partial \bar{\psi}} \\
&\rightarrow \bar{\psi}\,\Lambda^-\,\frac{\partial \mathcal{L}^o}{\partial \bar{\psi}}\bigg|_{\Lambda^+\frac{\partial \mathcal{L}^o}{\partial \psi}=0} \\
&= \bar{\psi}_+\left(i\gamma \cdot \partial - m\right)\psi \quad \text{where} \quad \gamma^+\left(i\gamma \cdot \partial - m\right)\psi = 0 \\
&= \bar{\psi}_+\, i\,\gamma^+\partial_+\,\psi_+ + \bar{\psi}_+\left(i\,\gamma^\perp\partial_\perp - m\right)\psi_- \quad (83)
\end{aligned}
$$

---

[15] x^+ is taken as the LF-time while $(x^-, x^\perp)$ indicate spatial coordinates. See, Refs. [33, 14, 5] for notation and discussion on the LF spinors. We note: $\Lambda^\pm = \frac{1}{2}\gamma^\mp\gamma^\pm$, $\gamma^+\psi_- = 0$, etc.

Here we used $\Lambda^{\pm}\,\gamma\cdot\partial = (\gamma^{\mp}\,\partial_{\mp}\,\Lambda^{\mp}+\gamma^{\perp}\cdot\partial_{\perp}\,\Lambda^{\pm})$ which shows that only ψ_{+} is dynamical and independent field. ψ_{-} carries no kinetic term and is a dependent field. In fact, on taking the variation of $\mathcal{L}^{o}$ with respect to the auxiliary field $\bar{\psi}_{-}$ we derive the constraint equation

$$\Lambda^{+}\frac{\partial\mathcal{L}^{o}}{\partial\bar{\psi}} = 0, \quad \text{or} \quad \gamma^{+}\left(i\,\gamma\cdot\partial - m\right)\psi = 0 \tag{84}$$

which gives ψ_{-}

$$\psi_{-} = \frac{1}{2i\partial_{-}}\left(i\,\gamma^{\perp}\partial_{\perp} + m\right)\gamma^{+}\,\psi_{+} \tag{85}$$

showing it to be dependent field component.

Consider now the *Yukawa theory* described by

$$\begin{aligned}
\mathcal{L} &= \bar{\psi}\left(i\gamma\cdot\partial - m\right)\psi + \frac{1}{2}\left(\partial_{\mu}\phi\right)^{2} - \frac{1}{2}M^{2}\phi^{2} + g\,\bar{\psi}\psi\,\phi \\
&= \bar{\psi}\left(\Lambda^{+} + \Lambda^{-}\right)\frac{\partial\mathcal{L}}{\partial\bar{\psi}} + \frac{1}{2}\left(\partial_{\mu}\phi\right)^{2} - \frac{1}{2}M^{2}\phi^{2} \\
&\rightarrow \bar{\psi}_{+}\,\Lambda^{-}\left.\frac{\partial\mathcal{L}}{\partial\bar{\psi}}\right|_{\Lambda^{+}\frac{\partial\mathcal{L}}{\partial\bar{\psi}}=0} + \frac{1}{2}\left(\partial_{\mu}\phi\right)^{2} - \frac{1}{2}M^{2}\phi^{2}\ .
\end{aligned} \tag{86}$$

The nondynamical component ψ_{-} is now determined from the constraint equation

$$\Lambda^{+}\frac{\partial\mathcal{L}}{\partial\bar{\psi}} \equiv \Lambda^{+}\left[\left(i\gamma\cdot\partial - m\right)\psi + S\right] = 0 \quad \text{or} \quad \gamma^{+}\left[\left(i\gamma\cdot\partial - m\right)\psi + S\right] = 0 \tag{87}$$

where $S = g\,\phi\,\psi$. We find

$$\psi_{-} \equiv \Lambda^{-}\psi = \psi_{-}^{o} - \frac{1}{2i\partial_{-}}\gamma^{+}\,S \tag{88}$$

where we define

$$\psi_{-}^{o} = \frac{1}{2i\partial_{-}}\left(i\gamma^{\perp}\partial_{\perp} + m\right)\gamma^{+}\,\psi_{+}\ . \tag{89}$$

Clearly,

$$\psi^{o} = \psi_{+} + \psi_{-}^{o}, \tag{90}$$

where $\psi_{+}^{o} \equiv \psi_{+} = \Lambda^{+}\psi$, satisfies the free field Dirac equation. Also

$$\psi = \psi_{+} + \psi_{-} = \psi^{o} - \frac{1}{2i\,\partial_{-}}\gamma^{+}\,S\ . \tag{91}$$

The *front form* Yukawa theory Lagrangian reads as

$$\begin{aligned}
\mathcal{L} &= \bar{\psi}_{+}\,\Lambda^{-}\left[\left(i\gamma\cdot\partial - m\right)\psi + S\right] + \cdots \\
&= \bar{\psi}_{+}^{o}\left(i\gamma\cdot\partial - m\right)\psi^{o} - \bar{\psi}_{+}^{o}\left(i\gamma\cdot\partial - m\right)\frac{1}{2i\,\partial_{-}}\gamma^{+}\,S + \bar{\psi}_{+}^{o}\,S + \cdots \\
&= \mathcal{L}^{o} + \mathcal{L}_{int}
\end{aligned} \tag{92}$$

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where

$$
\begin{aligned}
\mathcal{L}^o &= \bar{\psi}^o_+ \, (i\gamma \cdot \partial - m)\psi^o + \frac{1}{2}\,(\partial_\mu \phi)^2 - \frac{1}{2}\,M^2 \phi^2 \\
\mathcal{L}_{int} &= -\bar{\psi}^o_+ \, (i\gamma \cdot \partial - m)\,\frac{1}{2i\,\partial_-}\,\gamma^+\, S + \bar{\psi}^o_+\, S \\
&= -\bar{\psi}^o \, (i\gamma^\perp \, \partial_\perp - m)\,\frac{1}{2i\,\partial_-}\,\gamma^+\, S + \bar{\psi}^o\, \Lambda^-\, S \\
&\rightarrow \quad \bar{\psi}^o\, \Lambda^+\, S + \bar{\psi}^o\, \Lambda^-\, S = \quad \bar{\psi}^o\, S \\
&= g\,\bar{\psi}^o \left[\psi^o - \frac{1}{2i\,\partial_-}\,\gamma^+\, S \right]\phi \\
&= g\,\bar{\psi}^o\, \psi^o\, \phi - g^2\,\bar{\psi}^o\, \phi\, \frac{1}{2i\,\partial_-}\,\gamma^+\, \psi^o\, \phi \ . \tag{93}
\end{aligned}
$$

In order to re-express the first term, we have performed integrations by parts over the spatial coordinates x^-, $x^\perp$ in the Lagrangian; the $\gamma^+ \, \partial_+$ term drops out since $\gamma^{+\,2} = 0$. The interaction, when expressed in terms of the free field ψ^o, contains an additional instantaneous term. The LF quantization may be performed straightforwardly and Dyson-Wick perturbation theory expansion can be constructed. It is worth recalling that the LF fermionic propagator is also different from the one found in the *instant form* quantized theory. The instantaneous terms are necessary, for example, in restoring the Lorentz invariance in the computation of the meson nucleon scattering in the Yukawa theory. Ignoring it would lead to disagreement in the calculations of the nucleon self-energy in the LF and conventionally quantized theories. Their importance in LF quantized QCD in LC gauge was also discussed in our earlier paper.

We remark that the expression of $\gamma^+ S$ in Yukawa theory contains only the dynamical ψ_+ component. In the case of gauge theory, ψ_- would occur also on the right hand side of (88) if we do not use the LC gauge, since $\gamma^+ \gamma \cdot A, \psi = 2\, A_-\,\psi_- + \gamma^+\gamma^\perp\, A_\perp\,\psi_+$.

Abelian Higgs model

Next, we consider the derivation of the instantaneous interaction terms in the abelian Higgs model discussed in Section 2. From the Lagrangian written in LF coordinates it is clear that A_+ is a nondynamical since there is no corresponding kinetic term. It is also a dependent component. Consider the equation of motion for the gauge field

$$
-\partial \cdot \partial\, A_\mu + \partial_\mu \,(\partial \cdot A) = \frac{\partial \mathcal{L}}{\partial A^\mu} \ . \tag{94}
$$

We found significant simplifications in the fermionic sector of LF quantized gauge theory if we adopt the LC gauge. The underlying gauge symmetry in the Higgs model

allows one to adopt this gauge, $A_- = 0$. From the expression of the Lagrangian (4) it then follows that

$$(\partial \cdot A - M\,\eta)\,|_{A_-=0} = e\,\frac{1}{\partial_-}\,K^+ \quad \text{where} \quad K^+ = \frac{1}{e}\,\frac{\partial\mathcal{L}}{\partial A_+}\bigg|_{A_-=0,M=0} = (h\,\partial_-\eta - \eta\,\partial_- h)$$

(95)

Thus the free theory carries in it simultaneously the 't Hooft condition, as was also demonstrated in the Hamiltonian framework (and in the quantized theory). When the SSB is present and the mass of the gauge field is generated by the Higgs mechanism in our framework, the massive gauge field is described by the independent field components $A_\perp$ and η. We may define, as in the fermionic case, the dependent free field component A_+^o by the 't Hooft condition

$$\partial_- A_+^o = \partial_\perp A_\perp + M\,\eta \ .$$

(96)

It follows from (95) that

$$A_+ = A_+^o + e\,\frac{1}{(\partial_-)^2}\,K^+ \ .$$

(97)

Expressed in terms of the components $A_\perp, \eta, A_+^o$ and h the Lagrangian contains also instantaneous nonlocal interaction terms. They are indicated below on the right hand side of the arrow corresponding to the term which gives rise to it

$$M\,(A \cdot \partial)\,\eta \quad \rightarrow \quad -e\,M\,\eta\,\frac{1}{\partial_-}\,K^+$$

$$e(h\partial_\mu\eta - \eta\partial_\mu h)A^\mu \quad \rightarrow \quad e^2\,K^+\,\frac{1}{(\partial_-)^2}\,K^+$$

$$-\frac{1}{4}\,F_{\mu\nu}\,F^{\mu\nu} \quad \rightarrow \quad e\,M\,\eta\,\frac{1}{\partial_-}\,K^+ - e^2\,\frac{1}{2}\,K^+\,\frac{1}{(\partial_-)^2}\,K^+$$

(98)

where integrations by parts in the Lagrangian were freely used as in the fermionic case. We observe that the cubic nonlocal interaction terms cancel leaving behind only the quartic term.

LC gauge LF quantized QCD

In the fermionic piece we have now

$$S^i = \gamma^\mu\,A_\mu^a\,(t^a)^{ij}\,\psi^{oj} \quad \text{and} \quad \psi^i = \psi^{oi} - g\,\frac{1}{2i\,\partial_-}\,\gamma^+\,S^i\,|_{A_-^a=0} \ .$$

(99)

For the nonabelian gauge field theory we follow closely the above discussion for the Higgs model. We have

$$A_+^a = A_+^{oa} + g\,\frac{1}{(\partial_-)^2}\,j^{+a}$$

(100)

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where in the massless case we define $\partial_- A_+^{oa} = \partial_\perp A_\perp$ and

$$
\begin{aligned}
j^{+a} &= \left. \frac{1}{g} \frac{\partial \mathcal{L}}{\partial A_+^a} \right|_{A_-^a=0} = f_{abc} A_\perp^b \, \partial_- A_\perp^c + \bar{\psi}^i \, \gamma^+ \, (t^a)^{ij} \, \psi^j \\
&= f_{abc} A_\perp^b \, \partial_- A_\perp^c + \bar{\psi}^{oi} \, \gamma^+ \, (t^a)^{ij} \, \psi^{oj} \\
&\equiv [\, K^a + L^a \,] \; .
\end{aligned}
\tag{101}
$$

The field components A_+^a and ψ_-^i are again dependent variables. The fermionic piece contributes an instantaneous seagull interaction as in the Yukawa theory. There arises also another type of instantaneous interaction

$$
g^2 \, L^a \, \frac{1}{(\partial-)^2} \, [\, K^a + L^a \,] \; .
\tag{102}
$$

Similar contribution coming from the gauge field sector,

$$
-\frac{1}{4} \, F^{a\,\mu\nu} \, F_{a\,\mu\nu} = \frac{1}{2} \left[F_{a+-} \, F_{a+-} + 2 \, F_{a+\perp} \, F_{a-\perp} - \frac{1}{2} \, F_{a\perp\perp'} \, F_{a\perp\perp'} \right]
\tag{103}
$$

is found to be

$$
g^2 \, K^a \, \frac{1}{(\partial-)^2} \, [\, K^a + L^a \,] - \frac{1}{2} \, g^2 \, [\, K^a + L^a \,] \, \frac{1}{(\partial-)^2} \, [\, K^a + L^a \,] \; .
\tag{104}
$$

The interaction Hamiltonian in QCD follows: [14]

$$
\begin{aligned}
\mathcal{H}_{int} = -\mathcal{L}_{int} &= -g \, \bar{\psi}^i \gamma^\mu \, (t^a)^{ij} \, \psi^j \, A_\mu^a \\
&\quad + \frac{g}{2} \, f^{abc} \, (\partial_\mu A^a{}_\nu - \partial_\nu A^a{}_\mu) A^{b\mu} A^{c\nu} \\
&\quad + \frac{g^2}{4} \, f^{abc} f^{ade} A_{b\mu} A^{d\mu} A_{c\nu} A^{e\nu} \\
&\quad - \frac{g^2}{2} \, \bar{\psi}^i \gamma^+ \, \gamma^\mu \, A_\mu^a \, (t^a)^{ij} \, \frac{1}{i\partial_-} \, \gamma^\nu \, A_\nu^b \, (t^b)^{jk} \, \psi^k \\
&\quad - \frac{g^2}{2} \, (\frac{1}{i\partial_-} \, j_a^+) \, (\frac{1}{i\partial_-} \, j_a^+)
\end{aligned}
\tag{105}
$$

where

$$
j_a^+ = \bar{\psi}^i \gamma^+ (t_a)^{ij} \psi^j + f_{abc}(\partial_- A_{b\mu}) A^{c\mu}
\tag{106}
$$

and a sum over distinct quark and lepton flavors, not written explicitly, is understood in (105) and (106).

GWS model

In the electro-weak sector of the Standard model, "S" contains terms such as $\gamma^\mu Z_\mu \psi$ etc. Only in the LC gauge, with $A_- = Z_- = W_-^\pm = 0$, the $\gamma^+ S$ will contain solely the dynamical "$+$" component of the fermionic fields involved. The discussion in the GWS model in LC gauge follows closely the one given in QCD.

Appendix C

Feynman Rules and Propagators

The Dyson-Wick perturbation theory expansion on the LF can be realized in momentum space by employing the Fourier transform of the fields and the propagators discussed in Sections 2, 3, and in Ref. [14]. Many of the rules of the Feynman diagrams, for example, the symmetry factor $1/2$ for gluon loop, a minus sign associated with fermionic loops etc., are the same as those found in the conventional covariant framework. There are some differences: for example, the external quark line now carries a factor $\theta(p^+)\sqrt{m/p^+}$; the external boson line carries the factor $\theta(q^+)/\sqrt{2q^+}$ and the Lorentz invariant phase space factor is $\int d^2p^\perp dp^+ \, \theta(p^+)/(2p^+)$. The external massive vector boson line carries the polarization vector $E^{\mu(\alpha)}(q)$. Its properties and the sum over the polarization states are given Section 2. The notation for the quark field is as given in Refs. [33, 14]. The instantaneous interactions in electroweak theory may be found using Appendix B. The momentum space vertices can be derived straightforwardly employing the Fourier transforms of the fields given in the text and illustrated in Ref. [14] in QCD. The free propagators are

Fermionic propagator:

$$i\,\delta_{ij}\,\frac{N(p)}{p^2 - m^2 + i\epsilon}, \quad \text{with} \quad N(p) = (\not{p} + m) - (p^2 - m^2)\frac{\gamma^+}{2p^+}, \qquad \epsilon > 0,$$

where p_μ is the quark 4-momentum and i and j are color indices. The noncovariant second term on the right hand side is present only in the propagator of the dependent field ψ_-. Also $N(p) = (\not{p}_{on} + m)$ where $p_{on} : \left((m^2 + p_\perp^2)/2p^+, p^+, p^\perp \right)$.

Photon propagator:

$$i\,\frac{D_{\mu\nu}(q)}{q^2 + i\epsilon}, \quad \text{with} \quad D_{\mu\nu}(q) = \left(-g_{\mu\nu} + \frac{n_\mu q_\nu + q_\mu n_\nu}{n \cdot q} - \frac{q^2}{(n \cdot q)^2}n_\mu n_\nu \right),$$

where q_μ is the photon 4-momentum and n_μ is the gauge direction. We choose $n_\mu \equiv \delta_\mu^+$ and $n_\mu^* \equiv \delta_\mu^-$, the dual of n_μ.

Vector boson propagators:

$$\left\langle W_\mu^+(q)\, W_\nu^-(-q) \right\rangle = i\frac{K_{\mu\nu}(q)}{q^2 - m_W^2 + i\epsilon},$$

where

$$K_{\mu\nu}(q) = \left(-g_{\mu\nu} + \frac{n_\mu q_\nu + q_\mu n_\nu}{n \cdot q} - \frac{(q^2 - m_W^2)}{(n \cdot q)^2}n_\mu n_\nu \right),$$

where q_μ is the vector boson 4-momentum and n_μ is the gauge direction. We choose $n_\mu \equiv \delta_\mu^+$ and $n_\mu^* \equiv \delta_\mu^-$, the dual of n_μ. For the neutral Z vector boson m_W is substituted by m_Z.

The scalar fields $G^\pm, G^o$ and h have the standard covariant propagators $i/(q^2 - M^2)$ where $M = m_W, m_Z$ and m_h respectively.

It is worth recalling [14] the procedure for computing the discontinuity or imaginary parts of any Feynman diagram, employing the Cutkosky rules in our LF framework. For each cut, replace $1/(p^2 - m^2 + i\,\epsilon) \to -2\pi\,i\,\delta(p^2 - m^2)$ and then perform the loop integrals. We note that $(p^2 - m^2)\,\delta(p^2 - m^2) = 0$ such that last term in each of $N(p)$, $D_{\mu\nu}(q)$, and $K_{\mu\nu}(q)$ gives vanishing contribution.

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