

**A kinetic model for the one-dimensional electromagnetic solitons
in an isothermal plasma***

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Abstract

Two nonlinear second order differential equations for the amplitude of the vector potential and for the electromagnetic potential are derived, starting from the full Maxwell equations where the field sources are calculated by integrating in the momentum space the particle distribution function, which is an exact solution of the relativistic Vlasov equation. The resulting equations are exact in describing a hot one-dimensional plasma sustaining a relativistically intense, circularly polarized electromagnetic polarized electromagnetic radiation. The case of standing soliton-like structures in an electron-positron plasma is then investigated. It is demonstrated that at ultrarelativistic temperatures extremely large amplitude solitons can be formed in a strongly overdense plasma.

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Abstract

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intense, circularly polarized electromagnetic radiation. The case of standing soliton-like structures in an electron-positron plasma is then investigated. It is demonstrated that at ultrarelativistic temperatures extremely large amplitude solitons can be formed in a strongly overdense plasma.

I. INTRODUCTION

The interest in the theoretical investigations of an electron-positron ($e^- - e^+$) plasma and of the dispersive properties of the EM waves in it arises from numerous situations: it ranges from the astrophysical and cosmological applications to the laboratory experiments in connection with ultra-intense laser pulses interacting with matter.

The most recent reconstructions of the dynamics of the early Universe assume that between 10^{-2} and 1 sec after the Big Bang, matter was constituted by electrons (e^-), positrons (e^+) and photons in an almost thermal equilibrium at a temperature much higher than $m_e c^2$ [1]. Here, m_e and c are the electron rest mass and the speed of light in vacuum, respectively. Of particular interest is the possibility that at this stage of evolution of the Universe, the interaction between electromagnetic (EM) radiation and the $e^- - e^+$ plasma produces spatial density nonuniformities, since it is considered as a possible cause of the strongly inhomogeneous distribution of matter in the Universe as appears in our epoch [2]. In addition, it is believed that strong nonuniformities in the primordial Universe would have affected the primordial nucleosynthesis rate [3,4] and, as a consequence, the relative abundance of light [5,6], intermediate mass [7], and heavier elements [8], as well as the baryon-to-photon ratio, during its successive evolution. On the other side, $e^- - e^+$ plasmas are also thought to constitute pulsar and neutron star atmospheres [9], accretion disks [10], active galactic nuclei [11], and black holes [12]. Moreover, in space physics the $e^- - e^+$ pair plasmas play a key role in the model of the physical mechanisms of cosmological gamma ray bursts [13].

Coming to laboratory experiments, after the rapid development of the laser technology in the last ten years [14], femtosecond laser pulses with intensities up to 10^{21} W/cm^2 have be-

come available for investigating the laser-matter interaction under extreme conditions where the electron momentum largely exceeds $m_e c$ [15]. Moreover, it is foreseen that sometimes in the future intensities on the order of $10^{26-28} W/cm^2$ could be in principle available [16], entering the range of laser fields where an effective production of $e^- - e^+$ pairs is expected. However, during the interaction of a powerful laser pulse with ultrarelativistic electron beams (say, tens or hundreds of MeV electrons), presently available laser intensities (in the range of $10^{20-21} W/cm^2$) are already able to produce an appreciable fraction of $e^- - e^+$ pairs [17,18].

In many of the physical situations mentioned above, the thermal energy of the electrons and positrons is of the same order of or larger than $m_e c^2$, so that any "cold plasma" model fails in describing the strongly nonlinear interactions of EM waves with such relativistic plasmas. As a further example, taken from laboratory laser-plasma interaction physics, it has been demonstrated that the presence of a finite electron temperature, even if modelled in a very simple way, leads to the stable self-focusing of relativistically intense laser-beams in an electron-ion plasma [19].

Moreover, the most theoretically important is that in a pure $e^- - e^+$ plasma a finite temperature is a key ingredient in order for soliton-like structures to be sustained, as a result of the balance between the radiation pressure and the thermal pressure [20]. In such a system no charge separation is expected to occur due to the same inertia of the two plasma components.

Several papers have been published on the problem of the nonlinear propagation of EM waves in an $e^- - e^+$ plasma and in particular on the possibility of the existence of solitary structures in it [1,12,21-23]. In particular, bright soliton (EM field peaking) solutions have been found with the addition of a small positive ion concentration (in order to have electrostatic fields to balance the radiation pressure) [21], while dark soliton (density peaking) have been found in a cold [22] as well as in a warm [23] pure $e^- - e^+$ plasma. A set of localized solutions for $e^- - e^+$ as well as for $e^- - e^+$ -ion plasmas under the relaxation theory have been obtained in [24].

Recently, the possibility of existence of one-dimensional bright solitons of very high

amplitude in an overdense $e^- - e^+$ plasma has been demonstrated on the basis of a relativistic hydrodynamic formulation retaining the finite plasma temperature subjected to an adiabatic equation of state [20]. The choice of the adiabaticity conditions is consistent with the closure of the moment equations derived from the relativistic Vlasov equation by assuming the relativistic Maxwellian distribution function [25]. Strong plasma density and temperature inhomogeneities are then possible, which correspond to strong radiation concentrations and to a large frequency downshift.

In this paper we demonstrate that large amplitude one-dimensional bright solitons can exist in an overdense $e^- - e^+$ plasma, that is for $\omega < \sqrt{2}\omega_{pe}$, even on the basis of an isothermal hydrodynamic model derived by assuming *a priori* a distribution function which is an exact solution of the relativistic Vlasov equation. To our knowledge, it is the first derivation of soliton-like distributions of EM radiation based on a relativistic kinetic model. This type of exact solutions of the Maxwell-Vlasov equations represents an extension of the well known BGK solutions [26,27], that describe travelling electrostatic waves in plasmas as well as the magnetostatic equilibria of collisionless plasma-magnetic configurations [28–32].

We derive numerical solutions of the relevant second order differential equation for the vector potential amplitude and discuss the results, with particular interest in their comparison with those of the adiabatic model [20]. In Sect.II we give the particle distribution function and discuss under which conditions it is an exact solution of the relativistic Vlasov equation. The sources for the EM field are then calculated and the Maxwell equations are written explicitly for the general case of a multicomponent plasma. The equations are then reduced to the case of a one-dimensional $e^- - e^+$ plasma. Localized stationary solutions for arbitrary amplitude, frequency, and temperature are found in Sect.III. Sect.IV is devoted to concluding remarks.

II. DERIVATION OF THE FIELD EQUATIONS FROM THE KINETIC PLASMA MODEL

Let us consider the following distribution function for the charged particles of j -th species:

$$f_j(W_j, \mathbf{P}_{j\perp}) = \frac{N_{0j}}{2m_j K_1(\beta_j^{-1})} \delta(\mathbf{P}_{j\perp}) \exp\left(-\frac{W_j}{T_j}\right) \quad (1)$$

where we have defined the total particle energy (rest mass, kinetic, and potential)

$$W_j(\mathbf{r}, t) = m_j \gamma_j + q_j \phi(\mathbf{r}, t), \quad (2)$$

the particle generalized momentum

$$\mathbf{P}_j(\mathbf{r}, t) = \mathbf{p}_j + q_j \mathbf{A}(\mathbf{r}, t), \quad (3)$$

and the relativistic factor

$$\gamma_j = \left(1 + \frac{p_j^2}{m_j^2}\right)^{\frac{1}{2}}. \quad (4)$$

Furthermore, in the above equations the functions $\phi(\mathbf{r}, t)$ and $\mathbf{A}(\mathbf{r}, t)$ represent the electrostatic and vector potentials, respectively, $\beta_j = T_j/m_j$ is the ratio of the thermal energy to the rest energy of the particle of j -th species, m_j , q_j , N_{0j} , \mathbf{p}_j , T_j are the rest mass, electric charge, unperturbed density, momentum, equilibrium temperature of the j -th species, and $K_1(\xi)$ is the modified Bessel function of first order and argument ξ

Eq.(1) describes a highly anisotropic particle distribution function with a finite constant parallel temperature and a transverse beam-like distribution in the momentum space with a zero perpendicular energy spread. This is physically meaningful whenever the particle thermal spread transverse to the radiation propagation direction (or in the plane of the wave electric field) is negligible with respect to the particle motion under the action of the EM field.

The δ -function in Eq.(1) assures that the \perp -component of the generalized momentum (*i.e.*, perpendicular to the direction of the spatial gradients) be preserved. In our model this

condition is imposed *a priori*. For the sake of brevity of the notations, we have used and will use the physical units where the speed of light is unity.

If we substitute Eqs.(1-4) into the relativistic Vlasov equation

$$\frac{\partial f_j}{\partial t} + \frac{\mathbf{p}_j}{m_j \gamma_j} \cdot \nabla f_j + q_j \left[\mathbf{E}(\mathbf{r}, t) + \frac{\mathbf{p}_j}{m_j \gamma_j} \times \mathbf{B}(\mathbf{r}, t) \right] \cdot \frac{\partial f_j}{\partial \mathbf{p}_j} = 0 \quad (5)$$

for the j -th distribution function, several terms cancel out and we remain with

$$\frac{1}{T_j} \delta(\mathbf{P}_{j\perp}) \left(\frac{\partial \phi}{\partial t} - \frac{\partial \mathbf{A}}{\partial t} \cdot \frac{\mathbf{p}_j}{m_j \gamma_j} \right) + \left[\nabla \phi - (\nabla \mathbf{A}) \cdot \frac{\mathbf{p}_j}{m_j \gamma_j} \right] \cdot \frac{\partial}{\partial \mathbf{P}_{j\perp}} [\delta(\mathbf{P}_{j\perp})] = 0. \quad (6)$$

It is easy to verify that if we consider a one-dimensional geometry, where all the physical quantities depend on one spatial coordinate only (say x), assume the circular polarization for the EM radiation, and take advantage of the conservation of $\mathbf{P}_{j\perp}$, then Eq.(6), and therefore Eq.(5), are exactly satisfied by stationary EM energy distributions. As a consequence, the distribution function of Eq.(1) is an exact solution of the one-dimensional kinetic equation and it can be used to calculate the consistent charge density and current density distributions which enter the Maxwell equations as sources for the fields. To this aim, let us begin to calculate the lowest order moments of Eq.(1), the particle density

$$N_j(\mathbf{r}, t) = \int f_j(W_j, \mathbf{P}_{j\perp}) d^3 \mathbf{p}_j = N_{0j} \frac{K_1(\gamma_{\perp j} \beta_j^{-1})}{K_1(\beta_j^{-1})} \gamma_{\perp j} \exp \left(-\frac{\varphi_j}{\beta_j} \right), \quad (7)$$

and the transverse component of the current density

$$\mathbf{J}_{j\perp}(\mathbf{r}, t) = q_j \int \mathbf{v}_{\perp} f_j(W_j, \mathbf{P}_{j\perp}) d^3 \mathbf{p}_j = -|q_j| N_{0j} \frac{K_0(\gamma_{\perp j} \beta_j^{-1})}{K_1(\beta_j^{-1})} \mathbf{a}_{j\perp} \exp \left(-\frac{\varphi_j}{\beta_j} \right), \quad (8)$$

where normalized field variables have been used, that is $q_j \phi(\mathbf{A}_{\perp})/m_j \rightarrow \varphi_j(\mathbf{a}_{j\perp})$, $\gamma_{\perp j} = \sqrt{1 + a_{j\perp}^2}$, and $K_n(\xi)$ is the modified Bessel function of order n -th and argument ξ . We observe that the particle density and the current density distributions in general differ from what one would obtain by assuming the Boltzmann equilibrium. It is possible to show that the Boltzmann distribution is recovered only under the simultaneous limits of both small temperature and small radiation amplitude.

The corresponding normalized one-dimensional equation for the the vector potential reads

$$\mathbf{a}_{\perp xx}'' - \mathbf{a}_{\perp tt}'' = \quad (9)$$

$$= \mathbf{a}_{\perp} \left\{ \frac{K_0(\sqrt{1+a_{\perp}^2} \lambda_e^{-1})}{K_1(\lambda_e^{-1})} \exp\left(\frac{\varphi}{\lambda_e}\right) + \rho Z \frac{K_0[\sqrt{1+\rho^2 Z^2 a_{\perp}^2} (\rho \lambda_i)^{-1}]}{K_1[(\rho \lambda_i)^{-1}]} \exp\left(-\frac{Z\varphi}{\lambda_i}\right) \right\},$$

where the new dimensionless variables, independent of the species, have been defined: $e\mathbf{A}_{\perp}(\phi)/m_e \rightarrow \mathbf{a}_{\perp}(\varphi)$, $\omega_{pe}t(x) \rightarrow t(x)$. Moreover, $\lambda_e = T_e/m_e = \beta_e$, $\lambda_i = T_i/m_e$, $\rho = m_e/m_i$, and Z is the ion charge. By introducing the complex amplitude $a_{\perp}(x, t) = a_y(x, t) + ia_z(x, t)$, which suitably describes the circular polarization of the e.m. field, and looking for localized solution ($a_x = 0$) of the form $a_{\perp}(x, t) = a(x)e^{i\omega t}$, where ω is the field angular frequency, Eq.(9) becomes

$$a_{xx}'' + \omega^2 a(x) = \quad (10)$$

$$= a \left\{ \frac{K_0(\sqrt{1+a^2} \lambda_e^{-1})}{K_1(\lambda_e^{-1})} \exp\left(\frac{\varphi}{\lambda_e}\right) + \rho Z \frac{K_0[\sqrt{1+\rho^2 Z^2 a^2} (\rho \lambda_i)^{-1}]}{K_1[(\rho \lambda_i)^{-1}]} \exp\left(-\frac{Z\varphi}{\lambda_i}\right) \right\}.$$

In the same variables, the equation for the electrostatic potential reads

$$\varphi_{xx}'' = \sqrt{1+a^2} \frac{K_1(\sqrt{1+a^2} \lambda_e^{-1})}{K_1(\lambda_e^{-1})} \exp\left(\frac{\varphi}{\lambda_e}\right) - \quad (11)$$

$$- \sqrt{1+\rho^2 Z^2 a^2} \frac{K_1[\sqrt{1+\rho^2 Z^2 a^2} (\rho \lambda_i)^{-1}]}{K_1[(\rho \lambda_i)^{-1}]} \exp\left(-\frac{Z\varphi}{\lambda_i}\right).$$

Eqs.(10,11) constitute a closed set of one-dimensional relativistic equations for the fields interacting with a hot, two-component plasma, whose macroscopic state has been consistently derived from a solution of the kinetic Vlasov Eq.(5). Once they are solved, Eqs.(7,8) can be used to calculate the corresponding particle density and current density distributions.

Eqs.(10,11) admit the Hamiltonian:

$$H(a, a'_x, \varphi, \varphi'_x; \omega, \lambda_e, \lambda_i) = \frac{1}{2} \left[(a'_x)^2 + \omega^2 a^2 - (\varphi'_x)^2 \right] + W(a, \varphi) = \text{constant}, \quad (12)$$

where the constant is zero for localized solutions with $a, a'_x, \varphi, \varphi'_x \rightarrow 0$ for $|x| \rightarrow \infty$. The function W reads

$$W(a, \varphi) = -\lambda_e \left[1 - \sqrt{1+a^2} \frac{K_1(\sqrt{1+a^2} \lambda_e^{-1})}{K_1(\lambda_e^{-1})} \exp\left(\frac{\varphi}{\lambda_e}\right) \right] - \quad (13)$$

$$-\frac{\lambda_i}{Z} \left[1 - \sqrt{1 + \rho^2 Z^2 a^2} \frac{K_1[\sqrt{1 + \rho^2 Z^2 a^2} (\rho \lambda_i)^{-1}]}{K_1[(\rho \lambda_i)^{-1}]} \exp\left(-\frac{Z\varphi}{\lambda_i}\right) \right].$$

Moreover, exponentially decaying solutions of Eqs.(10,11) exist only for

$$\Delta\omega^2 = \frac{K_0(\lambda_e^{-1})}{K_1(\lambda_e^{-1})} + \rho Z \frac{K_0[(\rho \lambda_i)^{-1}]}{K_1[(\rho \lambda_i)^{-1}]} - \omega^2 > 0. \quad (14)$$

In the zero temperature limit, Eq.(14) gives the well known results $\omega < \sqrt{1 + \rho Z}$, corresponding to the condition of an overdense plasma. The requirement that, in order to trap the radiation, the background plasma should be opaque to the radiation frequency is physically well understandable. One may ask a question of how this condition is dynamically achieved in a real experiment, for example during the interaction of a relativistically intense laser pulse with a plasma. Indeed, particle-in-cell numerical simulations [33] show that during the propagation of a relativistically intense laser pulse in an underdense plasma, a non-negligible fraction of the laser energy is lost due to its trapping into quasi-stationary localized density depressions which are formed behind the pulse. In these regions the radiation frequency turns out to be lower than the laser frequency. The mechanism which produces the frequency down-shift has been explained in [34] and can be summarized as follows: during the nonlinear interaction the pulse is strongly distorted and depleted by Stimulated Raman Scattering, leading to the reduction of its amplitude. Since the time scale of the depletion is much longer than the period of the laser light, we can assume that the process occurs at constant photon number, the adiabatic invariant. Then the ratio of the laser energy (proportional to the square of the field amplitude) to the photon energy (proportional to their frequency) remains constant and as a consequence parts of the laser pulse acquire a frequency which is below the plasma frequency, becoming trapped.

III. LOCALIZED SOLUTIONS IN AN $E^- - E^+$ PLASMA

Let us apply our model to the case of an $e^- - e^+$ plasma (with $T_{e^-} = T_{e^+} = T$), where the masses and moduli of the charges of the two species are equal, that is $\rho = Z = 1$. Since the inertia of charged particles constituting the plasma is the same, no charge separation is

expected to occur. Therefore, no electrostatic potential is excited during the evolution of the system ($\phi = 0$ at any time). We therefore remain with a single nonlinear second order ordinary differential equation for $a(x)$

$$a''_{xx} + \omega^2 a(x) = 2a \frac{K_0(\sqrt{1+a^2} \lambda^{-1})}{K_1(\lambda^{-1})}, \quad (15)$$

where $\lambda_{e-} = \lambda_{e+} = \lambda$. Solutions of Eq.(15) in the form of localized concentrations of the e.m. field are found under the condition that for large $|x|$ -values they should be exponentially vanishing; oscillating functions should be avoided. By linearizing Eq.(15), the equation $a''_{xx} - \Delta\omega^2 a(x) \approx 0$ is found and the above mentioned condition becomes $\Delta\omega^2 = 2K_0(\lambda^{-1})/K_1(\lambda^{-1}) - \omega^2 > 0$. In Fig.1 the function $\Delta\omega^2 = 0$ is plotted in the form of λ vs ω (thick solid line): localized solutions are found in the region to the left of this curve. It is seen that the existence of soliton-like solutions is permitted in high temperature, strongly overdense plasmas. It means that, in order to trap super-intense radiation (upper-left side of the plot in Fig.1), the background plasma density should be much higher than the critical one. The critical density for an EM wave of frequency ω is defined as $N_c = m_e \omega^2 / (4\pi e^2)$.

It is easily verified that Eq.(15) admits the Hamiltonian of the form

$$H(a, a'_x; \omega, \lambda) = \frac{1}{2} [(a'_x)^2 + \omega^2 a^2] + V(a; \lambda) = \text{constant}, \quad (16)$$

where the constant is zero for localized solutions (*i.e.* with $a(x), a'_x \rightarrow 0$ for $|x| \rightarrow \infty$). Moreover, $V(a; \lambda) = 2\lambda(N - 1) = 2\lambda \left[\sqrt{1+a^2} \frac{K_1(\sqrt{1+a^2} \lambda^{-1})}{K_1(\lambda^{-1})} - 1 \right]$. Here and in what follows the particle density N is normalized to the unperturbed density N_∞ . Real phase space trajectories are given by

$$a'_x = \left\{ -2V(a; \lambda) - \omega^2 a^2 \right\}^{\frac{1}{2}}, \quad (17)$$

for $V(a; \lambda) < -\frac{1}{2} \omega^2 a^2 < 0$. In particular, $N_0 = 1 - (\omega^2 a_0^2) / (4\lambda)$, where, a_0 and N_0 are the values of the normalized field amplitude and of the normalized particle density at $x = 0$, respectively.

In Fig.1 a bundle of oblique lines represents the function $\lambda = (\omega^2 a_0^2) / [4(1 - N_0)]$ for different values of a_0 and N_0 . Each group of lines refers to a given value of a_0 , from 0.1 up

to 100, from the bottom to the top. Inside each group, N_0 decreases moving to lower lines. The considered central density values are 0.99, 0.9, 0.7, 0.5, 0.3, and 0.1. Moving along a line, the temperature and the radiation frequency change in order to maintain constant a_0 and N_0 . For given frequency ω and amplitude a_0 values, in the limiting case of $\lambda \rightarrow \infty$ the plasma density tends to be uniform, $N_0 \rightarrow 1$.

In addition, Eq.(15) can be led to quadrature giving

$$\pm x = \int_{a(0)}^{a(x)} \frac{da}{\{-2V(a; \lambda) - \omega^2 a^2\}^{\frac{1}{2}}}. \quad (18)$$

The particle density and the complex amplitude of the current density can be calculated in terms of $a(x)$, that is

$$N(x) = \sqrt{1 + a^2} \frac{K_1(\sqrt{1 + a^2} \lambda^{-1})}{K_1(\lambda^{-1})}, \quad (19)$$

$$j(x) = a \frac{K_0(\sqrt{1 + a^2} \lambda^{-1})}{K_1(\lambda^{-1})}, \quad (20)$$

respectively. We notice that the plasma density is a positive definite quantity, as it should be. This guarantee of density positivity is one of the important consequences of the finite temperature. In zero temperature models unphysical negative density solutions often appear.

The finite value of a which makes the r.h.s. of Eq.(17) equal to zero, that is a_+ such that $a'_x = 0$, gives the maximum amplitude of the corresponding localized solution. In Fig.2A a_+ has been plotted as a function of the temperature for several values of the radiation frequency, $\omega = 0.1 - 1.1$, illustrating the temperature interval in which soliton-like solutions exist. Also, the density perturbation, $N - 1$ (B), and the fluid velocity, $v = j/N$ (C) are reported. It is seen that at low temperatures, an almost full plasma density evacuation can occur, even if the amplitude of the soliton is small. This is the consequence of the fact that in a plasma where the charged particles have all the same mass, their inertia is the same and therefore no charge separation is expected to occur. On the other side, given a small radiation pressure perturbation, it can be balanced only by the thermal pressure, which in a cold plasma is also quite small. An equilibrium with low temperature and small amplitude

radiation concentration is possible. In Fig.3 the case of a soliton in a low temperature ($\lambda = 0.01$) plasma is presented. The field amplitude (A), the plasma density (B), and the fluid velocity (C) distributions are plotted as functions of x . It is possible to assume $N = 0$ in Eq.(18) and integrate it analytically. It turns out that the radiation profile is approximately

$$a(x) = \frac{2\lambda^{1/2}}{\omega} \cos(\omega x), \quad (21)$$

which is plotted also in Fig.3A, by a dotted line. It represents a cosine-soliton in the region of full cavitation. That the cavitation may occur at low temperatures is also understandable by taking the limit $\lambda \rightarrow 0$ in Eq.(19). The density distribution becomes

$$N(x) = (1 + a^2)^{1/4} \exp \left[-\frac{\sqrt{1 + a^2} - 1}{\lambda} \right], \quad (22)$$

which manifests a strong dependence on the background temperature. In Fig.4 the same plots as in Fig.3 are shown for an ultra-relativistic temperature plasma, with $\lambda = 30$. Here, due to the extreme high temperature, the density depletion is only partial.

In the small amplitude limit, it is possible to obtain an analytical solution to Eq.(15). It becomes a cubic nonlinear Schroedinger equation, that is

$$a''_{xx} - \Delta\omega^2 a(x) \approx -\frac{a^3}{\lambda}, \quad (23)$$

where the “frequency shift” is

$$\Delta\omega^2 = 2 \frac{K_0(\lambda^{-1})}{K_1(\lambda^{-1})} - \omega^2, \quad (24)$$

which should be positive in order to have localized (non periodic) solutions. The solution of Eq.(23) writes

$$a(x) = \sqrt{2\lambda\Delta\omega^2} \operatorname{sech}(\sqrt{\Delta\omega^2} x), \quad (25)$$

IV. CONCLUDING REMARKS

In the present paper we have developed a relativistic theory of transverse soliton dynamics in a hot multi-component plasma, deriving the EM field equations with the source terms calculated on the basis of an exact solution of the relativistic Vlasov equation. Although we have considered a very particular case of a one-dimensional standing distribution of circularly polarized EM field, the resulting equations are exact in describing a hot plasma sustaining a relativistically intense EM radiation. In this respect Eqs.(10,11) allow one to investigate soliton-like structures at arbitrary particle temperatures and field intensities for any choice of the plasma constituents.

The distribution function in Eq.(1) describes a strongly anisotropic population with a longitudinal temperature and no transverse thermal energy spread with the particle dynamics in the plane of the electric field being dominated by the EM radiation. Our analysis refers to a spatially uniform distribution of plasma temperature, corresponding to an isothermal assumption on the equation of state of the system. In [20] the case of an adiabatic closure of the relativistic hydrodynamic equations has been investigated, indicating the possibility of the creation of a highly inhomogeneous temperature distribution.

The case of an $e^- - e^+$ plasma has then been considered, for which it is essential to retain a finite particle temperature. Indeed, the thermal motion opposes the radiation pressure allowing the establishment of an equilibrium and then the existence of soliton-like structures. Similarly to what has been found in [20], equilibria with extremely high field intensities in strongly overdense plasmas have been demonstrated. While, in contrast to the adiabatic case, no lower limit occurs on the temperature in order to have solitons. However, it is to be noted that the hydrodynamic model in [20] predicts that, although the unperturbed temperature (that taken at infinity) has a lower limiting value, in the region of the soliton the temperature may decrease appreciably even by orders of magnitude.

A further point to be stressed is that the present analysis predicts the possibility of full plasma cavitation in an extended spatial region at sufficiently small plasma temperatures.

Indeed, in these zones the vacuum solution of the field Eq.(15) results in cosine-solitons.

We believe that the present paper, besides being to our knowledge the first investigation of EM soliton based on a relativistic kinetic model, is particularly important in that it demonstrates, in parallel with [20], that also in a strongly overdense isothermal $e^- - e^+$ plasma large amplitude EM solitons can exist. Moreover, such EM concentrations are always accompanied by appreciable density inhomogeneities, what can be thought as a possible seed of primordial inhomogeneous distribution of matter.

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FIGURES

FIG. 1. The function $\Delta\omega^2 = 0$ is plotted in the form of λ *vs* ω (thick full line). The curve represents the maximum background temperatures at which a soliton-like solution can exist for the given radiation frequency. The straight lines represents the loci of points of given normalized peak field amplitude, a_0 , and central plasma density, N_0 , expressed in the form of λ *versus* ω/ω_p . Four groups of lines are shown corresponding to $a_0 = 0.1$ (dotted lines), 1 (dashed lines), 10 (dot-dashed lines), and 100 (continuous lines). Within each group of lines, N_0 decreases moving towards lower lines, corresponding to the values 0.99, 0.9, 0.7, 0.5, 0.3, and 0.1.

FIG. 2. The peak amplitudes of the localized solutions a_+ (*A*), and the corresponding values of the plasma density perturbation $N - 1$ (*B*), and of the fluid velocity $v = j/N$ (*C*), are plotted *vs* the background temperature, for different frequencies of the e.m. radiation, $\omega = 1.1$ (*a*), 0.9 (*b*), 0.7 (*c*), 0.5 (*d*), 0.3 (*e*), and 0.1 (*f*).

FIG. 3. The spatial distribution of the field amplitude $a(\xi)$ (*A*), of the plasma density $N(\xi)/N_0$ (*B*), and of the fluid velocity $v = j/N$ (*C*) are displayed for a low temperature plasma with $\lambda = 10^{-2}$ and $\omega = 0.1$.

FIG. 4. The same quantities as in Fig.3 for an ultrarelativistic plasma with $\lambda = 30$ and $\omega = 0.1$.







