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Leptonic Unitarity Triangle and CP-violation

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Abstract

The area of the unitarity triangle is a measure of CP-violation. We introduce the leptonic unitarity triangles and study their properties. We consider the possibility of reconstructing the unitarity triangle in future oscillation and non-oscillation experiments. A set of measurements is suggested which will, in principle, allow us to measure all sides of the triangle, and consequently to establish CP-violation. For different values of the CP-violating phase, δ_D, the required accuracy of measurements is estimated. The key elements of the method include determination of $|U_{e3}|$ and studies of the $\nu_\mu - \nu_\mu$ survival probability in oscillations driven by the solar mass splitting Δm^2_{sun}. We suggest additional astrophysical measurements which may help to reconstruct the triangle. The method of the unitarity triangle is complementary to the direct measurements of CP-asymmetry. It requires mainly studies of the *survival* probabilities and processes where oscillations are averaged or the coherence of the state is lost.

1 Introduction

Measurement of CP-violation in leptonic sector is one of the main challenges in particle physics, astrophysics and cosmology.

For three neutrinos (similarly to the quark sector [1]) there is a unique complex phase in the lepton mixing matrix, δ_D, which produces observable CP-violating effects [2]. (If neutrinos are Majorana particles, two additional CP-violating phases exist. These phases, the so-called Majorana phases, do not appear in the oscillation patterns.) The phase δ_D leads to CP-asymmetry [3], $P(\nu_\alpha \to \nu_\beta) \neq P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$, as well as T-asymmetry [4], $P(\nu_\alpha \to \nu_\beta) \neq P(\nu_\beta \to \nu_\alpha)$, of the oscillation probabilities (see also [5] and references therein).

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Measurements of the CP- and T- asymmetries provide a *direct* method of establishing CP-violation. There are a number of studies of experimental possibilities to measure the asymmetries. It was realized that in the 3ν-schemes of neutrino mass and mixing which explain the atmospheric and solar neutrino data, the CP-violation and T-violation effects are small and it will be difficult to detect them [6]. The smallness is due to small values of U_{e3} (restricted by CHOOZ result) and Δm^2_{sun} (responsible for the solar neutrino conversion). Still, the effect can be seen in the new generation of the long baseline (LBL) experiments provided that the LMA-MSW is the solution of the solar neutrino problem and that $U_{e3} > 0.05$ [7, 8, 9].

Two types of LBL experiments sensitive to δ_D are under consideration [10]: the experiments with superbeams [8, 9] and neutrino beams from muon storage rings (the neutrino factories) [11]. Analysis shows [8, 12] that for $|U_{e3}| > 0.05$ and $\Delta m^2_{sun} = 5 \times 10^{-5}$ eV^2, neutrino factories can discriminate between $\delta_D = 0$ and $\delta_D = \frac{\pi}{2}$ at the 3σ level [13] while according to [8] superbeams are able to distinguish at the 3σ level $\delta_D = 0$ from $\delta_D = \frac{\pi}{9}$. In these experiments, the sensitivity to δ_D decreases linearly with Δm^2_{sun}. So, the present uncertainty in Δm^2_{sun} results in an order of magnitude uncertainty in evaluation of sensitivity to δ_D in the future neutrino factories and superbeam experiments. If Δm^2_{sun} is smaller than 2×10^{-5} eV^2, the direct methods will not be sensitive to δ_D [7]. Moreover, neutrino factories and superbeams are very expensive and technically difficult, interpretation of their results can be rather complicated and ambiguous. In view of these difficulties, we need to explore any alternative way to search for CP-violation.

Notice that apart from the asymmetries, the phase δ_D can be determined also from measurements of CP-conserving quantities, the oscillation probabilities themselves, which depend on δ_D [14].

The alternative method to establish CP-violation is to measure the area of *unitarity triangle*. This method is well elaborated in the quark sector. Indeed, the area of the unitarity triangle, S, is related to the Jarlskog invariant, J_{CP}, which is a parameterization independent measure of CP-violation, as

$$S = \frac{1}{2} J_{CP}. \tag{1}$$

So, to establish CP-violation it is sufficient to show that the longest side of the triangle, is smaller than the sum of the other two.

The problem is to measure lengths of the sides of the triangle. As we will see, the method of the unitarity triangle differs from measurements of asymmetries and may have certain advantages from the experimental point of view.

Previously, some general properties of the unitarity triangles for leptonic sector (geometric features, test of unitarity) have been discused in [15], [16], [17].

In this paper we will consider the possibility to reconstruct the leptonic unitarity triangle. In sect. 2, we introduce the leptonic unitarity triangles and study their properties. We estimate the accuracy with which the sides of the triangle should be measured to establish CP-violation. In sect. 3, we describe a set of oscillation measurements which would in principle allow us to reconstruct the triangle. Additional astrophysical measurements which would allow us to realize the method are suggested in sect. 4. Discussions and conclusions are given in sect. 5.

2 Leptonic unitarity triangles

In the three-neutrino schemes the flavor neutrino states, $\nu_f \equiv (\nu_e, \nu_\mu, \nu_\tau)$, and the mass eigenstates $\nu_{mass} \equiv (\nu_1, \nu_2, \nu_3)$, are related by the unitary MNS (Maki-Nakagawa-Sakata [18]) matrix *:

$$U_{MNS} = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix}. \tag{2}$$

The unitarity implies

$$\begin{aligned} U_{e1}U^*_{\mu 1} + U_{e2}U^*_{\mu 2} + U_{e3}U^*_{\mu 3} = 0, \\ U_{e1}U^*_{\tau 1} + U_{e2}U^*_{\tau 2} + U_{e3}U^*_{\tau 3} = 0, \\ U_{\tau 1}U^*_{\mu 1} + U_{\tau 2}U^*_{\mu 2} + U_{\tau 3}U^*_{\mu 3} = 0. \end{aligned} \tag{3}$$

In the complex plane, each term from the sums in (3) determines a vector. So, the Eqs. (3) correspond to three unitarity triangles. The CP-violating phase, δ_D, vanishes if and only if phases of all elements of matrix (2) are factorizable: $U_{\alpha i} = e^{i(\sigma_\alpha + \gamma_i)}|U_{\alpha i}|$. In this case $U_{\alpha i}U^*_{\beta i} = e^{i(\sigma_\alpha - \sigma_\beta)}|U_{\alpha i}||U_{\beta i}|$, and therefore the unitarity triangles shrink to segments.

To construct the unitarity triangle, one needs to measure the absolute values of the elements of two rows (or equivalently two columns) in the mixing matrix. The area of

*The mixing of three flavor states (two light neutrinos and heavy neutral lepton from the third generation) have been discussed in [19].

the triangle is given by the Jarlskog invariant, J_{CP} Eq. (1). The area is non-zero only if $\sin\delta_D \neq 0$.

2.1 $e-\mu$ triangle; properties

We will consider the triangle formed by the e- and μ-rows of the matrix (2) (see Eq. (3-a)). (Up to now, there is no direct information about the elements of the third row. Moreover, even in future, both creation of intense ν_τ beams and detection of ν_τ seem to be difficult.)

To reconstruct the $e-\mu$ triangle three quantities should be determined independently:

$$|U_{e1}U^*_{\mu 1}|, \qquad |U_{e2}U^*_{\mu 2}|, \qquad |U_{e3}U^*_{\mu 3}|. \tag{4}$$

The form of the triangle depends on the yet unknown value of $|U_{e3}|$ and on the specific solution of the solar neutrino problem. In what follows, we will consider mainly the LMA-MSW solution which provides the best fit for the solar neutrino data.

In Figs. 1 and 2, we show examples of the unitarity triangles for different values of U_{e3} and δ_D. In these figures we have normalized the sides of the triangles in such a way that the length of the first side equals one:

$$x = 1, \quad y = \frac{|U_{e2}U^*_{\mu 2}|}{|U_{e1}U^*_{\mu 1}|} \quad \text{and} \quad z = \frac{|U_{e3}U^*_{\mu 3}|}{|U_{e1}U^*_{\mu 1}|} \ . \tag{5}$$

We use the standard parameterization of the MNS mixing matrix [20] in terms of the rotation angles θ_{12}, θ_{13}, θ_{23} and the phase δ_D. We take values of θ_{12} and θ_{23} from the regions allowed by the solar and atmospheric neutrino data.

In Fig. 1 we present the triangles which correspond to $\sin^2 2\theta_{13} = 0.12$ (the upper bound from the CHOOZ experiment for $\Delta m^2_{atm} = 3\times 10^{-3}$ eV^2). The arcs show 10% uncertainty in measurements of the sides y and z. From Fig. 1, one can conclude that for maximal CP-violation, $\delta_D = 90^\circ$, the existence of CP-violation can be established at the 3σ-level or even better if the sides of the triangle are measured with 10% accuracy. For $\delta_D = 60^\circ$, the confidence level is approximately 2σ. No statement can be made for $\delta_D \leq 45^\circ$ unless the accuracy of measurements of the sides will be better. These estimates should be considered as tentative ones. In order to make precise statements one needs to perform careful analysis taking into account, in particular, correlations of the errors.

The triangles shrink for smaller values of $\sin^2 2\theta_{13}$ (Fig. 2). According to Fig. 2 which corresponds to $\sin^2 2\theta_{13} = 0.03$, for $\delta_D = 90^\circ$ CP-violation might be established at $\sim 2\sigma$ level. No conclusion can be made for $\delta_D < 70^\circ$.

The form of the triangle is also sensitive to variations of the angle θ_{12} within the allowed LMA region. In Fig. 3, we have set $\theta_{12} = \frac{\pi}{4}$ and $\sin^2 2\theta_{13} = 0.18$. As follows from this figure with 10% uncertainty in determination of the sides, CP-violation can be established for $\delta_D = 90^o$ and $\delta_D = 60°$.

Note that $y \sim O(1)$ and z is the smallest side, although its length may not be much smaller than others. So, CP-violation implies that

$$|U_{e1}U^*_{\mu 1}| < |U_{e2}U^*_{\mu 2}| + |U_{e3}U^*_{\mu 3}|. \tag{6}$$

Similar triangles can be obtained for the LOW and VAC solutions. The unitarity triangle is different in the case of the SMA-MSW solution. Taking $\tan^2\theta_{12} = \tan^2\theta_{sun} = 0.0016$, $\sin\theta_{23} = 1/\sqrt{2}$ and $\sin\theta_{13} = 0.15$, we find $y = 0.25$, $z = 0.96$. Now y is the smallest side, are the two other sides have comparable lengths. Note that in spite of small mixing of the electron neutrino the smallest side is not very small. Even in this case a moderate accuracy in determination of the sides would allow us to establish CP-violation.

In general, to establish CP-violation, one needs to construct the triangle without using the unitarity conditions. However, if we assume that only three neutrino species take part in the mixing and that there are no other sources of CP-violation apart from the MNS-matrix, we can use some equalities which follow from unitarity. In particular, we can use the independent normalization conditions:

$$\sum_{i=1,2,3} |U_{ei}|^2 = 1 \ , \qquad \sum_{i=1,2,3} |U_{\mu i}|^2 = 1. \tag{7}$$

In this case, the CP-violation effect can be mimicked at some level by the 4th (sterile) neutrino. To eliminate such a possibility, one should check the normalization conditions experimentally.

Thus, to find the sides of the triangle we should determine moduli of four mixing matrix elements:

$$|U_{e2}|, \quad |U_{\mu 2}|, \quad |U_{e3}|, \quad |U_{\mu 3}|. \tag{8}$$

They immediately determine the second and third sides. The two other elements, $|U_{e1}|$ and $|U_{\mu 1}|$, and consequently the first side, can be found from the normalization conditions (7). For the first side we have $|U^*_{e1}U_{\mu 1}| = \sqrt{(1-|U_{e2}|^2-|U_{e3}|^2)(1-|U_{\mu 2}|^2-|U_{\mu 3}|^2)}$. Taking into account this correlation in determination of the sides of the triangle one can estimate accuracy of measurements of the elements (8) needed to establish CP-violation via the inequality (6). Let us introduce

$$A \equiv |U_{e2}||U_{\mu 2}| + |U_{e3}||U_{\mu 3}| - \sqrt{(1-|U_{e2}|^2-|U_{e3}|^2)(1-|U_{\mu 2}|^2-|U_{\mu 3}|^2)} \tag{9}$$

which is a measure of CP violation. CP is conserved if $A = 0$. For the most optimistic cases, where U_{e3} is close to the CHOOZ bound and $\delta_D = 90°$, we find $A = 0.10 - 0.13$.

Suppose the elements $|U_{\alpha i}|$ are measured with accuracies $\Delta|U_{\alpha i}|$. Assuming that the errors $|\Delta U_{\alpha i}|$ are uncorrelated, we can write the error in the determination of A as

$$\Delta A = \sqrt{\sum_{\alpha=e,\mu,i=2,3} \left(\frac{dA}{d|U_{\alpha i}|}\right)^2 (\Delta|U_{\alpha i}|)^2}, \tag{10}$$

where

$$\frac{dA}{d|U_{e2}|} = |U_{\mu 2}| + \frac{|U_{e2}||U_{\mu 1}|}{|U_{e1}|}, \qquad \frac{dA}{d|U_{e3}|} = |U_{\mu 3}| + \frac{|U_{e3}||U_{\mu 1}|}{|U_{e1}|}, \tag{11}$$

$$\frac{dA}{d|U_{\mu 2}|} = |U_{e2}| + \frac{|U_{e1}||U_{\mu 2}|}{|U_{\mu 1}|}, \qquad \frac{dA}{d|U_{\mu 3}|} = |U_{e3}| + \frac{|U_{e1}||U_{\mu 3}|}{|U_{\mu 1}|}. \tag{12}$$

As an example, let us choose the oscillation parameters used in Fig. 1 and $\delta_D = 90°$. Then from Eqs. (11, 12) we find $dA/d|U_{e2}| = 0.82$, $dA/d|U_{e3}| = 0.77$, $dA/d|U_{\mu 2}| = 2.0$ and $dA/d|U_{\mu 3}| = 1.9$. Note that for muonic elements the derivatives are larger by factor of 2. This is a consequence of the appearance of the relatively small element $|U_{\mu 1}|$ in denominators of (12). So, the muonic elements should be measured with the accuracy two times better than the electronic elements.

For our example we find from Eq. (10) that $\Delta A < 0.065$, which would allow to establish deviation of A from zero at the 2σ level, if $\Delta|U_{e2}| = \Delta|U_{e3}| < 0.03$ and $\Delta|U_{\mu 2}| = \Delta|U_{\mu 3}| < 0.02$. This, in turn, requires the following upper bounds for relative accuracies of measurements of the matrix elements: 6% for $|U_{e2}|$, 17% for $|U_{e3}|$, and 3% for $|U_{\mu 2}|$ and $|U_{\mu 3}|$. Since

$$\frac{\Delta|U_{\mu 2}|}{|U_{\mu 2}|} = \left(\frac{|U_{\mu 1}|}{|U_{\mu 2}|}\right)^2 \frac{\Delta|U_{\mu 1}|}{|U_{\mu 1}|}$$

and $U_{\mu 1} \simeq 0.5 U_{\mu 2}$, the required 3% accuracy in $|U_{\mu 2}|$ corresponds to 12 % uncertainty in $U_{\mu 1}$. If there are correlations between $\Delta|U_{\alpha i}|$, the situation may become better. So, the above estimations can be considered as the conservative ones.

2.2 Present status

At present, we cannot reconstruct the triangle: knowledge of the mixing matrix is limited to the elements of the first row (from the solar neutrino data and CHOOZ/Palo Verde experiments) and the third column (from the atmospheric neutrino data). To reconstruct the triangle one needs to know at least one element from the block $U_{\beta i}$, where $\beta = \mu, \tau, \quad i = 1, 2$.

That is, one should measure the distribution of the ν_μ (or/and ν_τ) in the mass eigenstates with split by the solar Δm^2. Using the unitarity condition we can estimate only the ranges for these matrix elements. Clearly, present data are consistent with any value of the CP-violating phase and, in particular, with zero value which corresponds to degenerate triangles.

Let us summarize our present knowledge of the relevant matrix elements.

1). The values of the mixing parameters $|U_{e1}|$ and $|U_{e2}|$ can be obtained from studies of solar neutrinos. Neglecting small effect due to U_{e3}, for the LMA-MSW solution we obtain

$$\frac{|U_{e2}|}{|U_{e1}|} = |\tan\theta_{sun}| = 0.39 - 0.77, \qquad (95\% \ \ \text{C.L.}) \tag{13}$$

and then using the normalization condition:

$$|U_{e1}| \sim [1 + \tan^2\theta_{sun}]^{-1/2} = 0.79 - 0.93, \qquad (95\% \ \ \text{C.L.}). \tag{14}$$

2). The absolute value of $|U_{e3}|$ is restricted from above by the CHOOZ[21] and Palo Verde [22] experiments. The 2ν analysis of the CHOOZ data [21], gives for the best fit value of Δm^2_{atm}

$$|U_{e3}| < 0.20, \qquad (90\% \ \ \text{C.L.}). \tag{15}$$

For lower values of Δm^2_{atm}, the bound is weaker: $|U_{e3}| < 0.22$.

3). The admixture of the muon neutrino in the third mass eigenstate, $|U_{\mu 3}|$, is determined by the atmospheric neutrino data. Again, neglecting effects due to non-zero U_{e3}, we can write

$$4|U_{\mu 3}|^2(1 - |U_{\mu 3}|^2) = \sin^2 2\theta_{atm}, \tag{16}$$

where $\sin^2 2\theta_{atm}$ can be extracted, *e.g.*, from analysis of the zenith angle distribution of the μ-like events in terms of the $\nu_\mu - \nu_\tau$ oscillations. Using the Super-Kamiokande data, we find

$$|U_{\mu 3}| = 0.707^{+0.12}_{-0.14}, \qquad (90\%\text{C.L.}). \tag{17}$$

4). At present, there is no direct information about $|U_{\mu 1}|$ and $|U_{\mu 2}|$. To measure these elements, one needs to study the oscillations of muon neutrinos driven by Δm^2_{sun}. The normalization condition allows us to impose a bound on a combination of these elements:

$$|U_{\mu 1}|^2 + |U_{\mu 2}|^2 = 1 - |U_{\mu 3}|^2 = (0.33 - 0.67). \tag{18}$$

So, to determine $|U_{\mu 1}|$ and $|U_{\mu 2}|$ separately we need to measure a combination of these elements which differs from the normalization condition (18).

3 Reconstructing the unitarity triangle

Let us consider the possibility to determine the triangle in the forthcoming and future oscillation experiments. We suggest a set of oscillation measurements with certain configurations (base-lines, neutrino energies and features of detection) which will allow us to measure the moduli of the relevant matrix elements (see Eqs. (4, 5)).

In general, for 3ν-system the oscillation probabilities depend not only on moduli of the mixing matrix elements, (8) we are interested in, but also on other mixing parameters including the unknown relative phases of the mixing matrix elements, δ_x. Therefore, the problem is to select configurations of oscillation measurements for which the dominant effect is determined by relevant moduli and corrections which depend on unknown elements and phases are negligible or sufficiently small.

The hierarchy of mass splittings: $\Delta m^2_{atm} \gg \Delta m^2_{sun}$ helps to solve the problem. We use

$$\epsilon \equiv \frac{\Delta m^2_{sun}}{\Delta m^2_{atm}} \sim 0.02 \tag{19}$$

as an expansion parameter, where the estimation corresponds to the best fit values of the mass squired differences. Another small parameter in the problem is $|U_{e3}|$.

In what follows, we suggest a set of measurements for which the oscillation probabilities depend mainly on the relevant moduli:

$$P_{\alpha\beta} = P_{\alpha\beta}(|U_{ei}|, |U_{\mu i}|) + \Delta P_{\alpha\beta}(\delta_x), \qquad \alpha, \beta = e, \mu \ , \tag{20}$$

where $\Delta P \ll P$. We estimate corrections, $\Delta P_{\alpha\beta}(\delta_x)$, due to unknown mixing elements and phases.

It is convenient to study the dynamics of oscillations in the basis of states obtained through rotation by the atmospheric mixing angle: $(\nu_e, \nu'_\mu, \nu'_\tau)$, where

$$\nu'_\mu = \frac{1}{\sqrt{1-|U_{e3}|^2}}(U_{\tau 3}\nu_\mu - U_{\mu 3}\nu_\tau), \qquad \nu'_\tau = \frac{1}{\sqrt{1-|U_{e3}|^2}}(U^*_{\tau 3}\nu_\tau + U^*_{\mu 3}\nu_\mu). \tag{21}$$

Projections of these states onto the mass eigenstates equal

$$\langle \nu_e|\nu_1\rangle = U^*_{e1}, \quad \langle \nu'_\mu|\nu_1\rangle = -\frac{U^*_{e2}}{\sqrt{1-|U_{e3}|^2}}, \quad \langle \nu'_\tau|\nu_1\rangle = -\frac{U_{e1}U^*_{e3}}{\sqrt{1-|U_{e3}|^2}} \tag{22}$$

and

$$\langle \nu_e|\nu_3\rangle = U^*_{e3}, \quad \langle \nu'_\mu|\nu_3\rangle = 0, \quad \langle \nu'_\tau|\nu_3\rangle = \sqrt{1-|U_{e3}|^2}. \tag{23}$$

Note that in the limit $U_{e3} = 0$, the state ν'_τ coincides with mass eigenstate ν_3, whereas $\nu'_\mu = -\sin\theta_{12}\nu_1 + \cos\theta_{12}\nu_2$.

In matter, the system of three neutrinos $(\nu_e, \nu'_\mu, \nu'_\tau)$ has two resonances associated with the two different Δm^2. The corresponding resonance energies for the typical density in the mantle of the Earth, are

$$E^R_{23} = 7 \text{ GeV}, \qquad E^R_{12} = 0.15 \text{ GeV}. \tag{24}$$

These energies determine the typical energy scales of the problem as well as the energies of possible experiments. Also there are two length scales in the problem which correspond to the oscillation lengths:

$$l_{12} \equiv \frac{4\pi E}{\Delta m^2_{sun}} = 5 \cdot 10^4 \text{ km} \left(\frac{E}{\text{GeV}}\right), \quad l_{23} \equiv \frac{4\pi E}{\Delta m^2_{atm}} = 10^3 \text{ km} \left(\frac{E}{\text{GeV}}\right) . \tag{25}$$

These numbers have been obtained for the best fit values of the mass squared differences.

Let us consider possibilities to determine the moduli of relevant elements of mixing matrix (8) in turn.

3.1 $|U^*_{e3}U_{\mu 3}|$

In principle, this product can be directly measured in studies of the $\nu_\mu - \nu_e$ oscillations driven by Δm^2_{atm}. Let us consider a relatively short baseline experiment in vacuum. The transition probability can be written as

$$P_{\mu e} = 4|U^*_{e3}U_{\mu 3}|^2 \sin^2 \frac{\Delta m^2_{atm} L}{4E} + \Delta P_{\mu e}, \tag{26}$$

where $\Delta P_{\mu e}$ is the correction due to existence of the Δm^2_{sun} splitting : $\Delta P_{\mu e} \to 0$ when $\Delta m^2_{sun} \to 0$. Thus, if the original flux is composed of pure ν_μ (or pure ν_e), detecting the appearance of ν_e (or ν_μ), one can measure immediately $|U^*_{e3}U_{\mu 3}|$ provided that $\Delta P_{\mu e}$ is small enough. Note that $\Delta P_{\mu e}$ depends on mixing matrix elements $U_{\alpha 1}$, $U_{\alpha 2}$, $(\alpha = e, \mu)$, both on their absolute values and on phases which are unknown. So, we cannot predict $\Delta P_{\mu e}$ and the only way to proceed is to find conditions for experiment at which this value is small. An alternative method would be independent measurement of $|U_{e3}|$ and $|U_{\mu 3}|$.

For neutrino energies, $E > 100$ MeV (which are of practical interest) the oscillation length in vacuum, l_{23}, is more than several hundred kilometers. This means that the experiment should be a long-baseline one, and therefore oscillations will occur in the matter of the Earth.

In a medium with constant density† the probability can be written as

$$P_{\mu e} = \left|(U^m_{e3})^* U^m_{\mu 3}\left(e^{i\Phi^m_{32}} - 1\right) + (U^m_{e1})^* U^m_{\mu 1}\left(e^{i\Phi^m_{12}} - 1\right)\right|^2, \tag{27}$$

where $U^m_{\alpha i}$ are the mixing matrix elements in matter and Φ^m_{ij} is the oscillation phase difference of $i-$ and $j-$ eigenstates.

In the *vacuum limit* (one may consider a hypothetical configuration of experiment where neutrino beam propagates mainly in atmosphere or in a tunnel), $U^m_{\alpha i} = U_{\alpha i}$ and $\Phi^m_i = \Phi_i$. The first term in (27) corresponds to the mode of oscillation we are interested in, and the second term is due to the Δm^2_{sun} splitting. The main correction follows from the interference of these two terms.

For the correction we find

$$\Delta P_{\mu e} \approx -2\epsilon \left|U^*_{e1} U_{\mu 1} U_{e3} U^*_{\mu 3}\right| \Phi_{32}[\sin(\delta_x - \Phi_{32}) - \sin\delta_x], \tag{28}$$

where δ_x is the unknown phase of the product of four mixing matrix elements. In derivation of (28), we have used the smallness of the phase Φ_{12}:

$$\Phi_{12} = \epsilon \Phi_{32}, \tag{29}$$

assuming that $\Phi_{32} = O(1)$ (which maximizes the effect of oscillations). Then the relative correction is of the order of

$$\frac{\Delta P_{\mu e}}{P_{\mu e}} \sim \epsilon \frac{\sin 2\theta_{sun}}{|U_{e3}|}. \tag{30}$$

For the best fit values of the solar oscillation parameters (LMA-MSW solution) and $U_{e3} = 0.2$ we get $\Delta P_{\mu e}/P_{\mu e} \sim 0.1$. That is, the product $|U^*_{e3} U_{\mu 3}|^2$ can be measured with accuracy not better than 10% for maximal possible U_{e3}. Consequently, the accuracy in the determination of $|U^*_{e3} U_{\mu 3}|$ cannot be better than 5%.

There are two possibilities to improve the accuracy: 1) The main oscillation term and the interference term have different dependences on Φ_{32} and therefore on E/L. So, in principle one can disentangle these terms by studying the energy dependence of the effect. 2) The sign of the interference term can be changed varying E/L. Therefore, the correction can be suppressed by averaging over energy, especially if δ_x is small.

Note that for other solutions of the solar neutrino problem (LOW, SMA, VAC), Δm^2_{sun} is much smaller and the correction is negligible.

†For simplicity we will consider matter with constant density. Density variation effects do not change our conclusions.

In the matter the dependence of the oscillation probabilities on mixing matrix elements becomes more complicated. However, there are two limits in which the dominant term of $P_{\mu e}$ can be reduced approximately to the form (26): (i) low energy limit $E \ll E_{13}^R$ in which matter corrections are small and (ii) short base-line limit $L \ll l_{13}^m$ where "vacuum mimicking" condition is satisfied [23].

Let us consider first the *low energy* case, $E \sim (200 - 500)$ MeV. The relative corrections due to matter effect to the main term in (26) are of the order of

$$\frac{l_{23}}{l_0} = \frac{2\sqrt{2} G_F n_e E}{\Delta m_{atm}^2} \ , \tag{31}$$

where $l_0 \equiv \sqrt{2}\pi / G_F n_e$ is the refraction length. For $E \sim 200$ MeV, we have $\epsilon \sim 0.02$, while for $E \sim 1$ GeV, the corrections reach 10%. Moreover, the matter effect is of order 1 for the correction term driven by Δm_{sun}^2.

At low energies the mixing in the heaviest eigenstate is only weakly affected by matter, so that in the first approximation we can take $U_{e3}^m \approx U_{e3}$, $U_{\mu 3}^m \approx U_{\mu 3}$. For $E \sim 200$ MeV the oscillation length due to $2-3$ level splitting is ~ 200 km, and therefore the optimal baseline would be $L \sim (100 - 200)$ km.

The energies $E \sim 200$ MeV are in the resonance interval for Δm_{sun}^2. This means that the electron neutrino has comparable admixtures in the two light eigenstates: $U_{e2}^m \sim U_{e1}^m \sim 1/\sqrt{2}$. The oscillation length is of the order of the vacuum oscillation length (for the LMA-MSW solution $l_{12} \sim 5 \cdot 10^3$ km), and therefore the oscillation phase due to the 1-2 level splitting is small: $\Phi_{12}^m \sim 2\pi L / l_{12} \sim \epsilon \ll 1$. These features simplify the analysis of the correction term. Indeed, using (21, 22, 23)), we find

$$(U_{e1}^m)^* U_{\mu 1}^m \Phi_{12}^m \approx U_{e1}^* U_{\mu 1} \Phi_{12}. \tag{32}$$

Therefore, the relative correction $\Delta P_{\mu e} / P_{\mu e}$ which appears due to the interference of the term (32) with $(U_{e3}^*)^m U_{\mu 3}^m$ in (27) can be written as

$$\frac{\Delta P_{\mu e}}{P_{\mu e}} \sim \epsilon \frac{\sin 2\theta_{sun}}{|U_{e3}|} \frac{|U_{\tau 3}|}{|U_{\mu 3}|} \approx \epsilon \frac{\sin 2\theta_{sun}}{|U_{e3}|}. \tag{33}$$

Here we have taken into account the relation between phases (29). So, the expression for the correction is basically reduced to that in the vacuum oscillation case given by Eq. (30).

Similar considerations hold for the *antineutrino* channel. We find that not only the main term but also the orders of magnitude of the corrections coincide with those for the vacuum case.

Let us consider the case of *high* energies, $E \sim (5-10)$ GeV, and relatively short baselines, $L \sim 700$ km, for which the "vacuum mimicking" condition, $L \ll l_{13}^m$, is satisfied. Notice that these energies are in the range of the resonance due to Δm^2_{atm} (24).

In the limit where the Δm^2_{sun} splitting can be neglected the probability (for the constant density case) becomes

$$P_{\mu e} = \frac{4|U^*_{e3}U_{\mu 3}|^2}{R^2_{13}} \sin^2 \frac{1}{2}\Phi_{32} R_{13} + \Delta P_{\mu e}, \tag{34}$$

where

$$R^2_{13} \equiv \left(\cos 2\theta_{13} - \frac{l_{23}}{l_0}\right)^2 + \sin^2 2\theta_{13}. \tag{35}$$

The oscillation phase in matter, Φ^m_{32}, can be written in terms of the phase in vacuum, Φ_{32}, as

$$\Phi^m_{32} = \Phi_{32} R_{13}, \qquad \Phi_{32} \equiv 2\pi L/l_{32}. \tag{36}$$

For $\Phi^m_{32} \ll 1$ the expansion of the probability (34) in powers of Φ^m_{32} leads to

$$P_{\mu e} = 4|U^*_{e3}U_{\mu 3}|^2 \Phi^2_{32} \left(1 - \frac{R^2_{13}}{3}\Phi^2_{32}\right). \tag{37}$$

Note that the first (leading) term in (37) reproduces the vacuum probability (vacuum mimicking).

The correction depends on energy. In the resonance $R^2_{13} = \sin^2 2\theta_{13}$, and therefore according to (37) corrections are below 10%, even if $\Phi^2_{32} \sim 1$. Below the resonance: for $l_{23}/l_0 \to 0$, we have $R^2_{13} \to 1$ and to have a small correction Φ^2_{32} should be small, thus leading to suppression of the oscillation effect. Therefore, for such a type of experiment, the optimal range of energies is $E = (5-10)$ GeV, and the optimal baseline ~ 700 km. Above the resonance where $R^2_{13} \to \infty$, the oscillation effect is even more suppressed.

Let us evaluate the correction to (37), $\Delta P_{\mu e}$, due to the solar mass splitting. For high energies both oscillation phases Φ^m_{12} and Φ^m_{32} are small, so that from Eq. (27) we obtain the interference term

$$\Delta P_{\mu e} = 2Re\left[(U^m_{e3})^* U^m_{\mu 3} \Phi^m_{32} U^m_{e1} (U^m_{\mu 1})^* \Phi^m_{12}\right]. \tag{38}$$

Considering explicitly the mixing in the $\nu_e - \nu'_\tau$ system we find

$$(U^m_{e3})^* U^m_{\mu 3} \Phi^m_{32} \approx U^*_{e3} U_{\mu 3} \Phi_{32}. \tag{39}$$

The mixing of the electron neutrino in the first eigenstate is strongly suppressed by matter effect:

$$U^m_{e1}(U^m_{\mu 1})^* \approx U_{e1} U^*_{\mu 1} \frac{l_0}{l_{12}}\ , \tag{40}$$

where $l_0/l_{12} \ll 1$. As follows from the level crossing scheme, in the resonance region the phase difference between the two light eigenstates is

$$\Phi_{12}^m \approx -\Phi_{32}. \tag{41}$$

Plugging the expressions (39), (40) and (41) into Eq. (38) we find

$$\Delta P_{\mu e} \approx 2Re\left[U_{e3}^* U_{\mu 3} U_{e1} U_{\mu 1}^*\right] \Phi_{32}^2 \frac{l_0}{l_{12}}. \tag{42}$$

The relative correction can then be written as

$$\frac{\Delta P_{\mu e}}{P_{\mu e}} \approx \epsilon \frac{\sin 2\theta_{sun}}{|U_{e3}|} \frac{E_{23}^R}{E}. \tag{43}$$

Near the resonance the relative correction is similar to that in the low energy limit or in vacuum case. The correction can be further suppressed if $E > E_{13}^R$. At the same time, with the increase of energy, the phase of oscillations (Φ_{32} in Eq. (37)) decreases, and therefore the number of events decreases quadratically.

Let us consider the antineutrino channel. At high energies, $\bar{\nu}_{2m} \approx \bar{\nu}'_\mu$ and $\bar{\nu}_e$ mixes with $\bar{\nu}'_\tau$ in the $\bar{\nu}_{1m}$ and $\bar{\nu}_{3m}$ eigenstates. In the limit $\bar{\nu}_{2m} = \bar{\nu}'_\mu$, the standard vacuum expression for the probability $P_{e\mu}$ (Eq. (37) with $R_{13} \to \bar{R}_{13}$) is reproduced.

The correction is related to the admixture of $\bar{\nu}_e$ in $\bar{\nu}_{2m}$ which is determined by $\sin\theta_{12}^m$ and is strongly suppressed by matter. A straightforward calculation give

$$(\bar{U}_{e1}^m)^* \bar{U}_{\mu 1}^m \approx -\frac{\bar{U}_{\mu 3}\bar{U}_{e3}^*}{\bar{R}_{13}} + \bar{U}_{e1}^* \frac{U_{\mu 1}}{\bar{R}_{12}} \frac{\cos\theta_{13}^m}{\cos\theta_{13}}, \tag{44}$$

where the correction is given by the second term. Taking into account that $\Phi_{12}^m = \Phi_{12} R_{12} = -\Phi_{32} R_{12} \epsilon$ one can find

$$\frac{\Delta \bar{P}_{\mu e}}{\bar{P}_{\mu e}} \approx \epsilon \frac{\sin 2\theta_{sun}}{U_{e3}} \frac{|\bar{U}_{\tau 3}|}{|\bar{U}_{\mu 3}|} \frac{\cos\theta_{13}^m}{\cos\theta_{13}} \approx \epsilon \frac{\sin 2\theta_{sun}}{|U_{e3}|}. \tag{45}$$

Note that in contrast to the neutrino case the correction does not change with energy. So, using neutrino beam seems to be more promising because for neutrinos relative corrections decrease with increase of energy.

We now discuss the sensitivity of upcoming and planned experiments to the product $|U_{e3}^* U_{\mu 3}|$. It was shown [24] that combining the data from the MINOS and ICARUS experiments, one can obtain an upper bound $\sin^2 2\theta_{13} < 0.01$ at the 95% C. L. This would correspond to $|U_{e3}^* U_{\mu 3}| < (0.03 - 0.04)$ which is about 4 times stronger than the present bound: $|U_{e3}^* U_{\mu 3}| < 0.15$. The searches for the $\nu_\mu - \nu_e$ oscillation will be performed in phase I

of the JHF project. The sensitivity to the product $|U_{e3}^* U_{\mu 3}|$ can reach to 0.02 [8]. Therefore, if $|U_{e3}^* U_{\mu 3}|$ is at the border of the present upper bound it will be measured with about 15 % accuracy. Neutrino factories will be sensitive to $|U_{e3}^* U_{\mu 3}|$ down to 10^{-3}. However, for $|U_{e3}^* U_{\mu 3}| <$ few $\times 10^{-3}$ the correction due to non-zero value of Δm^2_{sun} (in the case of LMA-MSW solution) will be comparable with the main term (see [26] for related discussion). For other solutions the corrections are negligible.

3.2 $|U_{e3}|$

Independent determination of $|U_{e3}|$ seems to be important in view of the difficulties associated with the direct measurements of $|U_{e3}^* U_{\mu 3}|$ discussed in sect. 3.1. Knowledge of $|U_{e3}|$ is also needed for a precise determination of $|U_{e1}|$, $|U_{e2}|$ and other mixing elements.

The survival probability for ν_e-oscillations in vacuum can be written as

$$P_{ee} = \left| |U_{e3}|^2 \left(e^{i\Phi_{32}} - 1 \right) + 1 + |U_{e1}|^2 \left(e^{i\Phi_{12}} - 1 \right) \right|^2 . \tag{46}$$

Note that, in contrast to the conversion case, the probability amplitude depends on the required moduli of the matrix elements. A similar analysis holds for antineutrinos.

For low (reactor) energy experiments the matter effects are negligible and the probability equals

$$P_{ee} = 1 - 4(1 - |U_{e3}|^2)|U_{e3}|^2 \sin^2 \frac{\Phi_{32}}{2} + \Delta P_{ee} \ . \tag{47}$$

Here the correction ΔP_{ee} due to the Δm^2_{sun} splitting can be evaluated as

$$\Delta P_{ee} = 2|U_{e1}|^2 |U_{e3}|^2 \Phi_{12} \sin \Phi_{32} - \frac{1}{4} \Phi_{12}^2 \sin^2 2\theta_{sun}. \tag{48}$$

The relative correction is small: $\Delta P_{ee}/P_{ee} < 2$ %, so that in principle, $|U_{e3}|$ can be determined with better than 1% accuracy. Experimental errors in the measurement of P_{ee} will dominate.

Let us comment on the experimental prospects for measuring $|U_{e3}|$. A new reactor experiment, Kr2Det, has been proposed which will be able to set the bound $|U_{e3}| < 0.07$ at the 90 % C. L. [27]. This bound can be used to estimate the sensitivity. If, *e.g.*, $|U_{e3}| = 0.2$, one would expect that the experiment will give $|U_{e3}|^2 = 0.040 \pm 0.005$. Consequently, $|U_{e3}|$ itself will be determined with about 6% accuracy. In fact, the situation can be slightly better. If $|U_{e3}|$ is near its upper bound, one can study spectrum distortion and therefore to perform a more accurate determination of $|U_{e3}|$.

It is not clear if future measurements allow us to measure $|U_{e3}|$ precisely enough to reconstruct the third side of the triangle. But certainly, they will contribute to a more precise determination of $|U_{e1}|$ and $|U_{e2}|$.

3.3 $|U_{\mu 3}|$

The present analysis of the atmospheric neutrino data in terms of 2ν- mixing gives the bound (17) on $|U_{\mu 3}|$. Note that at this accuracy, the effects of non-zero U_{e3} and subleading modes driven by Δm^2_{sun} are unimportant. Indeed, in [28], it is shown that the allowed region for $|U_{\mu 3}|$ does not change considerably as $|U_{e3}|$ varies between zero and its maximal possible value. To reconstruct the unitarity triangle, we need a more precise measurement for $|U_{\mu 3}|$, which requires 3ν analysis taking into account the effect of non-zero $|U_{e3}|$.

The element $|U_{\mu 3}|$ can be measured in ν_μ-disappearance due to oscillations driven by Δm^2_{atm}. The ν_μ-survival probability in a uniform medium equals:

$$P_{\mu\mu} = \left| |U^m_{\mu 3}|^2 \left(e^{i\Phi^m_{32}} - 1\right) + 1 + |U^m_{\mu 1}|^2 \left(e^{i\Phi^m_{12}} - 1\right)\right|^2 . \tag{49}$$

Again, there are two limits in which the dominant term of this probability reduces to the vacuum oscillation probability plus small corrections: (i) the low energy limit ($E \ll E^R_{13}$), (ii) and the high energy case ($E \geq E_R$) with a small baseline for which the vacuum mimicking condition is satisfied.

Let us consider first the *high energy* limit. The dynamics is particularly simple, if $E > E^R_{13}$. In this case $\nu_{1m} \approx \nu'_\mu$, whereas the states ν_e and ν'_τ strongly mix in ν_{2m} and ν_{3m}. For the phases we have $\Phi^m_{12} \approx \Phi_{32}$ and $\Phi^m_{32} \approx \Phi_{32} R_{13} \ll 1$. Neglecting the admixture of ν'_τ in ν_{1m} and ν_{2m} (which is smaller than $|U_{e3}|\epsilon$) we obtain

$$|U^m_{\mu 1}|^2 \approx \frac{|U_{\tau 3}|^2}{1 - |U_{e3}|^2}, \qquad |U^m_{\mu 3}|^2 \approx \frac{|U_{\mu 3}|^2}{1 - |U_{e3}|^2} \cos^2\theta^m_{13} \ , \tag{50}$$

where $\cos^2\theta^m_{13} = [1 + (\cos 2\theta_{13} - l_{13}/l_0)/R_{13}]/2$. Inserting these matrix elements into (49) we can reduce the probability to the form

$$P_{\mu\mu} = 1 - (1 - |U_{\mu 3}|^2)|U_{\mu 3}|^2 \Phi^2_{32} + \Delta P_{\mu\mu} \ , \tag{51}$$

where $\Delta P_{\mu\mu}$ is the correction due to matter effects, non-zero value of U_{e3} and Δm^2_{sun}. Recalling $\Phi^m_{32} \ll 1$, we can write

$$\Delta P_{\mu\mu} = \left[|U_{e3}|^2 |U_{\mu 3}|^2 - 2|U_{\mu 3}|^2 (1 - |U_{\mu 3}|^2)(R_{13}\cos^2\theta^m_{13} - |U_{e3}|^2)\right] \Phi^2_{32}. \tag{52}$$

At the resonance, $R_{13}\cos^2\theta^m_{13} \approx |U_{e3}|$ and the corrections are strongly suppressed. However the resonance region is rather narrow. Above the resonance, $R_{13}\cos^2\theta^m_{13} \approx |U_{e3}|^2/R_{13}$, and as follows from (52), the relative corrections can be estimated as $\Delta P_{\mu\mu}/P_{\mu\mu} \sim |U_{e3}|^2 < 0.04$. Moreover, the corrections are calculable and can be taken into account once $|U_{e3}|^2$ is measured

even with a reasonable accuracy. Corrections, which depend on $U_{\mu 1}$ and $U_{\mu 2}$, are suppressed by the ratio ϵ at $E \sim E^R_{13}$.

In the antineutrino channel we find

$$|\bar{U}^m_{\mu 3}|^2 \approx \frac{|U_{\mu 3}|^2}{1-|U_{e3}|^2}\left(1-\frac{|U_{e3}|^2}{a^2_{13}}\right), \tag{53}$$

where $a_{13} \approx 1-2|U_{e3}|^2 + l_{13}/l_0$, $\bar{\Phi}^m_{32} \approx \Phi_{32}$, and

$$|U^m_{\mu 1}|^2 \approx \frac{|U_{\mu 3}|^2}{1-|U_{e3}|^2}\left[\frac{|U_{e3}|^2}{a^2_{13}} - \frac{|U_{\tau 3}|}{|U_{\mu 3}|}\frac{\sin 2\theta_{sun}}{\bar{R}_{12}}\frac{|U_{e3}|}{a_{13}}\cos\delta_x\right]. \tag{54}$$

Here δ_x is the unknown phase of the product of matrix elements. We can estimate $\bar{\Phi}^m_{12} = \bar{\Phi}^m_{13} - \bar{\Phi}^m_{23} \approx \bar{\Phi}^m_{13} + \Phi_{32} \approx \Phi_{32}(1-\bar{R}_{13})$. Using the first term in the Eq. (54) and keeping the lowest order terms in $|U_{e3}|^2$, we find

$$\bar{P}_{\mu\mu} = 1 - \Phi^2_{32}\left((1-|U_{\mu 3}|^2)|U_{\mu 3}|^2 + |U_{e3}|^2|U_{\mu 3}|^2(1-a^{-2}_{13})(1+a^{-2}_{13}-2|U_{\mu 3}|^2)\right). \tag{55}$$

As in the neutrino case, we assumed $\bar{\Phi}^m_{13} \ll 1$. According to (55) the correction to the standard expression for 2ν probability (the first term in the brackets) is of the order of $|U_{e3}|^2$ with coefficient smaller than 1. Note that, the unknown phases are not involved in the Eq. (55), so the corrections will be calculable once $|U_{e3}|$ is measured.

The relative corrections which depend on unknown phases originate from the interference of the term proportional to $|\bar{U}^m_{\mu 1}|^2$ (the third term in Eq. (49) replacing $\Phi^m_{ij} \to \bar{\Phi}^m_{ij}$ and $|U^m_{\mu i}| \to |\bar{U}^m_{\mu i}|$) and the main term. They can be estimated as

$$\frac{\Delta\bar{P}_{\mu\mu}}{1-\bar{P}_{\mu\mu}} \sim 2\epsilon \sin 2\theta_{sun}|U_{e3}|. \tag{56}$$

Note that in contrast to the case of determination of $|U^*_{e3}U_{\mu 3}|$, the relative corrections are suppressed by $|U_{e3}|$ because, in this case, the main term is larger; it corresponds to the dominant mode of oscillations (*i.e.,* $P_{\mu\mu} \sim 1$, while $P_{\mu e} \ll 1$).

Let us consider the *low energy* experiment with $E \sim E^R_{12} \sim (200-500)$ MeV. In this case the Δm^2_{atm}-driven oscillations are in the quasi-vacuum regime ($U^m_{\mu 3} \approx U_{\mu 3}$, $\Phi^m_{32} \approx \Phi_{32}$) and the base-line can be relatively small: $L \sim l_{12} \sim 100$ km. On the other hand, the oscillations driven by Δm^2_{sun} are in the vacuum mimicking regime: $\Phi^m_{12} \ll 1$. It can be shown that,

$$|U^m_{\mu 1}|^2 \approx \frac{|U_{\tau 3}|^2}{1-|U_{e3}|^2}\sin^2\theta^m_{12}, \qquad \Phi^m_{12} \approx R_{12}\Phi_{12} = \epsilon R_{12}\Phi_{13}, \tag{57}$$

where $\sin^2\theta^m_{12} = [1-(\cos 2\theta_{12} - l_{12}/l_0)/R_{12}]/2$, and R_{12} is the resonance factor for the (1 - 2) system:

$$R_{12} = \sqrt{\left(\cos 2\theta_{12} - \frac{l_{12}}{l_0}\right)^2 + \sin^2 2\theta_{12}}. \tag{58}$$

Inserting the matrix element (57) into (49), we can reduce the probability to the form of Eq. (51) with

$$\Delta P_{\mu\mu} = 2\epsilon |U_{\tau 3}|^2 |U_{\mu 3}|^2 \Phi_{32} \sin \Phi_{32} R_{12} \sin^2 \theta_{12}^m \ . \tag{59}$$

Let us consider last two factors in this expression. In the resonance, $R_{12} \sin^2 \theta_{12}^m \approx R_{12}/2 = \sin 2\theta_{sun}/2$, but above it $R_{12} \sin^2 \theta_{12}^m \to l_{12}/l_0$ and the correction increases with energy too. Below the resonance $R_{12} \sin^2 \theta_{12}^m \to \sin^2 \theta_{sun}$.

Thus, in the resonance region and below it, the correction is small and of the order of $\Delta m^2_{sun}/\Delta m^2_{atm}$. The corrections due to admixture of ν'_τ in the lowest mass eigenstate (which we have neglected) are of the order $|U_{e3}|^2$. As in the high energy limit, the relative corrections are restricted to $\Delta P_{\mu\mu}/P_{\mu\mu} < 0.04$, and moreover, the dominant part of these corrections can be calculated in terms of $|U_{e3}|$.

In the antineutrino channel, similar consideration gives the following corrections which can be calculated in terms of the moduli of the matrix elements:

$$\Delta \bar{P}_{\mu\mu} = \epsilon \frac{|U_{\tau 3}|^2 |U_{\mu 3}|^2}{(1 - |U_{e3}|^2)^2} \Phi_{32} \sin \Phi_{32} (\bar{R}_{12} - \sqrt{\bar{R}_{12}^2 - \sin^2 2\theta_{sun}}). \tag{60}$$

The relative corrections are of the order of $\Delta m^2_{sun}/\Delta m^2_{atm}$. The corrections which depend on unknown phases, are further suppressed ($\sim |U_{e3}|\epsilon$).

Studying the disappearance of ν_μ, the MINOS experiment will determine Δm^2_{atm} and $(1 - |U_{\mu 3}|^2)|U_{\mu 3}|^2$ with 10 % accuracy at the 99% C.L. after 10 kton-years of data taking [29, 24]. Much higher precision can be achieved in phase I of JHF: the oscillation parameters ($(1 - |U_{\mu 3}|^2)|U_{\mu 3}|^2$ and Δm^2_{atm}) will be determined with 1% uncertainty [8]. Thus, there are good perspectives to determine $|U_{\mu 3}|$ with precision better than 2 - 4 %.

Notice that the future atmospheric neutrino experiment, MONOLITH, can measure $\sin^2 2\theta_{23}$ with uncertainty of 8% [30].

3.4 $|U_{e1}|$ and $|U_{e2}|$

The values of $|U_{e1}|$ and $|U_{e2}|$ can be obtained from the solar neutrino data. To first approximation, due to the low energies of solar neutrinos the matter effect on $|U_{e3}|$ is negligible and the solar neutrino conversion driven by Δm^2_{atm} will produce only an averaged oscillation effect. In this case the survival probability equals [31]

$$P_{ee} = (1 - |U_{e3}|^2)^2 P_2(\tan^2 \theta_{sun}, \Delta m^2_{sun}) + |U_{e3}|^4, \tag{61}$$

where

$$\tan^2\theta_{sun} = \frac{|U_{e2}|^2}{|U_{e1}|^2} \tag{62}$$

and P_2 is the two neutrino oscillation (survival) probability determined from the solution of the two neutrino ($\nu_e' - \nu_e$) evolution equation with the oscillation parameters $\tan^2\theta_{sun}$, Δm^2_{sun} and the effective potential $(1-|U_{e3}|^2)V_e$.

Precise measurements of $|U_{e3}|^2$ will be performed by the KamLAND experiment for which P_{ee} is given by (61) with P_2 determined by the oscillation formula in vacuum:

$$P_2 = 1 - \frac{4|U_{e1}|^2|U_{e2}|^2}{(1-|U_{e3}|^2)^2}\sin^2\frac{\Phi_{12}}{2}\ . \tag{63}$$

The expected error in determination of $\sin^2 2\theta_{sun}$, and therefore the combination $|U_{e1}|^2|U_{e2}|^2$ is around 5% [32]. Then using the measured value of $|U_{e1}||U_{e2}|$ and the normalization condition, $|U_{e1}|^2+|U_{e2}|^2 = 1-|U_{e3}|^2$, we can find $|U_{e1}|$ and $|U_{e2}|$, separately. The accuracy can be better than (2 - 3)% .

3.5 $|U_{\mu 1}|$ and $|U_{\mu 2}|$

The determination of $|U_{\mu 1}|$ and $|U_{\mu 2}|$ is the most challenging part of the method. Note that in contrast to $|U^*_{e3}U_{e\mu}|$ (see sect. 3.1), it is not possible to measure the combinations $|U^*_{e1}U_{\mu 1}|$ or $|U^*_{e2}U_{\mu 2}|$, directly from the oscillation experiments. Indeed, in vacuum the $\nu_\mu - \nu_e$ transition probability is determined by the product $Re\left[U^*_{\mu 1}U_{e1}U_{\mu 2}U^*_{e2}\right]$ which depends not only on the absolute values of the matrix elements but also on their phases. (For example, in the case that $\Delta m^2_{sun}L/E$ is not resolved, the probability $P_{e\mu}$ is determined by the combination $|U^*_{\mu 1}U_{e1}+U^*_{\mu 2}U_{e2}|$.) Therefore we will consider the possibility to measure separately $|U_{\mu 1}|$ and $|U_{\mu 2}|$, so that the second side of the triangle can be found using the electron matrix element $|U_{e2}|$ obtained in other experiments. In fact, it is sufficient to measure some combination of $|U_{\mu 1}|$ and $|U_{\mu 2}|$ which differs from the normalization condition (18). This requires an experiment sensitive to the splitting between the first and second levels associated with Δm^2_{sun} which appears usually as a subdominant mode. To suppress the leading effect and the interference of the leading and sub-leading modes, the oscillations driven by Δm^2_{atm} should be averaged out. This condition necessitates the following experimental configuration:

1). The energy of beam should be low: $E < 1$ GeV.

2). The baseline should be large: $L \gg l_{23}$ (in contrast to configurations considered in the previous subsections). Moreover, to avoid suppression of the subdominant mode we need L to be of the order of the oscillation length due to the (1 - 2) splitting.

At $E < 0.5$ GeV, we have $l_{23} \sim 500$ km, and consequently, to reach averaging the baseline can be ~ 2000 km. In this case $\Phi_{12}^m \sim O(1)$.

To produce muons, we need $E > 100$ GeV. For these energies matter effects on (1 - 2) mixing are non-negligible and moreover, since the baseline is large, no vacuum mimicking will occur.

The experiment we have arrived at, seems even more difficult than that for direct measurements of the CP-asymmetries [8]. However, our proposed experiment measures quantities different from asymmetries, and moreover, only one beam, neutrino or antineutrino, is sufficient.

Let us consider the $\nu_\mu - \nu_\mu$ oscillation (*disappearance*) experiment with $E \sim E_{12}^R \sim (200 - 500)$ MeV and $L \geq 2000$ km. At these energies the influence of the matter effect on flavor mixing in the third mass eigenstate is small so that we can take $U_{e3}^m \approx U_{e3}$ and also $U_{\mu 3}^m \approx U_{\mu 3}$. (The corrections are of order of ϵ.) Therefore, the normalization condition gives $|U_{e1}^m|^2 + |U_{e2}^m|^2 = 1 - |U_{e3}^m|^2 \approx 1 - |U_{e3}|^2$. The mixing is reduced to the mixing in 2ν-system, so that matrix elements in matter can be obtained by substituting $U_{e1} \to U_{e1}^m$, $U_{e2} \to U_{e2}^m$.

The general form of the probability in a medium with constant density is given by Eq. (51). Let us calculate $|U_{\mu 1}^m|$. First we express the vacuum value of $U_{\mu 1}$ in terms of U_{e1}, U_{e2} and mixing in the third state, $U_{\alpha 3}$ ($\alpha = e, \mu$). To do this, we use $U_{\mu 1} = \Sigma_\alpha \langle \nu_\mu | \nu'_\alpha \rangle \langle \nu'_\alpha | \nu_1 \rangle$ and the relations (21, 22, 23). Then in the expression for $U_{\mu 1} = U_{\mu 1}(|U_{e1}|, |U_{e2}|, |U_{\alpha 3}|)$ we substitute $U_{ei} \to U_{ei}^m$ ($i = 1, 2$). A straightforward calculation gives

$$U_{\mu 1}^m = -\frac{1}{1 - |U_{e3}|^2} \left[|(U_{e2}^m)^* U_{\tau 3}| + U_{e1}^m U_{e3}^* U_{\mu 3} \right]. \tag{64}$$

Mixing elements in matter can be written as

$$|U_{e1}^m|^2 = |U_{e1}|^2 \frac{\cos^2 \theta_{12}^m}{\cos^2 \theta_{12}} = \frac{|U_{e1}|^2}{R_{12}} \frac{R_{12} + \cos 2\theta_{12} - l_{12}/l_0}{1 + \cos 2\theta_{12}} \tag{65}$$

(here $\cos 2\theta_{12} \equiv 2|U_{e1}|^2/(1 - |U_{e3}|^2) - 1$),

$$|U_{e2}^m|^2 = 1 - |U_{e1}^m|^2 - |U_{e3}|^2 \tag{66}$$

and

$$(U_{e1}^m)^* U_{e2}^m \approx \frac{1}{R_{12}} U_{e1}^* U_{e2}. \tag{67}$$

Using these equations we can express $|U_{\mu 1}^m|$ in terms of the mixing parameters in vacuum as

$$|U_{\mu 1}^m|^2 = \frac{1}{R_{12}} |U_{\mu 1}|^2 + F, \tag{68}$$

where

$$F = \frac{1}{(1-|U_{e3}|^2)}\left[|U_{\tau 3}|^2 f_+ + |U_{e3}|^2|U_{\mu 3}|^2 f_-\right] \tag{69}$$

and

$$f_\pm = \frac{R_{12} - 1 \pm l_{12}/l_0}{2R_{12}}. \tag{70}$$

Note that in the vacuum limit $f_\pm \to 0$, $R_{12} \to 1$ and $F \to 0$. At the resonance, $R_{12} \to \sin 2\theta_{12}$ and above the resonance where $E \gg E_R^{12}$

$$|U_{\mu 1}^m|^2 \to F \approx \frac{|U_{\tau 3}|^2}{1-|U_{e3}|^2}. \tag{71}$$

In this case the dependence of $|U_{\mu 1}^m|^2$, and consequently of the probability, on $|U_{\mu 1}|$ disappears in agreement with our result for the high energy version of the experiment in sect. 3.3. The survival probability can be written as

$$P_{\mu\mu} \approx |U_{\mu 3}|^4 + (1-|U_{\mu 3}|^2)^2 - 4\left(\frac{|U_{\mu 1}|^2}{R_{12}} + F\right)\left(1 - |U_{\mu 3}|^2 - F - \frac{|U_{\mu 1}|^2}{R_{12}}\right)\sin^2\frac{\Phi_{12}^m}{2}\ . \tag{72}$$

Let us underline that $F \equiv F(|U_{e1}|^2, |U_{\alpha 3}|^2)$ is a known function of $|U_{e1}|^2$ and $|U_{\alpha 3}|^2$ and it can be determined once these elements are measured. The contribution of the $|U_{\mu 1}|$-dependent terms to the probability is about 10%. Therefore to determine $|U_{\mu 1}|^2$ precisely enough, the probability should be measured with better that 1% accuracy.

The correction to the formula (72) due to matter effects are of the order ϵ.

For antineutrinos the probability is given by expression (72) substituting, $l_{12}/l_0 \to -l_{12}/l_0$, $R_{12} \to \bar{R}_{12}$, $\Phi_{12}^m \to \bar{\Phi}_{12}^m$ (obviously, $|U_{\alpha i}| = |\bar{U}_{\alpha i}|$). Note that in this case, above the resonance ($E > E_{12}^R$) we get

$$|\bar{U}_{\mu 1}^m|^2 \to F \approx \frac{|U_{e3}|^2|U_{\mu 3}|^2}{1-|U_{e3}|^2}, \tag{73}$$

and again the dependence on $|U_{\mu 1}|$ disappears.

In general the aforementioned conditions (to measure $|U_{\mu 1}|$) are fulfilled for the sub-GeV atmospheric neutrinos reaching the detector through nadir angles between 30° (for which the baseline is tangent to the core) and 80° (with $L \simeq 2000$ km). Indeed, for such neutrinos the phase of oscillations driven by Δm^2_{sun} is of order 1: $\Delta m^2_{sun} L/2E \sim V_e L \sim O(1)$, while $\Delta m^2_{atm} L/2E \gg 1$. However, due to the presence of both electron and muon neutrinos in the initial flux, the number of observable events, *e. g.* μ-like events, depends both on survival and on the conversion probabilities ($P_{e\mu}$ and $P_{\mu e}$). One can easily show that for conversion probabilities, the effects of interference terms, which depend on unknown phases, are non-negligible. So, it is not clear, whether atmospheric neutrino data can help to measure $|U_{\mu 2}|$.

4 Do alternative methods exist?

A straightforward (and similar to what we do in quark sector) way to determine the elements of the MNS matrix (and therefore the sides of the unitarity triangle) is to study the charged current interactions of neutrino *mass eigenstates*, ν_i. Indeed, the cross-section of the interaction

$$\nu_i + X \to l + Y,$$

where l is a charged lepton, is proportional to $|U_{li}|^2$. In particular, measuring the number of electrons and muons produced by the ν_1-beam one can immediately find the ratio $|U_{e1}|/|U_{\mu 1}|$. To perform such a measurement one needs to create a beam of pure neutrino mass eigenstate energetic enough to produce the charged lepton, l. There are several ways to produce (in principle) a pure mass eigenstate beam: (i) via adiabatic conversion, (ii) due to spread of the wave packets and (iii) as a consequence of neutrino decay. In general, one can also use a beam of several mass eigenstates provided that they are *incoherent*. Processes induced by such a beam will be determined by the moduli of matrix elements. Effective loss of coherence occurs due to averaging of oscillation of neutrinos from far objects (for which $\Delta m^2_{sun} L/2E \gg 1$). We will consider these possibilities in turn.

4.1 Adiabatic conversion of neutrinos in matter

In a medium with high density (larger or much larger than the resonance density) mixing can be suppressed. That is, the flavor state, produced at such a density, coincides with the eigenstate of the instantaneous Hamiltonian: $\nu_f \approx \nu_{im}$. If the density decreases slowly to zero along the path of neutrino, such that the adiabaticity condition is fulfilled, the neutrino state will always coincide with the same eigenstate: $\nu(t) \approx \nu_{im}(t)$. As a result, when the neutrino exits the layer (at zero density), it will coincide with the mass eigenstate $\nu(t_f) \approx \nu_{im} = \nu_i$.

This happens for solar neutrinos with energies 5 - 14 MeV in the case of the LMA-MSW solution. The electron neutrinos produced in the center of the Sun are converted to ν_2-state. So, by studying the interactions of neutrinos from the Sun we can measure $|U_{e2}|$.

Obviously, usual solar neutrinos cannot produce muons. Measurements of $|U_{\mu 1}|$ and/or $|U_{\mu 2}|$ will be possible, if high energy neutrinos ($E > m_\mu$) appear in the center of the Sun and propagate adiabatically to the surface. Such a possibility can be realized if massive dark matter particles, WIMPs, are trapped inside the Sun and annihilate emitting neutrinos.

Suppose that the dark matter is composed of neutralinos, χ. The neutralinos annihilate

into the Standard Model particles: $\chi\chi \to \mathrm{W^+W^-}, \mathrm{ZZ}, q\bar{q}$ etc., which in turn decay producing neutrinos and antineutrinos. The energy spectra and the flavor composition of neutrino fluxes (as well as the absolute value of the flux) depend on the parameters of the SUSY model. Generically, one expects an asymmetric flavor composition. Indeed, neutralinos annihilate preferably into $b\bar{b}$, $\tau\bar{\tau}$ (and if they are massive enough also into $t\bar{t}$, W^+W^- and $Z\bar{Z}$). Moreover, muons, pions and kaons are absorbed or lose a substantial fraction of their energy before decay. In contrast, the τ-leptons decay before appreciable energy loss. As a result, one expects an excess of ν_τ and approximately equal fluxes of ν_e and ν_μ:

$$F^0(\nu_\tau) = F^0(\bar{\nu}_\tau) > F^0(\nu_\mu) = F^0(\bar{\nu}_\mu) \approx F^0(\nu_e) = F^0(\bar{\nu}_e). \tag{74}$$

At high energies ($E > 100$ GeV) the inelastic interactions inside the Sun are very important and due to differences in the cross-sections one expects different energy spectra for neutrinos and antineutrinos. However, for $E_\nu < 50$ GeV the effect of inelastic interaction is smaller than 10%.

Note that in contrast to the case of usual solar neutrinos (for which pure ν_e flux is generated), WIMPs produce a neutrino flux with a *complex* flavor composition. This creates two problems: (i) one needs to know the flavor composition which is subject to various uncertainties; (ii) the final flux is a mixture of mass eigenstates (and is not a pure mass eigenstate).

Another problem is that only for rather low energies, the adiabaticity condition is fulfilled in the resonance channel. For $E > 1$ GeV neutrinos cross two resonance regions inside the sun: the high density (h)-resonance associated with Δm^2_{atm} at density $\rho < 30\ g/\mathrm{cm}^3$ and the low density (l)-resonance associated with Δm^2_{sun} at $\rho < 0.5 g/\mathrm{cm}^3$ for the LMA-MSW solution. For definiteness we will consider the scheme with normal mass hierarchy in which both resonances are in the neutrino channels.

The jump probability at the resonance which characterizes the adiabaticity violation can be written as $P_c \approx \exp(-\gamma \sin^2\theta)$, where

$$\gamma = 14 \left(\frac{\Delta m^2}{10^{-5}\mathrm{eV}^2}\right)\left(\frac{1\mathrm{GeV}}{E}\right) , \tag{75}$$

where we have used the density profile of the Sun in [34]. (The above formula is valid only for weak violation of adiabaticity: $P_c \ll 1$.) For $E = 4$ GeV in the l-resonance associated with $\Delta m^2_{sun} = 5 \cdot 10^{-5}$ eV^2 and $\tan^2\theta = 0.35$ we obtain $P_c \approx 0.01$. At the h-resonance violation of adiabaticity is negligible, provided that U_{e3} is not very small. We have $P_c \sim 10^{-7}$, for $|U_{e3}|^2 = 0.03$.

So, the adiabaticity violation effects are below 5 % for $E < 5$ GeV. In the antineutrino (non-resonant) channel the adiabaticity violation occurs at larger energies.

The effects of adiabaticity violation lead to the appearance of interference terms which depend on unknown complex phases. Therefore one needs to select low energy events. On the other hand, even for light neutralinos, $m_\chi \sim 50$ GeV, we expect that only about 30% (or less) of neutrinos have energies less than 5 GeV. This diminishes statistics and hence the efficiency of the method. Notice also that large underwater (ice) detectors have rather high energy thresholds, so detection of GeV neutrinos is may be problematic.

In what follows we present a simplified consideration (neglecting the inelastic interactions) to illustrate possibilities of the method and its shortcomings. A detailed analysis will be given elsewhere [35].

Using relations (74) we can write the ν_β flux ($\beta = e, \mu, \tau$) at the Earth as

$$F_\beta = \sum_\alpha P_{\beta\alpha} F^0_\alpha = F^0 + \Delta F^0 P_{\beta\tau}, \tag{76}$$

where $F^0 = F^0_\mu = F^0_e$ is the common flux of the electron and muon neutrinos, $\Delta F^0 \equiv F^0_\tau - F^0_\mu$ and $P_{\beta\alpha}$ is the $\nu_\alpha \to \nu_\beta$ conversion probability on the way from the production region in the center of the Sun to the detector on the Earth. (Here for simplicity we do not consider the Earth matter effect.) A similar expression can be written for the antineutrino channels.

Neutrinos from WIMP annihilation can be detected by large underwater and ice detectors via charged current interactions. In these detectors the rates of μ-like events will be measured. The detectors will not be able to identify the charge of the produced lepton and therefore we need to sum the neutrino and antineutrino fluxes in our analysis. Using Eq. (76) we can write the expression for the rate of the μ-like events as

$$N_\mu = N^0_\mu + \int d\Omega \Delta F^0 (P_{\tau\mu}\sigma_\mu + P_{\bar\tau\bar\mu}\bar\sigma_\mu), \tag{77}$$

where

$$N^0_\mu = \int d\Omega F^0 (\sigma_\mu + \bar\sigma_\mu) \tag{78}$$

is the rate without oscillations. In the above equations, σ_μ and $\bar\sigma_\mu$ are the cross-sections of the charged current interactions of neutrinos and antineutrinos, respectively. Here $\int d\Omega$ includes the integration over the neutrino energy, the angle between the neutrino and the produced muon and the energy of muon. One should also include the efficiency of detection and the energy resolution function.

Let us find the transition probability, $P_{\beta\tau}$, which determines according to (76) the oscillation effects. The general expressions for the probabilities $P_{\beta\alpha}$ are given in Ref. [36]. Here for illustrative purposes we will consider the case of pure adiabatic propagation in the Sun.

For $E < 5$ GeV and the relevant Δm^2 the oscillatory terms will be averaged out, in particular, due to finite energy resolution of the detector and change of the distance between the Sun and the Earth. Taking into account this averaging effect we find the $\nu_\tau \to \nu_\beta$ conversion probability in the adiabatic limit:

$$P_{\beta\tau} = \sum_{i=1,2,3} |U_{\beta i}|^2 |U^m_{\tau i}|^2, \tag{79}$$

where $U^m_{\tau i}$ is the mixing parameter in matter at the production region.

The density in the production region is much higher than the resonance densities, so that the mixing is strongly suppressed. Considering the level crossing diagram, it is easy to show that $\nu_{3m} \approx -\nu_e$, $\nu_{2m} \approx \nu'_\tau$ and $\nu_{1m} \approx -\nu'_\mu$. From these relations and the definition of ν'_μ and ν'_τ (21) we obtain

$$|U^m_{\tau 1}| = \frac{|U_{\mu 3}|}{\sqrt{1-|U_{e3}|^2}}, \qquad |U^m_{\tau 2}| = \frac{|U_{\tau 3}|}{\sqrt{1-|U_{e3}|^2}}, \qquad U^m_{\tau 3} \approx 0. \tag{80}$$

Inserting (80) into (79) we find the probability of the $\nu_\tau \to \nu_\mu$ conversion:

$$P_{\tau\mu} = \frac{1}{1-|U_{e3}|^2}\left[|U_{\mu 3}|^2(1-|U_{\mu 3}|^2) + (1-2|U_{\mu 3}|^2)|U_{\mu 2}|^2\right] . \tag{81}$$

Since the atmospheric mixing is close to maximal: $|U_{\mu 3}|^2 \sim 1/2$, the dependence of the probability $P_{\tau\mu}$ on $|U_{\mu 2}|^2$ is weak.

In the antineutrino channel, at high densities we have $\bar{\nu}_{1m} \approx \bar{\nu}_e$, $\bar{\nu}_{2m} \approx \nu'_\mu$ and $\bar{\nu}_{3m} \approx \bar{\nu}'_\tau$. Consequently, the mixing elements equal:

$$|\bar{U}^m_{\tau 1}| \approx 0, \qquad |\bar{U}^m_{\tau 2}| \approx \frac{|U_{\mu 3}|}{\sqrt{1-|U_{e3}|^2}}, \qquad |\bar{U}^m_{\tau 3}| \approx \frac{|U_{\tau 3}|}{\sqrt{1-|U_{e3}|^2}}. \tag{82}$$

Using (79) we find the probability of the $\bar{\nu}_\tau \to \bar{\nu}_\mu$ conversion:

$$\bar{P}_{\tau\mu} = \frac{|U_{\mu 3}|^2}{1-|U_{e3}|^2}\left[1-|U_{\mu 3}|^2-|U_{e3}|^2+|U_{\mu 2}|^2\right]. \tag{83}$$

Here $|U_{\mu 2}|^2$ appears with a relatively large coefficient.

In the adiabatic limit, the conversion probabilities do not depend on energy and the expressions for the rate of events can be written as

$$N_\mu \approx N^0_\mu + \frac{|U_{\mu 3}|^2(1-|U_{\mu 3}|^2)}{1-|U_{e3}|^2}\int d\Omega \Delta F^0(\sigma_\mu + \bar{\sigma}_\mu) +$$

$$\frac{|U_{\mu 2}|^2}{1-|U_{e3}|^2}\int d\Omega \Delta F^0[(1-2|U_{\mu 3}|^2)\sigma_\mu + |U_{\mu 3}|^2\bar{\sigma}_\mu]. \tag{84}$$

According to (84), the relative effect of the term proportional to $|U_{\mu 2}|^2$ is suppressed by smaller value of the antineutrino cross-section $\bar{\sigma}_\mu/\sigma_\mu \sim 1/2$.

The relative contribution to number of events from the term which depends on $|U_{\mu 2}|^2$ at low energies (see (84)) is

$$r \approx \frac{|U_{\mu 2}|^2 |U_{\mu 3}|^2}{N_\mu^0} \int d\Omega \Delta F^0 \bar{\sigma}_\mu. \tag{85}$$

For larger energies $E > 7$ GeV, in the neutrino channel the effects of the adiabaticity violation are $\gtrsim 10\%$, *i. e.* larger than the level of required accuracy in the determination of the mixing elements. In the antineutrino channel the adiabaticity violation is weaker. So if detector is able to identify the charge of lepton, and consequently, to select antineutrino events, one will be able to perform better measurements. In particular, events with higher energies can be studied.

Let us comment on the possibility to detect neutrinos from WIMP annihilation and to measure $|U_{\mu 2}|^2$. The event rates due to these neutrinos in a km^3-size detector can be as large as few 10^3 events/year [37], although the rate is very model dependent. If $\Delta F^0/F^0 \sim 0.2 - 0.5$, the contribution of the term sensitive to $|U_{\mu 2}|^2$ is about 10 %. Therefore $|U_{\mu 2}|^2$ can be determined with accuracy 10% at best, provided that all other involved parameters are known. In particular, one should know the original flux F_μ^0, and the difference of fluxes $F_\tau^0 - F_\mu^0$ as functions of energy.

There are several possible ways to deduce information about the ratio of original fluxes $\Delta F^0/F^0$:

1). Theoretical predictions: In principle, future high energy experiments at colliders (*e.g.* LHC), as well as results of the direct searches for dark matter particles will help to measure the mass and the composition (Higgsino-like *versus* gaugino-like) of neutralinos. This will allow to predict the *relative* neutrino fluxes from annihilation.

2). Information on *relative* neutrino fluxes from WIMP annihilation can be obtained by detecting neutrinos from WIMP annihilation in the Earth center.

These studies cannot determine the absolute value of the original flux (F^0). Once we obtained by the aforementioned methods the value of $\Delta F^0/F^0$, we can try to measure both $|U_{\mu 1}|^2$ and the original fluxes from studies of solar neutrinos themselves. If the detector is able to identify the flavor [38], we can compare the rates of μ-like with τ-like or e-like events to find the total flux and the value of $|U_{\mu 1}|^2$.

4.2 Spread of the wave packets

Bunches of neutrino mass eigenstates can be obtained as a result of difference in the group velocities. Neutrinos with a mass squared difference Δm^2 but the same energy E, produced in a source at the same time, will arrive at the detector with a time difference

$$\Delta t = 0.1\text{sec}\left(\frac{L}{10^{28}\ \text{cm}}\right)\left(\frac{\Delta m^2}{3\times 10^{-3}\ \text{eV}^2}\right)\left(\frac{100\ \text{MeV}}{E}\right)^2 . \tag{86}$$

Here L is the distance from the source. If the time during which neutrinos are produced at the source, τ_p, is considerably smaller than Δt ($\tau_p < \Delta t$), and if the energy spread is small enough (or the detector is able to select neutrinos of certain energy), neutrinos will arrive at the detector in bunches: the heavier neutrinos arrive after the light ones. We can measure the numbers of charged leptons produced by different bunches via the charged current interactions. Thus, the ratio of number of muons and τ-leptons produced by the first bunch gives $|U_{\mu 1}/U_{\tau 1}|$. Similarly, the second and third bunches give information about $|U_{l2}|$ and $|U_{l3}|$, respectively. The number of charged leptons of a given flavor, l, produced by the first and second bunches is proportional to $|U_{l1}|/|U_{l2}|$, etc.

According to (86), the time difference in arrival of the bunches for $\Delta m^2 = 3\times 10^{-3}$ eV^2, $E = 100$ MeV and $L = 10^{28}$ cm equals $\Delta t = 0.1$ sec. So, the duration of the neutrino pulse should be smaller than 0.1 second. Moreover, the number of events induced by a single pulse should be large enough. It is not clear if the required sources of neutrinos exist.

4.3 The decay of neutrinos

The neutrino decay provides another possibility to get pure beam of mass eigenstates. In the minimal extension of the Standard Model (in which neutrinos are massive and there are right-handed neutrinos) neutrinos can decay radiatively: $\nu_j \to \nu_i + \gamma$. However, the life time is extremely large: $\tau_\nu > 10^{45}$ s. In certain extensions of the SM the radiative decay or 3ν-decay may be much faster, however, according to astrophysical bounds, the lifetime of radiative decay must be much larger than the age of the Universe (see review [39]).

The decay which satisfies all the bounds and is relevant for our analysis is the Majoron decay [40, 41]:

$$\nu_i \to \bar{\nu}_j + J, \tag{87}$$

where ν_i and ν_j are mass eigenstates, and J is the Majoron.

Let us assume that $\tau_\nu \gg 10^{-3}$ sec, so the neutrinos from the Sun do not decay and the solar and atmospheric anomalies are explained by oscillations while neutrinos from very far

sources (*i.e.*, the gamma-ray bursters, the Active Galactic Nuclei and supernovae) decay before reaching the Earth. Then at the detectors the neutrino flux from the far source is composed only of the lightest neutrinos ν_1 and $\bar{\nu}_1$.

The γ-ray bursts may be accompanied by a flux of energetic neutrinos [42, 43]. Taking the distance of the γ-ray burster from the Earth to be of order 10^{28} cm, one finds that all heavy neutrinos will decay if the lifetime of neutrino at rest, τ_ν, satisfies the inequality

$$\tau_\nu \lesssim 10^{16}\text{sec}\,\frac{m_\nu}{E}. \tag{88}$$

Let us evaluate this bound both for hierarchical and quasi-degenerate neutrino mass spectrum setting $E \sim 1$ TeV. In the hierarchical case $m_1 \simeq 0$, $m_3 \sim 0.05$ eV and $m_2 \sim 0.007$ eV so from (88) we find that in order to let ν_3 decay τ_3 should be $\lesssim 10^2$ sec, while for ν_2, the bound is $\tau_2 < 10$ sec. For quasi-degenerate spectrum with $m_1 \simeq m_2 \simeq m_3 = 1$ eV, the bound is weaker: 10^4 sec.

It was estimated [43] that the flux of neutrinos with the TeV scale energies from an individual gamma burster at cosmological distance $z \sim 1$ produces $10^{-1} - 10$ muons in 1 km^3-size detectors. Since these neutrinos are correlated in time with the γ-ray bursts and aim at the same source, they can be distinguished from neutrinos produced by other sources. The rate of γ-ray bursts detectable on the Earth is $\sim 10^3$/year so the statistics are fairly high and we can deduce results based on studies of such neutrinos.

The large scale detectors cannot identify the charge of produced leptons, so in practice ν_1 and $\bar{\nu}_1$ signals will be summed up. Unfortunately, with present design for 1 km^3-size detectors, it is hardly possible to identify the flavor of the detected neutrinos, for the energy range which we are interested. However, there are methods which open a possibility to build a large detector with flavor identification [38].

Let us assume that these technical problems will be solved and that future detectors will be able to discriminate between different flavors. In the presence of the decay only the flux of the lightest neutrino, ν_1, will arrive at the Earth. Then the ratio of μ-like events to τ-like events produced by this flux equals

$$\frac{\mu-\text{like events}}{\tau-\text{like events}} = \frac{|U_{\mu 1}|^2}{|U_{\tau 1}|^2}, \tag{89}$$

where we have taken into account that for high energies, neutrinos of different flavors have nearly equal cross-sections: $\sigma(\nu_\mu) \simeq \sigma(\nu_\tau)$ and $\sigma(\bar{\nu_\mu}) \simeq \sigma(\bar{\nu_\tau})$.

Thus, if the detectors are able to identify τ-like events, we will be able to measure the ratio $|U_{\mu 1}/U_{\tau 1}|$. Using this ratio, the unitarity condition $|U_{e1}|^2+|U_{\mu 1}|^2+|U_{\tau 1}|^2 = 1$, and $|U_{e1}|^2$ determined by KamLAND, one can derive the value of $|U_{\mu 1}|$. Then $|U_{\mu 2}|^2 = 1-|U_{\mu 1}|^2-|U_{\mu 3}|^2$.

Similarly, for the ratio of $\mu-$like to $e-$like events we have

$$\frac{\mu-\text{like events}}{e-\text{like events}} = \frac{|U_{\mu 1}|^2}{|U_{e1}|^2}, \tag{90}$$

where we have used $\sigma(\nu_\mu) \simeq \sigma(\nu_e)$ and $\sigma(\bar{\nu}_\mu) \simeq \sigma(\bar{\nu}_e)$. This ratio can be used to determine $|U_{\mu 1}|^2$ immediately, once $|U_{e1}|^2$ is known. Unfortunately, identification of e-like events is very difficult.

Let us emphasize that the analysis based on (89) and (90) does not depend on astrophysical details (neutrino production mechanism, etc.). However, one should make sure that all heavy neutrinos have decayed on their way to the Earth. A check can be based on ratio of fluxes (eventually numbers of events). If neutrinos are stable we expect $F(\nu_e) : F(\nu_\mu) : F(\nu_\tau) \simeq 1 : 1 : 1$ [44], while in the case of decay $F(\nu_e) : F(\nu_\mu) : F(\nu_\tau) = |U_{e1}|^2 : |U_{\mu 1}|^2 : |U_{\tau 1}|^2 \simeq 1 : \frac{1}{2} : \frac{1}{2}$. The above analysis was based on assumption that neutrinos from all γ-ray bursters decay before reaching the Earth. It may happen, however, that due to spatial distribution of sources, the degree of decay can be different for different sources. From a single burst only few neutrinos can be detected, so studying neutrinos associated with only one γ-burst event, it is impossible to establish the existence of the decay. This can be done on the basis of observations of many bursts. The sources can be divided into two groups: close sources and far sources (the distance of the source can be measured by its redshift). Studying the flavor composition of neutrino fluxes from these two groups, one can check the stability of neutrinos. There are other measurements which can shed light on the decay rate of neutrinos [41, 45].

Note that this analysis does not depend on the solution of the solar problem.

4.4 Loss of coherence; averaged oscillations

Let us consider *stable* or meta-stable neutrinos produced by cosmological sources. For example, consider again the neutrinos with $E \sim 1$ TeV accompanying the γ-ray bursts [43]. For such neutrinos the oscillation length is much smaller than the distance from the source, $L \sim 10^{28}$ cm (even for the VAC solution of the solar neutrino problem, $\Delta m^2_{sun} L/E \gg 1$). Consequently, all the oscillatory terms in the probabilities will be averaged out. Furthermore, according to existing models of the bursters, the neutrinos are produced in the envelope of the star with density $\rho \sim 10^{-7}$ g cm^{-3} and radius $\sim 10^{13}$ cm and therefore the matter effects inside the source are negligible [46]. As a consequence, the oscillation probabilities for

neutrinos ($\nu_\alpha \to \nu_\beta$) and antineutrinos ($\bar{\nu}_\alpha \to \bar{\nu}_\beta$) take the form

$$P_{\alpha\beta} = \bar{P}_{\alpha\beta} = \sum_i |U_{\alpha i}|^2 |U_{\beta i}|^2. \tag{91}$$

(Here we used $|\bar{U}_{\alpha i}| = |U_{\alpha i}|$.) In particular,

$$P_{\mu\mu} = \sum_i |U_{\mu i}|^4 = K_{\mu\mu} - 2|U_{\mu 2}|^2 |U_{\mu 1}|^2 \tag{92}$$

and

$$P_{e\mu} = \sum_i |U_{\mu i}|^2 |U_{ei}|^2 = K_{e\mu} - |U_{\mu 2}|^2 (|U_{e1}|^2 - |U_{e2}|^2), \tag{93}$$

where $K_{\mu\mu}$ and $K_{e\mu}$ are known functions of $|U_{e1}|$, $|U_{e2}|$, $|U_{e3}|$, $|U_{\mu 3}|$ and do not depend on $|U_{\mu 1}|^2$ and $|U_{\mu 2}|^2$. The probability P_{ee} does not depend on $|U_{\mu 1}|^2$ and $|U_{\mu 2}|^2$.

The probabilities (91), (92), (93) have the following properties which play a key role in our calculations:

(i) $P_{\alpha\beta} = P_{\beta\alpha}$;

(ii) probabilities for neutrinos and antineutrinos are equal;

(iii) the probabilities do not depend on energy.

Let us calculate the number of charged current events produced by neutrinos from γ-ray bursters in the detectors. We assume that the source produces (differential) fluxes of electron neutrinos, F_e^0, and antineutrinos, $\bar{F}_e^0$, as well as muon neutrinos, F_μ^0, and antineutrinos, $\bar{F}_\mu^0$, whereas the fluxes of τ-neutrinos and τ-antineutrinos are negligible. Using the properties of the oscillation probabilities listed above and summing up neutrino and antineutrino contributions we can write for the number of μ-like events

$$(\mu\text{−like events}) = P_{\mu\mu} \left(\int F_\mu^0 \sigma dE + \int \bar{F}_\mu^0 \bar{\sigma} dE \right) + P_{\mu e} \left(\int F_e^0 \sigma dE + \int \bar{F}_e^0 \bar{\sigma} dE \right) , \tag{94}$$

where $\sigma = \sigma(\nu_e) \simeq \sigma(\nu_\mu) \simeq \sigma(\nu_\tau)$ and $\bar{\sigma} = \sigma(\bar{\nu}_e) \simeq \sigma(\bar{\nu}_\mu) \simeq \sigma(\bar{\nu}_\tau)$ are the cross-sections for neutrinos and antineutrinos. Similar expressions can be written for the number of e-like and τ-like events. Notice that the oscillation probabilities factorize out of the integrals over energy.

For the ratios of event numbers we can write

$$\frac{\mu\text{−like events}}{e\text{−like events}} = \frac{P_{\mu\mu} A + P_{\mu e}}{P_{ee} + P_{e\mu} A} \tag{95}$$

and

$$\frac{\tau\text{−like events}}{e\text{−like events}} = \frac{P_{\tau\mu} A + P_{\tau e}}{P_{ee} + P_{e\mu} A} , \tag{96}$$

where

$$A \equiv \frac{\int F^0_\mu \sigma dE + \int F^0_{\bar{\mu}} \bar{\sigma} dE}{\int F^0_e \sigma dE + \int F^0_{\bar{e}} \bar{\sigma} dE}$$

and the probabilities are defined in (91). The ratios in (95) and (96) are functions of A and $|U_{\mu 2}|$. Presumably all other mixing parameters will be measured by terrestrial experiments described in sect. 3.1 - 3.4. The astrophysical information (and uncertainties) is contained in A and it will be probably difficult (if possible) to predict this quantity. So basically we should consider A as an unknown parameter. If future detectors are able to identify flavor [38], we can determine two ratios (95) and (96) and A and $|U_{\mu 2}|$. Additional cross checks of results can be done if the detector is able to identify the charge of the produced lepton.

Note that this method works for all solutions of the solar neutrino problem.

5 Discussions and conclusions

Reconstruction of the unitarity triangle is the way to establish CP-violation alternative to the one based on the direct measurements of the CP- or T- asymmetries.

Properties of the leptonic unitarity triangles have been studied. Our estimates show that for maximal allowed value of $|U_{e3}|$ and maximal CP violation ($\delta_D = 90^o$) a precision better than 10% in measurements of the sides of the triangle will allow us to establish CP-violation at the 3σ level.

We have considered the possibility to reconstruct the triangle in future oscillation experiments. For this in the three neutrino context, one needs to measure the absolute values of the mixing matrix elements: $|U_{e2}|$, $|U_{e3}|$, $|U_{\mu 2}|$, $|U_{\mu 3}|$. The elements of the first side can be obtained from normalization. In general, the oscillation probabilities depend both on these absolute values and on the unknown relative phases. We suggest configurations of experiments: channels of neutrino oscillations, neutrino energies, baselines, and averaging conditions, for which corrections to the probabilities which depend on unknown phases are sufficiently small. We estimate that for value of $|U_{e3}|$ at the present upper bound and $\delta_D = 90^o$, the elements $|U_{e2}|$, $|U_{e3}|$, $|U_{\mu 2}|$, $|U_{\mu 3}|$ should be measured with better than 3 - 5 % accuracy to establish CP-violation.

We have found that

1). The third side of the triangle, $|U^*_{e3} U_{\mu 3}|$, can be directly measured in $\nu_\mu - \nu_e$ oscillations driven by Δm^2_{atm}. In this case, the relative corrections to the probability which depend on the unknown phase, $\Delta P_\delta / P$, are as large as $|U_{e3}|^{-1} \epsilon \sim 10\%$. This substantially restricts

the application of the method. Relative corrections could be suppressed at high energies: $E > E_{13}^R$ by an additional factor E_{13}^R/E. An alternative way to determine $|U_{e3}^* U_{\mu 3}|$ would be independent measurements of $|U_{e3}|$ and $|U_{\mu 3}|$ in different experiments.

2). The element $|U_{e3}|$ can be measured in studies of the $(\nu_e \to \nu_e)$ survival probability at low energy (reactor) experiments. The uncontrolled corrections to the probability are of order of ϵ.

3). $|U_{\mu 3}|$ can be determined by measurements of the $\nu_\mu - \nu_\mu$ survival probability in the ν_μ oscillations driven by Δm^2_{atm}. Both in the low energy ($E < 500$ MeV) and in the high energy ($E > 5$ GeV) regimes the relative uncontrolled corrections to the probability are of the order of ϵ.

4). The ratio of elements $|U_{e1}|$ and $|U_{e2}|$ can be measured rather precisely by KamLAND and by further solar neutrino studies. Then $|U_{e1}|$ and $|U_{e2}|$ can be obtained using the normalization condition and the value of $|U_{e3}|$ (measured in other experiments).

5). The determination of $|U_{\mu 1}|$ and $|U_{\mu 2}|$ is the most difficult part of the program. These quantities could be measured studying the ν_μ disappearance at low energies ($E < 500$ MeV) in experiments with a base-line $L > 2000$ km. The uncontrolled corrections, here, are relatively small ($< \epsilon$).

The reconstruction of the unitarity triangle requires a series of measurements which differ from direct measurements of CP- and T-asymmetries. Indeed,

- information on the absolute values of matrix elements follows mainly from studies of the *survival* probabilities;
- both neutrino and antineutrino beams give the similar results, so that one can work with a neutrino beam or an antineutrino beam or with some combination of them;
- averaging of the oscillatory terms and the loss of coherence help for determination of the relevant parameters.
- the method works better (in a sense that uncontrolled corrections are smaller) for smaller value of ratio $\epsilon = \Delta m^2_{sun}/\Delta m^2_{atm}$ which determines relative uncontrolled corrections.

These factors inhibit direct observations of CP-violation. In this sense, the method of the unitarity triangle is complimentary to the direct determination of CP-violation from measurements of asymmetries.

Clearly, the proof of feasibility of the method requires further studies, and especially, calculations of the effects in specific experiments. In any case, realization of the program will not be easy.

The ideal direct way to measure the absolute values of matrix elements would be experiments with beams of pure mass eigenstates of neutrinos. Charged current interactions of these beams will help to determine immediately the value of $|U_{\alpha i}|^2$ (or the ratios $|U_{\alpha i}|^2/|U_{\beta i}|^2$ if the absolute value of the neutrino flux is not known). However the masses of neutrinos are extremely small (much smaller than the quark case), and therefore neutrinos are produced and propagate in the flavor states, *i.e.,* the coherent states of mass eigenstates. Separation of the mass eigenstates is a non-trivial problem. We have made some suggestions based on spread of the neutrino wave packets, neutrino decay, adiabatic conversion and loss of the coherence due to averaging of oscillations.

In this connection we have considered several possibilities related to high energy extra-terrestrial neutrinos:

1). Neutrinos from possible annihilation of WIMP's in the center of the sun or the earth.

2). Decaying neutrinos from cosmological sources.

The oscillation probabilities for stable neutrinos from far sources (for which $\Delta m^2_{sun} L/2E \gg 1$) dependent only on the absolute values of the mixing matrix elements. We have considered the neutrinos accompanying gamma-ray bursts, as an example.

These possibilities require large ($\sim$ 1km^3-size) underwater (ice) detectors which enable us to identify the flavor of the interacting neutrino.

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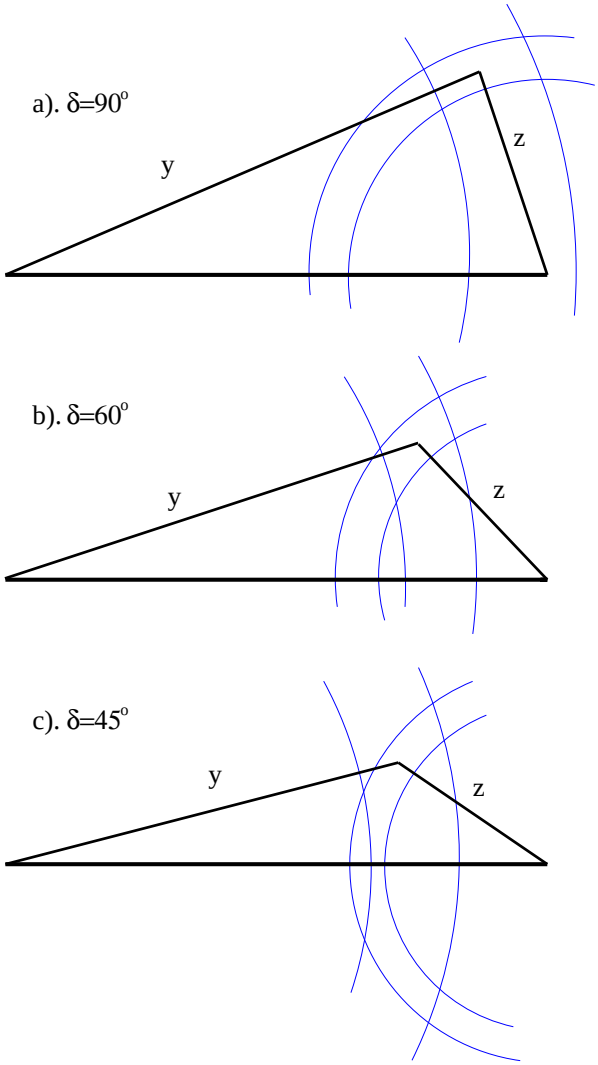

Figure 1: The unitarity triangles for different values of the CP-violating phase δ_D. For mixing angles, we take $\tan^2\theta_{12} = 0.3$, $\sin^2 2\theta_{23} = 1$ and $\sin^2 2\theta_{13} = 0.12$. The arcs show the 10 % uncertainties in determination of y and z.

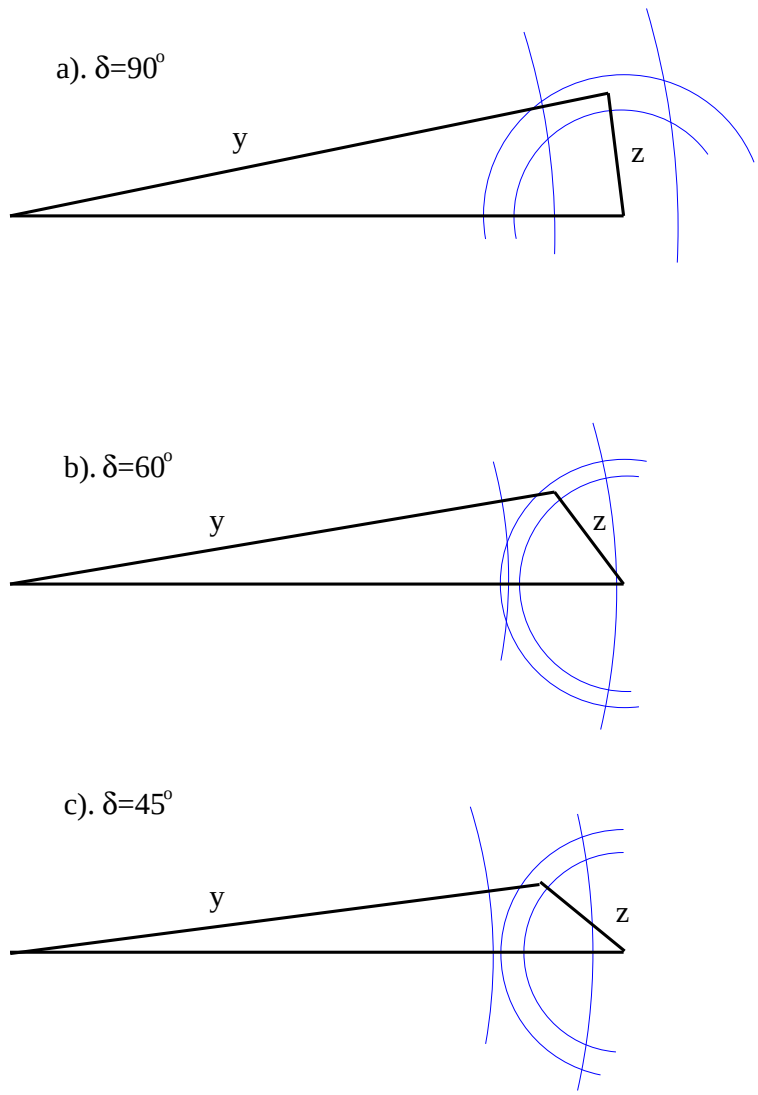

Figure 2: The same as Fig. 1 for $\sin^2 2\theta_{13} = 0.03$.

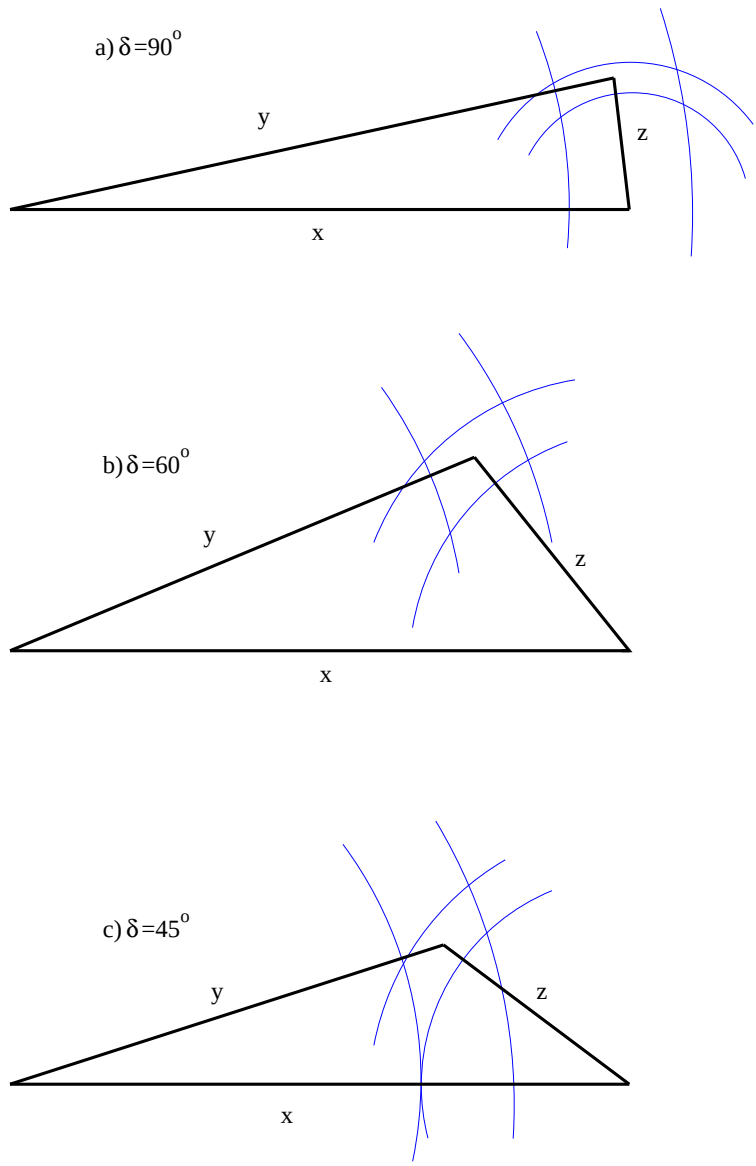

Figure 3: The same as Fig. 1 for $\sin^2 2\theta_{13} = 0.18$ and $\tan\theta_{12} = 1$.