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Positron polarisation and low energy running at a Linear Collider

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The physics potential of an e^+e^- linear collider can be significantly enhanced if both the electron and positron beams are polarised. Low energy running at the Z-resonance or close to the W-pair threshold is particularly attractive with polarised positrons. This note discusses the experimental aspects and physics opportunities of both low energy running and positron polarisation.

I. INTRODUCTION

An e^+e^- linear collider offers many possible options to enhance the baseline program [1, 2]. Already in its basic running mode the electron beam will be polarised to around 80% with a strained photocathode technology similar to that used at the SLC. As two additional options it should also be possible to polarise the positrons and to run at lower energies around the Z-pole and the W-pair threshold.

Positron polarisation enables some genuinely new measurements, especially in Supersymmetry [3]. In addition, polarised positrons improve the measurement of the beam polarisation due to favourable error propagation and due to the possibility of measuring the polarisation directly using $e^+e^- \to W^+W^-$ events [4].

Electroweak tests, already performed at LEP and SLC can be repeated with much higher precision with the linear collider running at low energy [5, 6]. The largest progress can be achieved in the measurement of the effective weak mixing angle using the left right asymmetry. This measurement will be completely limited by polarisation systematics unless positron polarisation is available. In the measurement of the W-mass the background can be controlled by measuring the left-right asymmetry near the W-pair threshold. Also this requires a fairly accurate polarisation measurement that can be helped by positron polarisation.

II. POSITRON POLARISATION

Simultaneous electron and positron beam polarisation results in six principal advantages: (1) higher effective polarisation, (2) suppression of background, (3) enhancement of event rates, (4) increased sensitivity to non-standard couplings, (5) fixing quantum numbers of new particles and (6) improved accuracy in measuring the polarisation [3, 7].

The fact that highly polarised electron beams are achievable in a linear collider has already been demonstrated at the SLC, and there is every reason to expect that electron polarisations $\mathcal{P}_{e^-}$ in excess of 80% will be possible at future linear colliders. Furthermore, methods for achieving 40–60% positron polarisation $\mathcal{P}_{e^+}$ have been proposed and are currently under development.

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A. Technical issues concerning positron polarisation

Compared to polarised electron sources, the technical hurdles for positron polarisation are significant. A fundamental difference is that the production of each positron requires 10-100 MeV photons, rather than the few eV photons per electron at an electron photocathode. In addition, the yields are typically an order of magnitude worse for positrons than for photoproduced electrons. Nevertheless, three different technical approaches for polarised positron production have been discussed in the literature: 1) bremsstrahlung pair production with a polarised electron beam, 2) Compton backscattering of photons from a high energy polarised electron beam, with subsequent photo-production of positrons, and 3) polarised photon production using a high energy electron beam in a helical undulator, with subsequent photo-production of positrons.

The first method, where for example a 50 MeV electron beam is incident on a 0.1 radiation length target, and where the produced positrons with energy larger than 25 MeV are captured and transported, would in principle produce a positron polarisation of 50% [8]. But the efficiency is low and a beam power of order 1.5 MW would be required. As this power is comparable to the expensive (of order 1 G$) CEBAF beam, the first method has been deemed impractical. The second technique is attractive because the positron polarisation would be controllable pulse-to-pulse by changing the circular polarisation of the laser. However, a dedicated high current 6 GeV electron linac and a complex laser system consuming a tremendous amount of laser power would be needed - in one design, a system of about 50 CO_2 lasers using a "wall plug" power of about 20 MW, in order to achieve 50-60% positron polarisation [9]. The third method would use $\sim$ 200 meters of helical undulator magnet through which the full-energy electron beam is passed, producing polarised photons. A collimated fraction of the photon beam ($\sim$ 20%) is directed onto a target, and positrons of energy larger than 15 MeV are retained producing a 60% polarised beam [10, 11]. A low emittance electron beam is required, and hence the post-collision beam probably cannot be used. TESLA proposes a similar design also for the non-polarised positron source [12]. For high energy running the same electron beam is used for positron creation and for physics. This scheme reduces the energy of the colliding e^- beam by $\mathcal{O}(1\%)$ and increases the energy spread from 0.05% to 1.5% which is considered acceptable. Alternately, separate electron bunches could be used to produce positrons preserving the energy spread of the colliding beam, but at the cost of ultimate luminosity. For GigaZ running one part of the electron arm is used to produce the ~ 50 GeV physics beam while the other part accelerates a ~ 200 GeV beam to produce the positrons. Technical issues arise regarding the construction of this first-of-a-kind undulator and the associated photon collimation and positron capture systems, and the cost of such a positron source will certainly be high.

B. Physics benefits of positron polarisation at high energies

In the limit of vanishing electron mass, SM processes in the s–channel are initiated by electrons and positrons polarised in the same direction, i.e. $e^+_L e^-_R$ (LR) or $e^+_R e^-_L$ (RL), where the first (second) entries denote helicities of the corresponding particles. This result follows from the vector nature of γ or Z couplings (helicity–conservation). In the following the convention will be used that, if the sign is explicitly given, + (−) polarisation corresponds to R (L) chirality with helicity $\lambda = +\frac{1}{2}$ ($\lambda = -\frac{1}{2}$) for both electrons and positrons. For these processes positron polarisation provides no fundamentally new information. However, choosing the suitable beam polarisation can significantly enhance rates and suppress background. In theories beyond the SM both (LL) and (RR) configurations for s-channel contributions are also allowed and so the polarisation of both beams offers a powerful tool, in addition to enhancing rates and suppressing SM backgrounds, for analysing the coupling structure of the underlying theory.

A short overview of the polarisation effects of a future linear collider is given in [3]. Since, however, one of the main advantages of having positron polarisation is related to the study of SUSY particles two examples for SUSY processes will be discussed here in more detail.

Higgs physics

Higgs production at a LC occurs mainly via WW fusion, $e^+e^- \to H\nu\bar{\nu}$, and Higgsstrahlung $e^+e^- \to HZ$. For a light Higgs of about $m_H \leq 130$ GeV both processes have comparable cross sections at a LC with $\sqrt{s} = 500$ GeV. Beam polarisation can help to measure the HZZ and the HWW coupling separately, e.g. via suppression of the WW background (and the signal of WW fusion) and enhancement of the HZ contribution with right polarised electrons and left polarised positrons. Furthermore, beam polarisation reduces considerably the error when determining the Higgs couplings.

Electroweak physics

At a LC it is possible to test the SM and its prediction for couplings and mixing angles with unprecedented accuracy. a) high $\sqrt{s}$: In order to test the SM with high precision one can carefully study triple gauge boson couplings in the process $e^+e^- \to W^+W^-$ by measuring the angular distribution and polarisation of the $W^\pm$'s.

Simultaneously fitting all of the couplings using unpolarised cross sections results in a strong correlation between the $\gamma-$ and $Z-$couplings whereas polarised beams are well suited to separate these couplings. However, the statistical error in the gauge couplings is small compared to the error due to the experimental uncertainty of $\mathcal{P}_{e^-}$. Simultaneously polarised e^+ and e^- beams reduce this error significantly in the polarisation measurement.
b) At GigaZ, $\sqrt{s} = m_Z$, an order–of–magnitude improvement in the accuracy of the determination of $\sin^2\theta^\ell_{\rm eff}$ may well be possible when using the Blondel Scheme, as discussed in section III B 2.

QCD physics

a) The LC offers the possibility of testing QCD at high energy scales with very high accuracy. Besides the improvement of rates and suppression of e.g. WW background in QCD in general the simultaneous polarisation of both beams leads in particular to a precise determination of top properties as well as to extreme limits of top flavour changing neutral (FCN) couplings.
b) The LC can also collide electrons with electrons and the possibility of $\gamma\gamma$ and $e^-\gamma$ collisions is under active study. All three "novel" collision modes could be used to study polarised structure functions (PSF) of photons. However, even the standard e^+e^- mode can be used to gain information on PSF if one uses highly polarised e^+ and e^- beams in the process $e^+e^- \to \gamma\gamma + e^+e^- \to$ Di-jets $+\, e^+e^-$. Since effects of depolarisation tend to be large at the $e\gamma$ vertex one needs highly polarised e^- and e^+ beams to get first experimental hints on polarised PSF.

Alternative Theories

a) Using simultaneous polarisation of both beams increases the effective polarisation from e.g. 80% to 95% (using the configuration (80%, 60%)) and leads to a higher effective luminosity. Beam polarisation is therefore a helpful tool to enlarge the discovery reach of Z', W' and to discriminate between different contact interactions.
b) In the direct search for extra dimensions, $e^+e^- \to \gamma G$, beam polarisation enlarges the discovery reach for the scale M_D, and is in particular crucial for enhancing the signal (S) and suppressing the dominant background (B) $e^+e^- \to \nu\bar{\nu}\gamma$. The main background is dominated by left–handed couplings and consequently the ratio $\frac{S}{\sqrt{B}}$ increases by a factor 2.1 when using $(\mathcal{P}_{e^-}, \mathcal{P}_{e^+}) = (+0.8, 0)$ and by a factor 4.4 when using $(+0.8, -0.6)$ for the study described in [13].

Supersymmetry

a) Simultaneous polarisation of both beams is absolutely needed for establishing the partnership between electron states and selectron states in Supersymmetry (SUSY), where the scalar particles get associated chiral quantum numbers of their SM partners: $e^-_{L,R} \leftrightarrow \tilde{e}^-_{L,R}$, $e^+_{L,R} \leftrightarrow \tilde{e}^+_{R,L}$.

The s–channel only allows incoming particle/antiparticle pairs $e^-_L e^+_R$ and $e^-_R e^+_L$ due to helicity conservation, whereas in the t–channel all possible beam configurations are allowed:

$$\text{s– and t–channel} : \; e^-_L e^+_R \to \tilde{e}^-_L \tilde{e}^+_L, \tilde{e}^-_R \tilde{e}^+_R, \quad \text{and} \quad e^-_R e^+_L \to \tilde{e}^-_L \tilde{e}^+_L, \tilde{e}^-_R \tilde{e}^+_R \;, \tag{1}$$

$$\text{t–channel} : \; e^-_L e^+_L \to \tilde{e}^-_L \tilde{e}^+_R, \quad \text{and} \quad e^-_R e^+_R \to \tilde{e}^-_R \tilde{e}^+_L \;. \tag{2}$$

Polarised cross sections including ISR and beamstrahlung for the different selectron pairs at $\sqrt{s} = 400$ GeV close to the production threshold are shown in Fig. 1 a). When using $\mathcal{P}_{e^-} = -80\%$ and variable $\mathcal{P}_{e^+}$, one sees that even for $\mathcal{P}_{e^+} = -40\%$ (LL) the highest rates are those for the pair $\tilde{e}^-_L \tilde{e}^+_R$. Its rate is more than a factor two larger than for all other pairs. The two particles can now be separated via charge identification. For these analyses the simultaneous polarisation of the positron beam is absolutely needed. It should be noted that this test of properties is not possible when running at energies far above the corresponding production threshold due to the event kinematics[14]. b) In SUSY models all coupling structures consistent with Lorentz invariance should be considered. Therefore it is also possible to get appreciable event rates for polarisation configurations that are unfavourable for SM processes. Therefore one example in R–parity violating SUSY, $e^+e^- \to \tilde{\nu} \to e^+e^-$, is studied, which is characterised by the exchange of a scalar particle in the direct channel. The main background to this process is Bhabha scattering. A study [15] was made for $m_{\tilde{\nu}} = 650$ GeV, $\Gamma_{\tilde{\nu}} = 1$ GeV, with an angle cut of $45^0 \le \Theta \le 135^0$ and a lepton–number violating coupling $\lambda_{131} = 0.05$ in the R–parity violating Lagrangian $\mathcal{L}_{\not R} \sim \sum_{i,j,k} \lambda_{ijk} L_i L_j E_k$. Here L_i denotes the lepton doublet super fields while E_k corresponds to the lepton singlets. In Fig. 1 b) the resonance curve for the process, including the complete SM–background is given. The cross section $\sigma(e^+e^- \to e^+e^-)$ including $\sigma(e^+e^- \to \tilde{\nu} \to e^+e^-)$ gives i) 7.17 pb (including Bhabha–background: 4.50 pb) for the unpolarised case, ii) 7.32 pb (including Bhabha–background: 4.63 pb) for $P_{e^-} = -80\%$ and iii) 8.66 pb (including Bhabha–background: 4.69 pb) for $P_{e^-} = -80\%$, $P_{e^+} = -60\%$. This means that only the electron polarisation enhances the signal only slightly by about 2%, whereas the simultaneous polarisation of both beams with $(-80, -60)$ produces a further increase by about 20%. This configuration of beam polarisations, which strongly suppresses pure SM processes, allows one to perform fast diagnostics for this R–parity violating process.
c) Simultaneously polarised e^- and e^+ beams lead to the additional enhancements of rates for specific processes with a corresponding suitable choice of beam polarisation. Such an enhancement can not be reached if only

electron beams are polarised even if 100% polarisation were provided and it can be decisive for the discovery of SUSY particles e.g. in extended SUSY models when particularly small rates have to be taken into account.

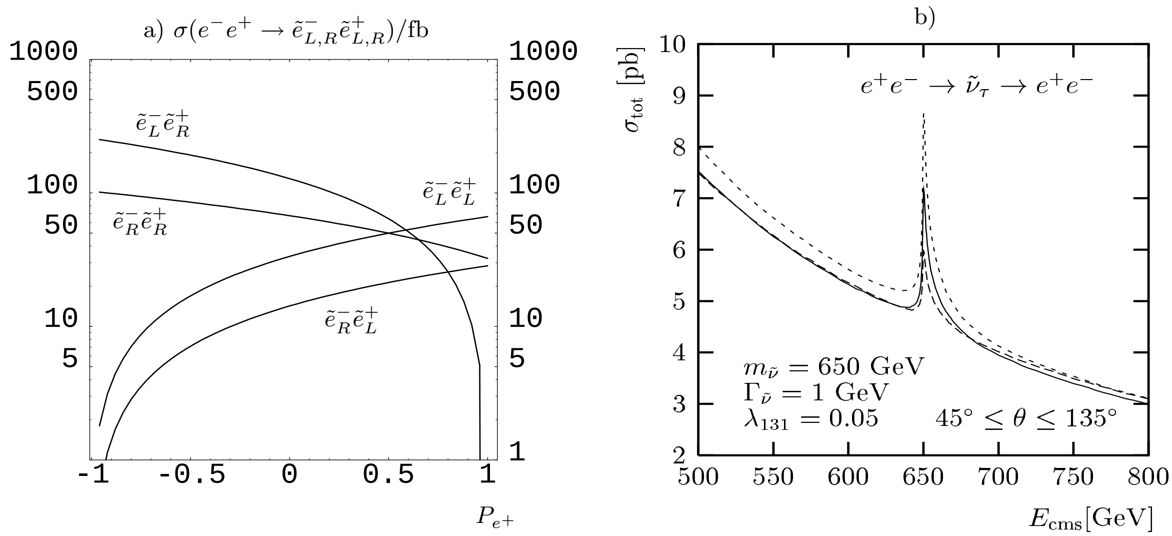

FIG. 1: a) Production cross sections as a function of P_{e^+} for $\sqrt{s} = 400$ GeV, $P_{e^-} = -0.8$, $m_{\tilde{e}_R} = 137.7$ GeV, $m_{\tilde{e}_L} = 179.3$ GeV, $M_2 = 156$ GeV, $\mu = 316$ GeV and $\tan\beta = 3$, $\mu = 316$ GeV and $\tan\beta = 3$. ISR corrections and beamstrahlung are included [14]. b) Sneutrino production in R–parity violating model: Resonance production of $e^+e^- \to \tilde{\nu}$ interfering with Bhabha scattering for different configurations of beam polarisation: unpolarised case (solid), $P_{e^-} = -80\%$ and $P_{e^+} = +60\%$ (long-dashed), $P_{e^-} = -80\%$ and $P_{e^+} = -60\%$ (short-dashed) [15]

Furthermore beam polarisation plays a decisive role in the discovery of new particles (by enhancing their production cross sections) and in particular in the determination of SUSY parameters. The analysis of polarised cross sections for the process $e^+e^- \to \tilde{t}_1\bar{\tilde{t}}_1$ leads to a precise determination of the stop mixing angle, see Sect. III D 2. The determination of the fundamental SUSY parameters (even if CP–violating) can be derived from the chargino and neutralino sector when measuring masses and polarised cross sections (see e.g. Ref. [16] and references therein.) In this context the simultaneous polarisation of both beams will lead to a higher accuracy. Moreover after the determination of the MSSM parameters it will be possible to test experimentally at a LC the fundamental SUSY prediction that the Yukawa couplings, $g_{\tilde{W}}$ and $g_{\tilde{B}}$, are identical to the SU(2) and U(1) gauge couplings g and g' very accurately.

Determining the model parameters with high accuracy and exploiting numerous consistency relations which are based on analytical calculations provide a powerful tool to illuminate the underlying structure of the supersymmetric model.

To summarize, the clean and fundamental nature of e^+e^- collisions in a linear collider is ideally suited for the search for new physics, and the determination of both Standard Model and New Physics couplings with high precision. Polarisation effects will play a crucial role in these processes. We have presented numerous examples that simultaneous polarisation of both beams can significantly expand the accessible physics opportunities for a complete reconstruction of the underlying theory with high accuracy.

III. LOW ENERGY RUNNING

In principle all measurements done at LEP and SLC can be repeated at the linear collider, however with much smaller statistical errors. In around 100 running days 10^9 Z-decays can be collected, about 50 times the LEP or 2000 times the SLC statistics. Two areas are of special interest for the linear collider: electroweak and B-physics. For all measurements related to Z-couplings there exists no real alternative to Z-pole running and the W-pair threshold region seems to be the best place for a precise W-mass measurement. The situation is more complex for B-physics. In 10^9 hadronic Z decays there are about $4 \cdot 10^8$ b-hadrons. This data sample is comparable to the e$^+$e$^-$ B-factories running at the $\Upsilon(4S)$ and much smaller than the samples expected at the TEVATRON or the LHC. However, contrary to the $\Upsilon(4S)$ all b-hadron species are produced and contrary to the hadron machines all events can be reconstructed. In addition, the b-quarks are highly polarised at production and, due to the large forward-backward asymmetry with polarised beams, the production-charge tagging can be done with good purity from the b-direction only. A more detailed discussion of B-physics with Z-running at a linear collider can be found in [17].

A. Machine issues

The present linear collider designs can deliver luminosities of $\mathcal{L} \sim 5 \cdot 10^{33}\mathrm{cm}^{-2}\mathrm{s}^{-1}$ at $\sqrt{s} \sim m_{\mathrm{Z}}$ and $\mathcal{L} \sim 10^{34}\mathrm{cm}^{-2}\mathrm{s}^{-1}$ at $\sqrt{s} \sim 2m_{\mathrm{W}}$.

The energy loss due to beamstrahlung for colliding particles is around $0.05\% - 0.1\%$ at m_{Z} and about a factor of four higher at $2m_{\mathrm{W}}$. The depolarisation in the interaction region is basically negligible. Sacrificing some luminosity the beamstrahlung can be reduced substantially, for example by a factor three for a luminosity loss of a factor two.

Apart from the beamstrahlung there are several other effects that influence the precision of the measurements:

- The mean energies of the two beams have to be measured very precisely. A precision relative to the Z mass of around 10^{-5} might be needed.
- The beam energy spread of the machine plays a crucial role in the measurement of the total width of the Z. If the shape of the distribution is known it can be measured from the acolinearity of Bhabha events in the forward region as long as the energies of the two colliding particles are not strongly correlated.
- With the high luminosities planned, the Z-multiplicity in a train becomes high. This can influence Z-flavour tagging or even Z counting.

The two main designs, X-band and superconducting, differ in some aspects relevant for running at energies around the Z-pole [1, 2]. For the X-band design a bunch train contains 190 bunches with 1.4 ns bunch spacing. In this case more than half of the Z-bosons are produced in the same train with at least one other Z, and the ability to separate events must be studied in detail. A TESLA train contains 2820 bunches with 337 ns bunch spacing. In this case event overlap is not a problem, but the requirements for the data acquisition system are higher. The smaller wakefields in the superconducting machine should reduce the beam energy spread, and the larger bunch spacing should result in a smaller energy difference between the bunches in a train. If the beam spectrometer has some ability to identify individual bunches, however, this latter effect is not a serious problem for either design.

In order not to inhibit the electroweak precision measurements already in the LC design it has to be assured that suitable space in the beam delivery system for very precise beam energy measurement and polarimetry is provided or that the beam energy measurement is directly incorporated into the final focus magnet system. A measurement of these quantities behind the IP will also be very helpful.

B. Electroweak observables

There are four classes of electroweak observables that can be measured at a linear collider during low energy running at the Z-resonance or W-pair threshold:

- observables related to the partial widths of the Z, measured in a Z-resonance scan;
- observables sensitive to the effective weak mixing angle;
- observables using quark flavour tagging;
- the W boson mass and width.

Table I summarises the present precision and the expectations for the linear collider for these quantities.

1. Observables from the Z-line scan

From a scan of the Z-resonance curve the following quantities are measured:

- the mass of the Z (m_{Z});
- the total width of the Z (Γ_{Z});

	LEP/SLC/Tev [18]	GigaZ
$\sin^2\theta_{\mathrm{eff}}^{\ell}$	0.23146 ± 0.00017	± 0.000013
Z lineshape observables:		
m_{Z}	91.1875 ± 0.0021 GeV	± 0.0021 GeV
$\alpha_s(m_{\mathrm{Z}}^2)$	0.1183 ± 0.0027	± 0.0009
$\Delta\rho_\ell$	$(0.55 \pm 0.10) \cdot 10^{-2}$	$\pm 0.05 \cdot 10^{-2}$
N_ν	2.984 ± 0.008	± 0.004
heavy flavours:		
$\mathcal{A}_{\mathrm{b}}$	0.898 ± 0.015	± 0.001
R_{b}^0	0.21653 ± 0.00069	± 0.00014
W boson:		
m_{W}	80.451 ± 0.033 GeV	± 0.007
Γ_{W}	2.114 ± 0.076 GeV	± 0.004

TABLE I: Possible improvement in the electroweak physics quantities at a linear collider. The W-boson mass will likely be known to better than 15 MeV after the LHC begins. For α_s and $\Delta\rho_\ell$, $N_\nu = 3$ is assumed.

- the hadronic pole cross section ($\sigma_0^{\mathrm{had}} = \frac{12\pi}{m_{\mathrm{Z}}^2}\frac{\Gamma_e\Gamma_{\mathrm{had}}}{\Gamma_Z^2}$);
- the ratio of the hadronic to the leptonic width of the Z ($R_\ell = \frac{\Gamma_{\mathrm{had}}}{\Gamma_l}$).

From these parameters two interesting physics quantities, the radiative correction parameter normalising the Z leptonic width, $\Delta\rho_\ell$, and the strong coupling constant, α_s, can be derived.

All observables are already at LEP systematics limited, so that the improvement in the statistical errors is not an issue. From LEP, m_{Z} is known to $2 \cdot 10^{-5}$, while the other three parameters are all known to about 10^{-3}. To improve on α_s and especially on $\Delta\rho_\ell$, it would be best to improve all three parameters. This implies that one has to scan for Γ_{Z}, needs the hadronic and leptonic selection efficiencies for R_ℓ, and in addition the absolute luminosity for σ_0^{had}. Due to the better detectors and the higher statistics that can be used for cross checks, the errors on the selection efficiency and the experimental error on the luminosity might be improved by a factor three relative to the best LEP experiment [19] while it is not clear whether the theory error on the luminosity can be improved beyond its present value of 0.05%. These errors would improve the precision on R_ℓ by a factor of four and σ_0^{had} by 30%.

For an improvement in Γ_{Z} a very precise point to point measurement of the beam energy is needed while the absolute calibration can be obtained from the m_{Z} measurement at LEP. With a Møller spectrometer a precision of 10^{-5} of the beam energy relative to m_{Z} is realistic, leading to a potential improvement of a a factor two in Γ_{Z}. However, because the second derivative of a Breit-Wigner curve at the maximum is rather large, Γ_{Z} and σ_0^{had} are significantly modified due to beamstrahlung and beam energy spread. For the TESLA parameters the fitted Γ_{Z} is increased by about 60 MeV and σ_0^{had} is decreased by 1.8%, where the majority comes from the beamspread. The beamspread thus needs to be understood to about 2% in order not to limit the precision of $\Delta\rho_\ell$. There is the potential to achieve this precision with the acolinearity measurement of Bhabha events [20] or to use at least five scan points and fit to the beamspread, but both options need further studies.

2. The effective weak mixing angle

If polarised beams are available, the quantity that is by far most sensitive to the weak mixing angle is the left-right asymmetry:

$$\begin{aligned} A_{\mathrm{LR}} &= \frac{1}{\mathcal{P}}\frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} \\ &= \mathcal{A}_{\mathrm{e}} \\ &= \frac{2v_e a_e}{v_e^2 + a_e^2}\ , \\ v_e/a_e &= 1 - 4\sin^2\theta_{\mathrm{eff}}^{\ell}\ , \end{aligned} \tag{3}$$

independent of the final state.

Details of this measurement are reported in [5, 21]. With 10^9 Zs, an electron polarisation of 80% and no positron polarisation the statistical error is $\Delta A_{\mathrm{LR}} = 4 \cdot 10^{-5}$. The error from the polarisation measurement is $\Delta A_{\mathrm{LR}}/A_{\mathrm{LR}} = \Delta\mathcal{P}/\mathcal{P}$. At SLC $\Delta\mathcal{P}/\mathcal{P} = 0.5\%$ has been reached [22]. With some optimism a factor two improvement is possible leading to $\Delta A_{\mathrm{LR}} = 3.8 \cdot 10^{-4}$. This is already more than a factor five improvement relative to the final SLD result and almost a factor four compared to the combined LEP/SLD average on $\sin^2\theta^{\ell}_{\mathrm{eff}}$.

If positron polarisation is available there is the potential to go much further using the Blondel scheme [23]. The total cross section with both beams being polarised is given as $\sigma = \sigma_{\mathrm{unpol}}\left[1 - \mathcal{P}_{e^+}\mathcal{P}_{e^-} + A_{\mathrm{LR}}(\mathcal{P}_{e^+} - \mathcal{P}_{e^-})\right]$. If all four helicity combinations are measured A_{LR} can be determined without polarisation measurement as

$$A_{\mathrm{LR}} = \sqrt{\frac{(\sigma_{++} + \sigma_{-+} - \sigma_{+-} - \sigma_{--})(-\sigma_{++} + \sigma_{-+} - \sigma_{+-} + \sigma_{--})}{(\sigma_{++} + \sigma_{-+} + \sigma_{+-} + \sigma_{--})(-\sigma_{++} + \sigma_{-+} + \sigma_{+-} - \sigma_{--})}}\ .$$

Figure 2 shows the error of A_{LR} as a function on the positron polarisation. For $\mathcal{P}_{e^+} > 50\%$ the dependence is relatively weak. For 10^9 Zs a positron polarisation of 20% is better than a polarisation measurement of 0.1% and electron polarisation only.

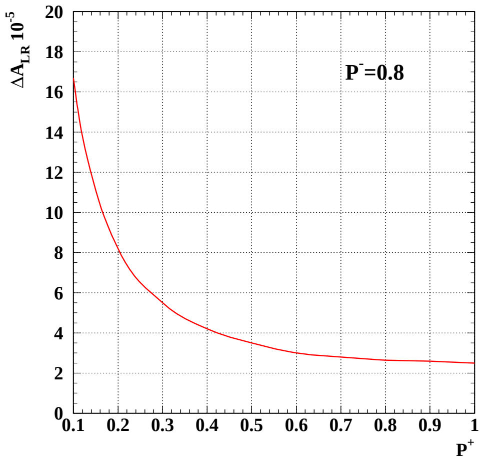

FIG. 2: Error of A_{LR} as a function of the positron polarisation for a luminosity corresponding to 10^9 unpolarised Zs.

However, polarimeters for relative measurements are still needed. The crucial point is the difference between the absolute values of the left- and the right-handed states. If the two helicity states for electrons and positrons are written as $\mathcal{P}_{e^\pm} = \pm|\mathcal{P}_{e^\pm}| + \delta\mathcal{P}_{e^\pm}$ the dependence is $\mathrm{d}A_{\mathrm{LR}}/\mathrm{d}\delta\mathcal{P}_{\mathrm{e}^\pm} \approx 0.5$. One therefore needs to understand $\delta\mathcal{P}_{e^\pm}$ to $< 10^{-4}$. If polarimeters with at least two channels with different analysing power are available, not only the analysing powers but also some internal asymmetries in the polarimeters can be fitted to the data.

Due to $\gamma - Z$ interference, the dependence of A_{LR} on the beam energy is given by $\mathrm{d}A_{\mathrm{LR}}/\mathrm{d}\sqrt{\mathrm{s}} = 2 \cdot 10^{-2}/\,\mathrm{GeV}$. The difference $\sqrt{s} - m_{\mathrm{Z}}$ thus needs to be known to $\sim 10\,\mathrm{MeV}$ to match the measurement with electron polarisation only, and to $\sim 1\,\mathrm{MeV}$ if polarised positrons are available, the same precision as for the foreseen Γ_{Z} improvement. For the same reason, beamstrahlung shifts A_{LR} by $\sim 9 \cdot 10^{-4}$ (TESLA design), so its uncertainty should only be a few percent. If beamstrahlung is identical in the Z-scan that calibrates the beam energy it gets absorbed in the energy calibration, so that practically no corrections are needed for A_{LR}. How far the beam parameters can be kept constant during the scan and how well the beamstrahlung can be measured still needs further studies. However, for A_{LR} only the beamstrahlung and not the beamspread matters. If the beamstrahlung cannot be understood to the required level in the normal running mode one can still go to a mode with lower beamstrahlung increasing the statistical error or the running time.

For the interpretation of the data it will be assumed that $\Delta A_{\mathrm{LR}} = 10^{-4}$ is possible, corresponding to $\Delta\sin^2\theta^{\ell}_{\mathrm{eff}} = 0.000013$. However, it has to be kept in mind that this error will increase by a factor four if no positron polarisation is available.

3. Observables with tagged quarks

Using quark tagging in addition to the observables already discussed, the partial widths and forward-backward asymmetries for b- and c-quarks can also be measured. These observables are sensitive to vertex corrections at the $Zq\bar{q}$ vertex and to new Born-level effects that alter the SM relations between quarks and leptons. Especially the $Zb\bar{b}$ vertex is very interesting, since the b-quark is the isospin-partner of the heavy top quark that plays a special role in many extensions of the SM. It should be noted that the leading vertex corrections are enhanced by the square of the top quark mass.

Up to now only estimates for b-quark observables exist [5, 24]. For the ratio of the Z partial widths to b-quarks and to hadrons (R_{b}) an improvement of a factor five to the LEP/SLD average is possible. This improvement is due to the much better b-tagging compared to LEP, which allows for a higher purity and a smaller energy-dependence reducing the hemisphere correlations.

The forward-backward asymmetry with unpolarised beams measures the product of the coupling parameters for the initial state electrons and the final state quarks $A^q_{\mathrm{FB}} = \frac{3}{4}\mathcal{A}_{\mathrm{e}}\mathcal{A}_{\mathrm{q}}$ while the left-right forward-backward asymmetry with polarised beams is sensitive to the quark couplings only, $A^q_{\mathrm{LR,FB}} = \frac{3}{4}\mathcal{P}\mathcal{A}_{\mathrm{q}}$. For this reason, a factor 15 improvement on $\mathcal{A}_{\mathrm{b}}$ relative to the LEP/SLC result is possible if polarised positrons are available. With polarised electrons only the improvement is limited to a factor six due to the polarisation error. To keep systematics under control also here the improved b-tagging capabilities are essential.

4. Observables at the WW-threshold

The measurement of the W mass and width using a polarised threshold scan at a linear collider has been investigated in [25]. The study considered a dedicated scan with 100 fb^{-1} of total integrated luminosity taken at several scan points near threshold with polarised electron and positron beams. It was demonstrated that the W mass could be measured with a precision of 5 MeV. This error includes the statistical error and systematic errors arising from event selection efficiency, background normalisation, luminosity normalisation and absolute polarisation determination all of which can be determined, if needed, from the data themselves. Therefore with respect to these error sources, the mass determination is experimentally robust. Point-to-point errors were assumed to be negligible; this seems a reasonable assumption. Errors from uncertainties on the background spin model were explored; this topic deserves further study, but methods to alleviate and/or control such an error as discussed in [2] can be applied.

Nevertheless, in order to exploit this terrific potential, one will need to control the absolute beam energy, the measurement of the luminosity spectrum, and theoretical uncertainties of the cross-section near threshold to sufficient precision. Based on studies of solutions to these issues (for example as discussed in [2]), it looks reasonable to be cautiously optimistic that such error sources can be controlled *by design*, leading to an estimated overall error on the W mass of around 7 MeV for an exposure of 100 fb^{-1} (see also the discussion in [26]).

A similar scan would yield an error on the W-width of 3–4 MeV with an additional systematic error coming from the beam-spread which is 80 MeV for the TESLA design. Hence if the beam-spread can be measured with an error of order 1%, the W-width could indeed be measured to around 4 MeV.

C. Beam Energy Requirements

To fully exploit the increased statistical precision available for precision electroweak measurements at a future linear collider, very precise determinations of the centre-of-mass collision energy are required. For the Z-pole running, this requirement is driven by the precision needed to convert the measured value of A_{LR} into the theoretically useful quantity $\sin^2\theta^{\ell}_{\mathrm{eff}}$. An uncertainty in $\sqrt{s}$ of 3 MeV would contribute an additional uncertainty to the determination of $\sin^2\theta^{\ell}_{\mathrm{eff}}$ which would match the total experimental precision expected using the Blondel scheme. A determination of $\sqrt{s}$ at the level of $\mathcal{O}(1\ \mathrm{MeV})$ is therefore required to keep this uncertainty from dominating the result. At the W-pair threshold, a 10 MeV measurement of $\sqrt{s}$ would lead to a 5 MeV uncertainty in m_{W}, again comparable to the total expected experimental precision.

One strategy to achieve this level of precision at the Z-pole is to calibrate the total cross-section observed at each scan point against the known lineshape parameters measured at LEP. This avoids the need to make an absolute measurement of $\sqrt{s}$ at the 1 MeV level, although having a device available to make relative measurements to a similar precision is probably still necessary from an operational standpoint. This strategy also requires that the luminosity collected at each scan point must be measured with a precision approaching that achieved at LEP1. Clearly this technique will not work directly at the W-pair threshold, although the precise knowledge of m_{Z} can be used to help calibrate the beam energy measurement.

A spectrometer built with the same design and philosophy as that employed at the SLC should be capable of achieving a 10^{-4} precision in $\sqrt{s}$, particularly if it can be cross-calibrated against the known value of m_{Z} at the Z-resonance. While this is just barely adequate for m_{W}, it is probably inadequate for the precision required for A_{LR}, particularly if the positrons are polarised. One possibility for improving this precision is to use the kinematics of Møller scattering off an internal gas jet target. This was studied for the beam energy measurement at LEP2, and a precision approaching 2 MeV appeared reasonable[27]. A detailed study of whether this approach is suitable for a low repetition rate linear collider is still needed, however. The kinematics of Compton scattering (potentially using the available polarimeter infrastructure) is also a possibility which warrants further study. A more detailed discussion of the requirements and prospects for beam energy measurements can be found in Ref. [28].

D. Physics opportunities at low energies

1. *Tests of the Standard Model*

Within the SM, the predictions for the electroweak precision observables are affected via loop corrections by contributions from the top quark mass, m_t, and the Higgs boson mass, M_H, where the loop corrections to m_{W} are usually contained in Δr [29] (see Ref. [6] for details.) The effective leptonic weak mixing angle, $\sin^2\theta^\ell_{\mathrm{eff}}$, is defined through the effective couplings g^f_V and g^f_A of the Z boson to fermions at the Z resonance, where the loop corrections enter through $g^f_{V,A}$ (see Ref. [6] for details.) The radiative corrections entering the predictions of M_W and $\sin^2\theta^\ell_{\mathrm{eff}}$ depend quadratically on m_t, while the leading dependence on M_H is only logarithmic. Comparing the theoretical prediction of the electroweak precision observables with their experimental value allows an indirect determination of the Higgs boson mass M_H. The uncertainty of this indirect determination arises from the theoretical uncertainties for the precision observables and from their experimental errors [26].

The current theoretical uncertainties arising from uncertainties in the input parameters are dominated by the uncertainty in the top quark mass, δm_t, and the uncertainty in the fine structure constant at the Z boson mass scale, $\delta\Delta\alpha$. At a linear collider the top quark mass will be measured to better than 200 MeV. With this precision the uncertainty in the predictions due to the top mass will be negligible. The status and prospects for $\Delta\alpha$ are reviewed in detail in [30]. Currently several methods to evaluate $\Delta\alpha$ are used all arriving at errors sufficient for the present data, but much too large for the GigaZ precision. However, at least two of the methods discussed in [30] have the potential to reduce the errors with additional experimental measurements of the $\mathrm{e}^+\mathrm{e}^-$ hadronic cross section and theoretical progress on the quark masses far enough to compete with the projected precision of GigaZ. Also the uncertainty in the prediction of $\sin^2\theta^\ell_{\mathrm{eff}}$ due to the Z-mass is similar to the experimental uncertainty with 10^9 Z-decays.

Assuming a future determination of $\Delta\alpha$ to a precision of $\delta\Delta\alpha = \pm 7\times 10^{-5}$ [31], a top quark mass measurement down to $\delta m_t = 130$ MeV and $\delta\alpha_s(M_Z) = 0.0010$ (from other GigaZ observables [19]), the future theory uncertainties including unknown higher-order corrections are given by [26]

$$\delta M_W(\text{theory}) = \pm 3.5 \text{ MeV}, \qquad \delta\sin^2\theta^\ell_{\mathrm{eff}}(\text{theory}) = \pm 3\times 10^{-5} \qquad (\text{future}). \tag{4}$$

The precisions for indirect determinations of M_H are summarised in Table II, see Ref. [26] for details. It becomes obvious that the inclusion of polarisation in combination with GigaZ drastically improves the indirect M_H determinations. By comparing the indirectly obtained value to the then known experimental value allows for a stringent test of the SM, becoming even more stringent by the use of polarisation.

	M_W	$\sin^2\theta^\ell_{\mathrm{eff}}$	all
now	106 %	60 %	58 %
LC (no low energy run, no pol.)	15 %	24 %	14 %
GigaZ (incl. polarisation)	12 %	8 %	8 %

TABLE II: *Cumulative expected precisions for the indirect determination of the Higgs boson mass, $\delta M_H/M_H$, taking into account the experimental errors and the theoretical uncertainties, eq. (4) [26]. The last column shows the indirect Higgs boson mass determination from the full set of precision observables.*

2. *Tests of Supersymmetry*

Supersymmetry (SUSY) is a prominent example of physics beyond the SM. If low-energy SUSY is realized in nature, it could well be discovered at the Tevatron or the LHC, and further explored at an $\mathrm{e}^+\mathrm{e}^-$ LC.

Similarly to the case of the SM, the predictions for M_W and $\sin^2\theta^\ell_{\rm eff}$ can also be employed for an indirect test of the MSSM. In contrast to the Higgs boson mass in the SM, the lightest CP-even MSSM Higgs boson mass, m_h, is not a free parameter but can be calculated from the other SUSY parameters. Comparing its prediction with the experimentally measured value will allow to set further constraints on the model. The precision observables M_W, $\sin^2\theta^\ell_{\rm eff}$ and m_h are particularly sensitive to the SUSY parameters of the scalar top and bottom sector and of the Higgs sector. This could allow to indirectly probe the masses of particles in supersymmetric theories that might not be accessible directly neither at the LHC nor at the LC.

As a specific example of indirect informations obtainable from the precision observables at GigaZ which could be complementary to direct experimental measurements at RunII of the Tevatron, the LHC and the LC the scalar top sector of the MSSM is considered here. If the lighter scalar top quark is within the kinematical reach of the LC, the process $e^+e^- \to \tilde{t}_1\bar{\tilde{t}}_1$ will allow to measure its mass, $m_{\tilde{t}_1}$, and the mixing angle in the stop sector, $\cos\theta_{\tilde{t}}$, with an accuracy of below the level of 1% [32]. These direct measurements can be combined with the indirect information on the mass of the heavier scalar top quark, $m_{\tilde{t}_2}$, from requiring consistency of the MSSM (here the case of the unconstrained MSSM with real parameters is considered) with a precise measurement of M_W, $\sin^2\theta^\ell_{\rm eff}$, and m_h. The evaluation of m_h has been performed with `FeynHiggs` [33] (based on Ref. [34]), while details about the evaluation and the theoretical errors of M_W and $\sin^2\theta^\ell_{\rm eff}$ can be found in Refs. [35, 36].

Fig. 3 [35] shows the allowed parameter space according to measurements of m_h, M_W and $\sin^2\theta^\ell_{\rm eff}$ in the plane of the heavier stop mass, $m_{\tilde{t}_2}$, and $|\cos\theta_{\tilde{t}}|$ for the accuracies at an LC with and without the GigaZ option and at the LHC. For $m_{\tilde{t}_1}$ the central value and experimental error of $m_{\tilde{t}_1} = 180 \pm 1.25$ GeV are taken for LC/GigaZ, while for the LHC an uncertainty of 10% in $m_{\tilde{t}_1}$ is assumed, see Ref. [35] for details about the other SUSY parameters. The central values for M_W and $\sin^2\theta^\ell_{\rm eff}$ have been chosen in accordance with a non-zero contribution to the precision observables from SUSY loops.

As one can see in Fig. 3, the allowed parameter space in the $m_{\tilde{t}_2}$–$|\cos\theta_{\tilde{t}}|$ plane is significantly reduced from the LHC to the LC, in particular in the GigaZ scenario. Using the direct information on $|\cos\theta_{\tilde{t}}|$ according to the analysis performed in Ref. [32] allows an indirect determination of $m_{\tilde{t}_2}$ with a precision of better than 5% in the GigaZ case. By comparing this indirect prediction for $m_{\tilde{t}_2}$ with direct experimental information on the mass of this particle, the MSSM could be tested at its quantum level in a sensitive and highly non-trivial way.

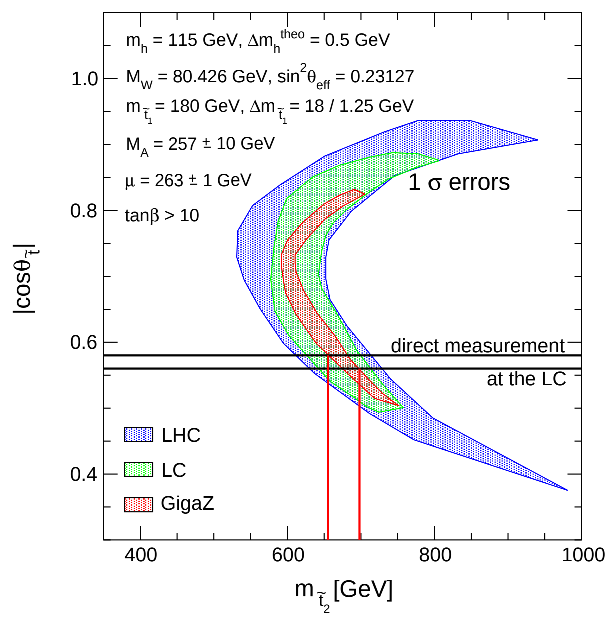

FIG. 3: Indirect constraints on the MSSM parameter space in the $m_{\tilde{t}_2}$–$|\cos\theta_{\tilde{t}}|$ plane from measurements of m_h, M_W, $\sin^2\theta^\ell_{\rm eff}$, m_t and $m_{\tilde{t}_1}$ in view of the prospective accuracies for these observables at an LC with and without GigaZ option and at the LHC. The direct information on the mixing angle from a measurement at the LC is indicated together with the corresponding indirect determination of $m_{\tilde{t}_2}$.

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