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Physics at the Light-Front*

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Physics at the Light-Front

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The light-front representation of quantum chromodynamics provides a frame-independent, quantum-mechanical representation of hadrons at the amplitude level, capable of encoding their multi-quark, hidden-color and gluon momentum, helicity, and flavor correlations in the form of universal process-independent hadron wavefunctions. The universality and frame-independence of the LCWF's thus allow a profound connection between diffractive dissociation, hard scattering exclusive processes such as elastic form factors, two-photon reactions, and heavy hadron decays. In this concluding talk of the ECT* International Conference On Light-Cone Physics: Particles And Strings (Trento 2001), I review recent calculations and new applications of light-front wavefunctions in QCD and other theories. I also review the distinction between the structure functions measured in deep inelastic lepton scattering and the quark distributions determined from light-front wavefunctions.

1. INTRODUCTION

Light-front quantization has become a standard tool for the analysis of nonperturbative problems in quantum field theory as well as in string [1] and M-theory [2]. The light-front method is especially useful for quantum chromodynamics, since it provides a physically appealing yet rigorous extension of many-body quantum mechanics to relativistic bound states: the multi-quark, hidden-color and gluon momentum, helicity, and flavor correlations of a hadron are encoded in the form of universal process-independent Lorentz-invariant wavefunctions. In higher dimension theories, the discretization of light-front momenta provides a way to describe the evolution of the world-sheet of strings [3].

The ECT* International Conference On Light-Cone Physics: Particles And Strings (Trento 2001) provided an outstanding forum to assess the continuing progress in this field. This report can only cover a portion of the many interesting developments in light-cone physics made this past year. I will also review a number of new directions and applications.

Quantization at fixed light-cone time $\tau = t + z/c$ [4] allows one to apply Heisenberg matrix mechanics to relativistic quantum field theory. The Heisenberg equation on the light-front,

$$H_{LC}|\Psi\rangle = M^2|\Psi\rangle \ , \tag{1}$$

can, at least in principle, be solved by diagonalizing the matrix $\langle n|H_{LC}|m\rangle$ on the free Fock basis: [5] The operator H_{LC} can be derived from the Lagrangian of the theory using canonical quantization. The eigenvalues $\{M^2\}$ of $H_{LC} = H^0_{LC} + V_{LC}$ then give the squared invariant masses of the bound and continuum spectrum of the theory. The projections $\{\langle n|\Psi\rangle\}$ of the eigensolution on the n-particle Fock states provide the light-front wavefunctions. Thus solving a quantum field theory is equivalent to solving a coupled many-body quantum mechanics problem:

$$\left[M^2 - \sum_{i=1}^{n} \frac{m^2_{\perp i}}{x_i}\right] \psi_n = \sum_{n'} \int \langle n|V_{LC}|n'\rangle \, \psi_{n'} \tag{2}$$

where the convolution and sum is understood over the Fock number, transverse momenta, light-cone momentum fractions $x_i = k_i^+/P_\pi^+ = (k^0 + k_i^z)/(P^0 + P^z)$ and spin projections of the intermediate states. Here $m^2_\perp = m^2 + k^2_\perp$. Remarkably, the formalism is frame-independent; *i.e.*, H_{LC} and the light-cone variables are independent of the total $P^+ = P^0 + P^z$ and $P_\perp$ of the system. A general review of light-front quantization methods is given in Ref. [5].

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The light-front wavefunctions $\psi_{n/p}(x_i, \vec{k}_{\perp i}, \lambda_i)$ of a hadron in QCD are the projections of the hadronic eigenstate on the free color-singlet Fock state $|n>$ at a given light-cone time $\tau = t + z/c$. They thus represent the ensemble of quark and gluon states possible when a hadron is intercepted at the light-front. The wavefunctions are Lorentz-invariant functions of the light-front momentum fractions x_i with $\sum_{i=1}^n x_i = 1$ and $\vec{k}_{\perp i}$ with $\sum_{i=1}^n \vec{k}_{\perp i} = \vec{0}_\perp$ and are independent of the bound state's physical momentum P^+, and $P_\perp$ [6]. The actual physical transverse momenta are $\vec{p}_{\perp i} = x_i \vec{P}_\perp + \vec{k}_{\perp i}$. The λ_i label the light-front spin S^z projections of the quarks and gluons along the quantization z direction. The physical gluon polarization vectors $\epsilon^\mu(k,\ \lambda = \pm 1)$ are specified in light-cone gauge by the conditions $k \cdot \epsilon = 0,\ \eta \cdot \epsilon = \epsilon^+ = 0$. The parton degrees of freedom are thus all physical; there are no Faddeev-Popov ghost terms or negative metric states. The spinors of the light-front formalism automatically incorporate the Melosh-Wigner rotation.

The wavefunctions derived from light-front quantization play a central role in QCD phenomenology, providing a frame-independent description of hadrons in terms of their quark and gluon degrees of freedom. A recent summary of their phenomenological applications can be found in Ref. [7]. For example, any spacelike and timelike form factors and matrix elements of local currents $\langle B|J^\mu(0)|A\rangle$, such as the semileptonic decay amplitudes of heavy hadrons have an exact representation as overlap momentum-space overlap integrals of light-front wavefunctions [8]. The light-front Fock representation is particularly important for semi-leptonic exclusive matrix elements such as $B \to D\ell\overline{\nu}$. The Lorentz-invariant description requires both the overlap of $n' = n$ parton-number conserving wavefunctions as well as the overlap of wavefunctions with parton numbers $n' = n-2$ which arises from the annihilation of a quark-antiquark pair in the initial wavefunction [9]. The skewed parton distributions which control deeply virtual Compton scattering can also be defined from diagonal and off-diagonal overlaps of light-front wavefunctions [10].

The key non-perturbative input for exclusive processes involving large momentum transfer is the gauge- and frame-independent hadron distribution amplitude [11,6] defined as the integral of the valence (lowest particle number) Fock wavefunction; *e.g.* for the pion $\phi_\pi(x_i, \Lambda) \equiv \int d^2k_\perp\, \psi^{(\Lambda)}_{q\overline{q}/\pi}(x_i, \vec{k}_{\perp i}, \lambda)$ where the global cutoff Λ is identified with the resolution Q. The distribution amplitude controls leading-twist exclusive amplitudes at high momentum transfer, and it can be related to the gauge-invariant Bethe-Salpeter wavefunction at equal light-cone time. The logarithmic evolution of hadron distribution amplitudes $\phi_H(x_i, Q)$ can be derived from the perturbatively-computable tail of the valence light-front wavefunction in the high transverse momentum regime [11,6]. The conformal basis for the evolution of the three-quark distribution amplitudes for the baryons [12] has been obtained by Braun *et al.* [13]. It is particularly important to understand the shape of the gauge- and process-independent meson and baryon valence-quark distribution amplitudes [6] $\phi_M(x, Q)$, and $\phi_B(x_i, Q)$. These quantities specify how a hadron shares its longitudinal momentum among its valence quarks; they control virtually all exclusive processes involving a hard scale Q, including form factors, Compton scattering, and photoproduction at large momentum transfer. New applications of the perturbative QCD factorization formalism were presented at this meeting in Ref. [14].

Progress in measuring the basic parameters of electroweak interactions and CP violation will require a quantitative understanding of the dynamics and phase structure of B decays at the amplitude level. Factorization theorems have now been proven which allow one to rigorously compute certain types of exclusive B decays in terms of the light-front wavefunctions and distribution amplitudes of B meson and the final state hadrons [15]. Recent progress in this field was reviewed at this meeting by Li [16].

2. STRUCTURE FUNCTIONS VERSUS LIGHT-FRONT WAVEFUNCTIONS

The quark and gluon distributions of hadrons, including the spin and transversity distributions

which enter hard inclusive reactions in QCD, can be defined as probability measures and density matrices of the light-front wavefunctions. For example, the quark distribution in a hadron H is

$$P_{\mathrm{q}/H}(x_{Bj}, Q^2) = \sum_n \int^{k_{i\perp}^2 < Q^2} \left[\prod_i dx_i \, d^2k_{\perp i} \right] \times |\psi_{n/H}(x_i, k_{\perp i})|^2 \sum_{j=q} \delta(x_{Bj} - x_j). \quad (3)$$

It has been conventional to identify the leading-twist structure functions $F_i(x, Q^2)$ measured in deep inelastic lepton scattering with the light-front probability distributions. For example, in the parton model, $F_2(x, Q^2) = \sum_q e_q^2 x P_{\mathrm{q}/H}(x, Q^2)$. However, Hoyer, Marchal, Peigne, Sannino, and I [17] have recently shown that the leading-twist contribution to deep inelastic scattering is affected by diffractive rescattering of a quark in the target, a coherent effect which is not included in the light-front wavefunctions, even in light-cone gauge. The effective gluon propagator in light-cone gauge $A^+ = 0$ is singular:

$$d_{LC}^{\mu\nu}(k) = \frac{i}{k^2 + i\varepsilon} \left[-g^{\mu\nu} + \frac{n^\mu k^\nu + k^\mu n^\nu}{n \cdot k} \right]. \quad (4)$$

It has a pole at $k^+ \equiv n \cdot k = 0$, which can be defined by an analytic prescription such as the Mandelstam-Liebbrandt prescription [18]. In final-state scattering involving on-shell intermediate states, the exchanged momentum k^+ is of $O(1/\nu)$ in the target rest frame, which enhances the second term of the light-cone gauge propagator. This enhancement allows rescattering to contribute at leading twist even in LC gauge. We have verified in Feynman and light-cone gauge that diffractive contributions to the deep inelastic scattering $\gamma^* p \to X p'$ cross sections, which leave the target intact, contribute at leading twist to deep inelastic scattering [17].

Diffractive events resolve the quark-gluon structure of the virtual photon, not the quark-gluon structure of the target, and thus they give contributions to structure functions which are not target parton probabilities. Our analysis of deep inelastic scattering $\gamma^*(q)p \to X$, when interpreted in frames with $q^+ > 0$, also supports the color dipole description of deep inelastic lepton scattering at small x_{bj}. For example, in the case of the aligned-jet configurations, one can understand $\sigma_T(\gamma^* p)$ at high energies as due to the coherent color gauge interactions of the incoming quark-pair state of the photon interacting, first coherently and finally incoherently, in the target. The distinction between structure functions and target parton probabilities is also implied by the Glauber-Gribov picture of nuclear shadowing [19]. In this framework, shadowing arises from interference of rescattering amplitudes involving diffraction channels and on-shell intermediate states. In contrast, the wave function of a stable target is strictly real since it does not have on energy-shell configurations. Thus nuclear shadowing is not a property of the light-front wavefunctions of a nuclear target; rather, it involves the total dynamics of the γ^*-nucleus collision. A strictly probabilistic interpretation of the deep inelastic cross section cross section is thus precluded.

3. COLOR TRANSPARENCY, DIFFRACTIVE PROCESSES, AND LIGHT-FRONT WAVEFUNCTIONS

One of the features of QCD which distinguishes it from traditional hadron physics is the fact that hadrons have interaction cross sections which fluctuate according to the size of its color dipole moment. Thus valence Fock states with small impact separation between the constituents will interact weakly and thus can transverse a nucleus with minimal interactions. At high energies the Fock states of a hadron with small transverse size interact weakly even in a nuclear target because of their small dipole moment. This is the basis of "color transparency" in perturbative QCD [20,21]. The amplitude for the diffractive dissociation of a hadron into jets at high energies is given by a transverse momentum derivative of its light-front wavefunction, so it is possible to measure the wavefunctions of a relativistic hadron by diffractively dissociating it into jets whose momentum distribution is correlated with the valence quarks' momenta [22,21,23,24]. Photon exchange measures a weighted sum of transverse

derivatives $\partial_{k_\perp}\psi_n(x_i, k_{\perp_i}, \lambda_i)$, and two-gluon exchange measures the second transverse partial derivative [25]. The diffractive dijet dissociation experiment E791 at Fermilab using 500 GeV incident pions on nuclear targets [26] has recently provided a remarkable confirmation of color transparency and thus the gauge theory interactions predicted by QCD. The measured longitudinal momentum distribution of the jets [27] is consistent with a pion light-front wavefunction of the pion with the shape of the asymptotic distribution amplitude, $\phi_\pi^{\rm asympt}(x) = \sqrt{3} f_\pi x(1-x)$. Data from CLEO [28] for the $\gamma\gamma^* \to \pi^0$ transition form factor also favor a form for the pion distribution amplitude close to the asymptotic solution to the perturbative QCD evolution equation [6]. The new EVA spectrometer experiment E850 at Brookhaven [29] has also reported striking effects of color transparency in quasi-elastic large momentum transfer proton-proton scattering in nuclei which at large momentum transfer is controlled by proton Fock states of small transverse size. The EVA experiment also finds a breakdown of color transparency at $\sqrt{s} \sim 5$ GeV and large θ_{cm}, which may reflect effects of the charm threshold [30].

4. THE ANOMALOUS GRAVITOMAGNETIC MOMENT, ANGULAR MOMENTUM CONSERVATION, AND LIGHT-FRONT WAVEFUNCTIONS

One can also express the matrix elements of the energy momentum tensor as overlap integrals of light-front wavefunctions [31]. An important consistency check of any relativistic formalism is to verify the vanishing of the anomalous gravitomagnetic moment $B(0)$, the spin-flip matrix element of the graviton coupling and analog of the anomalous magnetic moment $F_2(0)$. For example, at one-loop order in QED, $B_f(0) = \frac{\alpha}{3\pi}$ for the electron when the graviton interacts with the fermion line, and $B_\gamma(0) = -\frac{\alpha}{3\pi}$ when the graviton interacts with the exchanged photon. The vanishing of $B(0)$ can be shown to be exact for bound or elementary systems in the light-front formalism [31], in agreement with the equivalence principle [32].

The light-front formalism also provides a simple representation of angular momentum. See for example, Refs. [33,5,34,31,35]. The projection J_z is kinematical and is conserved separately for each Fock component: each light-front Fock wavefunction satisfies the sum rule: $J^z = \sum_{i=1}^n S_i^z + \sum_{j=1}^{n-1} l_j^z$. The sum over S_i^z represents the contribution of the intrinsic spins of the n Fock state constituents. The sum over orbital angular momenta $l_j^z = -\mathrm{i}\left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1}\right)$ derives from the $n-1$ relative momenta. This excludes the contribution to the orbital angular momentum due to the motion of the center of mass, which is not an intrinsic property of the hadron [31]. The numerator structure of the light-front wavefunctions in $k_\perp$ is determined by the angular momentum constraints. The spin properties of light-front wavefunctions provides a consistent basis for analyzing spin correlations and azimuthal spin asymmetries in both exclusive and inclusive reactions.

5. THE ROLE OF HIGHER PARTICLE-NUMBER FOCK STATES

The higher Fock states of the light hadrons describe the sea quark structure of the deep inelastic structure functions, including "intrinsic" strangeness and charm fluctuations specific to the hadron's structure rather than gluon substructure [36,37]. Ladder relations connecting state of different particle number follow from the QCD equation of motion and lead to Regge behavior of the quark and gluon distributions at $x \to 0$ [38].

Since the intrinsic heavy quarks tend to have the same rapidity as that of the projectile, they are produced at large x_F in the beam fragmentation region. The charm structure function measured by the EMC group shows an excess at large x_{bj}, indicating a probability of order 1% for intrinsic charm in the proton [37]. The presence of intrinsic charm in light-mesons provides an explanation for the puzzle of the large $J/\psi \to \rho\pi$ branching ratio and suppressed $\psi' \to \rho\pi$ decay [39]. The presence of intrinsic charm quarks in the B wave function provides new mechanisms for B decay. For example, Chang and Hou have considered the production of final states with

three charmed quarks such as $B \to J/\psi D\pi$ and $B \to J/\psi D^*$ [40]; these final states are difficult to realize in the valence model, yet they occur naturally when the b quark of the intrinsic charm Fock state $|b\overline{u}c\overline{c}\rangle$ decays via $b \to c\overline{u}d$. In fact, the J/ψ spectrum for inclusive $B \to J/\psi X$ decays measured by CLEO and Belle shows a distinct enhancement at the low J/ψ momentum where such decays would kinematically occur. Alternatively, this excess could reflect the opening of baryonic channels such as $B \to J/\psi \overline{p}\Lambda$ [41]. Recently, Susan Gardner and I have shown that the presence of intrinsic charm in the hadrons' light-front wave functions, even at a few percent level, provides new, competitive decay mechanisms for B decays which are nominally CKM-suppressed [42]. For example, the weak decays of the B-meson to two-body exclusive states consisting of strange plus light hadrons, such as $B \to \pi K$, are expected to be dominated by penguin contributions since the tree-level $b \to su\overline{u}$ decay is CKM suppressed. However, higher Fock states in the B wave function containing charm quark pairs can mediate the decay via a CKM-favored $b \to sc\overline{c}$ tree-level transition. Since they mimic the amplitude structure of "charming" penguin contributions [43], charming penguins need not be penguins at all [42].

6. GENERAL FEATURES OF LIGHT-FRONT WAVEFUNCTIONS

Even without explicit solutions, many features of the light-front wavefunctions follow from general principles and its equation of motion, Eq. (2). Every light-front Fock wavefunction has the form:

$$\psi_n = \frac{\Gamma_n}{M^2 - \sum_{i=1}^n \frac{m_{\perp i}^2}{x_i}} \tag{5}$$

where $\Gamma_n = \sum_{n'} \int V_{nn'} \psi_n$. Much of the dynamical dependence of a light-front wavefunction is controlled by its light-cone energy denominator. The maximum of the wavefunction occurs when the invariant mass of the partons is minimal; *i.e.*, when all particles have equal rapidity and are all at rest in the rest frame. In fact, Dae Sung Hwang and I [44] have noted that one can rewrite the wavefunction in the form:

$$\psi_n = \frac{\Gamma_n}{M^2[\sum_{i=1}^n \frac{(x_i - \hat{x}_i)^2}{x_i} + \delta^2]} \tag{6}$$

where $x_i = \hat{x}_i \equiv m_{\perp i}/\sum_{i=1}^n m_{\perp i}$ is the condition for minimal rapidity differences of the constituents. The key parameter is $M^2 - \sum_{i=1}^n m_{\perp i}^2/\hat{x}_i \equiv -M^2\delta^2$. We can also interpret $\delta^2 \simeq 2\epsilon/M$ where $\epsilon = \sum_{i=1}^n m_{\perp i} - M$ is the effective binding energy. This form shows that the wavefunction is a quadratic form around its maximum, and that the width of the distribution in $(x_i - \hat{x}_i)^2$ (where the wavefunction falls to half of its maximum) is controlled by $x_i\delta^2$ and the transverse momenta $k_{\perp_i}$. Note also that the heaviest particles tend to have the largest $\hat{x}_i$, and thus the largest momentum fraction of the particles in the Fock state, a feature familiar from the intrinsic charm model. For example, the b quark has the largest momentum fraction at small $k_\perp$ in the B meson's valence light-front wavefunction,, but the distribution spreads out to an asymptotically symmetric distribution around $x_b \sim 1/2$ when $k_\perp \gg m_b^2$.

We can also discern some general properties of the numerator of the light-front wavefunctions. $\Gamma_n(x_i, k_{\perp i}, \lambda_i)$. The transverse momentum dependence of Γ_n guarantees J_z conservation for each Fock state: For example, one of the three light-front Fock wavefunctions of a $J_z = +1/2$ lepton in QED perturbation theory is

$$\psi^{\uparrow}_{+\frac{1}{2}+1}(x, \vec{k}_\perp) = -\sqrt{2}\frac{(-k^1 + \mathrm{i}k^2)}{x(1-x)}\,\varphi\ , \tag{7}$$

where

$$\varphi(x, \vec{k}_\perp) = \frac{e/\sqrt{1-x}}{\frac{M^2 - (\vec{k}_\perp^2 + m^2)}{x - (\vec{k}_\perp^2 + \lambda^2)/(1-x)}}\ . \tag{8}$$

The orbital angular momentum projection in this case is $\ell^z = -1$. The spin structure indicated by perturbative theory provides a template for the numerator structure of the light-front wavefunctions even for composite systems. The structure of the electron's Fock state in perturbative QED shows that it is natural to have a negative contribution from relative orbital angular momentum which balances the S_z of its photon constituents.

We can also expect a significant orbital contribution to the proton's J_z since gluons carry roughly half of the proton's momentum, thus providing insight into the "spin crisis" in QCD.

The fall-off the light-front wavefunctions at large $k_\perp$ and $x \to 1$ is dictated by QCD perturbation theory since the state is far-off the light-cone energy shell. This leads to counting rule behavior for the quark and gluon distributions at $x \to 1$. Notice that $x \to 1$ corresponds to $k^z \to -\infty$ for any constituent with nonzero mass or transverse momentum. Explicit examples of light-front wavefunctions in QED are given in Ref. [31]. The above discussion suggests that an approximate form for the hadron light-front wavefunctions might be constructed through variational principles and by minimizing the expectation value of H_{LC}^{QCD}.

7. THE FORMULATION OF LIGHT-FRONT GAUGE THEORY

The light-front quantization of QCD in light-cone gauge has a number of advantages, including explicit unitarity, a physical Fock expansion, the absence of Faddeev-Popov ghost terms, and the decoupling properties needed to prove factorization theorems in high momentum transfer inclusive and exclusive reactions. The light-front framework for gauge theory in light-cone gauge $n \cdot A = A^+ = 0$, is a severely constrained dynamical theory with many second-class constraints [45–47]. These can be eliminated by constructing Dirac brackets, and the theory can be quantized canonically by the correspondence principle in terms of a *reduced number* of independent fields. The commutation relations among the field operators can be found by the Dirac method. A recent derivation of light-front quantized non-Abelian gauge theory in light-cone gauge is given in Ref. [47]. For example, the non-dynamical projections of the fermion and gauge field can be eliminated using nonlocal constraint equations. The removal of the unphysical components of the fields results in tree-level instantaneous gluon exchange and fermion exchange interaction terms. The interaction Hamiltonian of QCD can be expressed in a form resembling that of covariant theory with three- and four-point gauge interactions, except for the additional instantaneous four-point interactions which can be treated systematically.

The light-front quantized free gauge theory simultaneously satisfies the covariant gauge condition $\partial \cdot A = 0$ as an operator condition as well as the light-cone gauge condition. In our analysis [47] one imposes this condition linearly using a Lagrange multiplier, rather than a quadratic form. The numerator of the gauge propagator is doubly transverse:

$$D_{\mu\nu}(k) = -g_{\mu\nu} + \frac{n_\mu k_\nu + n_\nu k_\mu}{(n \cdot k)} - \frac{k^2}{(n \cdot k)^2}\, n_\mu n_\nu,$$

with

$$n^\mu D_{\mu\nu} = k^\mu D_{\mu\nu} = 0.$$

Thus only physical degrees of freedom propagate. The remarkable properties of $D_{\nu\mu}$ provide much simplification in the computations of loops. In the case of tree graphs, the term proportional to $n_\mu n_\nu$ cancels against the instantaneous gluon exchange term leading to Eq. (4). The renormalization constants in the non-Abelian theory can be shown to can be shown to satisfy the identity $Z_1 = Z_3$ at one loop order, as expected in a theory with only physical gauge degrees of freedom. Note that the one-loop correction to the three gluon vertex includes an instantaneous gluon exchange contribution since it is not one-particle irreducible. Thorn [48] gave the first computation of the renormalization constants of QCD in light-cone gauge. The QCD β function computed in the noncovariant light-cone gauge [47] agrees with the conventional result. Dimensional regularization and the Mandelstam-Leibbrandt prescription [18] for light-cone gauge were used to define the Feynman loop integration [49]. Ghosts only appear in association with the Mandelstam-Liebbrandt prescription. There are no Faddeev-Popov or Gupta-Bleuler ghost terms.

Bassetto, Griguolo, and Vian [50] have investigated the presence of multiple vacua and the use of new prescriptions for light-cone gauge. In previous work they showed that in two dimensions (where no UV singularities occur), both the Mandelstam-Liebbrandt and principal value pre-

scriptions for light-cone gauge are viable. In general they lead to different theories and to different results. Since pure Yang Mills theory in 1+1 dimensions is exactly solvable by geometric techniques without even fixing a gauge, Bassetto et al. [51,50] could show that the result of the principal value prescription coincides with the exact solution, whereas the Mandelstam-Leibbrandt procedure misses the topological effects.

In an important new development, Paston, Prokhvatilov and Franke, have shown that one can introduce a Pauli Villars spectrum of ghost fields to regulate the ultra-violet divergences of non-Abelian gauge theory [52]. This provides an important computational tool for the defining nonperturbative QCD without dimensional regularization. They have also demonstrated the equivalence between Light-Front Hamiltonian and conventional Lorentz-covariant formulations of gauge theory for QED(1+1) and perturbatively to all orders for QCD(3+1).

One of the most interesting formal questions in light-front quantization is how solutions of quantum field theories which display spontaneous symmetry emerges despite the simplicity of the light-front vacuum [53]. Spontaneous symmetry breaking and other nonperturbative effects associated with the instant-time vacuum are hidden in dynamical or constrained zero modes on the light-front. An introduction is given in Ref. [54]. Significant progress in this area was reported at this meeting. Grange, Salmons, and Werner [55] have given a new analysis of spontaneous symmetry breaking of ϕ^4 theory in 1+1 dimensions. Pirner [56] has shown how zero modes on the light-cone in QCD can cause a phase transition which can be studied phenomenologically in deep inelastic lepton scattering. The relation of zero modes to the θ vacuum of the bosonized version of QCD(1+1) was also discussed at this meeting in talks by Przeszowski [57] McCartor [58], Martinovic [59].

8. NONPERTURBATIVE CALCULATIONS OF LIGHT-FRONT WAVEFUNCTIONS

The calculation of the light-front wavefunctions of hadrons from first principles is a central goal in QCD. A number of methods have been proposed, all of which have shown considerable progress.

DLCQ (discretized light-cone quantization) is a method which solves quantum field theory by directly diagonalization of the light-front Hamiltonian [60]. The DLCQ method has been used with success in solving a number of quantum field theories in low space-time dimensions [61] and has found much utility in string theory. Susskind and others have adopted DLCQ to define $M-$theory [2,62]. Harinandrath *et al.* [63] have shown that the $S-$matrix in DLCQ does have the correct continuum limit. When one imposes periodic boundary conditions in $x^- = t + z/c$, then the plus momenta become discrete: $k_i^+ = \frac{2\pi}{L} n_i, P^+ = \frac{2\pi}{L} K$, where $\sum_i n_i = K$ [64,60]. For a given "harmonic resolution" K, there are only a finite number of ways positive integers n_i can sum to a positive integer K. Thus at a given K, the dimension of the resulting light-front Fock state representation of the bound state is rendered finite without violating Lorentz invariance. The eigensolutions of a quantum field theory, both the bound states and continuum solutions, can then be found by numerically diagonalizing a frame-independent light-front Hamiltonian H_{LC} on a finite and discrete momentum-space Fock basis. Solving a quantum field theory at fixed light-front time τ thus can be formulated as a relativistic extension of Heisenberg's matrix mechanics. The continuum limit is reached for $K \to \infty$. This formulation of the non-perturbative light-front quantization problem is called "discretized light-front quantization" (DLCQ) [60]. Lattice gauge theory has also been used to calculate the pion light-front wavefunction [65].

The DLCQ method has been used extensively for solving one-space and one-time theories [5], including applications to supersymmetric quantum field theories [66] and specific tests of the Maldacena conjecture [67]. There has been progress in systematically developing the computation and

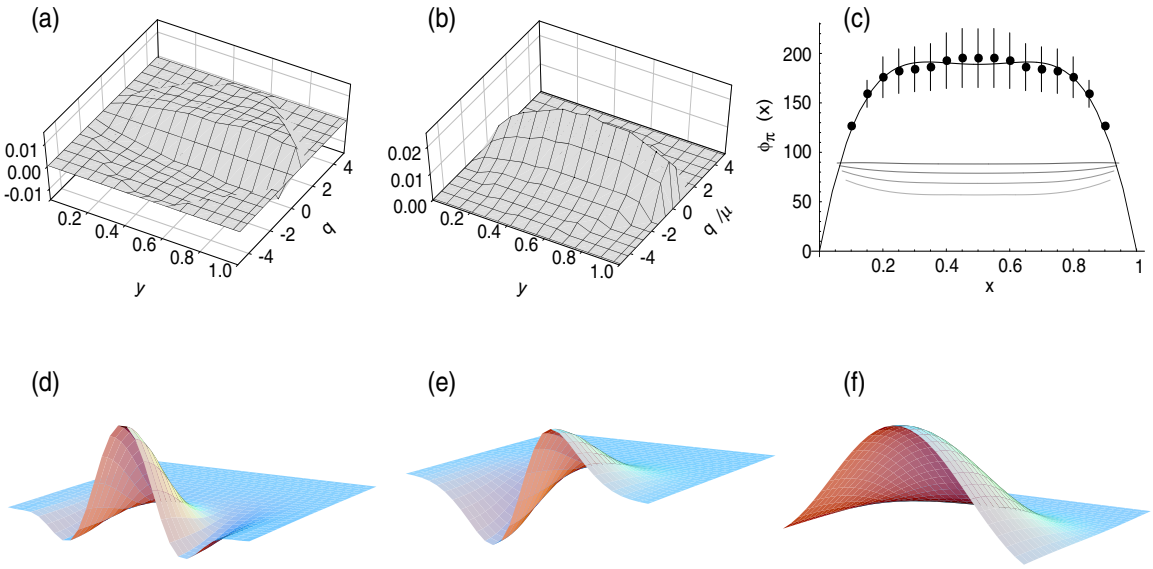

Figure 1. DLCQ results for the one-boson one-fermion wavefunction in a fermion system with (a) parallel and (b) antiparallel fermion helicity, as a function of longitudinal momentum fraction y and one transverse momentum component q_x in the $q_y = 0$ plane. The parameter values for the DLCQ resolution are $K = 29$, $N_\perp = 7$. Further details are given in Ref. [68]. (c) The distribution amplitude $\phi(x)$ of the pion for opposite helicity quarks computed from the transverse lattice by Dalley [69] at a transverse normalization scale $Q^2 \sim 1$ GeV^2. Grey curves correspond to DLCQ cut-offs $K = 6, 7, 8, 9$ (darker means larger K). Data points are the point-wise extrapolation of finite-K curves. The solid curve is a fit of the distribution amplitude to the conformal expansion. (d)-(e) Light-front wavefunctions for the first three eigensolutions of an effective 3+1 light-front equation generated from ϕ^3 field theory [70]. The vertical axis shows the value of the two-parton wavefunction. The front of the figure shows $\psi(x, \vec{k}_\perp = 0)$. (d) The ground state wavefunction peaks at $x = 1/2$ falling to zero $x = 0$ and $x = 1$. (e) The first excited state has a node in $\psi(x, \vec{k}_\perp = 0)$ at $x = 1/2$. The figure also shows the fall-off of the wavefunctions as a function of the magnitude of the transverse momentum as one goes from front to back. Further details are given in Ref. [70].

renormalization methods needed to make DLCQ viable for QCD in physical spacetime. For example, John Hiller, Gary McCartor, and I [68] have shown how DLCQ can be used to solve 3+1 theories despite the large numbers of degrees of freedom needed to enumerate the Fock basis. A key feature of our work is the introduction of Pauli Villars fields to regulate the ultraviolet divergences and perform renormalization while preserving the frame-independence of the theory. A recent application of DLCQ to a 3+1 quantum field theory with Yukawa interactions is given in Ref. [68]. Representative plots of the one-boson one-fermion light-front Fock wavefunction of the lowest mass fermion solution of the Yukawa (3+1) theory showing spin correlations and the presence of non-zero orbital angular momentum are shown in Fig. 1(a),(b). Hiller, McCartor and I have also shown that one can obtain exact analytic DLCQ solutions when the Pauli-Villars and normal particles are degenerate. This is valuable for checking DLCQ solutions and codes [71].

Pinsky, Trittman and Hiller [67] have shown how one can solve supersymmetric gauge theories by discretizing and diagonalizing the Q operators of DLCQ. The square of Q then provides H_{LC}. They have also used SDLCQ to give explicit checks of the Maldacena conjecture.

Dyson-Schwinger solutions [72] of hadronic Bethe-Salpeter wavefunctions can also be used to predict light-front wavefunctions and hadron distribution amplitudes by integrating over the relative k^- momentum. Roberts *et al.* [72] have made important progress treating meson states in QCD in terms of a Bethe-Salpeter/ Dyson-Schwinger approach. Much of the physics of chiral symmetry breaking can be simulated in this formalism without an explicit vacuum chiral condensate. A disadvantage of this approach is that the ladder approximation to the Bethe-Salpeter kernel is gauge dependent and breaks crossing symmetry. In addition, covariant gauges with ghost terms are required.

There has been important progress using the transverse lattice, essentially a combination of DLCQ in 1+1 dimensions together with a lattice in the transverse dimensions [73,69,74]. For example, Dalley and Burkardt have used the transverse lattice to compute the Isgur-Weis function measured in $B \to D\ell\overline{\nu}$ decays. Dalley [75] has also calculated the pion distribution amplitude from QCD using this method. A finite lattice spacing a can be used by choosing the parameters of the effective theory in a region of renormalization group stability to respect the required gauge, Poincaré, chiral, and continuum symmetries. The overall normalization gives $f_\pi = 101$ MeV compared with the experimental value of 93 MeV. An illustration is shown in Fig. 1(c). The resulting DLCQ/transverse lattice pion wavefunction compares well with the best fit to the diffractive di-jet data after corrections for hadronization and experimental acceptance [22]. The predicted form of $\phi_\pi(x, Q)$ is somewhat broader than but not inconsistent with the asymptotic form favored by the measured normalization of $Q^2 F_{\gamma\pi^0}(Q^2)$ and the pion wavefunction inferred from diffractive dijet production. However, there are experimental uncertainties from hadronization and theoretical errors introduced from finite DLCQ resolution, using a nearly massless pion, ambiguities in setting the factorization scale Q^2, as well as errors in the evolution of the distribution amplitude from 1 to 10 GeV^2. Instanton models also predict a pion distribution amplitude close to the asymptotic form [76].

Moments of the pion distribution amplitudes have been also computed in Euclidean lattice gauge theory [77,78]. The lattice results from Del Debbio *et al.* [78] imply a much narrower shape for the pion distribution amplitude than the asymptotic form or the distribution predicted by the transverse lattice. A new result for the proton distribution amplitude treating nucleons as chiral solitons has recently been derived by Diakonov and Petrov [79].

A systematic approach to QCD has been developed by Wilson *et al.* [80] and others which systematically eliminates higher Fock states in terms of new renormalization constants and effective interactions. Recent progress on the dynamics of effective gluons, renormalization, and the $k^+ \to 0$ behavior of the effective theory has been reported by Glazek [81], Brisudova [82] and Walhout [83]. One can also define a truncated theory by eliminating the higher Fock states in favor of an effec-

tive potential. [84] For example Pauli, *et al.* [85] and Karmanov *et al.*, [86] have shown how one can develop effective potentials which in principle can systematically incorporates the effects of higher Fock states. New methods have been developed by Mangin-Brinet *et al.* [87] and Fredirico *et al.*, [88] to solve the effective equations.

van Iersel, Bakker, and Pijlman [70] have shown how one can obtain bound state solutions and explicit light-front wavefunctions for theories with simple (3+1) field theory interactions. As an example, they consider a system consisting of two scalar particles of equal mass $m = 1$ exchanging a massless spinless particle, allowing for 2- and 3-particle Fock states. The $g\phi^3$ coupling strength is $\alpha = \frac{g^2}{16\pi m^2} = 17.36$. They solve the light-front bound state equation by making an expansion in basis functions – 8 cubic spline functions for the functional dependence in x and up to 3 Jacobi polynomials in the transverse momenta. Figure 1(d)-(e) shows the valence wavefunctions of the first three bound states. It is interesting to see how the node structure appears in the light-cone fraction x in the higher excited states.

There are other possible approaches to non-perturbative QCD using light-front quantization. For example, one can construct trial wavefunctions for the light-front wavefunctions which incorporate ladder relations between Fock states, angular momentum constraints, and perturbative QCD fall-off at high off-shell virtuality. The parameters would be determined variationally by minimizing the expectation value of H_{LC}. Lattice theory could also be adopted to light-cone coordinates $x^+, x^-, x_\perp$ rather than Cartesian coordinates, thus incorporating the good Lorentz boost properties of light-front quantization.

9. NEW DIRECTIONS

Light-cone quantization in light-cone gauge can be applied to the electroweak theory, including spontaneous symmetry breaking, thus providing a unitary as well as renormalizable theory of the Standard Model [89].

The light-front formalism can be used as an "event amplitude generator" for high energy physics reactions where each particle's final state is completely labeled in momentum, helicity, and phase [90]. The application of the light-front time evolution operator P^- to an initial state systematically generates the tree and virtual loop graphs of the T-matrix in light-front time-ordered perturbation theory in light-front gauge. Renormalized amplitudes can be explicitly constructed by subtracting from the divergent loops amplitudes with nearly identical integrands corresponding to the contribution of the relevant mass and coupling counter terms (the "alternating denominator method") [91].

In the usual treatment of classical thermodynamics, one considers an ensemble of particles $n = 1, 2, \ldots N$ which have energies $\{E_n\}$ at a given "instant" time t. The partition function is defined as $Z = \sum_n \exp -\frac{E_n}{kT}$. Similarly, in quantum mechanics, one defines a quantum-statistical partition function as $Z = tr \exp -\beta H$ which sums over the exponentiated-weighted energy eigenvalues of the system. In the case of relativistic systems, it is natural to characterize the system at a given light-front time $\tau = t + z/c$; *i.e.*, one determines the state of each particle in the ensemble as its encounters the light-front. Thus we can define a light-front partition function [90] $Z_{LC} = \sum_n \exp -\frac{p_n^-}{kT_{LC}}$ by summing over the particles' light-front energies $p^- = p^0 - p^z = \frac{p_\perp^2 + m^2}{p^+}$. The total momentum is $P^+ = \sum p_n^+$, $\vec{P}_\perp = \sum_n \vec{p}_{\perp n}$, and the total mass is defined from $P^+P^- - P_\perp^2 = M^2$. The light-front partition function should be advantageous for analyzing relativistic systems such as heavy ion collisions, since, like true rapidity, $y = \ell n \frac{p^+}{P^+}$, light-front variables have simple behavior under Lorentz boosts. The light-front formalism also takes into account the point that a phase transition does not occur simultaneously in t, but propagates through the system with a finite wave velocity.

10. Conclusions

There has been strong advances in light-cone physics, particularly in solving many formal problems of renormalization and implementation of spontaneous symmetry breaking within light-front quantized field theory. There have also

been important new developments using light-cone methods in string theory and supersymmetry. The Pauli-Villars method has now emerged as a systematic way to regulate ultraviolet divergences in light-front quantized non-Abelian gauge theory. The light-front formalism thus can provide a formalism for solving nonperturbative problems in QCD as rigorous as lattice gauge theory.

On the phenomenological side, the measurements from the Fermilab E791 experiment of diffractive dijet production in pion-nucleus collisions has not only verified color transparency, but also given us a direct look at the light-front wavefunction of the pion. It will be important to extend these measurements to the diffractive dissociation of real and virtual photons, nucleons, and even light nuclei.

The new applications of light-front wavefunctions to exclusive B decays makes even more urgent the need for bound state solutions of the light-front Hamiltonian for QCD in physical space-time dimensions. Strong progress has been made solving model $3+1$ field theories using DLCQ, light-front Tamm-Dancoff, effective light-front potential equations, and the Dyson-Schwinger approach. The transverse lattice method is now providing the first results for the pion distribution amplitude and the Isgur-Weis function for heavy hadron decays from QCD. Variational methods also look promising. Given these initial successes in (3+1) studies, we can be optimistic that with sufficient computational resources, first principle calculations of the spectrum and light-front wavefunctions of hadrons from light-front quantized QCD will soon emerge.

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