# Indirect Signatures of $C P$ Violation in the Processes $\gamma \gamma \rightarrow \gamma \gamma, \gamma Z$, and $Z Z$ 

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#### Abstract

This paper summarizes the utility of the processes $\gamma \gamma \rightarrow \gamma \gamma, \gamma Z$, and $Z Z$ at a future photon collider in searching for new signatures of $C P$ violation in quartic gauge boson self couplings. It is found that a $\gamma \gamma$ collider operating with energy $\sqrt{s_{e e}}=1 \mathrm{TeV}$ and integrated luminosity $L_{e e}=100 \mathrm{fb}^{-1}$ can probe new physics scales in the range $2-3 \mathrm{TeV}$.


## 1. Introduction

Future $\mathrm{e}^{+} \mathrm{e}^{-}$colliders will likely have the option of operating in $\gamma \gamma$ or ey collision modes [1]. These modes are reached by Compton scattering laser light off one or more of the incoming fermion beams, and then colliding the resulting high energy photons with the remaining fermion beam or with each other. There is a large potential for ey and $\gamma \gamma$ collisions to elucidate possible physics beyond the Standard Model; previous investigations have focused on anomalous couplings, searches for extra dimensions, properties of supersymmetry, and a broad host of other topics [2]. In this paper we will show that $\gamma \gamma \rightarrow \gamma \gamma, \gamma Z$, and $Z Z$ at a photon collider can provide sensitive tests of $C P$ violation in quartic gauge boson interactions. Assuming a center of mass energy $\sqrt{s_{e e}}=1 \mathrm{TeV}$ and an integrated luminosity $L_{e e}=100 \mathrm{fb}^{-1}$ for the parent $\mathrm{e}^{+} \mathrm{e}^{-}$collider, we will find that new physics scales of $\simeq 2-3 \mathrm{TeV}$ can be probed. A more complete summary of these results can be found in [3].

## 2. Results

Here we construct the most general set of operators that contribute to neutral gauge boson selfinteractions, subject to the following constraints. We consider only $C P$ odd $\mathrm{SU}(2) \times \mathrm{U}(1)$ invariant operators, and as the effects of $C P$-odd trilinear interactions have been extensively studied elsewhere [4], we concentrate upon genuinely quartic terms. We also make no assumption as to the mechanism of electroweak symmetry breaking. These restrictions lead us to the following set of seven dimension eight operators constructed from the $W$ and $B$ field strength tensors:

$$
\begin{align*}
& \mathcal{O}_{(B B)(B B)}=\left(B_{\mu v} B^{\mu v}\right)\left(B^{\rho \sigma} \tilde{B}_{\rho \sigma}\right), \quad \mathcal{O}_{(W W)(W W)}=\left(W_{a \mu v} W_{a}^{\mu \nu}\right)\left(W_{b}^{\rho \sigma} \tilde{W}_{b \rho \sigma}\right), \\
& \mathcal{O}_{(B B)(W W)}=\left(B_{\mu v} B^{\mu v}\right)\left(W_{a}^{\rho \sigma} \tilde{W}_{a \rho \sigma}\right), \quad \mathcal{O}_{(W W)(B B)}=\left(W_{a \mu v} W_{a}^{\mu v}\right)\left(B^{\rho \sigma} \tilde{B}_{\rho \sigma}\right), \\
& \mathcal{O}_{(B W)(B W)}=\left(B_{\mu \nu} W_{a}^{\mu v}\right)\left(B^{\rho \sigma} \tilde{W}_{a \rho \sigma}\right), \quad \mathcal{O}_{W B W B}=W_{a}^{\mu v} B_{v \rho} W_{a}^{\rho \sigma} \tilde{B}_{\sigma \mu}, \\
& \mathcal{O}_{B W B W}=B^{\mu v} W_{a v \rho} B^{\rho \sigma} \tilde{W}_{a \sigma \mu} . \tag{1}
\end{align*}
$$

Here $W_{a}^{\mu \nu}$ is the $\operatorname{SU}(2)$ field strength tensor and $B^{\mu \nu}$ the $\mathrm{U}(1)$ field strength tensor. As these operators are of dimension eight, they must be multiplied by a factor $1 / \Lambda_{\alpha}^{4}$, where $\Lambda_{\alpha}$ is the energy scale of the new physics that gives rise to them. The sensitivity to the above operators will be given in terms of the $\Lambda_{\alpha}$ that can be probed.

Each of these operators will give rise to a number of different $\gamma \gamma \gamma \gamma, \gamma \gamma \gamma Z$, and $\gamma \gamma Z Z$ structures, as well as other quartic operators involving three or more $Z$ bosons and various terms with

[^0]more than four gauge bosons. In this paper we will concentrate on those terms relevant for the scattering processes $\gamma \gamma \rightarrow \gamma \gamma, \gamma Z$, and $Z Z$; the other interactions cannot be probed in $2 \rightarrow 2$ scattering processes, and the resulting constraints obtainable on the operators of eq. (1) would be weakened. Denoting the operators to be studied by $\mathcal{O}_{i}^{\gamma \gamma}, \mathcal{O}_{i}^{\gamma Z}$, and $\mathcal{O}_{i}^{Z Z}$, each $\mathrm{SU}(2) \times \mathrm{U}(1)$ operator will have an expansion of the form
\[

$$
\begin{equation*}
\mathcal{O}_{\alpha}^{S U(2) \times U(1)}=a_{\alpha}^{i} \mathcal{O}_{i}^{\gamma \gamma}+b_{\alpha}^{i} \mathcal{O}_{i}^{\gamma Z}+c_{\alpha}^{i} \mathcal{O}_{i}^{Z Z}+\ldots \tag{2}
\end{equation*}
$$

\]

where the $a, b$, and $c$ are functions of the weak mixing angle and the ellipsis denotes the neglected terms. We will present distributions for a representative operator $\mathcal{O}_{i}^{\gamma X}$, and then estimate the search reaches that the process $\gamma \gamma \rightarrow \gamma X$ provides for the $\mathcal{O}_{\alpha}^{S U(2) \times U(1)}$.

We first consider the process $\gamma\left(k_{1}\right)+\gamma\left(k_{2}\right) \rightarrow \gamma\left(p_{1}\right)+\gamma\left(p_{2}\right)$. In the SM, this process is dominated at high energies by the amplitudes $\mathcal{M}_{ \pm \pm \pm \pm}^{S M}, \mathcal{M}_{ \pm \mp \pm \mp}^{S M}$, and $\mathcal{M}_{ \pm \mp \mp \pm}^{S M}$. These amplitudes are primarily imaginary, although the real parts are non-negligible; a detailed discussion of the $\gamma \gamma \rightarrow \gamma \gamma$ amplitudes can be found in [5]. The only non-vanishing $C P$ violating amplitudes are $\mathcal{M}_{ \pm \pm \mp \mp}^{C P}$; the combined constraints of Bose symmetry, oddness under $P$ and $T$, and crossing symmetries imply that a large subset of the $C P$ violating amplitudes vanish. To increase sensitivity to the anomalous interactions we should attempt to define an observable with interference between $\mathcal{M}_{ \pm \pm \mp \mp}^{C P}$ and one of the dominant SM terms. Unfortunately, no observable exists which contains such an interference with one of the large imaginary SM amplitudes. We can, however, construct the following observable which contains an interference with one of the real pieces of the SM amplitudes:

$$
\begin{equation*}
A_{\gamma \gamma}=\frac{\int_{0}^{2 \pi} \int_{0}^{2 \pi} d \phi_{1} d \phi_{2}\left[\left(\frac{d \sigma}{d \Omega}\right) \delta\left(\phi_{1}-\phi_{2}-\pi / 4\right)-\left(\frac{d \sigma}{d \Omega}\right) \delta\left(\phi_{1}-\phi_{2}+\pi / 4\right)\right]}{\int_{0}^{2 \pi} \int_{0}^{2 \pi} d \phi_{1} d \phi_{2}\left[\left(\frac{d \sigma}{d \Omega}\right) \delta\left(\phi_{1}-\phi_{2}-\pi / 4\right)+\left(\frac{d \sigma}{d \Omega}\right) \delta\left(\phi_{1}-\phi_{2}+\pi / 4\right)\right]} \tag{3}
\end{equation*}
$$

where $\phi_{1}, \phi_{2}$ denote the angles of photon linear polarization.
We show the integrated asymmetry versus $\sqrt{s}$ for $\mathcal{O}_{1}^{\gamma \gamma}$ in Figure 1 for a "typical" value of $\Lambda_{\alpha}$. In presenting these results we have assumed an $\mathrm{e}^{+} \mathrm{e}^{-}$integrated luminosity $L=500 \mathrm{fb}^{-1}$. At this luminosity the asymmetry is much smaller than the associated errors, and only a change in the total counting rate is statistically significant. Such an effect can arise from a variety of sources, and the identification of $C P$ violation requires higher luminosities to observe a non-vanishing asymmetry.

To estimate the value of $\Lambda_{\alpha}$ that can be probed at a $\gamma \gamma$ collider we have performed a combined least-squares fit to the total cross section and asymmetry. We have assumed standard statistical errors and an additional $1 \%$ luminosity error in the integrated cross section. The fit was performed with $\sqrt{s_{e e}}=1 \mathrm{TeV}$; approximate results for other energies can be obtained by scaling these numbers. Search reaches for four of the $\mathrm{SU}(2) \times \mathrm{U}(1)$ operators are also presented in Figure 1; those for $\mathcal{O}_{(W W)(B B)}$ and $\mathcal{O}_{(B W)(B W)}$ are identical to that for $\mathcal{O}_{(B B)(W W)}$, while the reach for $\mathcal{O}_{W B W B}$ is the same as that for $\mathcal{O}_{B W B W}$. We see that with an integrated luminosity of $100 \mathrm{fb}^{-1}$, $\Lambda_{\alpha} \simeq(0.8-1.8) \sqrt{s_{e e}}$ can be probed in $\gamma \gamma \rightarrow \gamma \gamma$.

We next consider the effects of these anomalous operators on the process $\gamma\left(k_{1}\right)+\gamma\left(k_{2}\right) \rightarrow$ $\gamma\left(p_{1}\right)+Z\left(p_{2}\right)$. The structure of the SM amplitudes for $\gamma \gamma \rightarrow \gamma Z$ is similar to that for $\gamma \gamma \rightarrow \gamma \gamma$, and even more pronounced. At high energies the process is dominated by the imaginary parts of the amplitudes $\mathcal{M}_{ \pm \pm \pm \pm}^{S M}, \mathcal{M}_{ \pm \mp \pm \mp}^{S M}$, and $\mathcal{M}_{ \pm \mp \mp \pm}^{S M}$; all other amplitudes are completely negligible [6]. However, the $C P$ violating amplitudes are quite different than those of $\gamma \gamma \rightarrow \gamma \gamma$; the arguments that led to a vanishing of a large number of $C P$ violating $\gamma \gamma \rightarrow \gamma \gamma$ amplitudes no longer hold in this process.

As before, we must construct an observable that contains an interference between a $C P$ odd amplitude with one of the dominant SM terms. The asymmetry considered for $\gamma \gamma \rightarrow \gamma \gamma$ contains only interference with the real parts of the SM amplitudes, and is therefore unacceptable. For interference effects between imaginary $C P$ odd and $C P$ even amplitudes, we must consider the following asymmetry which is measurable with circularly polarized beams:

$$
\begin{equation*}
A_{\gamma Z}=\frac{\left(\frac{d \sigma}{d \Omega}\right)_{+}-\left(\frac{d \sigma}{d \Omega}\right)_{-}}{\left(\frac{d \sigma}{d \Omega}\right)_{+}+\left(\frac{d \sigma}{d \Omega}\right)_{-}} \tag{4}
\end{equation*}
$$

The subscripts $\pm$ denote the initial polarization states of the laser and fermion beams, which we will now discuss. Setting $P_{t 1}=P_{t 2}=0$, we are left with four parameters describing the inital


Figure 1: Integrated asymmetry (left) for the anomalous $\gamma \gamma$ coupling versus $\sqrt{s_{e e}}(\mathrm{GeV})$, with $\Lambda_{1}=2 \mathrm{TeV}$ and $L_{e e}=500 \mathrm{fb}^{-1}$. The bars correspond to the statistical errors. Search reaches (right) for each $\operatorname{SU}(2) \times$ $\mathrm{U}(1)$ operator as a function of integrated luminosity at the $95 \% \mathrm{CL}$, assuming $\sqrt{s_{e e}}=1 \mathrm{TeV}$.
state polarization: $P_{e 1}, P_{e 2}, P_{l 1}$, and $P_{l 2}$. We will set $\left|P_{e}\right|=0.9$ and $\left|P_{l}\right|=1.0$, and label our inital states as $\left(P_{e 1}, P_{l 1}, P_{e 2}, P_{l 2}\right)$. In the SM, there are six independent states: $(++++),(++$ $+-),(++--),(+-+-),(-++-)$, and $(+---)$, where, for example, $(+-+-)$ means $P_{e 1}=0.9$, $P_{l 1}=-1.0, P_{e 2}=0.9$, and $P_{l 2}=-1.0$. States obtained by an overall sign flip are identical in the SM; for example, $(+-+-)$ and $(-+-+)$ lead to the same observables. This is not true when $C P$ violating interactions are present. The asymmetry of eq. (11), where the subscript + refers to a given inital state and - to the state obtained by flipping the signs of the polarizations, will vanish in the SM and be non-zero in the presence of the anomalous couplings.

The four largest asymmetries for $\mathcal{O}_{1}^{\gamma Z}$ are presented in Fig. 2 as a function of $\cos (\theta)$. At $L=$ $500 \mathrm{fb}^{-1}$ and $\Lambda_{1}=2 \mathrm{TeV}$, the "symmetric" asymmetries are statistically significant throughout the entire angular region, while the "antisymmetric" asymmetries are significant in the outer regions.


Figure 2: "Symmetric" asymmetries and "antisymmetric" asymmetries for $\mathcal{O}_{1}^{\gamma Z}$, with $\Lambda_{1}=2 \mathrm{TeV}$, $L_{e e}=500 \mathrm{fb}^{-1}$, and $\sqrt{s_{e e}}=1 \mathrm{TeV}$. The bars indicate the corresponding statistical errors.

To estimate the value of $\Lambda_{\alpha}$ that can be probed at a $\gamma \gamma$ collider we have performed a combined least-squares fit to the normalized binned cross section, binned asymmetry, and total cross section, with $\sqrt{s_{e e}}=1 \mathrm{TeV}$ for the two polarization states $(+-+-)$ and $(-++-)$. These two choices are chosen to illustrate the sensitivities obtainable from both symmetric and asymmetric initial polarizations; the search reaches from the remaining polarization states are similar. The results are presented in Figure 3. Here, we have included only five of the $\mathrm{SU}(2) \times \mathrm{U}(1)$ operators; $\mathcal{O}_{(B B)(W W)}$ and $\mathcal{O}_{(W W)(B B)}$ differ only in the sign of their asymmetries, as do $\mathcal{O}_{B W B W}$ and $\mathcal{O}_{W B W B}$, and hence have identical discovery regions. We see that $\Lambda_{\alpha} \simeq(1.0-2.8) \sqrt{s_{e e}}$ can be probed with an integrated luminosity of $100 \mathrm{fb}^{-1}$. Remembering that our operators are of dimension eight, and that the anomalous amplitudes therefore scale as $\mathcal{M} \sim s^{2} / \Lambda^{4}$, we see that the process $\gamma \gamma \rightarrow \gamma Z$ is quite sensitive to the $C P$ violating operators under consideration.


Figure 3: Search reach for $\Lambda_{\alpha}^{\gamma Z}$ for polarization state $(+-+-)$ (left), and $(-++-)$ (right). All of the quoted sensitivities are at the $95 \% \mathrm{CL}$ and assume $\sqrt{\overline{s_{e e}}}=1 \mathrm{TeV}$.

Finally, we discuss the measurement of $C P$ violation in $\gamma\left(k_{1}\right)+\gamma\left(k_{2}\right) \rightarrow Z\left(p_{1}\right)+Z\left(p_{2}\right)$. The SM amplitudes for this process are similar to those for $\gamma \gamma \rightarrow \gamma \gamma$ and $\gamma \gamma \rightarrow \gamma Z$ in that they are completely dominated at high energies by the imaginary parts of $\mathcal{M}_{ \pm \pm \pm \pm}^{S M}, \mathcal{M}_{ \pm \mp \pm \mp}^{S M}$, and $\mathcal{M}_{ \pm \pm \mp \pm}^{S M}[7]$; the relevant asymmetry that contains interference between the large imaginary SM amplitudes and the anomalous amplitudes is again given by the expression in eq. (4). However, Bose symmetry, oddness under $P$ and $T$, and crossing symmetries imply the vanishing of $\mathcal{M}_{ \pm \mp \pm \mp}^{C P}$ and $\mathcal{M}_{ \pm \mp \mp \pm}^{C P}$; this results in the vanishing of the "antisymmetric" asymmetry for the initial state ( -++- ) seen in $\gamma \gamma \rightarrow \gamma Z$.

The binned asymmetries for the symmetric initial polarizations of the $\mathcal{O}_{i}^{Z Z}$ are similar to those shown for $\gamma \gamma \rightarrow \gamma Z$, and will not be presented here. We have performed a combined leastsquares fit to the normalized binned cross section, binned asymmetry, and total cross section, with $\sqrt{S_{e e}}=1 \mathrm{TeV}$, for the polarization state $(+-+-)$ to estimate the value of $\Lambda_{\alpha}$ that can be probed in this process at a photon collider. The search reaches obtainable from the other symmetric polarization states are similar. The results are presented in Fig. 4. We only display results for the operators $\mathcal{O}_{(B B)(B B)}, \mathcal{O}_{(B W)(B W)}, \mathcal{O}_{(B B)(W W)}$, and $\mathcal{O}_{B W B W}$; the sensitivity to $\mathcal{O}_{(B B)(B B)}$ and $\mathcal{O}_{(W W)(W W)}$ is identical at high energies. Similarly, $\mathcal{O}_{(B B)(W W)}$ and $\mathcal{O}_{(W W)(B B)}$ differ only in the sign of their asymmetries, as do $\mathcal{O}_{B W B W}$ and $\mathcal{O}_{W B W B}$, and hence yield identical search reaches. As in $\gamma \gamma \rightarrow \gamma Z, \Lambda_{\alpha}$ in the range $(1.0-2.8) \sqrt{s_{e e}}$ can be probed with $L_{e e}=100 \mathrm{fb}^{-1}$.


Figure 4: Sensitivity to $\Lambda_{\alpha}$ for the polarization state ( +-+- ) at the $95 \% \mathrm{CL}$, assuming $\sqrt{s_{e e}}=1 \mathrm{TeV}$.

## 3. Discussion and Conclusion

In summary, the processes $\gamma \gamma \rightarrow \gamma \gamma, \gamma Z$, and $Z Z$ are sensitive probes of $C P$ violating gauge boson self couplings. The examination of these processes nicely complements previous studies that have focused primarily on $W$ boson, top quark, or Higgs production.

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