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# **Gauged Supergravities and de Sitter Space and Cosmology**

Renata Kallosh *et al.*

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*Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309*

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# Gauged Supergravities, de Sitter Space and Cosmology

Renata Kallosh,<sup>1</sup> Andrei Linde,<sup>1</sup> Sergey Prokushkin<sup>1</sup> and Marina Shmakova<sup>2</sup>

<sup>1</sup>*Department of Physics, Stanford University,  
Stanford, CA 94305-4060, USA and*

<sup>2</sup>*CIPA, 366 Cambridge Avenue, Palo Alto, CA 94306,  
and SLAC, Stanford University, Stanford, CA 94309*

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## Abstract

We have studied scalar potentials  $V$  of gauged N=8,4,2 supergravities in d=4. Extrema of these potentials may correspond to de Sitter, anti de Sitter and Minkowski vacua. We have found that all known de Sitter extrema correspond to unstable maximum/saddle points with negative curvature  $|V''| = 2V$  for the fields canonically normalized at the extremum. This is equivalent to the relation  $|m^2| = |R|/2 = 6H^2$  for the tachyonic mass  $m$ , the curvature scalar  $R$ , and the Hubble constant  $H$ . This prevents the use of de Sitter extrema for slow-roll inflation in the early universe, which would require  $|V''| \ll V$ . Moreover, in all models that we were able to analyse the potential is unbounded from below. On the other hand, barring the question how realistic such models could be, one can use them for the description of the accelerated expansion of the universe at the present epoch. This is related to a novel possibility of the fast-roll inflation with  $|V''| \gtrsim V$ . We also display some potentials that have flat directions with vanishing cosmological constant, and discuss their possible cosmological implications.

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## I. INTRODUCTION

Recent cosmological observations based on the study of supernova [1] and of the anisotropy of the cosmic microwave background radiation [2] suggest that soon after the big bang our universe experienced a stage of a very rapid accelerated expansion (inflation) [3, 4, 5, 6]. Moreover, observations indicate that few billion years after the big bang the universe entered a second stage of accelerated expansion. The rate of acceleration now is many orders of magnitude smaller than during the stage of inflation in the early universe. For a discussion of the recent cosmological observations and their theoretical interpretation see e.g. [7].

Most of the inflationary models are based on the assumption that the energy-momentum tensor during inflation is dominated by potential energy density of a scalar field,  $T_{\mu\nu} \approx g_{\mu\nu}V(\phi)$  with  $V > 0$  [8, 9]. This means, in particular, that  $\dot{\phi}^2/2 \ll V(\phi)$ . The limiting case  $\dot{\phi} = 0$  corresponds to de Sitter space with a positive cosmological constant. The current cosmological acceleration can be explained either by a positive vacuum energy  $V$  (cosmological constant) or by a slowly rolling scalar field in a near de Sitter background with  $\dot{\phi}^2/2 \ll V(\phi)$  (quintessence) [10].

Much of the recent progress in theory of all fundamental interactions is related to supersymmetry. Therefore it would be very desirable to find de Sitter or near-de Sitter solutions in supersymmetric theories. In N=1 theories many choices are available and therefore the motivation for a particular choice has to come from a more fundamental theory, either directly from M/string theory in 11/10 dimensions or perhaps from an effective 4d theory with higher supersymmetry.

This was recently discussed in the context of 4d N=2 supersymmetry in [11] where a hybrid hypersymmetric model of inflation/acceleration was proposed. On the other hand, it may be difficult to obtain 4d de Sitter space directly from M/string theory. The problems have been recently reviewed in [12, 13].

The basic problem seems to originate from the compactification of M/string theory on internal space with the finite volume. However, some 4d N=8,4,2 gauged supergravity theories are known to have de Sitter solutions with spontaneous breaking of supersymmetry [14, 16, 17, 18]. These versions of 4d supergravity are related to 11d supergravity with a non-compact internal 7d space [19]. M-theory has some solutions with warped products of de Sitter space with hyperbolic spaces and generalized cylinders,  $dS_4 \times \mathcal{H}^{p,q,r}$ . For conceptual problems associated

with non-compact internal space and attempts to overcome them we refer to [12] and references therein.

Here we will take a simple attitude to potentials of gauged supergravities: we will consider only 4d theories where all kinetic terms have correct sign and therefore these theories from the 4d perspective do not have any conceptual problems whatsoever. We will study the properties of their potentials with regard to cosmology. If we find any potentials interesting for cosmology, we may come back to the problems of accommodating these 4d supergravity models in the framework of M-theory with non-compact internal space.

The physical motivation for our study of potentials of supergravity originates in cosmology. A standard *slow-roll inflation* resulting in many e-foldings and scale independence of the spectrum will be reviewed. A new concept of *fast-roll inflation* will be proposed. It is partially motivated by the properties of supergravity potentials and also by the present epoch acceleration with small number of e-foldings.

We start with reminding some well known facts about the relation between de Sitter space and expanding universe in Section II. Section III explains a standard *slow-roll inflation* and introduces a new concept of a *fast-roll inflation*. This defines the properties of the potentials interesting for cosmology.

In Sections IV and V we study the potentials of N=8 and N=4, 2 gauged supergravities. In Section VI we discuss some potentials of N=8 supergravity which have interesting Minkowski vacua with flat directions. We discuss the main results and perspectives in Conclusions.

## II. DE SITTER VERSUS ANTI DE SITTER

Anti de Sitter space was at full attention of high-energy physics community for the last 10 years from the time when it was realized that near horizon the geometry of stringy BPS black holes and branes tends to anti de Sitter space. It was known also for a very long time that the potentials of gauged supergravities have anti de Sitter critical points which are at the top of the inverted potentials and correspond to the maximum of these potentials and the tachyons are present. Still when the negative (mass)<sup>2</sup> is limited in a certain way corresponding to Breitenlohner-Freedman bound, the anti de Sitter solution is stable. For  $adS_4$  the bound is

$$m^2 \geq -\frac{9}{4}H^2, \tag{1}$$

where  $H^{-1}$  defines the radius of the throat of the hyperboloid in the flat 5d space

$$X_0^2 - X_1^2 - X_2^2 - X_3^2 + X_4^2 = H^{-2} . \quad (2)$$

The hyperboloid in the 5d flat space defining de Sitter space  $dS_4$  is

$$X_0^2 - X_1^2 - X_2^2 - X_3^2 - X_4^2 = -H^{-2} . \quad (3)$$

The difference between the two hyperboloids is that the right hand side has an opposite sign. Besides, the *adS* hyperboloid has two ‘time directions’ whereas *dS* one has only one ‘time direction’. The symmetry group for the *adS* hyperboloid is  $SO(3, 2)$  whereas for the *dS* hyperboloid it is  $SO(4, 1)$ .

To explain shortly the relation of de Sitter space to cosmology we will first present here the Friedmann-Robertson-Walker (FRW) metric describing the expanding Friedmann universe:

$$ds^2 = dt^2 - a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right] , \quad (4)$$

where  $k = \pm 1, 0$  for a closed, open or flat expanding Friedmann universe, respectively. Here  $a(t)$  is the time-dependent scale factor. For a spatially flat universe with  $k = 0$  the metric can be represented in a form

$$ds^2 = dt^2 - a^2(t) d\vec{x}^2 . \quad (5)$$

Closed Friedmann universe with  $k = 1$  can be represented in the form

$$ds^2 = dt^2 - a^2(t) [d\chi^2 + \sin^2(\chi)(d\theta^2 + \sin^2 \theta d\phi^2)] , \quad 0 \leq \chi \leq 2\pi . \quad (6)$$

Finally, an open Friedmann universe with  $k = -1$  can be represented in the form

$$ds^2 = dt^2 - a^2(t) [d\chi^2 + \sinh^2(\chi)(d\theta^2 + \sin^2 \theta d\phi^2)] , \quad 0 \leq \chi \leq \infty . \quad (7)$$

The relation between the metric of expanding universe and de Sitter space is the following. First, one can consider the hyperboloid (3) in a coordinate system which spans the half of the hyperboloid with  $X_0 + X_4 > 0$ . The choice

$$X_0 = H^{-1} \sinh Ht + \frac{1}{2} H e^{Ht} \vec{x}^2, \quad X_4 = H^{-1} \cosh Ht - \frac{1}{2} H e^{Ht}, \quad X_i = e^{Ht} x_i, i = 1, 2, 3$$

leads to the metric

$$ds^2 = dt^2 - e^{2Ht} d\vec{x}^2 . \quad (8)$$

This is a FRW spatially flat metric with the exponential scale factor  $a(t) = e^{Ht}$ . We may choose the coordinates which span the entire hyperboloid :

$$\begin{aligned} X_0 &= H^{-1} \sinh Ht , & X_1 &= H^{-1} \cosh Ht \cos \chi , & X_2 &= H^{-1} \cosh Ht \sin \chi \cos \theta , \\ X_3 &= H^{-1} \cosh Ht \sin \chi \sin \theta \cos \phi , & X_4 &= H^{-1} \cosh Ht \sin \chi \sin \theta \sin \phi , \end{aligned}$$

which results in the metric

$$ds^2 = dt^2 - H^{-2} \cosh^2 Ht [d\chi^2 + \sin^2(\chi)(d\theta^2 + \sin^2 \theta d\phi^2)] , \quad 0 \leq \chi \leq 2\pi . \quad (9)$$

This is a FRW metric of the closed Friedmann universe with the exponential scale factor  $a(t) = H^{-1} \cosh Ht$ . An open Friedmann universe appears from de Sitter space if the coordinate system is chosen as follows:

$$\begin{aligned} X_0 &= H^{-1} \sinh Ht \cosh \chi , & X_1 &= H^{-1} \cosh Ht , & X_2 &= H^{-1} \sinh Ht \sinh \chi \cos \theta , \\ X_3 &= H^{-1} \sinh Ht \sinh \chi \sin \theta \cos \phi , & X_4 &= H^{-1} \sinh Ht \sinh \chi \sin \theta \sin \phi . \end{aligned}$$

These coordinates do not cover the entire hyperboloid. The metric is

$$ds^2 = dt^2 - H^{-2} \sinh^2 Ht [d\chi^2 + \sinh^2(\chi)(d\theta^2 + \sin^2 \theta d\phi^2)] , \quad 0 \leq \chi \leq \infty . \quad (10)$$

This is a FRW metric of the open Friedmann universe with the exponential scale factor  $a(t) = H^{-1} \sinh Ht$ .

### III. DE SITTER SPACE, SLOW-ROLL INFLATION AND FAST-ROLL INFLATION

De Sitter space has a direct relation to inflationary cosmology [3, 4, 5, 6, 8, 9], which we will briefly describe now. Consider a theory of a scalar field  $\phi$  with potential  $V(\phi)$ . The Friedmann equation for the scale factor of the universe looks as follows:

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{\rho(\phi)}{3} \quad (11)$$

in units  $M_p = 1$ . Here  $k = \pm 1, 0$  for a closed, open or flat universe correspondingly,  $\rho(\phi) = V(\phi) + \dot{\phi}^2/2 + (\partial_i \phi)^2/2$  is the energy density of the scalar field.

Let us assume first that  $V(\phi) = V = \text{const} > 0$ , and the field  $\phi$  is constant and homogeneous,  $\dot{\phi} = \partial_i \phi = 0$ . Then

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{V}{3}. \quad (12)$$

The solutions of this equation describe de Sitter space with

$$H = \sqrt{\frac{V}{3}}. \quad (13)$$

Note that at very large times  $t \rightarrow \infty$  all 3 types of de Sitter metric for the flat, closed and open universe lead to the same exponential scale factor

$$e^{Ht}, \quad \cosh Ht \rightarrow e^{Ht}, \quad \sinh Ht \rightarrow e^{Ht}. \quad (14)$$

Therefore we will concentrate on the simplest case of the flat universe with  $a(t) = a(0)e^{Ht}$ . Flatness of the universe is a standard prediction of most of the inflationary models, where typically the term  $\frac{k}{a^2}$  can be neglected as compared with  $\rho/3$  after inflation [3, 4, 5, 6, 8, 9]. This means that inflationary theory predicts that our universe at present cannot be in the anti-de Sitter regime, because Eq. (11) does not have any solutions for  $\rho < 0$  in the flat universe.

De Sitter space can describe late stages of the evolution of our universe if the universe has nonvanishing vacuum energy  $V \lesssim 10^{-120} M_p^4$ . However, in order to use de Sitter-like stages for a description of inflation in the early universe one should find how de Sitter stage ends and the usual hot Friedmann universe emerges.

One possibility is to study models where the potential  $V(\phi)$  has a very flat maximum, as in the new inflation scenario [4]. Consider, for example, the model where  $V(\phi)$  has a maximum at  $\phi = 0$ , such that in the vicinity of the maximum

$$V(\phi) = V_0 - \frac{m^2 \phi^2}{2}. \quad (15)$$

Suppose that initially the field  $\phi = \phi_0$  was homogeneous, small, and had small velocity, so that  $m^2 \phi^2/2, \dot{\phi}^2/2 \ll V_0$ . Then the Hubble constant  $H^2 = \rho/3 = \frac{1}{3}(V_0 - m^2 \phi^2/2 + \dot{\phi}^2/2)$  practically did not change until the field rolled down to the point with  $m^2 \phi_1^2 \sim V_0$ . Therefore the universe continues expanding as  $e^{Ht}$  until the field  $\phi$  rolls from  $\phi_0$  to

$$\phi_1 \sim \sqrt{V_0}/m. \quad (16)$$

The motion of a homogeneous field  $\phi$  is described by equation

$$\ddot{\phi} + 3H\dot{\phi} = -V' = m^2 \phi. \quad (17)$$

Now let us assume that  $|V''| = |m^2| \ll H^2 = V/3$ . In this case one can show that the field moves very slowly, so that one can neglect  $\ddot{\phi}$  as compared to  $3H\dot{\phi}$ , and the growing solution for the field  $\phi$  is given by  $\phi = \phi_0 \exp \frac{m^2 t}{3H}$ . This is the standard slow-roll solution for the scalar field during inflation. The slow-roll regime continues at least until the field rolls down below  $\phi_1$ , i.e. during the time

$$\Delta t = \frac{3H}{m^2} \log \frac{\phi_1}{\phi_0}. \quad (18)$$

This leads to inflation by a factor of

$$e^{H\Delta t} \sim \left( \frac{\phi_1}{\phi_0} \right)^{3H^2/m^2} \sim \left( \frac{\phi_1}{\phi_0} \right)^{V/|V''|}. \quad (19)$$

Thus one can obtain an exponentially large degree of inflation for

$$|V''| \ll |V|, \quad (20)$$

which is one of the two well-known inflationary slow-roll conditions:  $\eta = \frac{|V''|}{|V|} \ll 1$  and  $\epsilon = \frac{1}{2} \left( \frac{V'}{V} \right)^2 \ll 1$  [9]. The last condition is automatically satisfied at the top of the effective potential.

The slow-roll conditions serve two purposes: They make the total expansion of the universe during the stage of inflation very large,  $e^{H\Delta t} \sim \left( \frac{\phi_1}{\phi_0} \right)^{|V|/|V''|}$ , and they ensure that the spectrum of adiabatic density perturbations produced during inflation is almost scale-independent. These density perturbations are produced due to quantum effects during inflation [21]; they are playing a critical role in the subsequent process of formation of the large-scale structure of the universe [8, 9]. The deviation from scale-independence (flatness) of the spectrum is characterized by the factor  $|n-1| = |2\frac{|V''|}{|V|} - 3\left(\frac{V'}{V}\right)^2|$  [9]. Recent observations of anisotropy of the cosmic microwave background radiation suggest that  $|n-1| \lesssim 0.1$  [2], which implies that

$$|V''| \lesssim 0.05V. \quad (21)$$

if the perturbations that we observe were produced during inflation when the field  $\phi$  was near the top of the potential.

The slow-roll condition  $|V''| \ll V$  implies that  $|m^2| \ll V_0$ , and, consequently,

$$\phi_1 \gg 1 \quad (22)$$

in Planck units (i.e.  $\phi_1 \gg M_p$ ). This is very similar to the standard requirement that appears in the simplest models of chaotic inflation with  $V(\phi) \sim \phi^n$  [5, 8]. To avoid this requirement



and still have slow-roll inflation one would need to make the potential very flat at the top, and very curved near the minimum of the potential, as in the original version of the new inflation scenario [4], or as in the hybrid inflation scenario [6].

One of the results of our paper is that the slow-roll condition  $V'' \ll V$  is not satisfied near any of the extrema of the potentials with  $V > 0$  in N=8 gauged supergravity that have been studied in the literature. As we will show, in all of the models of N=8 gauged supergravity studied so far one has  $V'' = 2V$ . Thus, in all known cases these models do not support slow-roll inflation near the extrema of the corresponding potentials.

Is it possible to have a *fast-roll inflation* with  $|V''| = |m^2| \geq H^2 = V/3$ ? In our investigation of this question we have found, much to our own surprise, that the answer to this question is positive.

For simplicity we will consider first the limiting case  $|m^2| \gg H^2$ . In this case one can neglect the term  $3H\dot{\phi}$  in the equation for the field  $\phi$ . Then the growing solution becomes

$$\phi = \phi_0 e^{mt} . \quad (23)$$

The duration of rolling from  $\phi_0$  to  $\phi_1$  is given by

$$\Delta t = m^{-1} \log \frac{\phi_1}{\phi_0} . \quad (24)$$

Until the field rolls down to  $\phi_1$  the energy density remains dominated by  $V_0$ . This leads to inflation by a factor of

$$e^{H\Delta t} \sim \left( \frac{\phi_1}{\phi_0} \right)^{H/m} . \quad (25)$$

Usually one does not expect the ratio  $\frac{\phi_1}{\phi_0}$  to be exponentially large, and therefore one could think that for  $H/m < 1$  the duration of inflation of this kind must be rather insignificant. Also, no long-wavelength perturbations of metric are generated in the regime  $m > H$  by the standard inflationary mechanism. That is why the possibility of a fast-roll inflation with  $m \geq H$  has not been thoroughly studied in the literature.

Meanwhile fast-roll inflation can be quite interesting, at least for a marginally fast-rolling regime with  $m \sim H$  (i.e.  $\sqrt{|V''|} \sim \sqrt{V}$ ). This is the regime that we will often encounter in our investigation of extrema of the potentials in N=8 gauged supergravity.

First of all, let us note that the initial value of the field  $\phi$  can be quite small. Formally, one may have  $\phi_0 = 0$ . In this case the factor  $\frac{\phi_1}{\phi_0}$  in Eq. (25) can be indefinitely large. In reality,

$\phi_0$  in this equation cannot be taken much smaller than the level of quantum fluctuations with momenta  $k < m$ , since such fluctuations also experience exponential growth, even in the absence of a homogeneous field  $\phi_0$ :  $\delta\phi_k(t) \sim \delta\phi_k(0)e^{\sqrt{m^2-k^2}t}$ . A typical initial amplitude of all quantum fluctuations with  $k < m$  participating in the exponential growth of the field  $\phi$  can be estimated by  $\delta\phi \sim Cm$ , where  $C = O(10)$  [24]. A typical time it takes for this field to grow up to  $\phi_1$  is given by [24]

$$\Delta t \sim m^{-1} \log \frac{C\phi_1}{m}. \quad (26)$$

This leads to inflation by a factor of

$$e^{H\Delta t} \sim \left( \frac{10\phi_1}{m} \right)^{H/m}. \quad (27)$$

Now that we have studied two limiting case, let us study a more general regime where  $m$  and  $H$  can be of the same order. To study this problem one should look for solutions of Eq. (17) in the form  $\phi = \phi_0 e^{i\omega t}$ . This yields

$$\omega = i \left( \frac{3H}{2} \pm \sqrt{\frac{9H^2}{4} + m^2} \right). \quad (28)$$

The solution with the sign - corresponds to the exponentially growing solution

$$\phi = \phi_0 \exp \left[ (Ht \cdot F(m^2/H^2)) \right], \quad (29)$$

where

$$F(m^2/H^2) = \sqrt{\frac{9}{4} + \frac{m^2}{H^2}} - \frac{3}{2}. \quad (30)$$

This immediately gives us the general result for the total expansion of the universe during inflation near the maximum of the potential:

$$e^{H\Delta t} \sim \left( \frac{\phi_1}{\phi_0} \right)^{1/F}. \quad (31)$$

One can easily check that this result coincides with our previously obtained results in the limiting cases  $m \gg H$  and  $m \ll H$ .

As an example, consider first the potentials with  $m = H$ . In this case one has  $F(1) = 0.3$ . In the theories with  $m \sim H$  one has  $\phi_1 \sim M_p = 1$ , so for  $\phi_0 \sim 10m^{-1}$  (i.e. for the initial value of the field provided by quantum fluctuations [24]) one has

$$e^{H\Delta t} \sim \left( \frac{10M_p}{m} \right)^{3.3}. \quad (32)$$

Clearly, this number can be quite significant.

To be more specific, consider the possibility that such models can be responsible for the present stage of accelerated expansion of the universe with the Hubble constant  $H \sim 10^{-60} M_p$ . Then inflation in an unstable state close to the maximum of the potential in such a theory can lead to expansion of the universe by a factor that can be as large as

$$e^{H\Delta t} \sim (10^{61})^{3.3} \sim 10^{200} \sim e^{460} . \quad (33)$$

This is more than sufficient to explain the observed single e-folding of accelerated expansion of the universe at the present epoch.

Meanwhile if one takes  $m \sim 10^2 \text{ GeV} \sim 10^{-16} M_p$ , which corresponds to the electroweak scale, one can obtain fast-roll inflation by a factor of

$$e^{H\Delta t} \sim (10^{17})^{3.3} \sim 10^{56} \sim e^{130} . \quad (34)$$

Efficiency of fast-roll inflation rapidly decreases once one considers the regime with  $H \ll m$ . An interesting example is provided by gauged N=8 supergravity, where, as we will see later,  $|V''| = 2V$ , i.e.  $m^2 = 6H^2$ . In terms of our potential  $V(\phi) = V_0 - \frac{m^2\phi^2}{2}$  this implies that the point  $\phi_1$ , which corresponds to  $V(\phi_1) = V_0/2$ , is given by  $\phi_1 = 1/\sqrt{2}$ . In dimensional units this is equivalent to having  $\phi_1 \sim 1.5 \times 10^{18} \text{ GeV}$ . In this model one has  $F(m^2/H^2) = F(6) = 1.37$ , and  $1/F = 0.73$ . Then, for  $m = \sqrt{6} H \sim 2 \times 10^{-60} M_p$  one finds

$$e^{H\Delta t} \sim (5 \cdot 10^{60})^{0.73} \sim 10^{44} \sim e^{100} . \quad (35)$$

Thus, fast-roll inflation in N=8 gauged supergravity may be responsible for up to 100 e-folds of exponential expansion of the universe with the Hubble constant similar to its present value  $H \sim 10^{-60} M_p$ .

On the other hand, for  $m = \sqrt{6} H \sim 10^2 \text{ GeV}$  one has

$$e^{H\Delta t} \sim 10^{13} \sim e^{28} . \quad (36)$$

To give a different example, one may consider a simplest model of spontaneous symmetry breaking with the potential

$$V(\phi) = \frac{\lambda}{4}(\phi^2 - v^2)^2 = -\frac{1}{2}m^2\phi^2 + \frac{m^2}{4v^2}\phi^4 + \frac{1}{4}m^2v^2 , \quad (37)$$

where  $m^2 = \lambda v^2$ , and  $\phi = v$  corresponds to the minimum of  $V(\phi)$  with symmetry breaking. The Hubble constant at  $\phi = 0$  in this model is given by  $H^2 = \frac{m^2 v^2}{12}$ , so that  $F(m^2/H^2) = F(12/v^2)$ . Fast-roll inflation in this model occurs for  $\phi \lesssim v/2$ . Assuming, e.g.,  $v = 1$  and  $m \sim 100$  GeV, one finds  $F^{-1}(12) = 0.44$ , and

$$e^{H\Delta t} \sim (10^{17})^{1/F} \sim 10^7, \quad (38)$$

whereas for  $v = 0.7$ , as in Polonyi model, one has

$$e^{H\Delta t} \sim 10^5. \quad (39)$$

Of course, adiabatic density perturbations are not produced during the fast-roll inflation. On the other hand, unless this stage of inflation is too prolonged, it does not erase density perturbations produced by a possible earlier stage of slow-roll inflation. Moreover, such inflation may induce perturbations in other scalar fields which may have mass  $m < H$  during the fast-roll inflation. Such perturbations lead to isocurvature perturbations of metric, which are disfavored by observations. However, under certain conditions such perturbations can be converted to the usual adiabatic perturbations [22].

More importantly, the stage of the fast-roll inflation may help us to solve the moduli problem and the gravitino problem, in the same way as thermal inflation [23]. In order to solve the moduli problem one would need to have inflation by a modest factor  $10^4 - 10^5$ , which dilutes the energy density of moduli and the density of gravitinos by a factor of  $10^{-12} - 10^{-15}$ . Then the scalar field driving inflation must decay to ultra-relativistic particles. It is quite possible to obtain inflation by a factor of  $10^4 - 10^5$  in the fast-roll model discussed above. And it is even easier to add few extra e-folds to the stage of thermal inflation. It was usually assumed that thermal inflation ends as soon as the field begins to fall from the top of the effective potential. This assumption is often correct for the models with the scale of spontaneous symmetry breaking much smaller than  $10^{18}$  GeV [23]. However, as we have seen above, one can have much more than  $10^5$  e-folds of inflation in the models with  $v \gtrsim 0.5$ . Note that potentials with  $v = O(1)$  often appear in phenomenological  $N=1$  supergravity.

Thus, if in a certain class of theories one has  $|V''| \sim V$ , one should not immediately discard such theories as candidates for the description of an accelerated (inflationary) stage of the evolution of the universe. Such theories can describe a prolonged stage of fast roll inflation if  $m \ll 1$  in Planck units, see Eqs. (27), (32). For  $|V''| \sim V$ , the requirement  $m \ll 1$  is

equivalent to the requirement that the extremum of the effective potential corresponds to the energy density much smaller than the Planck density,  $V \ll 1$ .

## IV. N=8 GAUGED SUPERGRAVITIES

### A. de Wit-Nicolai Potential

The ungauged N=8 supergravity of Cremmer and Julia [25] has a local  $SU(8)$  symmetry and a rigid  $SL(8, \mathbb{R})$  symmetry, equations of motion have a larger, non-compact  $E_{7(7)}$  symmetry. The 70 real scalars of N=8 supergravity parametrize the coset space  $E_7/SU(8)$  and can be described by an element  $\mathcal{V}(x)$  of the fundamental 56-dimensional representation of  $E_{7(7)}$ :

$$\mathcal{V}(x) = \begin{pmatrix} u_{ij}^{IJ}(x) & v_{ijKL}(x) \\ v^{klIJ}(x) & u_{KL}^{kl}(x) \end{pmatrix}. \quad (40)$$

Out of 133 fields 63 may be gauged using an  $SU(8)$  symmetry. De Wit and Nicolai [26] gauged the  $SO(8)$  subgroup of  $SL(8, \mathbf{R})$  symmetry of the ungauged supergravity. The  $SO(8)$  gauge coupling constant  $g$  is a new parameter which the gauged supergravity has, in addition to the gravitational constant. The local N=8 supersymmetry of gauged supergravity requires a non-trivial effective potential for the scalars. It is proportional to the square of the gauge coupling. The scalar and gravity part of de Wit-Nicolai action is

$$\int d^4x \sqrt{-g} \left( \frac{1}{2} R - \frac{1}{96} |A_\mu^{ijkl}|^2 - V \right), \quad (41)$$

where the building blocks for the scalar kinetic terms are:  $A_\mu^{ijkl} = -2\sqrt{2} (u_{IJ}^{ij} \partial_\mu v^{klIJ} - v^{ijIJ} \partial_\mu u_{IJ}^{kl})$ . De Wit-Nicolai nontrivial effective potential can be written as the difference of two positive definite terms:

$$V = -g^2 \left( \frac{3}{4} |A_1^{ij}|^2 - \frac{1}{24} |A_2^i{}_{jkl}|^2 \right), \quad (42)$$

$$A_1^{ij} = -\frac{4}{21} T_m^{ijm}, \quad A_2^i{}_{jkl} = -\frac{4}{3} T_l^{[ijk]}, \quad (43)$$

are some particular combinations of T-tensors:

$$T_l^{kij} = (u_{IJ}^{ij} + v^{ijIJ}) (u_{lm}^{JK} u_{KI}^{km} - v_{lmJK} v^{kmKL}). \quad (44)$$

The 56-bein  $\mathcal{V}(x)$  can be brought into the following form in the  $SU(8)$  unitary gauge by the  $SU(8)$  rotation

$$\mathcal{V}(x) = \exp \begin{pmatrix} 0 & \phi_{ijkl}(x) \\ \phi^{ijkl}(x) & 0 \end{pmatrix}, \quad (45)$$

where  $\phi^{ijkl}$  is a complex self-dual tensor describing the 35 scalars and 35 pseudo-scalars of  $\phi^{ijkl}$  of  $N = 8$  supergravity. The potential has an  $adS_4$  critical point where all scalars and pseudo-scalars vanish.

### B. $SO(p) \times SO(q) \times SO(r)$ sector

Compact gaugings of  $N=8$  supergravity do not give de Sitter solutions, however the non-compact and non-semi-simple gaugings with  $CSO(p, q, r)$  groups, suggested by Hull [16] do have de Sitter and Minkowski solutions. These are unitary 4d theories with positive definite kinetic terms.

Very recently somewhat more general non-compact and non semi-simple gaugings with potentials depending on two scalars were performed by Ahn and Woo in the  $SO(p) \times SO(q) \times SO(r)$  sectors of the theory in [18]. One starts with the subalgebra of the  $SL(8, \mathbb{R})$  algebra with the metric parametrized by  $\xi$  and  $\zeta$

$$\eta_{AB} = \begin{pmatrix} \mathbf{1}_{p \times p} & & \\ & \xi \mathbf{1}_{q \times q} & \\ & & \xi \zeta \mathbf{1}_{r \times r} \end{pmatrix}, \quad p + q + r = 8 \quad (46)$$

The  $CSO(p, q, r)$  group is a group contraction of the  $SO(p+r, q)$  group preserving a symmetric metric with  $p$  positive eigenvalues,  $q$  negative eigenvalues (for negative  $\xi$ ), and  $r$  zero eigenvalues. Such gaugings lead to a more complicated potential with two real scalar fields  $s$  and  $t$ .

$$V_{\xi, \zeta}^{p, q, r}(s, t) = g^2 \left( 4(\partial_s W_{\xi, \zeta}^{p, q, r}(s, t))^2 + 4(\partial_t W_{\xi, \zeta}^{p, q, r}(s, t))^2 - 6(W_{\xi, \zeta}^{p, q, r}(s, t))^2 \right) \quad (47)$$

where the superpotential  $W_{\xi, \zeta}^{p, q, r}(s, t)$  is:

$$W_{\xi, \zeta}^{p, q, r}(s, t) = \frac{1}{8} \left( p e^{\sqrt{\frac{4q}{p(p+q)}}s - \sqrt{\frac{r}{2(p+q)}}t} + q \xi e^{-\sqrt{\frac{4p}{q(p+q)}}s - \sqrt{\frac{r}{2(p+q)}}t} + r \xi \zeta e^{\sqrt{\frac{p+q}{2r}}t} \right). \quad (48)$$

and the kinetic terms for scalar fields have a canonical form. Gravitational and scalar part of the supergravity action for each model with  $p, q, r$  and  $\xi, \zeta$  is given by

$$S_{\xi, \zeta}^{p, q, r} = \int d^4x \sqrt{-g} \left( \frac{1}{2} R - \frac{1}{2} \partial^\mu s \partial_\mu s - \frac{1}{2} \partial^\mu t \partial_\mu t - V_{\xi, \eta}^{p, q, r}(s, t) \right). \quad (49)$$

Thus we have a family of models characterized by 3 discrete parameters  $p + q + r = 8$  and by two parameters  $\xi, \zeta$ . At  $r = 0$ ,  $p + q = 8$  the meaning of  $\xi$  can be inferred from the higher-dimensional interpretation of these models. It has been shown by Hull and Warner [19] that they can be obtained from 11d supergravity (M-theory). The general case of a compactification on a hyperboloid gives  $SO(p, q)$  gauging and a compactification on a sphere leads to  $SO(8)$  gauging of de Wit and Nicolai [26]. The corresponding hypersurface constraining the internal 7-manifold is

$$r^2 = \eta_{AB} z^A z^B = \sum_{A=1}^p (z^A)^2 + \xi \sum_{A=p+1}^8 (z^A)^2. \quad (50)$$

For  $\xi < 0$  the expression (50) represents a family of hyperboloids and for  $\xi > 0$  it is a family of ellipsoids. When also  $r$  null directions are present in the metric, the higher dimensional interpretation was not studied yet. But clearly, the new parameter  $\zeta$  plays an important role in specification of the model.

The extrema of the potential in these models have been studied in [18]. They have found 5 classes of models with  $\xi = \pm 1$  and  $\zeta = \pm 1$  and various sets of  $p, q, r$  which have de Sitter vacua at some critical values of scalar fields, which we denote by  $s_{cr}$  and  $t_{cr}$ .

We have investigated the nature of these critical points. We have found that they are always saddle points, the eigenvalues of the matrix of the second derivatives have one positive and one negative value. We have presented the results of the calculations in the Table 1 for  $g^2 = 1$ .

$p$	$q$	$r$	$\xi$	$\zeta$	$s_{cr}$	$t_{cr}$	$W(s_{cr}, t_{cr})$	$V(s_{cr}, t_{cr})$	$(m_1)^2$	$(m_2)^2$
1	2	5				-0.7522	$\frac{1}{2} \times 3^{-3/8}$		10.53	-5.264
1	4	3				0.7522	$-\frac{1}{2} \times 3^{-3/8}$		-5.264	3.51
2	1	5	1	-1	0	-0.7522	$\frac{1}{2} \times 3^{-3/8}$	$2 \times 3^{1/4}$	10.53	-5.264
2	3	3				0.7522	$-\frac{1}{2} \times 3^{-3/8}$		-5.264	3.51
3	2	3				0.7522	$-\frac{1}{2} \times 3^{-3/8}$		-5.264	3.51
4	1	3				0.7522	$-\frac{1}{2} \times 3^{-3/8}$		-5.264	3.51
1	3	4							-4.	4.
2	2	4	1	-1	0	0	0	2	-4.	4.
3	1	4							-4.	4.
3	1	4			0.4757	-0.5826	$\frac{1}{2} \times 3^{-3/8}$		-5.264	3.51
3	2	3			0.6017	-0.4513	$\frac{1}{2} \times 3^{-3/8}$		-5.264	3.51
3	3	2	-1	1	0.6728	-0.3364	$\frac{1}{2} \times 3^{-3/8}$	$2 \times 3^{1/4}$	-5.264	3.51
3	4	1			0.7192	-0.2202	$\frac{1}{2} \times 3^{-3/8}$		-5.264	3.51
5	1	2			-0.5014	0.5606	$-\frac{1}{2} \times 3^{-3/8}$		10.53	-5.264
5	2	1			-0.6565	0.367	$-\frac{1}{2} \times 3^{-3/8}$		10.53	-5.264
4	1	3							4.	-4.
4	2	2	-1	1	0	0	0	2	-4.	4.
4	3	1							4.	-4.
1	3	4			-0.4757	-0.5826	$-\frac{1}{2} \times 3^{-3/8}$		-5.264	3.51
1	5	2			0.5014	0.5606	$\frac{1}{2} \times 3^{-3/8}$		10.53	-5.264
2	3	3	-1	-1	-0.6017	-0.4513	$-\frac{1}{2} \times 3^{-3/8}$	$2 \times 3^{1/4}$	-5.264	3.51
2	5	1			0.6565	0.367	$\frac{1}{2} \times 3^{-3/8}$		10.53	-5.264
3	3	2			-0.6728	-0.3364	$-\frac{1}{2} \times 3^{-3/8}$		-5.264	3.51
4	3	1			-0.7192	-0.2202	$-\frac{1}{2} \times 3^{-3/8}$		-5.264	3.51
1	4	3							4.	-4.
3	4	1	-1	-1	0	0	0	2	4.	-4.
2	4	2							-4.	4.

Table 1. De Sitter critical points for  $p, q, r, \xi, \zeta$  models. One of the mass square eigenvalues  $(m_1)^2$  or  $(m_2)^2$  is negative, all critical points are saddles.



For each model we give the critical values of the fields, a superpotential and a potential. The eigenvalues of the square of the mass matrix are  $(m_1)^2$  and  $(m_2)^2$ . Their product is in all cases negative. Now we may also compare the tachyonic mass near the critical point with the value of the potential there. We find that in all 27 cases a remarkable relation takes place:  $-V'' = |m^2|_{tach} = 2V$ . The absolute value of the square of the tachyonic mass is twice the value of the potential in units in which  $M_p = 1$ . Using the curvature of the de Sitter space we can present these relations as

$$|m^2|_{tach} = 2V = \frac{1}{2}|R| = 6H^2. \quad (51)$$

The positive square mass eigenvalues at the critical point also have a simple relation to the potential and/or to the curvature. In models where  $W_{cr} = 0$  one has

$$m_{tach}^2 = -\frac{1}{2}|R|, \quad m_{pos}^2 = \frac{1}{2}|R|. \quad (52)$$

In models with  $W_{cr} \neq 0$

$$m_{tach}^2 = -\frac{1}{2}|R|, \quad m_{pos}^2 = |R| \quad \text{or} \quad m_{pos}^2 = \frac{1}{3}|R|. \quad (53)$$

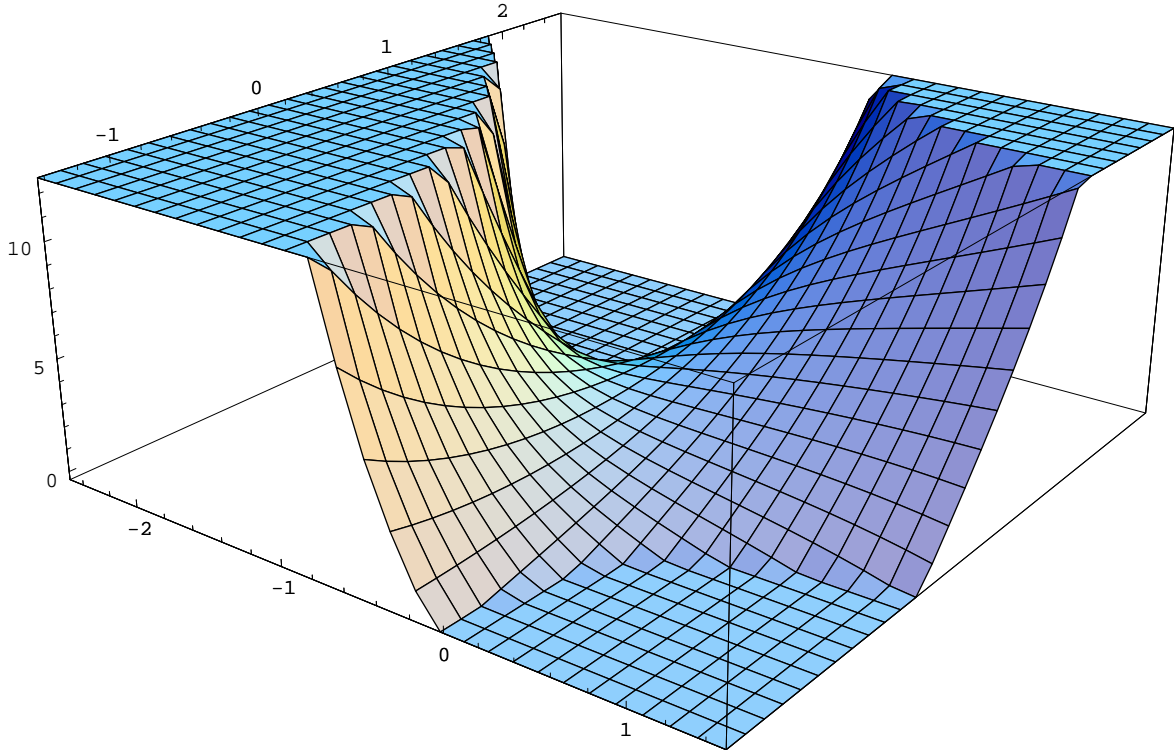


FIG. 1: An example of de Sitter saddle point in N=8 gauged supergravity.

Until now we described potentials with an extremum corresponding to de Sitter space. All of such extrema corresponded to saddle points of the type shown in Fig. 1. In all models of such type known so far, there is only one such saddle point, and the potential is unbounded from below, as we have checked. Whereas such potentials could play some role in the description of the present stage of exponential expansion of the universe, it is hard to see them playing any role in inflationary cosmology. The slow-roll condition is not satisfied, however, as we discussed in Sect. III, the fast-roll regime with  $|V''| = 2V$  may be acceptable in certain cases. The situation may change once one considers potentials involving many other scalar fields and/or quantum corrections.

### C. $SO(p) \times SO(q)$ sector

Now we will also study the nature of the critical points for potentials depending on one field in the models with  $SO(p, q)$  gauging with  $p + q = 8$  and  $r = 0$ ,  $\zeta = 0$  [16]. We will use the form of these models as given in [18], where the scalar field  $\phi$  has a canonical kinetic term. The relevant part of the action of N=8 gauged supergravity is:

$$S_{p,q,\xi} = \int d^4x \sqrt{-g} \left( \frac{1}{2} R - \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - V_{p,q,\xi}(\phi) \right). \quad (54)$$

The potential can be written with the help of a superpotential  $W_{p,q,\xi}$ :

$$W_{p,q,\xi} = \frac{1}{8} \left( p e^{\sqrt{\frac{q}{2p}} \phi} + q \xi e^{-\sqrt{\frac{p}{2q}} \phi} \right), \quad (55)$$

$$V_{p,q,\xi} = g^2 \left( 4(\partial_\phi W_{p,q,\xi}(\phi))^2 - 6W_{p,q,\xi}(\phi)^2 \right). \quad (56)$$

There are three cases of de Sitter vacua here.

- $p = q = 4$

$$V_{4,4,\xi} = -g^2 \left( e^{\sqrt{2}\phi} + 4\xi + \xi^2 e^{-\sqrt{2}\phi} \right). \quad (57)$$

De Sitter critical point is at negative  $\xi$  so that  $e^{-\sqrt{2}\phi_{cr}} = \frac{1}{|\xi|}$ . The value of the potential and of its second derivative at the critical points is:

$$V_{4,4,\xi}|_{cr} = 2g^2 |\xi|, \quad (V''_{4,4,\xi})_{cr} = -4g^2 |\xi|. \quad (58)$$

- $p = 5$  ,  $q = 3$

$$V_{5,3,\xi} = -\frac{3g^2}{8} \left( 5e^{\sqrt{\frac{6}{5}}\phi} + 10\xi e^{-\sqrt{\frac{2}{15}}\phi} + \xi^2 e^{-\sqrt{\frac{10}{3}}\phi} \right). \quad (59)$$

De Sitter critical point is at negative  $\xi$  and  $e^{-4\sqrt{\frac{2}{15}}\phi_{cr}} = \frac{3}{|\xi|}$ . The value of the potential and of its second derivative at the critical point is:

$$V_{5,3,\xi}|_{cr} = 2g^2 \cdot 3^{\frac{1}{4}} |\xi|^{\frac{3}{4}}, \quad (V''_{5,3,\xi})_{cr} = -4g^2 \cdot 3^{\frac{1}{4}} |\xi|^{\frac{3}{4}}. \quad (60)$$

- $p = 3$  ,  $q = 5$

$$V_{3,5,\xi} = -\frac{3g^2}{8} \left( e^{\sqrt{\frac{10}{3}}\phi} + 10\xi e^{\sqrt{\frac{2}{15}}\phi} + 5\xi^2 e^{-\sqrt{\frac{6}{5}}\phi} \right). \quad (61)$$

De Sitter critical point is at negative  $\xi$  and  $e^{-4\sqrt{\frac{2}{15}}\phi_{cr}} = \frac{3}{|\xi|}$ . The value of the potential and of its second derivative at the critical points is:

$$V_{3,5,\xi}|_{cr} = 2g^2 \cdot 3^{\frac{1}{4}} |\xi|^{\frac{5}{4}}, \quad (V''_{3,5,\xi})_{cr} = -4g^2 \cdot 3^{\frac{1}{4}} |\xi|^{\frac{5}{4}}. \quad (62)$$

Thus we find that in all 3 cases above the critical point is a maximum of the scalar potential and the tachyonic mass squared has twice the value of the potential:

$$-V'' = |m^2|_{tach} = 2V = \frac{1}{2}|R|. \quad (63)$$

## V. DE SITTER VACUA OF N=4 AND N=2 GAUGED SUPERGRAVITIES

The first de Sitter solutions of gauged supergravity have been discovered by Gates and Zwiebach [14], [15] in the framework of  $SU(2) \times SU(2)$  gauged version of the  $SO(4)$  N=4 theory. It seems that  $SO(4)$  gauged supergravities have to have two independent gauge couplings  $g_1$  and  $g_2$  corresponding to each  $SU(2)$ . However, it was found by Zwiebach [15] that it is not really the case due to the presence of scalar fields in front of the kinetic terms of the vector fields. These scalar fields acquire vacuum expectation values and this makes it necessary to rescale vector fields and gauged couplings so that the model has only one effective coupling constant. For the case corresponding to a positive cosmological constant  $g_1$  and  $g_2$  have to satisfy a relation  $g_{1eff} = -g_{2eff} = \sqrt{g_1 g_2}$ . It was shown that the value of cosmological constant depends only on this one effective coupling constant. More general N=4 supergravities were studied in superspace in [27].

Later de Roo and Wagemans [17] studied a more the general case of  $SU(2) \times SU(2)$  gauging with separate phases  $\alpha_{1,2}$  for each  $SU(2)$ . For  $\alpha = \alpha_1 - \alpha_2 = \frac{\pi}{2}$  the scalar potential proposed in [17] corresponds to the potentials from the papers of Gates and Zwiebach [14],[15] and Hull [16]. In all cases scalars parametrize the  $\frac{SU(1,1)}{U(1)}$  coset space.

We will also take into account that some supergravity actions have the Einstein term as  $\pm \frac{1}{2}R$ , which corresponds to  $\pm \frac{M_p^2}{2}R$  with  $M_p = 1$ , whereas some other actions have  $\pm \frac{1}{4}R$ , which corresponds to  $\pm \frac{M_p^2}{2}R$  with  $M_p = \frac{1}{\sqrt{2}}$ . In N=8 case in the previous action we had  $M_p = 1$  case, in [17] we have  $M_p = \frac{1}{\sqrt{2}}$  case. When we will compare the relation between tachyon mass and the potential, we will keep this in mind. We will perform the study of de Sitter solutions, starting with the potential from de Roo and Wagemans paper [17]:

$$V = -\frac{1}{2} (g_1^2 |\Phi_1|^2 + g_2^2 |\Phi_2|^2) - i g_1 g_2 (\Phi_1^* \Phi_2 - \Phi_2^* \Phi_1), \quad (64)$$

where scalar fields  $\Phi_1$  and  $\Phi_2$  are

$$\begin{aligned} \Phi_1 &= e^{i\alpha_1} \phi^1 + e^{-i\alpha_1} \phi^2, \\ \Phi_2 &= e^{i\alpha_2} \phi^1 + e^{-i\alpha_2} \phi^2, \end{aligned}$$

where  $\phi_1$  and  $\phi_2$  are  $SU(1,1)$  doublet of scalar fields from N=4 Weyl multiplet

$$\phi^1 = (\phi_1)^*, \quad \phi^2 = -(\phi_2)^*, \quad (65)$$

with the constraint:

$$\phi^a \phi_a = 1, \quad a = 1, 2. \quad (66)$$

The solution of this constraint gives:

$$\phi_1 = \frac{e^{i\beta}}{\sqrt{1 - |Z|^2}}, \quad \phi_2 = \frac{Z e^{i(\alpha_1 + \alpha_2)}}{\sqrt{1 - |Z|^2}}, \quad (67)$$

where

$$e^{i\beta} = \frac{e^{i\alpha} g_1^2 + e^{-i\alpha} g_2^2}{|e^{i\alpha} g_1^2 + e^{-i\alpha} g_2^2|}$$

and  $\alpha = \alpha_1 - \alpha_2$ . The only remaining independent scalar field is  $Z = X + iY$  and in terms of this field  $Z$  and parameter  $\alpha = \alpha_1 - \alpha_2$  the potential is:

$$V = -\frac{1}{2} \frac{1}{(1 - |Z|^2)} ((g_1^2 + g_2^2)(1 + |Z|^2) - 2|e^{i\alpha} g_1^2 + e^{-i\alpha} g_2^2|X) - 2g_1 g_2 \sin \alpha. \quad (68)$$

The critical point for this potential with the additional constraint  $|Z| < 1$ , required for the positivity of the kinetic terms for scalars, is:

$$\begin{aligned} X_{cr} &= Z_0 = \frac{g_1^2 + g_2^2 - 2|g_1 g_2 \sin(\alpha)|}{|e^{i\alpha} g_1^2 + e^{-i\alpha} g_2^2|} \\ &= \frac{g_1^2 + g_2^2 - 2|g_1 g_2 \sin \alpha|}{\sqrt{(g_1^2 + g_2^2 - 2g_1 g_2 \sin \alpha)(g_1^2 + g_2^2 + 2g_1 g_2 \sin \alpha)}} , \\ Y_{cr} &= 0. \end{aligned}$$

The potential at this point is  $V|_{cr} = -|g_1 g_2 \sin \alpha| - 2g_1 g_2 \sin \alpha$ . For  $2g_1 g_2 \sin \alpha < 0$  the potential is positive

$$V_{cr} = |g_1 g_2 \sin \alpha|. \quad (69)$$

To find a second derivative of the potential we have to find the scalars which have canonical kinetic terms at the critical point. Using (65) and (66) we will get kinetic terms at the critical point:

$$\frac{1}{2} \partial \phi^a \partial \phi_a = -\frac{1}{2} \frac{1}{(1 - Z_0^2)^2} \partial X \partial X - \frac{1}{2} \frac{1}{(1 - Z_0^2)} \partial Y \partial Y. \quad (70)$$

Thus  $x = \frac{1}{(1 - Z_0^2)} X$  and  $y = \frac{1}{(1 - Z_0^2)^{1/2}} Y$  are the fields over which we have to differentiate the potential. We find:

$$m_{xx}^2 = -4|g_1 g_2 \sin \alpha|, \quad (71)$$

$$m_{yy}^2 = -(g_1^2 + g_2^2 + 2|g_1 g_2 \sin \alpha|). \quad (72)$$

Thus we see (with account of normalization  $M_p^2 = \frac{1}{2}$  in [17]) that the tachyonic mass in the  $x$  direction compares with the potential as in all previous cases:

$$-M_p^2 V_{cr}'' = 2V_{cr}, \quad m_{tach}^2 = -\frac{1}{2}|R|. \quad (73)$$

However, the potential in  $y$  direction has a tachyonic mass which looks quite independent of the value of the potential at the critical point. This looks puzzling in view of the large amount of examples considered before. However, in a particular case of  $\alpha = \frac{\pi}{2}$  this puzzle is resolved as follows:

$$V_{cr} = |g_1 g_2|, \quad m_{xx}^2 = -4|g_1 g_2|, \quad m_{yy}^2 = -(g_1 + g_2)^2. \quad (74)$$

This is still puzzling, however here we have to remember that without looking at kinetic terms for vector fields, one can not make a definite judgement about the relation between  $g_1$  and  $g_2$ . But this analysis was performed by Zwiebach in [15] and he concluded that effectively one has to consider only the case of  $g_1 = -g_2$  for the de Sitter solution. This case gives us  $m_{yy}^2 = 0$ . In fact the potential depends only on the combination  $Z^2 = X^2 + Y^2$  and there is only one tachyon excitation and a flat direction. Thus suggests that if we would perform the analysis of the kinetic terms for vector fields for the theory with  $\alpha \neq \frac{\pi}{2}$  we would find again that for canonical kinetic terms we do not have an extra tachyon field.

Another form of potential of N=4 theory was given in [16] and has been recently discussed in [12]:

$$V = - \left( 4g_1g_2 + (g_1^2 + g_2^2) \cosh(a|\varphi|) + (g_1^2 - g_2^2) \frac{Re\varphi}{|\varphi|} \sinh(a|\varphi|) \right) \quad (75)$$

with  $W = \frac{\varphi}{|\varphi|} \tanh(\frac{a|\varphi|}{2})$ . For  $g_1 = -g_2$  we get the potential discussed in [12]

$$V = -\frac{1}{2}g^2(\cosh(a|\varphi|) - 2)$$

with  $g^2 = -4g_1g_2$ . The critical point is  $\varphi = 0$  and  $V|_{cr} = \frac{1}{2}g^2$ . The presence of the parameter  $a$  will not affect the properties of the system even though it seems that in this case  $V''|_{cr} = -\frac{a^2}{2}g^2$  and for very small  $a$  it is possible to get  $V|_{cr} \gg |V''|_{cr}$ . The properties of the potential are related to canonical kinetic terms and the rescaling of the scalar field  $\varphi$  in the potential will also lead to the rescaling of the kinetic terms therefore we conclude that there is no free adjustable parameter which can be used for a slow-roll condition.

The  $SU(2) \times SU(2)$  gauging of N=4 supergravity can be easily reduced to N=2 gauged supergravity with one vector multiplet gauging [20]. In this case  $g_1 = -g_2$  and the potential has a form:

$$V = 2g_1^2 - 4g_1^2 \frac{|Z|^2}{(1 - |Z|^2)} . \quad (76)$$

The critical point corresponds to  $Z = 0$  and  $V|_{cr} = 2g_1^2$ . Again we find that the tachyon mass is related to the potential as  $-M_p^2 V''_{cr} = 2V_{cr}$ .

To summarize, in all cases of N=4, 2 gauged supergravities we find tachyons with relation to the curvature of the de Sitter space of the form  $m_{tach}^2 = -\frac{1}{2}|R|$ .

## VI. MINKOWSKI VACUA AND A POSSIBILITY OF INFLATION ALONG FLAT DIRECTIONS

There is another class of potentials which should not be overlooked in our search for de Sitter solutions. For certain values of parameters the potentials have flat directions corresponding to Minkowski space with  $V(\phi) = 0$ . Existence of Minkowski or near-Minkowski ground state is a pre-requisite of a successful inflationary cosmology, so even though at the classical level such potentials do not have any de Sitter solutions, their existence is rather intriguing.

Several examples of such models have been presented in [18]. The corresponding results can be summarized by the following table.

$p$	$q$	$r$	$\xi$	$\zeta$	$s_{cr}$	$t_{cr}$	$W(s_{cr}, t_{cr})$	$V(s_{cr}, t_{cr})$	$m_1^2$	$m_2^2$
1	1	6	1	0	0	any	$\frac{1}{4} \exp \left[ -\sqrt{\frac{3}{2}} t_{cr} \right]$	0	$2 e^{-\sqrt{6} t_{cr}}$	0
2	1	5							0	0
2	2	4							0	0
2	3	3	0	1, 0, -1	any	any	$\frac{1}{4} \exp \left[ \frac{2s_{cr} - \sqrt{5} t_{cr}}{\sqrt{6}} \right]$	0	0	0
2	4	2							0	0
2	5	1							0	0

The class of theories in this table has a possibility to be related to M-theory avoiding the non-compactness problem. In all cases here  $\xi \geq 0$ , thus there is no negative components of the metric on the hypersurface (50). Instead, there are some non-compact  $U(1)$  directions, for example, in  $p = 1, q = 1, r = 1, \xi = 1, \zeta = 0$  case the internal space is  $\mathbf{S}^2 \times \mathbf{R}^6$  and the symmetry is  $SO(2) \times U(1)^{15}$ . However the flat directions  $\mathbf{R}^6$  could be compactified to  $\mathbf{T}^6$  as explained in [12].

In the case  $p = 1, q = 1, r = 1, \xi = 1, \zeta = 0$  the potential of the scalar fields  $s$  and  $t$  looks as follows:

$$V(s, t) = \frac{1}{8} e^{-\sqrt{2}(2s+\sqrt{3}t)} \left( 1 - e^{2\sqrt{2}s} \right)^2 = \frac{1}{2} e^{-\sqrt{6}t} \sinh^2(\sqrt{2}s). \quad (77)$$

This potential blows up at large negative  $t$  and at large  $|s|$ , it is even with respect to  $s$ , and it vanishes for all  $t$  along the valley  $s = 0$ , see Fig. 2. Its curvature is positive in all directions, no tachyons are present.

This potential is not of inflationary type, at least at the classical level. Indeed, there is no slow-roll regime of inflationary type in the theories with exponential potentials of type  $e^{c\phi}$  with  $c \geq 1$ ; such potentials are too steep. Meanwhile, the potential along the valley vanishes, so inflation does not appear for the fields moving along this valley as well. One could hope that the regime of acceleration may emerge when the fields oscillate in the valley and slowly drift towards positive  $t$ . However, the energy density of the oscillating field rapidly drops down. We have checked numerically, for various initial conditions for  $s$  and  $t$ , that the cosmological evolution in the theory with this potential is not inflationary.

A similar situation appears in the model with  $p = 2$ , described in the lower part of the table. In this model the potential vanishes (i.e. the cosmological constant is equal to zero) for all  $s$  and  $t$ . Once again, in this case one does not have an inflationary regime.

However, it might happen that the flat directions can be lifted due to quantum effects. Note that in these models supersymmetry is broken along the flat directions, so it does not protect the effective potential against radiative corrections.

In this respect it is instructive to remember the recent example related to P-term inflation in  $N=2$  [11], as well as very similar examples of D-term inflation [28] and F-term inflation in  $N=1$

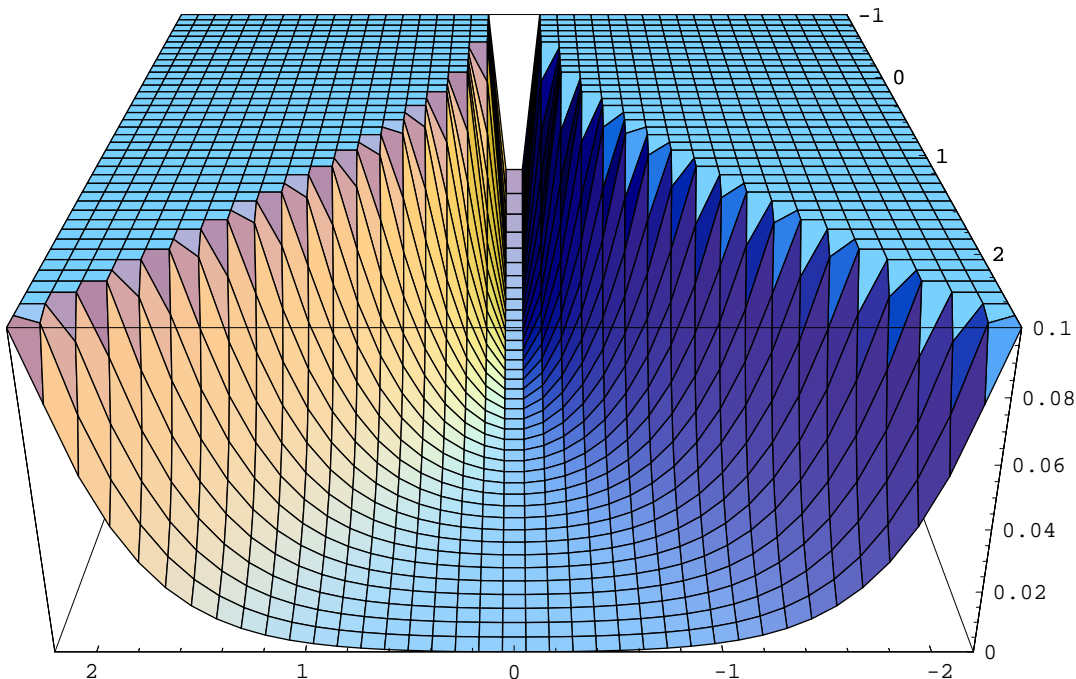


FIG. 2: A potential with a valley with  $V = 0$  corresponding to Minkowski space.



[29]. The effective potential in P-term inflation at the classical level has several Minkowski flat directions with unbroken supersymmetry. Perturbative effects cannot lift up these flat directions and give rise to inflation. However, once one takes into account possible FI terms, these flat directions are lifted up to a state with  $V > 0$ . They still remain flat, but they correspond to de Sitter vacuum with broken supersymmetry. Then the radiative corrections, which appear because of the supersymmetry breaking, make the effective potential  $V$  slightly tilted. This leads to a realization of the hybrid inflation scenario in  $N=2$ .

We do not know whether anything like that will happen in  $N=8$ , but this is a very interesting possibility that deserves separate investigation.

One should note also that in this paper we concentrated on the investigation of potentials with extrema at finite values of the scalar fields. However, one may also look for possible classical potentials that may have flat directions approaching de Sitter state with  $V > 0$  or Minkowski state with  $V = 0$  *only asymptotically*. From the point of view of cosmology such potentials are very interesting: They can describe inflation at large  $V(\phi)$  and quintessence at small  $V(\phi)$ .

## VII. CONCLUSIONS

In this paper we investigated the possibility to have de Sitter-like solutions describing inflation/accelerated expansion of the universe in  $N=8,4,2$  gauged supergravities. In each model that we have studied we have found that de Sitter state corresponds to a single unstable extremum of the scalar potential  $V$ . The (negative) curvature of the potential in the direction of the fastest descent in all of these models obeys the simple rule:  $|V''| = 2V$  in units  $M_p = 1$ . (Note that throughout the paper we are using the following definition of the Planck mass:  $M_p^2 = 8\pi/G$ , where  $G$  is the gravitational coupling constant.) This relation can be represented in the following way:  $|m^2| = |R|/2$ , where  $m^2$  is the tachyonic mass corresponding to the excitation in the direction of the fastest descent, and  $R$  is the curvature scalar,  $|R| = 12H^2$ .

It would be very interesting to find a simple geometric explanation of this result. In this paper we concentrated on the derivation of this result for a large class of models, and on investigation of its consequences.

One of the consequences is that the slow-roll inflationary regime is impossible near the extrema of the scalar potential in such models. Note, however, that this conclusion may change

when one takes into account all other scalar fields and other gaugings. Indeed, initially there are 35 scalars and 35 pseudoscalars in  $N=8$  supergravity, and many different ways to gauge supergravity are possible. We have studied only those models for which an explicit expression for the potential was known. In these models the potential depends only one or two scalar fields. Thus it may happen that some of our conclusions are not generic. It would be interesting, in particular, to look for potentials that have flat directions with  $|V''| \ll V$  asymptotically approaching Minkowski vacuum with  $V = 0$  or de Sitter vacuum with  $V \lesssim 10^{-120}$ , which would correspond to the present state of the universe.

If such a regime is possible, one may start looking for it already at the classical level. One may also try to find out whether the flat directions with  $V = 0$ , which are present in some versions of  $N = 8$  gauged supergravity, can be lifted up by quantum effects, and can be used for implementation of inflation. An example of how this possibility could be realized in  $N=2$  have been recently given in [11]; see also examples in  $N=1$  given in [28, 29].

On the other hand, if one is only interested in describing the present stage of quasi-exponential expansion of the universe in a state with  $V \sim 10^{-120}$ , the danger of the vacuum instability can be removed to a very distant future, hundreds of billions of years from the present epoch: The universe with  $|V''| = 2V \sim 10^{-120}$  may experience more than 100 e-folds of fast-roll inflation, which is more than sufficient to explain the present stage of accelerated expansion. Of course, such models require rather extreme values of parameters, but it is still interesting that such a regime is possible despite the expected strong instability of de Sitter space with  $|V''| = 2V$ .

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## Note Added

While we were preparing this paper for the submission we became aware of a related investigation by Paul Townsend, hep-th/0110072. He has found a model based on N=8 gauged supergravity with a sufficiently flat potential, which may lead to a marginally inflationary regime  $a(t) \sim t^3$ . Such potentials can be useful for the description of the present acceleration of the universe, as proposed in hep-th/0110072. It would be interesting to use such potentials for a description of inflation in the early universe. However, density perturbations produced in the universe with  $a(t) \sim t^3$  have a substantially non-flat spectrum,  $n$  is well below 1, whereas observations suggest  $|n - 1| \lesssim 0.1$ . One should note also that the model proposed in hep-th/0110072 is based on finding de Sitter solution in d=5 and making a subsequent reduction to d=4. In order to check whether this model is realistic it would be important to find out whether the corresponding 5d de Sitter solution may suffer from the same problem of instability as all 4d de Sitter solutions analysed in our paper.

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