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Two-Photon Processes at Intermediate Energies*

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Abstract

Exclusive hadron production processes in photon-photon collisions provide important tests of QCD at the amplitude level, particularly as measures of hadron distribution amplitudes and skewed parton distributions. The determination of the shape and normalization of the distribution amplitudes has become particularly important in view of their importance in the analysis of exclusive semi-leptonic and two-body hadronic B-decays. Interesting two-photon physics, including doubly-tagged $\gamma^*\gamma^*$ reactions, will be accessible at low energy, high luminosity e^+e^- colliders, including measurements of channels important in the light-by-light contribution to the muon g–2 and the study of the transition between threshold production controlled by low-energy effective chiral theories and the domain where leading-twist perturbative QCD becomes applicable. The threshold regime of hadron production in photon–photon and e^+e^- annihilation, where hadrons are formed at small relative velocity, is particularly interesting as a test of low energy theorems, soliton models, and new types of resonance production. Such studies will be particularly valuable in double-tagged reactions where polarization correlations, as well as the photon virtuality dependence, can be studied.

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1 Introduction

Two-photon annihilation $\gamma^*(q_1)\gamma^*(q_2) \to$ hadrons for real and virtual photons provide some of the most detailed and incisive tests of QCD. Among the processes of special interest are:

1. the total two-photon annihilation hadronic cross section $\sigma(s, q_1^2, q_2^2)$, which is related to the light-by-light hadronic contribution to the muon anomalous moment;

2. the formation of $C = +$ hadronic resonances, which can reveal exotic states such as $q\bar{q}g$ hybrids and discriminate gluonium formation [1, 2];

3. single-hadron processes such as $\gamma^*\gamma^* \to \pi^0$, which test the transition from the anomaly-dominated pion decay constant to the short-distance structure of currents dictated by the operator-product expansion and perturbative QCD factorization theorems;

4. hadron pair production processes such as $\gamma^*\gamma \to \pi^+\pi^-, K^+K^-, p\bar{p}$, which at fixed invariant pair mass measures the $s \to t$ crossing of the virtual Compton amplitude [3, 4]. When one photon is highly virtual, these exclusive hadron production channels are dual to the photon structure function $F_2^\gamma(x, Q^2)$ in the endpoint $x \to 1$ region at fixed invariant pair mass. The leading twist-amplitude for $\gamma^*\gamma \to \pi^+\pi^-$ is sensitive to the $1/x - 1/(1-x)$ moment of the $q\bar{q}$ distribution amplitude $\Phi_{\pi^+\pi^-}(x, Q^2)$ of the two-pion system [5, 6], the timelike extension of skewed parton distributions. In addition one can measure the pion charge asymmetry in $e^+e^- \to \pi^+\pi^-e^+e^-$ arising from the interference of the $\gamma\gamma \to \pi^+\pi^-$ Compton amplitude with the timelike pion form factor [7]. At the unphysical point $s = q_1^2 = q_2^2 = 0$, the amplitude is fixed by the low energy theorem to the hadron charge squared. As reviewed by Karliner in these proceedings [8], the ratio of the measured $\gamma\gamma \to \Lambda\bar{\Lambda}$ and $\gamma\gamma \to p\bar{p}$ cross sections is anomalous at threshold, a fact which may be associated with the soliton structure of baryons in QCD [9];

5. At large momentum transfer, the angular distribution of hadron pairs produced by photon-photon annihilation are among the best determinants of the shape of the meson and baryon distribution amplitudes $\phi_M(x, Q)$, and $\phi_B(x_i, Q)$ which control almost all exclusive processes involving a hard scale Q. The determination of the shape and normalization of the distribution amplitudes, which

are gauge-invariant and process-independent measures of the valence wavefunctions of the hadrons, has become particularly important in view of their importance in the analysis of exclusive semi-leptonic and two-body hadronic B-decays [10, 11, 12, 13, 14, 15]. There has also been considerable progress both in calculating hadron wavefunctions from first principles in QCD and in measuring them using diffractive di-jet dissociation.

Much of this important two-photon physics is accessible at low energy, high luminosity e^+e^- colliders such as the proposed PEP-N project, particularly for measurements of channels important in the light-by-light contribution to the muon g–2 and the exploration of the transition between threshold amplitudes which are controlled by low-energy effective theories such as the chiral Hamiltonian through the transition to the domain where leading-twist perturbative QCD becomes applicable. There have been almost no measurements of double-tagged events needed to unravel the separate q_1^2 and q_2^2 dependence of photon-photon annihilation. Hadron pair production from two-photon annihilation plays a crucial role in unraveling the perturbative and non-perturbative structure of QCD, first by testing the validity and empirical applicability of leading-twist factorization theorems, second by verifying the structure of the underlying perturbative QCD subprocesses, and third, through measurements of angular distributions and ratios which are sensitive to the shape of the distribution amplitudes. In effect, photon-photon collisions provide a microscope for testing fundamental scaling laws of PQCD and for measuring distribution amplitudes. It would also be interesting to measure the novel relativistic atomic coalescence processes, single and double muonium formation: $e^+e^- \to [\mu^+e^-]\mu^+e^-$ and $e^+e^- \to [\mu^+e^-][\mu^+e^-]$.

2 The Photon-to-Pion Transition Form Factor and the Pion Distribution Amplitude

The simplest and perhaps most elegant illustration of an exclusive reaction in QCD is the evaluation of the photon-to-pion transition form factor $F_{\gamma\to\pi}(Q^2)$ which is measurable in single-tagged two-photon $ee \to ee\pi^0$ reactions. The form factor is defined via the invariant amplitude $\Gamma^\mu = -ie^2 F_{\pi\gamma}(Q^2)\varepsilon^{\mu\nu\rho\sigma}p^\pi_\nu\varepsilon_\rho q_\sigma$. As in inclusive reactions, one must specify a factorization scheme which divides the integration regions of the loop integrals into hard and soft momenta, compared to the resolution scale $\widetilde{Q}$. At leading twist, the transition form factor then factorizes as a convolution of the $\gamma^*\gamma \to q\bar{q}$ amplitude (where the quarks are collinear with the final state pion) with the valence

light-cone wavefunction of the pion [4]:

$$F_{\gamma M}(Q^2) = \frac{4}{\sqrt{3}} \int_0^1 dx \phi_M(x, \widetilde{Q}) T^H_{\gamma \to M}(x, Q^2). \tag{1}$$

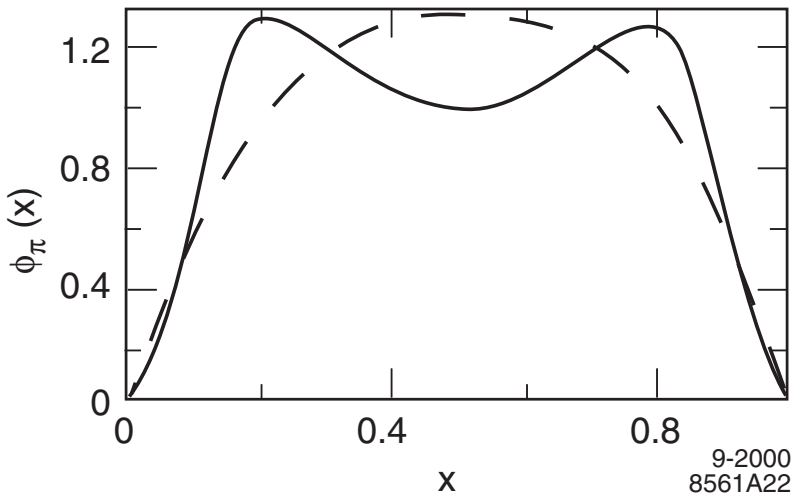
The hard scattering amplitude for $\gamma\gamma^* \to q\bar{q}$ is $T^H_{\gamma M}(x, Q^2) = [(1-x)Q^2]^{-1}\,(1 + \mathcal{O}(\alpha_s))$. For the asymptotic distribution amplitude $\phi_\pi^{\rm asympt}(x) = \sqrt{3} f_\pi x(1-x)$ one predicts [16]

$$Q^2 F_{\gamma\pi}(Q^2) = 2 f_\pi \left(1 - \frac{5}{3}\frac{\alpha_V(Q^*)}{\pi}\right)$$

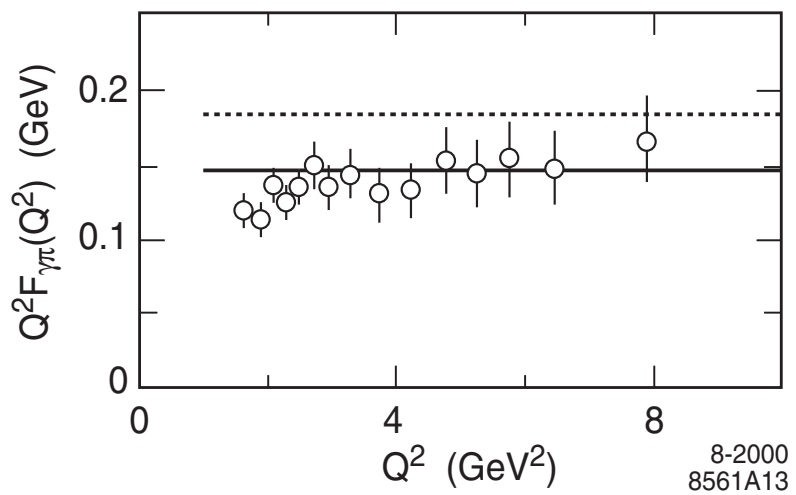
where $Q^* = e^{-3/2}Q$ is the estimated BLM scale for the pion form factor in the V scheme.

The PQCD predictions have been tested in measurements of $e\gamma \to e\pi^0$ by the CLEO collaboration [17] (see Figure 1 (b)). The flat scaling of the $Q^2 F_{\gamma\pi}(Q^2)$ data from $Q^2 = 2$ to $Q^2 = 8$ GeV^2 provides an important confirmation of the applicability of leading twist QCD to this process. The magnitude of $Q^2 F_{\gamma\pi}(Q^2)$ is remarkably consistent with the predicted form, assuming the asymptotic distribution amplitude and including the LO QCD radiative correction with $\alpha_V(e^{-3/2}Q)/\pi \simeq 0.12$. One could allow for some broadening of the distribution amplitude with a corresponding increase in the value of α_V at small scales. Radyushkin [18], Ong [19] and Kroll [20] have also noted that the scaling and normalization of the photon to pion transition form factor tends to favor the asymptotic form for the pion distribution amplitude and rules out broader distributions such as the two-humped form suggested by QCD sum rules [21]. More comprehensive analyses, which include consideration of next-to-leading order corrections and some higher-twist contributions dictated by vector meson spectra and QCD sum rules have been given by A. Khodjamirian [22], A. Schmedding and O. Yakovlev [23], and by A. P. Bakulev *et al.* [24].

When both photons are virtual, the denominator of T_H for the $\gamma\gamma^* \to \pi^0$ reaction becomes $(1-x)Q_1^2 + xQ_2^2$ [4, 19], and the amplitude becomes nearly insensitive to the shape of the distribution amplitude once it is normalized to the pion decay constant. Thus the ratio of singly virtual to doubly virtual pion production is particularly sensitive to the shape of $\phi_\pi(x, Q^2)$ since higher order corrections and normalization errors tend to cancel in the ratio.



(a)



(b)

Figure 1: (a) Transverse lattice results for the pion distribution amplitude at $Q^2 \sim 10\text{GeV}^2$. The solid curve is the theoretical prediction from the combined DLCQ/transverse lattice method [25]; the chain line is the experimental result obtained from dijet diffractive dissociation [26, 27]. Both are normalized to the same area for comparison. (b) Scaling of the transition photon to pion transition form factor $Q^2F_{\gamma\pi^0}(Q^2)$. The dotted and solid theoretical curves are the perturbative QCD prediction at leading and next-to-leading order, respectively, assuming the asymptotic pion distribution The data are from the CLEO collaboration [17].

3 Non-Perturbative Calculations of the Pion Distribution Amplitude

The distribution amplitude $\phi(x, \widetilde{Q})$ can be computed from the integral over transverse momenta of the renormalized hadron valence wavefunction in the light-cone gauge at fixed light-cone time [4]:

$$\phi(x, \widetilde{Q}) = \int d^2 \vec{k_\perp}\, \theta \left(\widetilde{Q}^2 - \frac{\vec{k_\perp}^{\,2}}{x(1-x)} \right) \psi^{(\widetilde{Q})}(x, \vec{k_\perp}), \tag{2}$$

where a global cutoff in invariant mass is identified with the resolution $\widetilde{Q}$. The distribution amplitude $\phi(x, \widetilde{Q})$ is boost and gauge invariant and evolves in $\ln \widetilde{Q}$ through an evolution equation [4]. Since it is formed from the same product of operators as the non-singlet structure function, the anomalous dimensions controlling $\phi(x, Q)$ dependence in the ultraviolet $\log Q$ scale are the same as those which appear in the DGLAP evolution of structure functions [28]. The decay $\pi \to \mu\nu$ normalizes the wave function at the origin: $\int_0^1 dx \phi(x, Q) = f_\pi/(2\sqrt{3})$. One can also compute the distribution amplitude from the gauge invariant Bethe-Salpeter wavefunction at equal light-cone time. This also allows contact with both QCD sum rules [29] and lattice gauge theory; for example, moments of the pion distribution amplitudes have been computed in lattice gauge theory [30, 31, 32]. Conformal symmetry can be used as a template to organize the renormalization scales and evolution of QCD predictions [28, 33]. For example, Braun and collaborators have shown how one can use conformal symmetry to classify the eigensolutions of the baryon distribution amplitude [34].

Dalley [25] and Burkardt and Seal [35] have calculated the pion distribution amplitude from QCD using a combination of the discretized light-cone quantization [36] method for the x^- and x^+ light-cone coordinates with the transverse lattice method [37, 38] in the transverse directions, A finite lattice spacing a can be used by choosing the parameters of the effective theory in a region of renormalization group stability to respect the required gauge, Poincaré, chiral, and continuum symmetries. The overall normalization gives $f_\pi = 101$ MeV compared with the experimental value of 93 MeV. Figure 1 (a) compares the resulting DLCQ/transverse lattice pion wavefunction with the best fit to the diffractive di-jet data (see the next section) after corrections for hadronization and experimental acceptance [26]. The theoretical curve is somewhat broader than the experimental result. However, there are experimental uncertainties from hadronization and theoretical errors introduced from finite DLCQ resolution, using a nearly massless pion, ambiguities in setting the factorization scale Q^2, as well as errors in the evolution of the distribution amplitude from 1 to 10 GeV^2. Instan-

ton models also predict a pion distribution amplitude close to the asymptotic form [39]. In contrast, recent lattice results from Del Debbio *et al.* [32] predict a much narrower shape for the pion distribution amplitude than the distribution predicted by the transverse lattice. A new result for the proton distribution amplitude treating nucleons as chiral solitons has recently been derived by Diakonov and Petrov [40]. Dyson-Schwinger models [41] of hadronic Bethe-Salpeter wavefunctions can also be used to predict light-cone wavefunctions and hadron distribution amplitudes by integrating over the relative k^- momentum. There is also the possibility of deriving Bethe-Salpeter wavefunctions within light-cone gauge quantized QCD [42] in order to properly match to the light-cone gauge Fock state decomposition.

4 Measurements of the Pion Distribution Amplitude by Di-jet Diffractive Dissociation

The shape of hadron distribution amplitudes can be measured in the diffractive dissociation of high energy hadrons into jets on a nucleus. For example, consider the reaction [43, 44, 45] $\pi A \to \mathrm{Jet}_1 + \mathrm{Jet}_2 + A'$ at high energy where the nucleus A' is left intact in its ground state. The transverse momenta of the jets balance so that $\vec{k}_{\perp i} + \vec{k}_{\perp 2} = \vec{q}_\perp < R^{-1}{}_A$. The light-cone longitudinal momentum fractions also need to add to $x_1 + x_2 \sim 1$ so that $\Delta p_L < R_A^{-1}$. The process can then occur coherently in the nucleus. Because of color transparency and the long coherence length, a valence $q\bar{q}$ fluctuation of the pion with small impact separation will penetrate the nucleus with minimal interactions, diffracting into jet pairs [43]. The $x_1 = x$, $x_2 = 1 - x$ dependence of the di-jet distributions will thus reflect the shape of the pion valence light-cone wavefunction in x; similarly, the $\vec{k}_{\perp 1} - \vec{k}_{\perp 2}$ relative transverse momenta of the jets gives key information on the second derivative of the underlying shape of the valence pion wavefunction [44, 45, 46]. The diffractive nuclear amplitude extrapolated to $t = 0$ should be linear in nuclear number A if color transparency is correct. The integrated diffractive rate should then scale as $A^2/R_A^2 \sim A^{4/3}$.

The E791 collaboration at Fermilab has recently measured the diffractive di-jet dissociation of 500 GeV incident pions on nuclear targets [26]. The results are consistent with color transparency, and the momentum partition of the jets conforms closely with the shape of the asymptotic distribution amplitude, $\phi_\pi^{\mathrm{asympt}}(x) = \sqrt{3} f_\pi x(1-x)$, corresponding to the leading anomalous dimension solution [4] to the perturbative QCD evolution equation.

The interpretation of the diffractive dijet processes as measures of the hadron distribution amplitudes has recently been questioned by Braun *et al.* [47] and by

Chernyak [48] who have calculated the hard scattering amplitude for such processes at next-to-leading order. However, these analyses neglect the integration over the transverse momentum of the valence quarks and thus miss the logarithmic ordering which is required for factorization of the distribution amplitude and color filtering in nuclear targets.

5 Exclusive Two-Photon Annihilation into Hadron Pairs

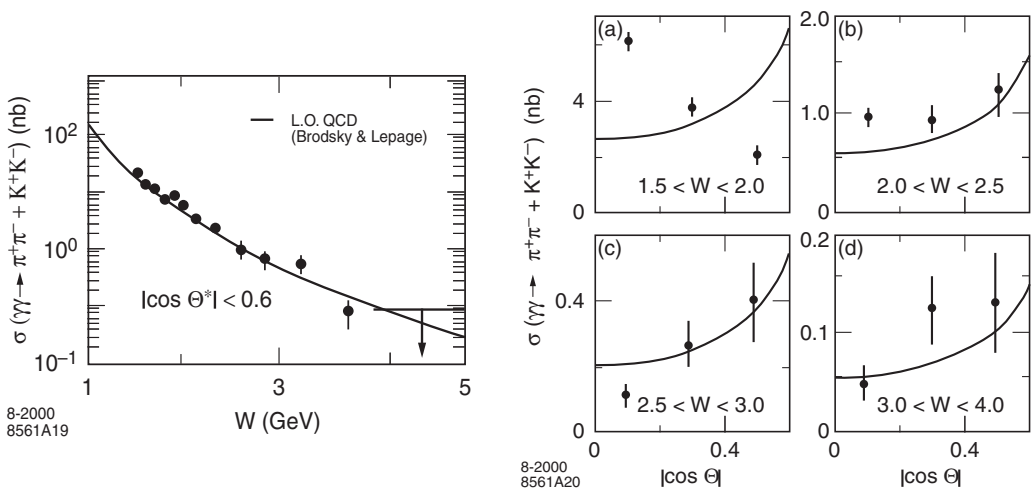


Figure 2: Comparison of the sum of $\gamma\gamma \to \pi^+\pi^-$ and $\gamma\gamma \to K^+K^-$ meson pair production cross sections with the scaling and angular distribution of the perturbative QCD prediction [3]. The data are from the CLEO collaboration [50].

Two-photon reactions, $\gamma\gamma \to H\bar{H}$ at large $s = (k_1 + k_2)^2$ and fixed $\theta_{\rm cm}$, provide a particularly important laboratory for testing QCD since these cross-channel "Compton" processes are the simplest calculable large-angle exclusive hadronic scattering reactions. The helicity structure, and often even the absolute normalization can be rigorously computed for each two-photon channel [3]. In the case of meson pairs, dimensional counting predicts that for large s, $s^4 d\sigma/dt(\gamma\gamma \to M\bar{M}$ scales at fixed t/s or $\theta_{\rm cm}$ up to factors of $\ln s/\Lambda^2$. The angular dependence of the $\gamma\gamma \to H\bar{H}$ amplitudes can be used to determine the shape of the process-independent distribution amplitudes, $\phi_H(x, Q)$. An important feature of the $\gamma\gamma \to M\bar{M}$ amplitude for meson pairs is that the contributions of Landshoff pitch singularities are power-law

suppressed at the Born level—even before taking into account Sudakov form factor suppression. There are also no anomalous contributions from the $x \to 1$ endpoint integration region. Thus, as in the calculation of the meson form factors, each fixed-angle helicity amplitude can be written to leading order in $1/Q$ in the factorized form $[Q^2 = p_T^2 = tu/s;\ \widetilde{Q}_x = \min(xQ, (l-x)Q)]$:

$$\begin{aligned} \mathcal{M}_{\gamma\gamma\to M\bar{M}} &= \int_0^1 dx \int_0^1 dy \\ &\phi_{\bar{M}}(y, \widetilde{Q}_y) T_H(x, y, s, \theta_{\rm cm} \phi_M(x, \widetilde{Q}_x), \end{aligned} \tag{3}$$

where T_H is the hard-scattering amplitude $\gamma\gamma \to (q\bar{q})(q\bar{q})$ for the production of the valence quarks collinear with each meson, and $\phi_M(x, \widetilde{Q})$ is the amplitude for finding the valence q and $\bar{q}$ with light-cone fractions of the meson's momentum, integrated over transverse momenta $k_\perp < \widetilde{Q}$. The contribution of non-valence Fock states are power-law suppressed. Furthermore, the helicity-selection rules [49] of perturbative QCD predict that vector mesons are produced with opposite helicities to leading order in $1/Q$ and all orders in α_s. The dependence in x and y of several terms in $T_{\lambda,\lambda'}$ is quite similar to that appearing in the meson's electromagnetic form factor. Thus much of the dependence on $\phi_M(x, Q)$ can be eliminated by expressing it in terms of the meson form factor. In fact, the ratio of the $\gamma\gamma \to \pi^+\pi^-$ and $e^+e^- \to \mu^+\mu^-$ amplitudes at large s and fixed θ_{CM} is nearly insensitive to the running coupling and the shape of the pion distribution amplitude:

$$\frac{\frac{d\sigma}{dt}(\gamma\gamma \to \pi^+\pi^-)}{\frac{d\sigma}{dt}(\gamma\gamma \to \mu^+\mu^-)} \sim \frac{4|F_\pi(s)|^2}{1 - \cos^2\theta_{\rm cm}}. \tag{4}$$

The comparison of the PQCD prediction for the sum of $\pi^+\pi^-$ plus K^+K^- channels with recent CLEO data [50] is shown in Figure 2. The CLEO data for charged pion and kaon pairs show a clear transition to the scaling and angular distribution predicted by PQCD [3] for $W = \sqrt{(s_{\gamma\gamma}} > 2$ GeV. It is clearly important to measure the magnitude and angular dependence of the two-photon production of neutral pions and $\rho^+\rho^-$ in view of the strong sensitivity of these channels to the shape of meson distribution amplitudes (see Figures 3 and 4). QCD also predicts that the production cross section for charged ρ-pairs (with any helicity) is much larger than for that of neutral ρ pairs, particularly at large $\theta_{\rm cm}$ angles. Similar predictions are possible for other helicity-zero mesons.

The analysis of exclusive B decays has much in common with the analysis of exclusive two-photon reactions [51]. For example, consider the three representative contributions to the decay of a B meson to meson pairs illustrated in Figure 5.

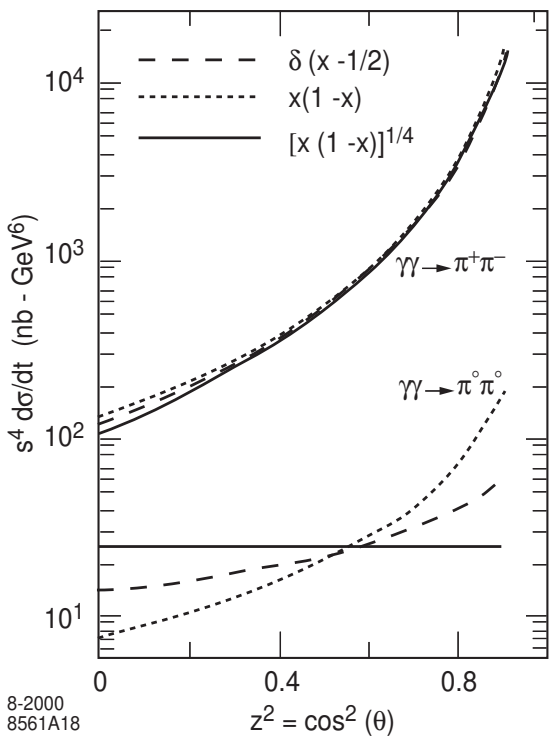


Figure 3: Predictions for the angular distribution of the $\gamma\gamma \to \pi^+\pi^-$ and $\gamma\gamma \to \pi^0\pi^0$ pair production cross sections for three different pion distribution amplitudes [3].

In Figure 5(a) the weak interaction effective operator $\mathcal{O}$ produces a $q\bar{q}$ in a color octet state. A gluon with virtuality $Q^2 = \mathcal{O}(M_B^2)$ is needed to equilibrate the large momentum fraction carried by the b quark in the $\bar{B}$ wavefunction. The amplitude then factors into a hard QCD/electroweak subprocess amplitude for quarks which are collinear with their respective hadrons: $T_H([b(x)\bar{u}(1-x)] \to [q(y)\bar{u}(1-y)]_1[q(z)\bar{q}(1-z)]_2)$ convoluted with the distribution amplitudes $\phi(x,Q)$ [4] of the incident and final hadrons:

$$\mathcal{M}_{\rm octet}(B \to M_1 M_2) = \int_0^1 dz \int_0^1 dy \int_0^1 dx \; \phi_B(x,Q) T_H(x,y,z) \phi_{M_1}(y,Q) \phi_{M_2}(z,Q).$$

Here $x = k^+/p_H^+ = (k^0+k^z)/(p_H^0+p_H^z)$ are the light-cone momentum fractions carried by the valence quarks.

There are a several features of QCD which are required to ensure the consistency of the PQCD approach: (a) the effective QCD coupling $\alpha_s(Q^2)$ needs to be under control at the relevant scales of B decay; (b) the distribution amplitudes of the hadrons need

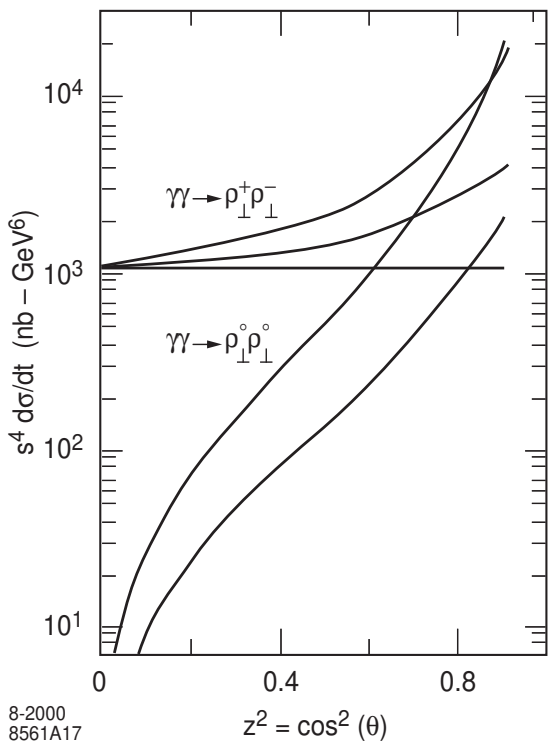


Figure 4: Predictions for the angular distribution of the $\gamma\gamma \to \rho^+\rho^-$ and $\gamma\gamma \to \rho^0\rho^0$ pair production cross sections for three different ρ distribution amplitudes as in Figure 3 [3].

to satisfy convergence properties at the endpoints; and (c) one requires the coherent cancelation of the couplings of soft gluons to color-singlet states. This property, color transparency [52], is a fundamental coherence property of gauge theory and leads to diminished final-state interactions and corrections to the PQCD factorizable contributions. The problem of setting the renormalization scale of the coupling for exclusive amplitudes is discussed in [16].

Baryon pair production in two-photon annihilation is also an important testing ground for QCD. The calculation of T_H for Compton scattering requires the evaluation of 368 helicity-conserving tree diagrams which contribute to $\gamma(qqq) \to \gamma'(qqq)'$ at the Born level and a careful integration over singular intermediate energy denominators [53, 54, 55]. Brooks and Dixon [56] have recently completed a recalculation of the proton Compton process at leading order in PQCD, extending and correcting earlier work. It is useful to consider the ratio $d\sigma/dt(\gamma\gamma \to \bar{p}p)/d\sigma/dt(e^+e^- \to \bar{p}p)$ since the power-law fall-off, the normalization of the valence wavefunctions, and much of

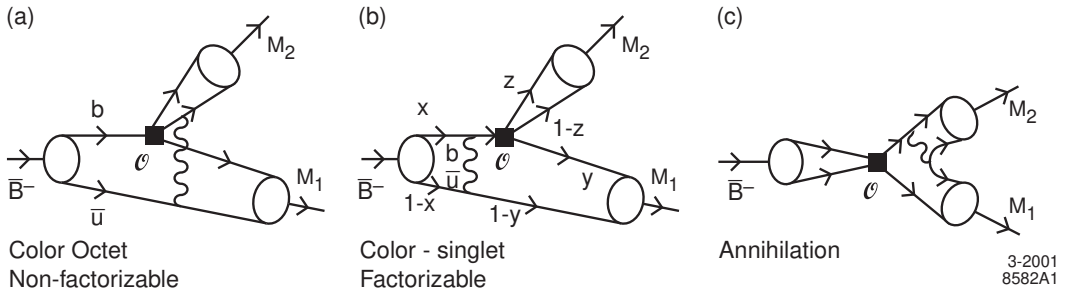


Figure 5: Three representative contributions to exclusive B decays to meson pairs in PQCD. The operators $\mathcal{O}$ represent the QCD-improved effective weak interaction.

the uncertainty from the scale of the QCD coupling cancel. The scaling and angular dependence of this ratio is sensitive to the shape of the proton distribution amplitudes. The perturbative QCD predictions for the phase of the Compton amplitude phase can be tested in virtual Compton scattering by interference with Bethe-Heitler processes [57].

It is also interesting to measure baryon and isobar pair production in two photon reactions near threshold. Ratios such as $\sigma(\gamma\gamma \to \Delta^{++}\Delta^{--})/\sigma(\gamma\gamma \to \Delta^{+}\Delta^{-})$ can be as large as $16:1$ in the quark model since the three-quark wavefunction of the Δ is expected to be symmetric [8]. Such large ratios would not be expected in soliton models [9] in which intermediate multi-pion channels play a major role.

Recently Pobylitsa *et al.* [58] have shown how the predictions of perturbative QCD can be extended to processes such as $\gamma\gamma \to p\bar{p}\pi$ where the pion is produced at low velocities relative to that of the p or $\bar{p}$ by utilizing soft pion theorems in analogy to soft photon theorems in QED. The distribution amplitude of the $p\pi$ composite is obtained from the proton distribution amplitude from a chiral rotation. A test of this procedure in inelastic electron scattering at large momentum transfer $ep \to p\pi$ and small invariant $p'\pi$ mass has been remarkably successful. Many tests of the soft meson procedure are possible in multiparticle e^+e^- and $\gamma\gamma$ final states.

6 Conclusions

The leading-twist QCD predictions for exclusive two-photon processes such as the photon-to-pion transition form factor and $\gamma\gamma \to$ hadron pairs are based on rigorous factorization theorems. The recent data from the CLEO collaboration on $F_{\gamma\pi}(Q^2)$ and the sum of $\gamma\gamma \to \pi^+\pi^-$ and $\gamma\gamma \to K^+K^-$ channels are in excellent agreement

with the QCD predictions. It is particularly compelling to see a transition in angular dependence between the low energy chiral and PQCD regimes. The success of leading-twist perturbative QCD scaling for exclusive processes at presently experimentally accessible momentum transfer can be understood if the effective coupling $\alpha_V(Q^*)$ is approximately constant at the relatively small scales Q^* relevant to the hard scattering amplitudes [16]. The evolution of the quark distribution amplitudes in the low-Q^* domain also needs to be minimal. Sudakov suppression of the endpoint contributions is also strengthened if the coupling is frozen because of the exponentiation of a double logarithmic series.

One of the formidable challenges in QCD is the calculation of non-perturbative wavefunctions of hadrons from first principles. The recent calculation of the pion distribution amplitude by Dalley [25] and by Burkardt and Seal [35] using light-cone and transverse lattice methods is particularly encouraging. The predicted form of $\phi_\pi(x, Q)$ is somewhat broader than but not inconsistent with the asymptotic form favored by the measured normalization of $Q^2 F_{\gamma\pi^0}(Q^2)$ and the pion wavefunction inferred from diffractive di-jet production.

Clearly much more experimental input on hadron wavefunctions is needed, particularly from measurements of two-photon exclusive reactions into meson and baryon pairs at the high luminosity B factories. For example, as shown in Figure 3, the ratio

$$\frac{d\sigma}{dt}(\gamma\gamma \to \pi^0\pi^0)/\frac{d\sigma}{dt}(\gamma\gamma \to \pi^+\pi^-)$$

is particularly sensitive to the shape of pion distribution amplitude. At fixed pair mass, and high photon virtuality, one can study the distribution amplitude of multi-hadron states [6]. Two-photon annihilation will provide much information on fundamental QCD processes such as deeply virtual Compton scattering and large angle Compton scattering in the crossed channel. I have also emphasized the interrelation between the wavefunctions measured in two-photon collisions and the wavefunctions needed to study exclusive B and D decays.

Much of the most interesting two-photon annihilation physics is accessible at low energy, high luminosity e^+e^- colliders, including measurements of channels important in the light-by-light contribution to the muon g–2 and the study of the transition between threshold production controlled by low-energy effective chiral theories and the domain where leading-twist perturbative QCD becomes applicable.

The threshold regime of hadron production in photon-photon and e^+e^- annihilation, where hadrons are formed at small relative velocity, is particularly interesting as a test of low energy theorems, soliton models, and new types of resonance production. Such studies will be particularly valuable in double-tagged reactions where

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