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On the theoretical uncertainties in the muon anomalous magnetic moment

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I present a fairly detailed discussion of various contributions to the anomalous magnetic moment of the muon a_μ. I try to give an unbiased evaluation of the validity of the SM prediction for this quantity and to point out some delicate issues involved in its calculation. I conclude that the theory uncertainties in the SM prediction for the muon anomalous magnetic moment are underestimated and a great deal of work will be required to reduce these uncertainties to the level required by experiment.

I. INTRODUCTION

Recently, the E821 experiment at Brookhaven National Laboratory reported a new value of the anomalous magnetic moment of the muon, based on the μ^+ data collected through 1999. Their result. averaged with previous measurements, leads to a new world average [1],

$$a_\mu^{\rm exp} = 116\ 592\ 020(150) \times 10^{-11}, \qquad (1)$$

that is claimed to be 2.6σ away from the Standard Model prediction:

$$a_\mu^{\rm th} = 116\ 591\ 597(67) \times 10^{-11}. \qquad (2)$$

There are numerous theoretical papers devoted to the interpretation of this apparent discrepancy as the direct signal of physics beyond the Standard Model (SM) (see [2] for examples).

Because of the potential importance of this result and because of the subtlety of certain of the SM contributions, it is important to carefully review the corresponding calculations. Such a review has been presented recently by A. Czarnecki and W. Marciano [3]; however, in my opinion, more attention should be paid to certain aspects of the problem.

This article is organized in the following way. First, I briefly describe all the contributions to the SM value for a_μ to remind the reader what went into the theory result quoted by the g-2 collaboration. I then concentrate on the hadronic contribution to photon vacuum polarization and discuss its evaluation based on both τ and e^+e^- data. After that I describe the hadronic light-by-light scattering contribution.

There are three major questions I would like to address in this article: 1) Is there a g-2 crisis? 2) What should be done to make a solid case for the crisis? 3) What are the odds that there *will be* a crisis after E821 reaches its projected accuracy of 40×10^{-11}. My analysis indicates that the theory uncertainties in a_μ are larger than indicated in Eq.(2) and that a great deal of sophisticated work will be required to reduce them.

I hope the information presented here will be useful both, for a person who is about to post a New Physics paper on the muon anomalous magnetic moment at the LANL archive, and for a person who wants to understand the down-to-earth physics issues involved in the calculation of a_μ.

Finally, I should apologize to the many experts on the anomalous magnetic moment of the muon who have done the hard work to achieve the accuracy of the SM prediction that we now have. Although I have not done any of the original theoretical work, a fresh look at these important calculations might be useful. I also hope that there are some points in my discussion that might be of interest even for the experts working in the field.

II. WHAT THEORETICAL INPUT IS IN THE 2.6 σ DISCREPANCY?

Let us first make clear to ourselves what went into the theoretical number that is 2.6 σ away from the result of the g-2 Collaboration. It is a common practice to write the muon anomalous magnetic moment as the sum of the QED, weak and hadronic contributions:

$$a_\mu^{\rm th} = a_\mu^{\rm QED} + a_\mu^{\rm weak} + a_\mu^{\rm hadr}, \qquad (3)$$

for which the following values have been used by the g-2 Collaboration:

$$a_\mu^{\rm QED} = 116\ 584\ 705.7(2.9) \times 10^{-11}, \qquad (4)$$
$$a_\mu^{\rm weak} = 152(4) \times 10^{-11}, \qquad (5)$$
$$a_\mu^{\rm had} = 6739(67) \times 10^{-11}. \qquad (6)$$

The first thing to notice here is the one per cent accuracy of the hadronic contribution to a_μ and I will discuss how trustworthy it is in detail in the rest of the paper. Before that, however, I would like to comment on the QED and weak contributions.

The pure QED contribution (which only includes

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muon, electron and τ) is known to five loops[1]. Note that the value of the fine structure constant obtained from the electron g-2 has been employed here but the result reported in Eq.(4) does not change significantly if more conservative values for the fine structure constant (for example, as obtained from the quantum Hall effect) are used to evaluate it.

At the current level of precision the five loop QED contribution is not yet relevant; roughly, it gives 5×10^{-11}. The four loop contribution, on the other hand is quite relevant; its contribution to Eq.(4) is $\sim 400 \times 10^{-11}$. I think it is appropriate to ask how well the four loop contribution is known. The answer to this question is not quite simple. The complete four loop calculation has been performed by a single group and has never been checked by an independent calculation. To see that there is a potential problem here, one can look at how the central value of the four loop contribution evolved in time . In [5] it was reported as $140(6)(\alpha/\pi)^4$ and was later revised [6] to $126.92(41)(\alpha/\pi)^4$. The shift, $\sim 40 \times 10^{-11}$, is a non-negligible change on the level of precision 100×10^{-11}. On the other hand, about 90 per cent of the full $\mathcal{O}(\alpha^4)$ result comes from a simple class of diagrams where the electron vacuum polarization is inserted into one of three photons in $\mathcal{O}(\alpha^3)$ electron light-by-light scattering diagrams. This particular contribution has been evaluated several times with the result $116(\alpha/\pi)^4$ (see e.g. [7]). Unfortunately, this fact alone does not tell us much. It is clear that 1) mistakes, in general, can happen and 2) we are talking here about one of the most heroic and sophisticated calculations *ever* performed in perturbative quantum field theory. For the rest of this article we will assume that $a_\mu^{\rm QED}$, as given in Eq.(4), is correct but it is important to keep in mind that even in the QED part of a_μ there is, potentially, some work to be done if we want to gain complete confidence in the SM prediction for this quantity.

The weak contribution to a_μ is currently known up to two loops [8,9]. The result is:

$$a_\mu^{\rm weak} = (195 - 43(4)) \times 10^{-11} = 152(4) \times 10^{-11}, \quad (7)$$

where the one-loop and the two-loop contributions are displayed separately. The two-loop correction seems to be too large for a "normal" electroweak correction. The reason for such a big second order effect is that large logarithms $\log(m_W/m_f)$ where m_f is the mass of a light fermion (muon, electron or any of the quarks) appear in the two loop diagrams for the first time [10]. These logarithms make up the bulk of the second order correction and they can be summed up [11] using renormalization group techniques. This has been done [11] and it did not change the value quoted above significantly. So, the weak corrections seem to be well established.

[1]More precisely, four loops are computed, the $\mathcal{O}(\alpha^5)$ contribution is estimated.

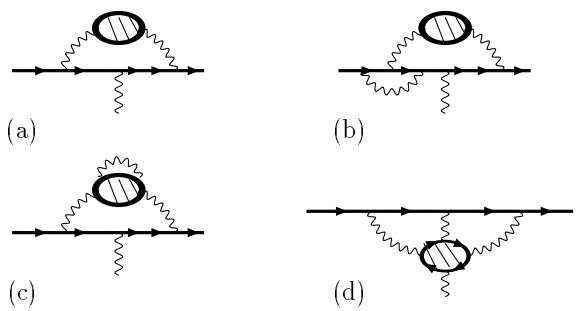

FIG. 1. Examples of hadronic contributions to g-2. a) Leading order hadronic vacuum polarization diagram; b) example of the next-to-leading order hadronic vacuum polarization diagram; c) the diagram that is *not* considered as part of the next-to-leading order hadronic vacuum polarization diagrams in the usual nomenclature; d) hadronic light-by-light.

Finally, hadronic contributions to the muon anomalous magnetic moment are usually separated into hadronic vacuum polarization (which is further separated into the leading and the next-to-leading order pieces) and hadronic light-by-light scattering contributions (see Fig.1). In Ref. [1], the following results have been used for these contributions:

$$a_\mu^{\rm hadr}(\text{lo v.p.}) = 6924(62) \times 10^{-11} \qquad [12], \quad (8)$$

$$a_\mu^{\rm hadr}(\text{nlo v.p.}) = -100(6) \times 10^{-11} \qquad [13], \quad (9)$$

$$a_\mu^{\rm hadr}(\text{lbl}) = -85(25) \times 10^{-11} \qquad [14, 16]. \quad (10)$$

The theory prediction, Eq.(2), is obtained by taking the sum of the QED, weak and hadronic contributions and *adding their errors in quadratures*:

$$a_\mu^{\rm th} = 116\ 591\ 597(67) \times 10^{-11}, \quad (11)$$

which leads to

$$\left[a_\mu^{\rm exp} - a_\mu^{\rm th}\right] \times 10^{11} = 426 \pm 150|_{\rm exp} \pm 67|_{\rm th} \equiv 426 \pm 165,$$

where at the last step the theoretical and experimental errors are combined in the quadratures. This difference is interpreted as the 2.6 σ deviation in [1].

Some comments about this result and its interpretation can be made immediately. First of all, the dominant source of the theory error is the hadronic contribution, in particular the vacuum polarization (later I will discuss the hadronic light-by-light scattering result in detail). A glance at the numbers in Eq.(8) shows clearly that one needs to control the calculation of hadronic vacuum polarization at the *one per cent level.* Hadronic vacuum polarization is derived by using the dispersion representation for the photon propagator which relates the hadronic vacuum polarization correction to a_μ and the annihilation cross section $e^+e^- \to$ hadrons:

$$a_\mu^{\rm hadr}(\text{lo v.p.}) = \frac{1}{4\pi^3} \int\limits_{4m_\pi^2}^{\infty} {\rm d}s\ K(s)\sigma^0(s)_{e^+e^- \to {\rm had}}. \quad (12)$$

For large s the function $K(s)$ behaves, to a good approximation, as m_μ^2/s and for this reason the contribution of the low s region is dominant and any "first principles" calculation becomes impossible. This forces one to rely on experimental data to evaluate $a_\mu^{\rm hadr}$(lo v.p.) and for this reason it is quite important to know exactly where the number in Eq.(8) comes from. In fact this number is taken from the calculation [12] based, in addition to $e^+e^- \to$ hadrons, on 1) using the data on τ decays supplemented by the conserved vector current (CVC) hypothesis and isospin symmetry; 2) sophisticated machinery of the finite energy QCD sum rules designed specifically to minimize the errors; 3) application of perturbative QCD down to energy scales of about 1.8 GeV (J/ψ and Υ families are treated separately, using experimental input). It is important to notice that the natural scale for both CVC and isospin symmetry violations is the one per cent; the smallness of the error of the result in Eq.(8) is the consequence of both, the quality of the τ data *and* the use of pQCD down to rather low energy energies.

My second comment is that the 2.6 σ deviation only appears if all the errors are combined in quadratures. If one combines all the theory errors linearly, one ends up in a somewhat different situation:

$$\left[a_\mu^{\rm exp} - a_\mu^{\rm th}\right] \times 10^{11} = 426 \pm 150|_{\rm exp} \pm 100|_{\rm th}. \qquad (13)$$

I would like to stress that it is not at all clear how these numbers should be combined to get the final error since it is a bit too strong an assumption to assign a Gaussian distribution to the theory error. It is true that it is unclear how to interpret Eq.(13) in terms of standard deviations; on the other hand it is equally unclear that this should be done since a glance at the content of the hep-ph archive through recent months is probably a good illustration of the fact that too many people take the word "standard deviation" too literally.

III. HADRONIC VACUUM POLARIZATION AND THE τ DATA

The use of the τ data for the evaluation of a_μ(lo v.p) is related to the fact that the integration over s in Eq.(12) saturates at $\sqrt{s} < 2$ GeV: about 70 per cent of the total hadronic contribution comes from the two pion final state at energies as low as $\sqrt{s} < 1$ GeV, and about 90 per cent from the energy region $\sqrt{s} \leq 2$ GeV. The accuracy of the data available until rather recently from e^+e^- machines was not quite adequate. It then looked natural to combine them with the data on τ decays obtained by the ALEPH collaboration to improve the calculation of $a_\mu^{\rm hard}$.

As I have already mentioned, the essential *theoretical* input one brings in with the τ data is the CVC hypothesis and the isospin symmetry. We know that, generically, these are rather good symmetries. We also know that these symmetries should be violated at the one per cent level (by e.g. electromagnetism or $(m_\pi/m_\rho)^2$) and therefore the real question here is in how well these violations can be controlled.

Both isospin and CVC are violated by the mass difference of the up and down quarks and also by the QED corrections. Let us discuss the transition from the τ data to $e^+e^- \to$ hadrons in some detail to expose potential problems here.

Imagine that we have a perfect measurement of the $\tau \to \nu_\tau \pi^0 \pi^-$ branching ratio. Starting from there, one identifies several effects that might affect the transition to $e^+e^- \to$ *hadrons*. They are: 1) the short distance QED corrections in τ decays; 2) the difference in the masses of charged and neutral pions and $m_u - m_d \neq 0$ effects; 3) the difference in the decay widths and the masses of neutral and charged ρ mesons; 4) the long-distance QED radiative corrections in τ decays.

Let us discuss these effects step by step. The short-distance QED corrections are Wilson coefficients of the four-fermion operators that describe τ decays; they are generated by exchanges of photons with the virtualities $m_\tau^2 \ll k^2 \ll m_W^2$. Because of this, the short-distance corrections are universal in a sense that, for a given four-fermion operator, they do not depend on subtle details of hadronic final state. Note, however, that due to the difference in relevant four-fermion operators for leptonic and hadronic τ decays, these short distance corrections are absent in $\tau \to \nu_\tau +$ leptons. The short-distance QED Wilson coefficient is [17]:

$$S_{\rm ew} = \left(1 + \frac{\alpha}{\pi} \log \frac{m_W}{\mu}\right) \approx 1.009, \qquad (14)$$

where μ is an arbitrary parameter; the numerical value corresponds to $\mu = m_\tau$. Since there is no similar renormalization factor in e^+e^- annihilation, the relation between τ decay width and the e^+e^- annihilation cross section reads, schematically:

$$\Gamma(\tau^- \to \nu_\tau \pi^0 \pi^-) \approx S_{\rm ew}^2 \sigma(e^+e^- \to {\rm hadrons}). \qquad (15)$$

The renormalization factor $S_{\rm ew}$ is taken into account when the τ data is used to predict $e^+e^- \to$ hadrons [12]; numerically, as can be seen from the numbers above, it amounts to renormalizing the τ data by about -2 per cent.

The second effect is the difference in the masses of charged and neutral pions. Since the pions are produced in the P-wave, both in $\tau^- \to \nu_\tau \pi^0 \pi^-$ and in $e^+e^- \to \pi^+\pi^-$, the rates are proportional to the third power of the velocity of the pions. In this case, the relation between the τ decay rate and the e^+e^- cross section is:

$$\Gamma(\tau^- \to \nu_\tau \pi^0 \pi^-) \approx \frac{\beta^3_{\pi_0\pi^-}}{\beta^3_{\pi^+\pi^-}} \sigma(e^+e^- \to {\rm hadrons}).$$

Since the mass difference of charged and neutral pions is relatively large, this correction turns out to be in a few per cent range in the threshold region. At around

the mass of the ρ meson, where the bulk of the contribution to $a_\mu^{\rm hard}$ comes from, this correction becomes much smaller (see [18]).

The third effect is the difference in the decay widths of charged and neutral ρ mesons. Close to the ρ resonance this effect can be estimated as:

$$\Gamma(\tau^- \to \nu_\tau \pi^0 \pi^-) \approx \frac{\Gamma^2_{\rho_0}}{\Gamma^2_{\rho_-}} \sigma(e^+e^- \to \text{hadrons}). \qquad (16)$$

One can check that in the vicinity of the resonance the effect of the difference in the widths almost cancels the phase space corrections due to the difference in the pion masses discussed above. Let me spell out more precisely how the widths difference is taken into account. Starting from the τ data and taking into account all the relevant corrections (e.g. the short distance QED correction $S_{\rm ew}$), one ends up with the distribution in invariant mass of π_0 and π^-. This distribution is fitted using some parameterization of the pion form factor to determine the mass and the width of the charged ρ. To compute $e^+e^- \to \pi^+\pi^-$, one uses the same parameterization of the pion form factor but with the mass and the width of the neutral ρ instead of similar parameters for the charged one.

The three effects discussed above are usually taken into account in the existing analyses, however, the fourth effect, the long distance QED corrections, seems to be more problematic. In computing long distance QED corrections one usually assumes that the pions can be treated as point-like particles. This assumption is not quite correct, since in the hard renormalization factors $S_{\rm ew}$ only photon virtualities down to the mass of the τ are included. It is quite clear that photons with virtualities from the mass of the τ down to, say, 1 GeV certainly resolve the pion and see its quark structure. The contribution of this momentum region is therefore treated not quite correctly in the existing estimates. At any rate, the most recent calculation [18] of the long distance QED corrections, performed using scalar QED for *point-like pions*, claimed that the long-distance QED corrections in $\tau \to \nu_\tau \pi_0 \pi$ add $+0.4$ per cent[2] to the short distance renormalization factor $S^2_{\rm ew}$.

The attitude to the techniques used to obtain this number (chiral power counting, point-like pions) can certainly vary from person to person, however, it is important to mention that a more complete study of the QED effects, including attempts to introduce hadronic structure, has been performed for the decay rate $\tau \to \nu_\tau \pi$ [19]. Since the two processes are similar, it is instructive to look at the results in [19]. Specifically, consider the QED corrections to the ratio:

$$R_{\tau/\pi} = \frac{\Gamma(\tau \to \pi\nu_\tau)}{\Gamma(\pi \to \mu\nu_\mu)}. \qquad (17)$$

The corrections, computed in various approximations, are [19]: a) short-distance QED[3]: -1 per cent; b) point-like pion (the QED corrections to the ratio are finite for point-like pions): $+1$ per cent; c) "best estimate" of [19] that includes hadronic structure and short distance corrections: $0.0 \div 0.25$ per cent. The short-distance QED and the point-like pions are the two extreme limits of the problem that can be easily understood and I consider their difference as the indication that the uncertainty in long distance QED corrections to τ decays can be of order 1 per cent. Other authors (see e.g. [3]) consider ± 0.5 per cent as a more reasonable estimate of this uncertainty. It is clear that only convincing complete calculation of the QED radiative corrections to $\tau \to \nu_\tau \pi^- \pi_0$ can tell us which of the two numbers is closer to the truth but in the absence of such a calculation I think it makes sense to have a conservative attitude.

Finally, we come to an important point that is often not well understood. Imagine that we have actually succeeded in computing the QED corrections to τ decays and have carefully taken into account all the isospin violating effects in transforming the τ data to $e^+e^- \to$ hadrons. Is this the end of the story? The usual answer here is yes, but the correct answer is *no*. This issue is related to the discussion above on how corrections due to the widths differences of charged and neutral ρ are implemented. Imagine that the masses and widths of neutral and charged ρ are obtained independently from the fits to e^+e^- and τ data. The pion form factor is defined as the $\gamma^*\pi^+\pi^-$ interaction vertex with all the QED interactions between the pions being switched off. Therefore, if one starts from the τ data, determines the pion form factor and uses this form factor with the masses and widths for the neutral ρ as obtained from $e^+e^- \to \pi^+\pi^-$, one obtains the *bare* pion form factor $\gamma^* \to \pi^+\pi^-$. One should then compute the final state QED interaction corrections and include them into the dispersion integral. It is also important to stress that these corrections are *not* included in what is usually called the next-to-leading order hadronic vacuum polarization corrections (see Fig.1c).

How large can these corrections be? To give a simple estimate I consider the $\pi^+\pi^-$ final state and assume that the pions are point-like particles. In this case the QED corrections are easy to compute. The corresponding calculation can be found in [20]:

[2] Another correction not considered in [18] but relevant for their analysis is the QED correction to leptonic decay mode $\tau \to \nu_\tau e \bar{\nu}_e$ which is used for the normalization of the data. Effectively, this correction [17] adds another $+0.4$ per cent to $S^2_{\rm ew}$.

[3] The short-distance QED corrections both to $\tau \to \pi\nu_\tau$ and $\pi \to \mu\nu_\mu$ are given by $S_{\rm ew}$. These corrections do not cancel out exactly because the appropriate μ (cf. Eq.(14)) is thought to be different in the two processes.

$$\sigma_{\pi^+\pi^-} = \frac{\alpha^2\pi}{s}\beta^3|F_\pi(s)|^2\left[1+\frac{\alpha}{\pi}\mathcal{F}_{fs}(\beta)\right], \qquad (18)$$

where $\beta = \sqrt{1-4m_\pi^2/s}$ and

$$\mathcal{F}_{fs}(\beta) = \frac{(1+\beta^2)}{\beta}\left\{4\mathrm{Li}_2\left(\frac{1-\beta}{1+\beta}\right)+2\mathrm{Li}_2\left(-\frac{1-\beta}{1+\beta}\right)\right.$$
$$\left.-3\log\left(\frac{2}{1+\beta}\right)\log\left(\frac{1+\beta}{1-\beta}\right)-2\log(\beta)\log\left(\frac{1+\beta}{1-\beta}\right)\right\}$$
$$-3\log\left(\frac{4}{1-\beta^2}\right)-4\log(\beta)$$
$$+\frac{1}{\beta^3}\left(5\frac{(1+\beta^2)^2}{4}-2\right)\log\left(\frac{1+\beta}{1-\beta}\right)+\frac{3}{2}\frac{(1+\beta^2)}{\beta^2}. \qquad (19)$$

The radiative correction, as described by this function, is plotted in Fig.2. First note that the radiative correction is rather large; in particular, it is significantly larger than the corresponding correction for the production of two fermions. Asymptotically, for $\beta \to 1$, the correction is $3\alpha/\pi$ and so one sees a compensation of the famous $1/\pi$ factor that is always present in simple estimates of the QED radiative corrections. At threshold, the correction is again significant because of Coulomb singularity. The bottom line is that the correction is relatively large everywhere.

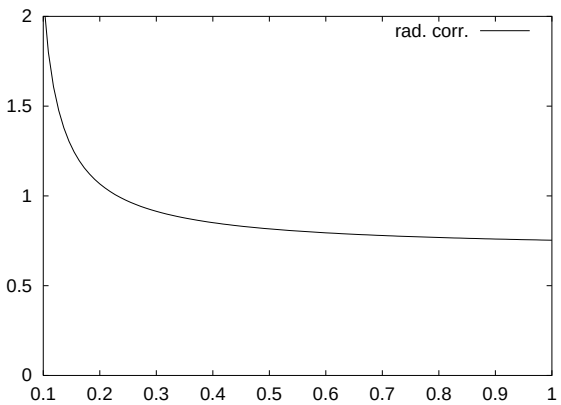

FIG. 2. QED corrections to the production of two pions, in per cent, in dependence on s, GeV^2.

If I use Eq.(18) in Eq.(12) and integrate it up to $\sqrt{s}=1$ GeV, I obtain the net increase in the contribution of the two pion final state to a_μ by slightly less than 1 per cent. This would add an additional $\sim 50\times 10^{-11}$ to hadronic vacuum polarization contribution to a_μ, if one evaluates it using the τ data.

Let me stress that I do not consider the above estimate to be an absolute prediction for the missing effect since certainly there are questions here of how well scalar QED for point-like pions actually describes the real world where the pions are not point-like and which part of the radiative correction is correctly accounted for by modeled (rather than measured) widths difference of charged and neutral ρ mesons. Rather, I think that this estimate should be considered as a counter-example to a popular statement that the QED corrections are always $\mathcal{O}(\alpha/\pi)$ and for this reason are insignificant.

At the very least, these considerations do imply that the error on $a_\mu^{\rm hadr}$ in [12] is too optimistic. Consider the following. The result of [12] is $a_\mu^{\rm hadr}(\text{lo v.p}) = (6924\pm 56|_{\rm exp}\pm 26|_{\rm th})\times 10^{-11}$. I guess everyone would agree that the long distance QED effects in $\tau\to\nu_\tau\pi^-\pi^0$ *and* in $e^+e^-\to\pi^+\pi^-$ can be 0.5 per cent *each*. We then get a one percent theory uncertainty from the two pion channel and this is $\pm 50\times 10^{-11}$. Certainly, this uncertainty is not completely taken into account in the theory uncertainty 26×10^{-11} from [12]. So, at the very least, the systematic uncertainty in the result of [12], used by the g-2 collaboration in their evaluation of the SM result, is smaller than it should be[4].

The possibility to use the τ data gave us a useful cross check on the accuracy of the e^+e^- data. However, it should be clear from the above discussion that the use of the τ data in the analysis of a_μ requires essential *theoretical* input which, at the required level of precision, is hard to justify or check. It may happen that a better theory will convincingly demonstrate that it is possible to control the transition from τ to e^+e^- data with the accuracy well below 1 per cent. In the absence of that, I believe that the use of the τ data for computing a_μ may turn out to be counter-productive. As will be seen from the discussion to follow, if one uses the e^+e^- data, one at least may try to minimize and *experimentally* control certain theoretical assumptions necessary to transform the raw data into hadronic vacuum polarization contribution to a_μ. I, personally, do not see how this can be done if one starts from the τ data.

IV. HADRONIC VACUUM POLARIZATION AND THE E^+E^- DATA

Let me now elaborate on the use of the e^+e^- data to compute the hadronic vacuum polarization contribution to a_μ. The point I would like to make here is that the e^+e^- data offers a relatively clean and, what is perhaps more important, verifiable approach to evaluating hadronic vacuum polarization with the required precision.

There are many papers where the $a_\mu^{\rm hadr}(\text{lo v.p})$ is evaluated from e^+e^- data and I will not discuss all of them (for the recent reviews and calculations see [22–24]).

For the purpose of the illustration I will use the result

[4] To be fair, I should perhaps say that in many other evaluations of hadronic vacuum polarization contribution to a_μ the QED corrections are also somewhat forgotten. It is only because of the exceptional precision of the result in [12] that I focus on that reference.

of [21][5] as well as more recent updates by one of these authors [22]. Their results are:

$$a_\mu^{\rm hadr}({\rm lo\ v.p}) = 7025(150)\times 10^{-11} \quad [1995],$$
$$= 6974(105)\times 10^{-11} \quad [2000]. \qquad (20)$$

By comparing these numbers to the τ-based result $6924(62)\times 10^{-11}$ [12] that has been used by the g-2 collaboration for the calculation of the SM prediction for the muon anomalous magnetic moment, one sees that the e^+e^- numbers are somewhat larger. However, it is not simply τ vs. e^+e^- that determines the difference of the two results.

To show this, let me note that there exists a dedicated analysis of the CVC hypothesis for various exclusive channels [25] based on comparison of the e^+e^- and τ data. For example, if one uses the e^+e^- data and CVC to predict the corresponding branching ratio $B(\tau^-\to\pi^0\pi^-\nu_\tau)$, one obtains [25]:

$$B(\tau^-\to\pi^0\pi^-\nu_\tau)|_{\rm CVC} = 24.52\pm 0.33,$$
$$B(\tau^-\to\pi^0\pi^-\nu_\tau)|_{\tau\ {\rm data}} = 25.32\pm 0.15, \qquad (21)$$

which implies that the τ data actually predicts a *larger* contribution of the ρ resonance, an opposite situation to what one sees in the final numbers for $a_\mu^{\rm hadr}$. The explanation for that is that the calculation [12] is more than just the τ data; it also involves e.g. the use of perturbative QCD down to 1.8 GeV; the approach the authors of [21] try to avoid. Since the old e^+e^- data at around 2 GeV is *significantly higher* than the pQCD results at that energies, this turns out to be an important part of the difference *in the central values* of [12] and [21].

Let me now say a few words about the composition of the errors in e^+e^- data. The errors are distributed as [21]:

$$2m_\pi < E < 0.8\ {\rm GeV}: \qquad \sim 100\times 10^{-11},$$
$$0.8\ {\rm GeV} < E < 1.41\ {\rm GeV}: \qquad \sim 50\times 10^{-11},$$
$$1.41\ {\rm GeV} < E < 3.10\ {\rm GeV}: \qquad \sim 50\times 10^{-11}.$$

Other errors seem to be negligible.

Another important benchmark [26] is how accurate different exclusive channels should be known if the final result for a_μ is to be known with the precision 100×10^{-11} (I assume that the errors from individual channels are combined in quadrature). The $e^+e^-\to\pi^+\pi^-$ final state should be known at the level of one per cent; $\omega\to 3\pi$, ϕ, 4π and the contribution from above 2 GeV should be known roughly at the 10 per cent level. To decrease the error to something like 30×10^{-11} all these uncertainties should be scaled down by a factor of three, approximately.

Some of these errors, most notably the error from the region below $\sqrt{s}\sim 1$ GeV will go down significantly due to new data from the VEPP-2M collider at Novosibirsk. Their final results are not made public yet, however the anticipated accuracy of 0.6 per cent is already known. The other improvement will, potentially, come from BEPC [27]. They are measuring the value of $R(s)$ at $\sqrt{s} > 2$ GeV. At energy regions below J/ψ, their preliminary results are accurate to within 7 per cent and they are about 15 per cent lower than the earlier results of Mark I and Gamma2 experiments and a bit higher than the pQCD results.

Let us now return to the two pion channel and the forthcoming Novosibirsk results. Since the 0.6 per cent accuracy is outstanding, it is important perhaps to spell out some delicate issues that might help to make it more believable.

The major point to realize about the difference in the use of e^+e^- data for the evaluation of a_μ as compared to τ data is that in principle when one uses the e^+e^- data one needs much less theoretical input. Clearly, there are certain things to be worked out, like QED corrections related to initial state radiation and vacuum polarization, but the part of the QED corrections that describes the interaction of π^+ and π^- in the final state is already in the data. Still there remains a potential problem that I discuss below. It is important to distinguish at this point the pion form factor as used for comparison with different models and for the determination of the mass and the width of the ρ meson where, by definition, the final state QED radiative corrections are not included and the cross section $\gamma^*\to\pi^+\pi^-$ for the purpose of a_μ calculation, where the final state QED radiative corrections should be kept intact. I believe that the two pion channel data from CMD2 will be analyzed this way [30].

Although appreciating the difference between F_π and $\gamma^*\to\pi^+\pi^-$ when the QED effects are considered is important, it is equally important to realize potential problems with the Novosibirsk analysis. First of all, the Monte Carlo event generator for the two pion channel that is used to analyze the data is based on point-like pions [31]. This might be a potential limitation.

Another problem is that the experimental analysis starts with imposing certain cuts to isolate *two pion* final state. The major requirement here is that the two pions are essentially back-to-back and therefore this cut excludes the $\gamma\pi^+\pi^-$ final state where the photon is radiated off one of the pions or an electron or positron at a relatively large angle. My estimates[6] show that the cuts ap-

[5] The renormalization group improved result quoted in [21] as the principal result can not be used together with higher order QED corrections to vacuum polarization contribution computed in [13]. For this reason, I quote below the result in [21] that is obtained *without* renormalization group improvement.

[6] I used the Monte Carlo event generator for $e^+e^-\to\pi^+\pi^-\gamma$

plied in the recent CMD2 measurement of $e^+e^- \to \pi^+\pi^-$ [29] remove from 50 to 80 per cent of $e^+e^- \to \pi^+\pi^-\gamma$. The degree of rejection is energy dependent (for smaller $\sqrt{s}$ fewer events are rejected, since the transverse momentum of photons is smaller) and it is also different for final and initial state radiation. At any rate, there are events that are rejected right up front by experimental cuts and that, potentially, should be put back since no independent measurement of $\pi^+\pi^-\gamma$ for large angle photons at the energy region around the ρ meson has been reported so far. The only way this can be done without doing the measurement of $\pi^+\pi^-\gamma$ is to use the Monte Carlo. One should realize, however, that in this way one puts back the large angle photon emission by using point-like pions and this is the most problematic region for the point-like pion approximation to begin with.

To see the importance of higher order QED corrections to the hadronic vacuum polarization contribution, one can can either recall the discussion of Schwinger's correction to the two pion channel in the previous Section or look at a much cleaner next-to-leading order hadronic vacuum polarization calculation [13] (see Fig.1b). Take any diagram that contributes to the two-loop QED correction to g-2 and insert the hadronic vacuum polarization in to either of the two virtual photon lines. This gives a correction $\Delta a_\mu^{\rm had}(\text{nlo v.p.}) = -100(6) \times 10^{-11}$ [13]. One particular contribution to this number comes from combined leptonic and hadronic vacuum polarization in the one-loop diagram. This one is interesting since, in some sense, it is related to the vacuum polarization insertions in $e^+e^- \to$ hadrons. This contribution is about 100×10^{-11} and one clearly sees how large the corresponding corrections can be.

Let me now discuss the question of what should be done in order to ensure a careful job on $e^+e^- \to \pi^+\pi^-$. The best thing, of course, is if one actually measures the $\pi^+\pi^-\gamma$ channel separately and checks that it actually matches the $\pi^+\pi^-$ channel as far as the acolliniarity angle of the two pions is concerned. This might be quite a tough measurement since at the very end one will have to disentangle the large angle final state radiation from the initial one.

However, if relatively energetic photons and pions are detected, one can make a study of the charge asymmetry of the produced pions [28]. In case when the hard photon is tagged, this effect comes from the interference between initial and final state radiation and is therefore linear in the final state radiation amplitude. Thus, the charge asymmetry of the produced pions in $\pi^+\pi^-\gamma$ events with all the particles emitted at relatively large angles, gives a direct handle on the amplitude of the final state radiation. I believe these kind of studies should be done to cross check the *model* (and, certainly, the scalar QED for the interaction of pions with photons *is* a model) used in the Monte Carlo event generators. There are some preliminary results from DAPHNE [32] and also old results from Novosibirsk on $\pi^+\pi^-\gamma$ channel [33] that seem to indicate that the point-like pion approximation works amazingly well in the energy range around 1 GeV; however it is still not completely conclusive.

At any rate, summarizing the use of e^+e^- data for a_μ predictions, I can say that, currently, the e^+e^--based evaluations of $a_\mu^{\rm hadr}$ have a somewhat higher central value and larger error bars than the value of $a_\mu^{\rm hadr}$ [12] used in [1] which implies that if one evaluates the SM prediction for a_μ using the e^+e^- data, the g-2 "crisis" becomes less acute. The precision of e^+e^--based evaluations will improve once the new data from the low energy e^+e^- machines is incorporated. It is important to realize that at this new level of precision new questions, primarily related to the correct treatment of QED radiative corrections, will start to appear. However, it seems to me that a program of measurements and analysis can be set up that makes it possible to control every step on the way from the e^+e^- data to the muon anomalous magnetic moment. This is the principal difference with the τ data.

V. THE LIGHT-BY-LIGHT SCATTERING CONTRIBUTION

The light-by-light is probably the most tricky thing in the muon g-2 calculation. The trouble is that, in contrast to hadronic vacuum polarization, there is no simple way to relate this contribution to anything observable. In this situation, one has to resort to models to describe low-energy hadron dynamics and then the question of the reliability of a certain model becomes central.

Before going into the discussion of the *delicate* issues related to hadronic light-by-light scattering, let me first clarify a misconception that, as it seems to me, is quite common in the current literature. The issue I want to address is what is the relevant scale for the loop momenta that determines the contribution of the light-by-light scattering diagrams to a_μ. It is usually said in the literature that this is the mass of the muon; for hadronic light-by-light, this statement is not correct.

To see this, it is useful to analyze a simple QED example by computing the light-by-light scattering contribution of the fermion of the mass M to the anomalous magnetic moment of the muon. Lets introduce the variable $x = M/m_\mu$ and consider the limit $M \gg m_\mu$ which is relevant for hadronic light-by-light (both the mass of the pion and the constituent quark masses are larger than the mass of the muon). The result of the QED calculation is then [34]:

$$a_\mu|_{x\gg 1} = \left(\frac{\alpha}{\pi}\right)^3 \left\{\frac{0.615}{x^2} + \frac{1}{x^4}\left(-0.2\ \log^2 x\right.\right.$$

[28]; I am grateful to G. Venanzoni for help with that.

$$-0.33\log x - 0.15) + \mathcal{O}(x^{-6})\Big\}. \qquad (22)$$

The question I would like to discuss is how the above result can be obtained using either the effective field theory technique or, equivalently, asymptotic expansion of the relevant Feynman diagrams in m/M.

Upon examination, one can easily identify three expansion regimes for a generic light-by-light scattering diagram. The first regime is the Taylor expansion of the diagram in the ratio of m_μ/M. In this regime, the momenta of all three virtual photons are of the order of the *heavy* fermion mass. In the language of effective field theories, this is the contribution that directly induces the anomalous magnetic moment operator in the effective Lagrangian:

$$\mathcal{L}_1 = c_1 \frac{m_\mu}{M^2}\ \bar{\psi}\sigma_{\mu\nu}\psi F^{\mu\nu}, \qquad (23)$$

where c_1 is some constant.

The second expansion regime is related to Euler-Heisenberg Lagrangian for photons. In this case, the momenta of all three virtual photons are of the order of the muon mass m_μ and the corresponding part of the effective Lagrangian is:

$$\mathcal{L}_2 = \frac{\alpha^2}{360\ M^4}\left[4\left(F^{\mu\nu}F_{\mu\nu}\right)^2 + 7\left(\frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}\right)^2\right]. \qquad (24)$$

This piece in the effective Lagrangian determines the strength of the low-energy photon-photon scattering.

The third expansion regime is the following: one of three virtual photons has the momentum of order m_μ, while two others have large $\sim M$ virtualities. Integrating out heavy degrees of freedom in this configuration induces the following term in the effective Lagrangian

$$\mathcal{L}_3 = \frac{1}{M^4}F_{\mu\nu}F_{\alpha\beta}\bar{\psi}\left[m_\mu\Gamma_1^{\mu\nu\alpha\beta} + \Gamma_2^{\mu\nu\alpha\beta\rho}D_\rho\right]\psi. \qquad (25)$$

Here D_ρ is the (QED) covariant derivative and $\Gamma_{1,2}$ are appropriate Lorentz tensors; their exact form is of no concern to us here. The Lagrangian $\mathcal{L}_3$, in principle, contributes to low energy muon-photon scattering.

An important point to notice is that only the *first* expansion regime (and therefore the effective operator $\mathcal{L}_1$) gives an $\mathcal{O}(M^{-2})$ contribution to the anomalous magnetic moment, whereas the other two *only* start to contribute at $\mathcal{O}(M^{-4})$. This trivial observation immediately implies that, no matter how small the muon mass is, *there is no low energy information one can use to determine the leading contribution to the muon anomalous magnetic moment coming from large momentum scales.* The only thing one can do (and, very roughly, this is what one usually does) is to still use the low energy effective Lagrangians $\mathcal{L}_2$ and $\mathcal{L}_3$ and compute the Feynman diagrams using hard momentum cut-off λ. If one takes this cut-off to be $\lambda \sim M$, one generates the contributions $m_\mu^2\lambda^2/M^4 \sim m_\mu^2/M^2$ even from higher dimensional operators; these contributions, however, have nothing to do with the correct result.

We therefore see that the typical scale for the loop momenta in hadronic light-by-light is set by the hadronic scale and not by the muon mass. Since the lightest hadron is the pion and since it contributes to the light-by-light diagrams, one can hope that the leading contribution is determined by momentum transfers of order m_π and this is small as compared to the scale of chiral symmetry breaking ~ 1 GeV. If this is true, this seems to exclude the possibility to use quarks and gluons as the relevant degrees of freedom for this calculation and, instead, one can argue that for the momentum transfer being that small, chiral perturbation theory should be quite reliable. For this reason, all the calculations of hadronic light-by-light scattering are based on chiral perturbation theory gauged with $U(1)$ electromagnetism. However, it turns out that the integrals *do not* converge at momentum scales around m_π and for this reason chiral perturbation theory alone can not produce a definite prediction. One then resorts to phenomenological models such as large N_c, extended Nambu-Jona-Lasinio, hidden local symmetry etc.

Within this framework, three major contributions to light-by-light scattering are distinguished by using large N_c and chiral power counting. The first is the box of charged pions, the second is the contribution of the neutral pseudoscalar boson through a transition $\gamma^*\gamma^* \to$ P $\to \gamma^*\gamma$ where $P = \pi^0, \eta, \eta'$. The third contribution is due to constituent quark loops.

The simplest calculation to be done is to compute the contribution of the quark loop in QED and a similar contribution with elementary pions in scalar QED. In this way one gets [14]:

$$a_\mu^{\rm hadr}({\rm lbl,\ pions}) = -44.58(23)\times 10^{-11}, \qquad (26)$$

$$a_\mu^{\rm hadr}({\rm lbl,\ quarks}) = 62(3)\times 10^{-11}, \qquad (27)$$

where the constituent masses for light quarks ($m_u = m_d = 0.3$ GeV and $m_s = 0.5$ GeV) have been used. One sees that the two contributions in no way match *since they differ by a sign.*

To improve on that result one would like to take into account a) the interactions between pions or quarks and b) the modifications of the pion-photon coupling for the off-shell photons. The self-interaction of pions is thought to be described by higher-derivative terms in the chiral Lagrangian. Generically, these terms generate corrections of the form $p_{\rm typ}^2/\Lambda_\chi^2$, where $p_{\rm typ}$ is the typical pion momenta and $\Lambda_\chi \sim 1$ GeV is the scale of the chiral symmetry breaking in three flavor QCD. Taking $p_{\rm typ} \sim m_\pi$ for the estimate, one expects ~ 1 per cent corrections but I doubt that this estimate is actually correct (see below). The trouble is that if one wants to go from the estimate to the calculation one would not be able to produce an unambiguous answer both within chiral perturbation the-

ory *and* the models used currently for the calculation of the hadronic light-by-light. For this reason the problem of self interactions between pions and quarks is ignored in the current literature.

Let us now consider the modification of the pion to photon coupling for the off shell photons. The simplest model is the vector-meson-dominance (VMD) model, that essentially postulates that photons interact with pions through a transition to the ρ (or any vector) meson. The propagator of the photon is then modified to be:

$$\frac{i}{q^2} \to \frac{i m_\rho^2}{q^2\left(m_\rho^2 - q^2\right)}. \tag{28}$$

How much do we expect the free pion result would change once the VMD modification of the photon propagator is introduced? If all the integrals converge at around m_π, as is usually assumed, one would expect the modification to be of the order $m_\pi^2/m_\rho^2 \sim 0.04$. In reality, the situation is quite different. First of all, there *is* a significant modification – the result with VMD is smaller by a factor $6-10$. This implies that original integrals actually converge at momentum scales *much higher* than the mass of the pion (what about chiral perturbation theory in such a situation?).

There is a good illustration of this fact in [14]. In the complete calculation one may try to consider the masses of the pion and the ρ meson as variable quantities and ask about asymptotic behavior of the results once these masses are changed. For example, by keeping the mass of the ρ meson fixed but changing the mass of the pion, one obtains the m_π^{-2} scaling law; this indicates that the integral over the pion box subdiagram is saturated at small momenta. On the contrary, changing the mass of the ρ meson gives the asymptotic behavior:

$$a_\mu^{\rm hadr}({\rm lbl}\ m_\pi, m_\rho) - a_\mu({\rm lbl}\ m_\pi, \infty) = 0.23\left(\frac{m_\mu}{m_\rho}\right)\left(\frac{\alpha}{\pi}\right)^3.$$

This shows that this contribution is quite large[7]. The final result for this contribution quoted in [14] is:

$$a_\mu^{\rm hadr}({\rm lbl,\ pions}) = \left(-0.03557 + 0.23\frac{m_\mu}{m_\rho}\right)\left(\frac{\alpha}{\pi}\right)^3 = -0.00355\left(\frac{\alpha}{\pi}\right)^3 = -4.5(8.1)\times 10^{-11}. \tag{29}$$

a reduction by one order in magnitude as compared to the point-like pion result Eq.(26).

A very similar pattern is observed if the VMD modification is applied to photon couplings to constituent quarks. The final result quoted for this contribution in [14] is

$$a_\mu^{\rm hadr}({\rm lbl,\ quarks}) = 9.7(11.1)\times 10^{-11}. \tag{30}$$

The last contribution to be considered is the one from the pseudoscalar meson pole (see Fig.3) and, in view of the smallness of the pion and quark contributions (after VMD), it turns out to be the dominant one. The result is [15]:

$$a_\mu^{\rm hadr}({\rm lbl,\ pole}) = -82.7(6.4)\times 10^{-11}. \tag{31}$$

Here, π^0, η and η' are contributions are taken into account. The π^0 contribution is about 70 per cent, with the rest being distributed equally between η and η'. Large contributions from η and η' look surprising and I comment on it at the end of this Section. The result in Eq.(31) is obtained by using some constraints on the $\gamma^*\gamma^*\pi^0$ interaction vertex from the measurement of the pion transition form factor by the CLEO collaboration [35].

The final result quoted in [15] is obtained by summing up Eqs.(29,30,31) and adding to it small (1.7×10^{-11}) axial-vector meson contribution. The result reads:

$$a_\mu({\rm lbl, total}) = (-79 \pm 15.4)\times 10^{-11}, \tag{32}$$

if the errors of the individual contributions are added in quadratures. If they are added linearly, the error on this contribution is $\pm 25\times 10^{-11}$.

Another calculation of the hadronic light-by-light contribution, based on Extented Nambu-Jona-Lasinio model, has been presented in [16]. The result quoted in [16] is[8]:

$$a_\mu({\rm lbl, total}) = (-92 \pm 32)\times 10^{-11} \tag{33}$$

and is therefore close to the result in Eq.(32) (see Ref. [36] for the comparison of [16] and [15]).

The final result for the light-by-light scattering contribution used by the g-2 collaboration is the arithmetic average of the two results; a similar procedure is applied to calculate the uncertainty. Even if the two results are to be trusted, a sensible thing to do is, perhaps, to take the uncertainty to be so large as to cover the whole range as allowed by individual results. We then have:

[7] This linear dependence might be an artifact of the fitting procedure. Using the data in [14] I can fit the pion box contribution to $a_\mu(m_\pi, m_\rho) - a_\mu(m_\pi, \infty) = (\alpha/\pi)^3\ 0.8\ m_\mu^2/m_\rho^2 \log^2 m_\rho/m_\mu$ which is more consistent with what one should expect from the Feynman integrals with two largely different scales involved.

[8] The calculation of Ref. [16] is performed with the cut off on the loop momentum and the sensitivity of the final result to the cut off is studied. The authors observe that in order to have a stable result for some of the contributions, the integration should be extended up to several GeV. The error of the final result in [16] is obtained if the errors on individual contributions are added linearly.

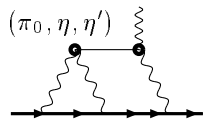

FIG. 3. The pseudoscalar contribution to g-2.

$$a_\mu(\text{lbl}, \text{total}) = -85(38) \times 10^{-11}, \tag{34}$$

and it is important to stress that the only thing this uncertainty is supposed to represent is the *model dependence*.

In view of the large value of the pseudoscalar pole contribution it is appropriate to discuss it in some detail. Again, it is instructive to ask the question about momentum flows in the corresponding diagrams to see potential troubles. The $\pi^0\gamma\gamma$ vertex for the on-shell pion and photons is given by:

$$\mathcal{L} = -\frac{\alpha}{8\pi f_\pi}\pi^0 \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}. \tag{35}$$

Considering this as a new vertex in the low-energy effective Lagrangian and inserting it into the diagram that describes its contribution to the anomalous magnetic moment of the muon, we observe that the result is divergent and this divergence is cut off by the pion transition form factor. It is easy to work out the corresponding contribution in the leading logarithmic approximation with the result:

$$a_\mu^{\text{hadr}}(\text{lbl}, \pi_0) \sim \left(\frac{\alpha}{\pi}\right)^3 \frac{m_\mu^2}{(4\pi f_\pi)^2} \int\limits_{m_\pi}^{m_\rho} \frac{\mathrm{d}k}{k} \log\frac{m_\rho}{k}, \tag{36}$$

where k is the momentum that runs along the π_0 line. In doing this estimate I assumed that the integrals are cut off from above by the hadronic scale comparable to the mass of the ρ meson.

Eq.(36) gives us useful information about the structure of the divergences and, hence, about momentum flow. First, the divergences are double logarithmic and not just single logarithmic, as it is sometimes claimed in the literature. Second, there are two divergences. One is associated with large virtualities of the photons, when the momentum that goes through the pion line is kept fixed. The second one is associated with large virtualities of the *pion*.

Currently, both of these divergences are cut off by adopting VMD prescription for the photon lines. This approach is supported by the phenomenological success of the VMD models in describing the $\gamma\gamma^* \to \pi^0$ transition form factor. Also, using this kind of the regularization one can compute the decay width $\pi^0 \to e^+e^-$ and similar and obtain a reasonable agreement with the data. For this reason it probably makes complete sense to use the VMD motivated regularization for the photon loop subdivergence. It is less clear if the same regularization makes any sense for the other divergence associated with highly virtual π^0. The point is that this kinematics is not related to any observable form factor and it is not clear what the highly virtual π_0 means. Ideally, this configuration should somehow match on to the quark box QCD diagram, but no one knows how to implement that in practice.

What also seems rather intriguing is the fact that the contributions of η and η' are quite large. Approximately, they are one fourth of the π_0 contribution in spite of huge difference in masses. For the sake of the argument, consider η meson. Its coupling to two photons is roughly the same as the $\pi^0\gamma\gamma$ coupling. Taking into account that the ratio of the masses is $m_\pi/m_\eta \sim 1/4$, one concludes that the suppression from the loop integral goes like $1/m$ when the mass is increased. A similar conclusion is reached when the η' contribution is analyzed.

Another point I would like to mention is that it is unclear if the quark loop contribution should be damped by introducing VMD modification of the photon propagator. Imagine that we want to set up a calculation in the effective field theory framework. To do that, we are supposed to introduce a factorization scale. Above this scale, we do the calculation with quarks and gluons and below this scale with hadrons. If one looks at the calculation from this perspective, then the calculation of the quark contribution to light-by-light scattering should be cut off in the infra-red; for technical reasons this is achieved by introducing the quark masses $\sim$ 300 MeV. On the other hand, the calculation with hadrons is regularized using VMD modification for the photon propagators which cuts off the integrals *from above*. In spite of the fact that the cut off is implemented differently in two parts of the calculation, in my opinion, the set up described above is internally consistent. On the other hand, it shows that the introduction of the VMD modified photon propagators into the quark contribution to light-by-light scattering is not quite logical. The quark contribution is supposed to describe physics at energy scales $2m_Q \leq k$. When, *in addition to quark masses*, the VMD is introduced into loop integrals, the integration is cut off at $k \leq m_\rho \sim 2m_Q$ and this is clearly *outside* the momentum range that the quark contributions is supposed to cover. For this reason, it seems to me that the quark contribution Eq.(27) *without* VMD suppression might be a more appropriate description for the contribution of the high energy region. If so, the result for hadronic light-by-light scattering might receive additional positive contribution.

Let me finally comment on the argument used to justify the application of the Extended Nambu-Jona- Lasinio model to the calculation of the hadronic light-by-light contribution to g-2. A possible check is to apply the same model to the calculation of hadronic vacuum polarization contribution to g-2 and see how well the result based on experimental data can be reproduced. This has been done in Ref. [37] where the claim is that within the class of models like the ones used in [14,16], the hadronic vacuum polarization contribution can be predicted to within

15 per cent. This fact, by itself, is nice but I am not sure how restrictive it is. Let me take three constituent quarks with masses 200 MeV and use the lowest order cross section for $e^+e^- \to q\bar{q}$ to compute the "hadronic" vacuum polarization to a_μ. Including only u, d and s quarks and integrating up to 2 GeV, I get the hadronic vacuum polarization contribution to a_μ to be $\sim 5000 \times 10^{-11}$, a perfectly reasonable number. Using the same "model" for computing hadronic light-by-light, I would have obtained the result close to 100×10^{-11}. This number is in the "correct" range but the *sign* is opposite. I think this shows my point – in light-by-light we are sensitive to much more detailed structure of the hadronic interactions than we can check using hadronic vacuum polarization and this fact seem to matter after all.

To summarize my discussion of hadronic light-by-light, I would like to stress that the major question here is the model dependence of the result and in this respect, in my opinion, the agreement between the two independent calculations [14,16] does not tell us much since the models used in these calculations are similar. As I have discussed above in detail, the momentum scales that control the hadronic light-by-light scattering contribution are neither the mass of the muon nor the mass of the pion, as is often assumed. For this reason, any low energy hadronic model that is used for such a calculation, should be accurate up to ~ 1 GeV and, as far as I understand, there are not too many such models on the market. It would certainly be very helpful if this problem would come under scrutiny of the low-energy hadron physics community.

VI. CONCLUSIONS AND FUTURE PROSPECTS

What can be expected for the muon anomalous magnetic moment in the future? The g-2 collaboration will improve the accuracy of their result to 40×10^{-11}. However, even with this accuracy the interpretation of this result will depend on our ability to estimate the hadronic contribution to g-2.

The analysis of $e^+e^- \to$ hadrons from Novosibirsk is in its final phase. This implies that soon the new e^+e^- based estimate of the hadronic vacuum polarization will be available. Hopefully, it will include a proper treatment of QED radiative corrections. With this new result, there will, probably, be no need to use the τ data, since the e^+e^- data will become sufficiently accurate.

The value of $R(s)$ will probably be re-measured by using radiative return by KLOE, CLEO and BaBar collaborations at existing facilities. It is hard to imagine that the radiative return based measurements will achieve a one per cent accuracy; $3-5$ per cent accuracy is, probably, within reach. This might be sufficient for the energy region above 1 GeV but it is not sufficient for the 2π channel. Therefore, to a large extent, the e^+e^- based interpretation of the muon anomaly, will hang on the new Novosibirsk data.

On the theory side, the four loop QED radiative corrections were not checked by an independent calculation. Certainly, these kinds of calculations are much less rewarding than model building and so it seems that the chances to have any progress here are slim.

Finally, the real bottleneck seems to be the hadronic light-by-light scattering contribution, because all the existing arguments that make one believe in the validity of theoretical estimates are, from my viewpoint, rather inconclusive. There is talk about getting some help from lattice field theory but it is difficult to believe in that. It would be of great help if the people who do low energy hadron physics phenomenology would come up with a *radically* different model (as compared to what is used now) to do the calculation of hadronic light-by-light. If this happens, there will be at least some indication on how large the real model dependence is.

From what I have said, it should be quite clear that so far there is really *no* g-2 crisis. For the purpose of illustration, consider the following estimate. Let us take the recent e^+e^- based re-evaluation of the hadronic vacuum polarization [22], which central value is about 50×10^{-11} higher than the result in [12]. Let me also use 38×10^{-11} as an uncertainty in the light-by-light. Finally, let me add all the theory errors linearly. I obtain:

$$\left[a_\mu^{\rm exp} - a_\mu^{\rm th}\right] \times 10^{11} = 377 \pm 150|_{\rm exp} \pm 156|_{\rm th}. \qquad (37)$$

Clearly, if one presents it in such a way there is no significant discrepancy. Again, to avoid misunderstanding, let me stress that I do not consider Eq.(37) as my "best" estimate of the current difference between the theory and experiment; rather, Eq.(37) shows that the theory uncertainty on the SM value of a_μ, if evaluated conservatively, is significant and a great deal of work will be required to reduce it to the level required by experiment.

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[1] H. N. Brown *et al.*, [Muon g-2 collaboration], Phys. Rev. Lett. **86**, 2227 (2001).
[2] http://phyppro1.phy.bnl.gov/g2muon/index.shtml
[3] A. Czarnecki and W. Marciano, hep-ph/0102122.
[4] See e.g., P. J. Mohr and B. N. Taylor, Rev. Mod. Phys. **72**, 351 (2000).

[5] T. Kinoshita, B. Nizic and Y. Okamoto, Phys. Rev. Lett. **52**, 717 (1984).
[6] T. Kinoshita, B. Nizic and Y. Okamoto, Phys. Rev. **D41**, 593 (1990).
[7] C. Chlouber and M. Samuel, Phys. Rev. **D16**, 3596 (1977).
[8] S. J. Brodsky and J. D. Sullivan, Phys. Rev. **156**, 1644 (1967);
R. Jackiw and S. Weinberg, Phys. Rev. **D5**, 2396 (1972);
K. Fujikawa, B.W. Lee and A. I. Sanda, Phys. Rev. **D6**, 2923 (1972);
I. Bars and M Yoshimura, Phys. Rev. **D6**, 374 (1972);
G. Altarelli, N. Cabbibo and L. Maiani, Phys. Lett. **B40**, 415 (1972);
W. A. Bardeen, R. Gastmans and B. E. Lautrup, Nucl. Phys. **B46**, 315 (1972).
[9] A. Czarnecki, B. Krause and W. Marciano, Phys. Rev. **D 52**, 2619 (1995); Phys. Rev. Lett. **76**, 3267 (1996).
[10] T. V. Kuhto, E. A. Kuraev, A. Schiller and Z. K. Silagadze, Nucl .Phys. **B371**, 567 (1992).
[11] S. Peris, M. Perrottet and E. de Rafael, Phys. Lett. **B355**, 523 (1995).
[12] M. Davier and A. Hocker, Phys. Lett. **B435**, 427 (1998).
[13] B. Krause, Phys.Lett. **B390**, 392 (1997).
[14] M. Hayakawa, T. Kinoshita and A. Sanda, Phys. Rev. **D54**, 3137 (1996).
[15] M. Hayakawa, T. Kinoshita, Phys. Rev. **D57**, 465 (1996).
[16] J. Bijnens, E. Pallante and J. Prades, Nucl. Phys. **B474**, 379 (1996).
[17] W. Marciano and A. Sirlin, Phys. Rev. Lett. **61**, 1815 (1988).
[18] V. Cirigliano, G. Ecker and H. Neufeld, hep-ph/0104267.
[19] R. Decker and M. Finkemeier, Nucl. Phys. **B438**, 17 (1995)
[20] J. Schwinger, in "Particles, Sources and Fields", volume III, Perseus Books, Reading, Massachusetts, 1989. See also M. Drees and K. Hikasa, Phys. Lett. **B252**, 127 (1990), where the misprint in Schwinger's formula is corrected.
[21] S. Eidelman and F. Jegerlehner, Z. Phys. **C67**, 585 (1995).
[22] F. Jegerlehner, hep-ph/0104304.
[23] W. Marciano and B. Lee Roberts, hep-ph/0105056.
[24] S. Narison, hep-ph/0103199.
[25] S. I. Eidelman and V. N. Ivanchenko, Nucl. Phys. Proc. Suppl. **40**, 131 (1995).
[26] F. Jegerlehner, hep-ph/9901386.
[27] For a review see, e.g. Z. Zhao, Int. Jour. Mod. Phys. **A15**, 3739 (2000).
[28] S. Binner, J.H. Kühn and K. Melnikov, Phys. Lett.**B459**, 279 (1999).
[29] R. R. Akhmetshin *et al.* [CMD-2 Collaboration], hep-ex/9904027.
[30] S. Eidelman and I. Logaschenko, private communication.
[31] A. B. Arbuzov, V. A. Astakhov, A. V. Fedorov, G. V. Fedotovich, E. A. Kuraev and N. P. Merenkov, JHEP **9710**, 006 (1997) [hep-ph/9703456].
[32] A. Denig, talk at Workshop e^+e^- *physics at intermediate energies*, Stanford, California, May 2001.
[33] S. I. Dolinsky *et al.*, Phys. Rept. **202**, 99 (1991).
[34] S. Laporta and E. Remiddi, Phys. Lett. **B301**, 401 (1993).
[35] CLEO Collaboratin J. Gronberg *et al.* [CLEO Collaboration], Phys. Rev. **D57**, 33 (1998).
[36] J. Bijnens, hep-ph/9710341.
[37] E. de Rafael, Phys. Lett. **B322**, 239 (1994).