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Counterterms in Supergravity

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Abstract

We examine the ultraviolet behaviour of supergravity theories as a function of dimension and number of supercharges. We do so by the computation of one- and two-loop physical on-shell four-point amplitudes.

For maximal supergravity, our computations prove the nonrenormalizability of supergravity for $D \geq 6$ (including the maximal $D = 11$ case) and give strong evidence for the existence of a five-loop counterterm in $D = 4$. For type I supergravity our results indicate similar patterns.

We shall also explore a remarkable relationship between gravity amplitudes and those of Yang-Mills theories. In many ways gravity calculations discover features which relate to the equivalent Yang-Mills features by a squaring procedure.

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1 Motivation

Supergravity is one of the key theories in understanding quantum gravity. In itself, it is almost certainly not a renormalizable theory, so it must appear as the low energy limit of another theory such as M-theory. However, supergravity and variations thereof will be the effective theories which describe quantum gravity at energies less than the (colossally large) Planck scale.

We shall study the ultraviolet behavior of supergravity theories for two reasons: firstly we wish to prove the conjectured bad behavior of these theories; secondly we hope to understand some features of the physics at the Planck scale. Adding counterterms is a well defined, but unpredictive at the Planck scale, way to regulate a theory. A physical regulator should in some sense provide the same regulation but within a predictive context. The symmetries and structure of the physical theory might well find themselves mirrored in the counterterm structure.

One of the themes of this TMR network has been the use of integrability in understanding two dimensional field theories. Unfortunately the enormous success in two dimensions has not, yet, continued to higher dimensions. However, the techniques we use have some formal similarities. We can construct S-matrix elements from the analytical nature of the amplitudes. However we fall a long way short of the exact S-matrices found in two dimensions. Nonetheless, we are able to construct enough of the S-matrix elements to determine large amounts of the ultra-violet structure of supergravity theories in dimension four or greater. In constructing amplitudes we attempt to use any and all information regarding the amplitude. Supersymmetry is one useful tool and in gravity theories with extended supersymmetry we can make more progress. For the maximal supergravity theory we can prove its nonrenormalizability in $D = 11$ and can conjecture the behavior in $D = 4$.

2 Technology

Our philosophy is to evaluate the physical, on-shell S-matrix from its analytic properties. As far as possible we shall only consider on-shell objects.

A key property of the S-matrix is *unitarity*. Also, within dimensional regularization the amplitude is analytic in the dimension. The optical theorem, a consequence of unitarity, states

$$2\,\mathrm{Im}T \;=\; T^\dagger T.$$

In perturbation theory, comparing both sides order by order relates, for example, the imaginary part of a one-loop amplitude to the product of tree amplitudes. In practical terms the imaginary part of a one loop amplitude is just the coefficient of a logarithm (or di- or polylogarithms) since,

$$\ln(-s_{ij}) = \ln(|s_{ij}|) \;-\; i\pi\Theta(s_{ij}),$$

where s_{ij} is one of the momentum invariants. Naively, the optical theorem only determines part of the one-loop amplitude, since the amplitude may contain rational functions $f(s_{ij})$ which have no imaginary part. However we can, by using dimensional regularization, determine these rational parts also. Within dimensional regularization the one-loop amplitude has a momentum weight of

-2ϵ (since $d^D p \longrightarrow d^{D-2\epsilon}p$). This implies that the rational functions must be replaced by terms such as $f(s_{ij})(-s_{ij})^{-2\epsilon}$. Since

$$(-s_{ij})^{-2\epsilon} \;=\; 1 - 2\epsilon \ln(-s_{ij}),$$

the amplitude will pick up imaginary parts at $O(\epsilon)$. We thus deduce that *knowledge of the cuts to all powers in ϵ will enable us to determine the amplitude.*

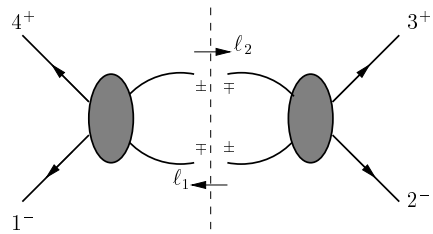

Figure 1: Sample t-channel cut in a one-loop amplitude.

Of course this can be a painful computational burden in many circumstances. (Although in practice it is not always necessary to determine the cuts to all orders in ϵ.) To see how this works consider the cut in a one-loop amplitude, as shown in fig. 1. Then the optical theorem states [1],

$$\begin{aligned}&\text{Disc } M^{1-loop}(1,2,3,4)\Big|_{s-cut} = \\ &i\int \sum_{\substack{\text{internal}\\ \text{particles}}} M^{\text{tree}}(-\ell_1^s,1,2,\ell_2^{s'})\, M^{\text{tree}}(-\ell_2^{s'},3,4,\ell_1^s)\end{aligned}$$

where the integral is over on-shell ℓ_i. We must use this carefully within dimensional regularization if we wish to determine the LHS correctly to all orders in ϵ. The RHS contains tree amplitudes. Normally we do not regard these as depending upon ϵ; however, the momenta ℓ_i must match the loop momenta on the LHS. These are in $D-2\epsilon$ dimensions so that the tree amplitudes should have the momenta ℓ_i in $D-2\epsilon$ and the others in D dimensions.

The analysis here is naturally merely indicative and the reader should refer elsewhere for the details of how this works and how it may be applied.

The optical theorem thus stated is a key ingredient to our calculational program; however, it is not the only important input. We also use some or all of the following features:

• Amplitudes may in principle be calculated using Feynman diagrams. This allows us to restrict the "function" space an amplitude may lie in.

• Supersymmetric theories generally have simpler amplitudes which can be easier to calculate.

• Field theory amplitudes may be calculated as the low energy limit of string theory amplitudes [2].

• Amplitudes should have factorization and collinear singularities when momentum invariants have specific limiting values [3].

3 $N = 8$ Supergravity Amplitudes

Maximal supergravity [4, 5] is a fascinating theory whose ultraviolet behavior had been suspected but until the last few years had defied definite calculation. We shall attempt to determine this

behavior.

The one-loop amplitude was calculated many years ago by Green and Schwarz and Brink [6] to be

$$\mathcal{M}_4^{\text{one-loop}} = \left(\frac{\kappa}{2}\right)^4 stu\, M_4^{\text{tree}}\Big(\mathcal{I}_4(s,t) + 2 \text{ perms}\Big)\,,$$

where $\mathcal{I}_4(s,t)$ is the D-dimensional scalar box integral (which may be easily evaluated). The one-loop amplitude is infinite in $D=8$ but not in other dimensions (when the external legs are on-shell). To find the first divergences in other dimensions we must go beyond a one-loop calculation.

Using the technology described previously we have obtained a remarkably simple result for the final form for the two-loop four graviton amplitude [7],

$$\begin{aligned}\mathcal{M}_4 = \left(\frac{\kappa}{2}\right)^6 stu\, M_4^{\text{tree}}\Big(& s^2\,\mathcal{I}_4^{\text{P}}(s,t) + s^2\,\mathcal{I}_4^{\text{P}}(s,u) \\ & + s^2\,\mathcal{I}_4^{\text{NP}}(s,t) + s^2\,\mathcal{I}_4^{\text{NP}}(s,u) + \text{cyclic}\Big)\,.\end{aligned}$$

where $\mathcal{I}_4^{\text{P}}$ and $\mathcal{I}_4^{\text{NP}}$ are two-loop scalar box integrals. They are the planar and nonplanar double box integrals, respectively. This amplitude has ultraviolet infinities in all dimension $D>6$. In particular there is a definite divergence in the maximal dimension $D=11$.

The two-loop ultraviolet divergences for $N=8$ supergravity in $D=7$, 9 and 11, are

$$\begin{aligned}\mathcal{M}_4^{D=7}|_{\text{pole}} &= \frac{1}{2\epsilon\ (4\pi)^7}\frac{\pi}{3}(s^2+t^2+u^2)\ \times \mathcal{F}, \\ \mathcal{M}_4^{D=9}|_{\text{pole}} &= \frac{1}{4\epsilon\ (4\pi)^9}\frac{-13\pi}{9072}(s^2+t^2+u^2)^2\ \times \mathcal{F}, \\ \mathcal{M}_4^{D=11}|_{\text{pole}} &= \frac{1}{48\epsilon\ (4\pi)^{11}} \times \frac{\pi}{5791500}\Big(438(s^6+t^6+u^6) - 53 s^2t^2u^2\Big) \times \mathcal{F},\end{aligned}$$

where $\mathcal{F} = (\kappa/2)^6 \times stuM_4^{\text{tree}}$. There are no sub-divergences because one-loop divergences are absent in odd dimensions when using dimensional regularization. For even dimensions

$$\begin{aligned}\mathcal{M}_4^{D=8}|_{\text{pole}} &= \frac{1}{2\ (4\pi)^8} \times \Big(-\frac{1}{24\,\epsilon^2} + \frac{1}{144\epsilon}\Big)(s^3+t^3+u^3) \times \mathcal{F}, \\ \mathcal{M}_4^{D=10}|_{\text{pole}} &= \frac{1}{12\epsilon\ (4\pi)^{10}}\frac{-13}{25920} \times stu\,(s^2+t^2+u^2) \times \mathcal{F}.\end{aligned}$$

The $1/\epsilon^2$ pole in $D=8$ is *precisely* that needed to cancel the $1/\epsilon^2$ pole obtained when the one-loop counterterm is used to calculate to two loops. The $1/\epsilon$ pole shows how the expected nonpredictive nature of renormalization occurs — new terms must be added to the Lagrangian order by order.

In all cases, for four graviton external states, the linearized counterterms take the form of derivatives acting on

$$t_8 t_8 R^4 \equiv t_8^{\mu_1\mu_2\cdots\mu_8}\, t_8^{\nu_1\nu_2\cdots\nu_8}\, R_{\mu_1\mu_2\nu_1\nu_2}\, R_{\mu_3\mu_4\nu_3\nu_4}\, R_{\mu_5\mu_6\nu_5\nu_6}\, R_{\mu_7\mu_8\nu_7\nu_8}\,,$$

plus the appropriate $N=8$ completion [8]. This expression also appears as the one-loop counterterm for $N=8$ supergravity in $D=8$. This particular tensor is well known from string theory amplitudes [9], appears in the string effective action [10] and is one of the higher-dimensional analogs of

the Bel-Robinson tensor [11]. It is consistent with $N = 8$ supersymmetry, which may not allow other possibilities.

For example, the $D = 11$ counterterm is a linear combination of the two tensors

$$T^A = t_8 t_8 \cdot \partial_{\alpha\gamma\eta} R \partial^{\alpha\gamma\eta} R \partial_{\beta\delta\rho} R \partial^{\beta\delta\rho} R,$$
$$T^B = t_8 t_8 \cdot \partial_{\alpha\gamma\eta} R \partial^{\alpha\delta\eta} R \partial_{\beta\gamma\rho} R \partial^{\beta\delta\rho} R.$$

In each case the indices on the curvatures are contracted with the t_8 tensors, and the indices on the derivative are contracted with each other. The $D = 11$ counterterm is

$$-\frac{1}{48\epsilon\,(4\pi)^{11}} \times \frac{\pi}{5791500}\left(\frac{2575}{12}T^A + \frac{53}{6}T^B\right).$$

4 Higher-loop conjecture

To determine the behavior for $D \leq 6$ we need to go beyond two loops. As yet this remains a very challenging calculation. In order to specify the precise form of the conjecture at L loops one would need to investigate cuts with up to $(L+1)$ intermediate particles. Nevertheless, some of the integral coefficients and numerators can be obtained from iterated two-particle cuts. If we assume these pieces of the amplitude are representative of the full result, we can conjecture the ultraviolet structure. Of course in the absence of definite calculations this remains very much a guess — however, a guess which we expect will prove correct.

By examining some of the cuts we can identify the most divergent pieces as

$$\int (d^D p)^L \frac{(p^2)^{2(L-2)}}{(p^2)^{3L+1}}.$$

This integral will be finite when

$$D < \frac{10}{L} + 2 \qquad\qquad (L > 1).$$

The results of this analysis are summarized in table 1. In particular, in $D = 4$ no three-loop divergence appears — contrary to expectations from a superspace analysis [12, 13] — and the first R^4-type counterterm occurs at five loops. The divergence will have the same kinematical structure as the $D = 7$ two-loop divergence, but with a different nonvanishing numerical coefficient. (The possibility that duality symmetries might rescue maximal supergravity in four dimensions is discussed in ref. [21].)

5 Non-maximal Supergravity

It is interesting to compare the structures found between types II and type I supergravity (and their lower dimensional descendants.) We have examined the one-loop structures for dimensions $4 \leq D \leq 10$ [14]; however here we shall restrict the presentation to the features of the $D = 8$ case.

Dimension	Loop	Degree	Counterterm
8	1	log.	R^4
7	2	log.	$\partial^4 R^4$
6	3	quad.	$\partial^6 R^4$
5	4	quad.	$\partial^6 R^4$
4	5	log.	$\partial^4 R^4$

Table 1: The relationship between dimensionality and the number of loops at which the *first* ultraviolet divergence should occur in the $N = 8$ supergravity four-point amplitude. The form of the associated counterterm assumes the use of dimensional regularization.

In $D = 8$ power counting indicates that the counterterms will be of the form R^4. There are seven independent R^4 tensors [15] (in $D < 8$ these are no longer independent). [1]

$$\begin{aligned}
T_1 =& (R_{p,q,r,s}R_{p,q,r,s})^2,\\
T_2 =& (R_{p,q,r,s}R_{p,q,r,t})(R_{p',q',r',s}R_{p',q',r',t}),\\
T_3 =& R_{p,q,r,s}R_{p,q,t,u}R_{t,u,v,w}R_{r,s,v,w},\\
T_4 =& R_{p,q,r,s}R_{p,q,t,u}R_{r,t,v,w}R_{s,u,v,w},\\
T_5 =& R_{p,q,r,s}R_{p,q,t,u}R_{r,v,t,w}R_{s,v,u,w},\\
T_6 =& R_{p,q,r,s}R_{p,t,r,u}R_{t,v,u,w}R_{q,v,s,w},\\
T_7 =& R_{p,q,r,s}R_{p,t,r,u}R_{t,v,q,w}R_{u,v,s,w}.
\end{aligned}$$

On shell the combination

$$-\frac{T_1}{16} + T_2 - \frac{T_3}{8} - T_4 + 2T_5 - T_6 + 2T_7$$

vanishes (or rather is a total divergence), being proportional to the Euler form.

In order to calculate the appropriate $N = 8$ counterterm we evaluate the (on-shell) amplitude and we find it factorizes in the following way:

$$M^{N=8,D=8} = \frac{1}{\epsilon} \times \left(\frac{\kappa}{2}\right)^4 \frac{i}{(4\pi)^4}\, \frac{1}{2}\, K_1 \times K_1,$$

where

$$K_1 = tu(\epsilon_1 \cdot \epsilon_2)(\epsilon_3 \cdot \epsilon_4) + 2(\epsilon_1 \cdot \epsilon_2)\Big(t(\epsilon_3 \cdot k_1 \epsilon_4 \cdot k_2) + u(\epsilon_3 \cdot k_2 \epsilon_4 \cdot k_1)\Big) \;+\; \text{cyclic terms.}$$

The counterterm necessary to cancel this infinity is

$$\frac{1}{\epsilon}\left(\frac{\kappa}{2}\right)^4 \frac{i}{(4\pi)^4}\frac{1}{4}\Big[-\frac{T_1}{16} + T_2 - \frac{T_3}{8} - 0 \times T_4 + 2T_5 - T_6 - 2T_7\Big].$$

The other case is $N = 4$ supergravity. By this we mean the type I supergravity in $D = 10$ and its dimensional descendants. There is of course both a matter multiplet and a gravity multiplet (which

[1] The Riemann tensor is undistinguished from the Weyl tensor in our R^4 terms for on-shell four point amplitudes.

contains the graviton). The $N=8$ multiplet is a sum of these so only one is independent from the $N=8$ case. For the graviton amplitude with states in the matter multiplet circulating in the loop the infinity is

$$M^{N=4,D=8} = \frac{1}{\epsilon} \times \left(\frac{\kappa}{2}\right)^4 \frac{i}{(4\pi)^4} \times \frac{1}{720} K_1 \times K_2$$

where

$$\begin{aligned} K_2 &= -\epsilon_1 \cdot \epsilon_2 \epsilon_3 \cdot \epsilon_4 (3t^2 + 5tu + 3u^2) + \cdots \\ &+ 2\epsilon_1 \cdot \epsilon_2 \Big(3s\epsilon_3 \cdot k_4 \epsilon_4 \cdot k_3 + t\epsilon_3 \cdot k_1 \epsilon_4 \cdot k_2 + u\epsilon_3 \cdot k_2 \epsilon_4 \cdot k_1 \Big) + \cdots \\ &- 12 (k_2 \cdot \epsilon_1 k_1 \cdot \epsilon_2 k_4 \cdot \epsilon_3 k_3 \cdot \epsilon_4 + k_3 \cdot \epsilon_1 k_4 \cdot \epsilon_2 k_1 \cdot \epsilon_3 k_2 \cdot \epsilon_4 + k_4 \cdot \epsilon_1 k_3 \cdot \epsilon_2 k_2 \cdot \epsilon_3 k_1 \cdot \epsilon_4) \end{aligned}$$

where $+\cdots$ indicates the necessary terms we must add. We have organized K_2 according to the number of $\epsilon_i \cdot \epsilon_j$. The counterterms necessary to cancel this are

$$-\frac{1}{\epsilon} \left(\frac{\kappa}{2}\right)^4 \frac{i}{(4\pi)^4} \frac{1}{11520} \Big(-3T_1 + 24T_2 - 6T_3 + 4T_4 + 0 \times T_5 + 0 \times T_6 + 32T_7 \Big) .$$

We can relate this also to specific tensors contracted against R^4. The tensor t_8 can be split into two pieces $t_{(12)}$ and $t_{(48)}$ $t_8 = \frac{1}{2}\Big(t_{(12)} + t_{(48)}\Big)$ each having the same symmetry properties as t_8. The tensors $t_{(12)}$ and $t_{(48)}$ contain 12 and 48 quartic monomials in the δ's respectively and are the only two tensors which have the same symmetry properties of t_8 in eight dimensions [9].

$$\begin{aligned} t_{(12)}^{ijklmnpq} &= -\Big((\delta^{ik}\delta^{jl} - \delta^{il}\delta^{jk})(\delta^{mp}\delta^{nq} - \delta^{mq}\delta^{np}) \\ &\quad + (\delta^{km}\delta^{ln} - \delta^{kn}\delta^{lm})(\delta^{pi}\delta^{qj} - \delta^{pj}\delta^{qi}) \\ &\quad + (\delta^{im}\delta^{jn} - \delta^{in}\delta^{jm})(\delta^{kp}\delta^{lq} - \delta^{kq}\delta^{lp}) \Big), \\ t_{(48)}^{ijklmnpq} &= \Big(\delta^{jk}\delta^{lm}\delta^{np}\delta^{qi} + \delta^{jm}\delta^{nk}\delta^{lp}\delta^{qi} + \delta^{jm}\delta^{np}\delta^{qk}\delta^{li} + [i \leftrightarrow j] + [k \leftrightarrow l] + [m \leftrightarrow n] \Big), \end{aligned}$$

where $[i \leftrightarrow j]$ denotes antisymmetrization with respect to i and j. From these tensors we can define

$$A = \frac{1}{4} t_{(12)} t_{(12)} \cdot R^4 \quad , \quad B = \frac{1}{4} t_{(12)} t_{(48)} \cdot R^4$$
$$C = \frac{1}{4} t_{(48)} t_{(48)} \cdot R^4$$

where the $\cdot$ denotes the usual contraction of the upper and lower eight indices. (We can also express these tensor contractions as traces [16], $t_8 t_{(12)} \cdot R^4 = 48 t_8 \, \mathrm{Tr}(R^4)$, etc.).

In terms of these combinations the $N=8$ counterterm of the type $t_8 t_8 R^4$ is just

$$\frac{1}{768} \Big(A + 2B + C \Big) ,$$

and the $N=4$ matter contributions is proportional to $\Big(2A + C\Big)$.

We have obtained very similar results for $D=10$. In $D=10$ the $N=8$ supergravity amplitude vanishes (on-shell) but the two $N=4$ components do not. For these the counterterm also factorizes in the form $\sim K_1 \times L_i$, where the tensors L_i contain two more powers of momenta than the K_i.

6 Relationships between gravity and Yang-Mills

Analyzing the structure in amplitudes can reveal strong parallels between gravity and Yang-Mills calculation. In many ways the gravity results appear as the "square" of those for Yang-Mills. In fact, calculations in $N = 4$ Yang-Mills have been used as an initial step in the $N = 8$ calculation [17]. The extension to $N = 8$ involved to some extent repeating the calculation whilst squaring the algebra.

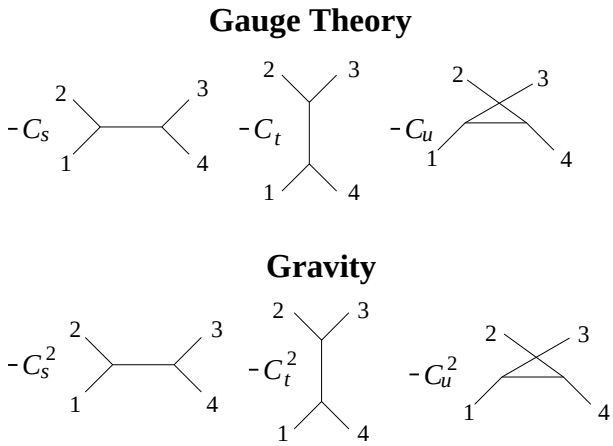

Figure 2: The tree amplitudes can be arranged to display a simple squaring relationship between the Yang-Mills and gravity cases

Relationships between the tree amplitudes of Yang-Mills were obtained from the low-energy limit of string theory by Kawai, Lewellen and Tye [18]. They found a series of algebraic relationships between the two sets of tree amplitudes. At the four-point level,

$$M_4^{\text{tree}}(1,2,3,4) = -is_{12} A_4^{\text{tree}}(1,2,3,4)\, A_4^{\text{tree}}(1,2,4,3)\,,$$

where M_4 is the gravity amplitude and A_4 is the Yang-Mills color-ordered tree amplitude. The KLT relationships have proved extremely useful; however, the trees can also be rearranged as in fig. 2, where we have rearranged the amplitudes to be in the form

$$\text{kinematic polynomial} \times \text{ pole structure.}$$

This can be done so that the relationship between gravity and Yang-Mills is clear [19] — we keep the pole structure and square the multiplying polynomial C_i. Note that there is some freedom in this since we can move terms between the different coefficients to some extent.

When we consider the one-loop amplitudes we find a very similar relationship. The amplitude can be written as a coefficient times an integral function. For the case of $N = 8$ supergravity the comparison to $N = 4$ super Yang-Mills theory is shown in fig. 3. We can again see that the coefficient of the integral is squared.

For the two loop case we find the situation shown in fig. 4. (Overall factors of stA^{tree} and $[stA^{\text{tree}}]^2$ have been omitted for clarity.) Whilst calculating the supergravity amplitude this relationship was postulated which allowed an anasatz for the supergravity amplitude to be quickly made. With a specific ansatz in hand, checking the cuts was relatively straightforward.

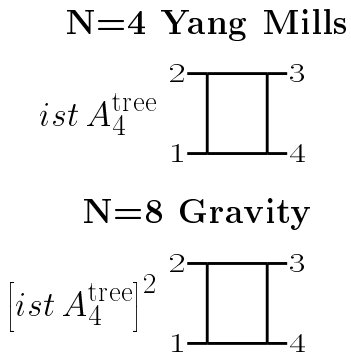

Figure 3: The simple relationship between one-loop amplitudes of maximal $N = 4$ super-Yang-Mills and $N = 8$ supergravity.

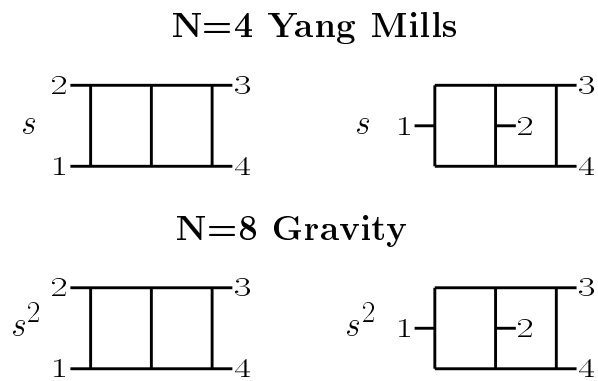

Figure 4: A simple relationship between the amplitudes of maximal SUSY and SUGRA persists to two loops

We suspect the relationship between the perturbative S-matrices of gravity and gauge theories is rather a deep one, although our understanding of it is limited at present.

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