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LIGHT-FRONT-QUANTIZED QCD IN LIGHT-CONE GAUGE *

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Abstract

The light-front (LF) quantization of QCD in light-cone gauge has a number of remarkable advantages, including explicit unitarity, a physical Fock expansion, the absence of ghost degrees of freedom, and the decoupling properties needed to prove factorization theorems in high momentum transfer inclusive and exclusive reactions. We present a systematic study of LF-quantized gauge theory following the Dirac method and construct the Dyson-Wick S-matrix expansion based on LF-time-ordered products. The gauge field is shown to satisfy the Lorentz condition as an operator equation as well as the light-cone gauge condition. Its propagator is found to be transverse with respect to both its four-momentum and the gauge direction. The propagator of the dynamical ψ_+ part of the free fermionic field is shown to be causal and to not contain instantaneous terms. The interaction Hamiltonian of QCD can be expressed in a form resembling that of covariant theory, except for additional instantaneous interactions which can be treated systematically. The renormalization factors are shown to be scalars and we find $Z_1 = Z_3$ at one loop order. The running coupling constant and QCD β function are also computed in the noncovariant light-cone gauge. Some comments on the relationship of our LF framework to the analytic effective charge and renormalization scheme defined by the pinch technique are made. LF quantization thus provides a consistent formulation of gauge theory, despite the fact that the hyperplanes $x^{\pm} = 0$ used to impose boundary conditions constitute characteristic surfaces of a hyperbolic partial differential equation.

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1 Introduction

The quantization of relativistic field theory at fixed light-front time $\tau = (t - z/c)/\sqrt{2}$, which was proposed by Dirac [1] half a century ago, has found important applications [2, 3, 4, 5] in both gauge theory and string theory [6]. The light-front (LF) quantization of QCD in its Hamiltonian form provides an alternative to lattice gauge theory for the computation of nonperturbative quantities such as the spectrum and the light-front Fock state wavefunctions of relativistic bound states [3]. LF variables have also found natural applications in other contexts, such as in the quantization of (super-) string theory and M-theory [6]. Light-front quantization has been employed in the nonabelian bosonization [7] of the field theory of N free Majorana fermions and was used in the demonstration of the asymptotic freedom of the Yang-Mills theory beta-function [8]. The requirement of the microcausality[9] implies that the LF framework is more appropriate for quantizing [10] the self-dual (chiral boson) scalar field.

Since LF coordinates are not related to the conventional coordinates by a finite Lorentz transformation, the descriptions of the same physical result may be different in the equal-time (*instant form*) and equal-LF-time (*front form*) formulations of the theory. This was in fact found to be the case in a recent study [11, 12] of some soluble two-dimensional gauge theory models, where it was also demonstrated that LF quantization is very economical in displaying the relevant degrees of freedom, leading directly to a physical Hilbert space. The corresponding Fock representation is boost independent since the *front form* has seven kinematical Poincaré generators [1], including Lorentz boost transformations, compared to only six in the *instant form* framework. LF-time-ordered perturbation theory is much more economical than equal-time-ordered perturbation theory, since only graphs with particles with positive LF momenta $p^+ = (p^0 + p^3)/\sqrt{2}$ appear. LF-time-ordered perturbation theory has also been applied [13, 14] to massive fields. It was used in the analysis of the evolution of deep inelastic structure functions [15] and the evolution of the distribution amplitudes which control hard exclusive processes in QCD [16].

It has been conventional to apply LF quantization to gauge theory in light-cone (l.c.) gauge $A^+ = A_- = (A^0 + A^3)/\sqrt{2} = 0$, since the transverse degrees of freedom of the gauge field can be immediately identified as the dynamical degrees of freedom, and ghost fields can be ignored in the quantum action of the nonabelian gauge theory [16, 17, 18]. The light-front (LF) quantization of quantum chromodynamics in l.c. gauge thus has a number of remarkable advantages, including explicit unitarity, a physical Fock expansion, and the complete absence of ghost degrees of freedom. In addition, the decoupling of gluons to propagators carrying high momenta and the absence of collinear divergences in irreducible diagrams in the l.c. gauge are important tools for proving the leading-twist factorization of soft and hard gluonic corrections in high momentum transfer inclusive and exclusive reactions [16]. On the negative side,

any noncovariant gauge brings in the breaking of manifest rotational invariance, instantaneous interactions, and, apparently, a more difficult renormalization procedure [17, 18, 19].

In this paper we will discuss the LF quantization of QCD gauge field theory in l.c. gauge employing the Dyson-Wick S-matrix expansion [20] based on LF-time-ordered products [21]. We shall first study the gauge-fixed quantum action of the theory on the LF. The LF Hamiltonian will then be constructed following the Dirac method [22, 23] which allows one to self-consistently identify the independent fields and their commutation relations in the presence of the l.c. gauge condition and other constraints.

The LF framework is a severely constrained dynamical theory with many second-class constraints. These can be eliminated by constructing Dirac brackets, and the theory can be quantized canonically by the correspondence principle in terms of a *reduced number* of independent fields. The commutation relations among the field operators are also found by the Dirac method, and they are used to obtain the momentum space expansions of the fields. For example, the nondynamical projection of the fermion field can be eliminated using a nonlocal constraint equation. The resulting propagator for the dynamical component $\psi_+(x)$ of the free fermionic field on the LF will be shown to be causal. The Fourier transform of the field ψ can be written in terms of a convenient form of the light-front free Dirac spinor. Next, the gauge-field quantization of the *massless* field in the l.c. gauge in the *front form* theory is studied following the Dirac procedure. Using the derived commutators, we find that LF quantized gauge theory simultaneously satisfies the covariant gauge condition $\partial \cdot A = 0$ as an operator condition as well as the light-cone gauge condition. The Fourier transform of the free theory gauge field and its propagator in momentum space then follow straightforwardly. The latter contains both covariant and noncovariant terms. The removal of the unphysical components of the gauge field results in [2, 24] tree-level instantaneous interaction terms which can be evaluated systematically (See Sections 5 and 6). The instantaneous light-cone gauge interactions of the light-front Hamiltonian are incorporated into nonperturbative approaches such as DLCQ [25].

The QCD interaction Hamiltonian is constructed in Section 5 where we restore in the expression the dependent components A_+ and ψ_-. It then takes a form close to that of covariant gauge theory without ghost terms, plus instantaneous interactions which are straightforward to handle in the Dyson-Wick perturbation theory.

The renormalization procedure in our framework is illustrated in Section 6 by considering the nonabelian YM gauge theory. The renormalization factors in our framework are demonstrated to be Lorentz scalars, independent of the direction of the gauge vector. The renormalized running coupling and $\beta-$function at one loop are also computed. The results are compared with those found [17, 18, 26] in the conventional l.c. gauge equal-time framework. Gluon self-energy coming from quark loops is also computed. A calculation of the electron-muon scattering amplitude in QED is used to

show the relevance of instantaneous interactions for recovering the Lorentz invariance. We recall that the Dyson-Wick expansion has been used [27] to renormalize two-dimensional scalar field theory on the LF with nonlocal interactions. The complete renormalization of QCD in our framework will be considered in a forthcoming paper.

2 QCD Action in Light-Cone Gauge

The LF coordinates are defined as $x^\mu = (x^+ = x_- = (x^0 + x^3)/\sqrt{2},\, x^- = x_+ = (x^0 - x^3)/\sqrt{2},\, x^\perp)$, where $x^\perp = (x^1, x^2) = (-x_1, -x_2)$ are the transverse coordinates and $\mu = -, +, 1, 2$. The coordinate $x^+ \equiv \tau$ will be taken as the LF time, while x^- is the longitudinal spatial coordinate. We can of course choose a convention where the role of x^+ and x^- interchanged. The equal-x^+ quantized theory already contains the information on the equal-x^- commutator [11, 12]. The LF components of any tensor, for example, the gauge field, are similarly defined, and the metric tensor $g_{\mu\nu}$ may be read from $A^\mu B_\mu = A^+B^- + A^-B^+ - A^\perp B^\perp$. Also k^+ indicates the longitudinal momentum, while $k^-$is the corresponding LF energy.

The quantum action of QCD in l.c. gauge is described in standard notation by the following Lagrangian density

$$\mathcal{L}_{QCD} = -\frac{1}{4}F^{a\mu\nu}F^a{}_{\mu\nu} + B^aA^a{}_- + \bar{c}^a\mathcal{D}^{ab}_- c^b + \bar{\psi}^i(i\gamma^\mu D^{ij}{}_\mu - m\delta^{ij})\psi^j \tag{1}$$

Here ψ^j is the quark field with color index $j = 1..N_c$ for an $SU(N_c)$ color group, $A^a{}_\mu$ the gluon field, $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a{}_\mu + gf^{abc}A^b{}_\mu A^c{}_\nu$ the field strength, $\mathcal{D}^{ac}{}_\mu = (\delta^{ac}\partial_\mu + gf^{abc}A^b{}_\mu)$, $D^{ij}{}_\mu\psi^j = (\delta^{ij}\partial_\mu - igA^a{}_\mu t^{aij})\psi^j$, $t^a \equiv \lambda^a/2$, $a = 1..(N_c{}^2 - 1)$, the gauge group index, and $\bar{c}^a, c^a$ are anti-commuting ghost fields. In writing the quantum action we introduce auxiliary Lagrange multiplier fields $B^a(x)$ and add to the Lagrangian the *linear* gauge-fixing term $(B^aA^a{}_-)$, which is a traditional procedure. In addition we are required to also add ghost terms such that the action (1) becomes invariant under BRS symmetry [28] transformations.

It is worth recalling the corresponding procedure for implementing a covariant gauge-fixing condition. For example, in Feynman gauge one adds the term $(B^a\partial^\mu A_{a\mu} + B^aB^a/2)$ to the Lagrangian. The quadratic B^aB^a term is allowed on dimensional considerations. However in the case of l.c. gauge, the auxiliary field B^a carries canonical dimension three and as such a quadratic term is not allowed in (1). We mention yet another example: the quantum action for constructing [10] the quantized theory of the self-dual scalar field (chiral boson) in two-dimensional space-time. One starts by adding the traditional linear term $B\partial_-\phi$ to the free scalar field Lagrangian. Its LF quantization can be performed without any violation of the principle of microcausality, in contrast to that occurs in the conventional treatment. The quantized theory is found to be trivial indicating that the traditional Lagrange multiplier method breaks

down at the quantum level. However, if we add to the theory an additional B^2 term, which is allowed on dimensional considerations, the LF quantization of the improved theory does produce a satisfactory description [10] of the quantized left- and right-movers. The theory also contains as a special case the well known Floreanini-Jackiw action [29], giving a plausible reason for the success of that model.

3 Spinor Field Propagator on the LF

For completeness and for fixing notation which will be needed latter, we shall include here the derivation of the spinor field propagator on the LF as given in Ref. [24]. The $\gamma^\pm$ defined by $\gamma^\pm = (\gamma^0 \pm \gamma^3)/\sqrt{2}$ satisfy $(\gamma^+)^2 = (\gamma^-)^2 = 0$, $\gamma^0\gamma^+ = \gamma^-\gamma^0$, $\gamma^+\gamma^- + \gamma^-\gamma^+ = 2I$, $\gamma^{+\dagger} = \gamma^-$ and it follows that $\Lambda^\pm = \frac{1}{2}\gamma^\mp\gamma^\pm = \frac{1}{\sqrt{2}}\gamma^0\gamma^\pm$ are hermitian projection operators: $(\Lambda^\pm)^2 = \Lambda^\pm$, $\Lambda^+\Lambda^- = \Lambda^-\Lambda^+ = 0$, $\gamma^0\Lambda^+ = \Lambda^-\gamma^0$. The spinor field on the LF [30] is decomposed naturally into $\pm$ projections $\psi_\pm = \Lambda^\pm\psi$ and $\psi = \psi_+ + \psi_-$, $\bar\psi = \psi^\dagger\gamma^0 = \bar\psi_+ + \bar\psi_-$, $\gamma^\pm\psi_\mp = 0$, $\psi^\dagger{}_\pm\gamma^\pm = 0$ *etc.*

The quark field term in LF coordinates reads

$$\begin{aligned}\bar\psi^i(i\gamma^\mu D^{ij}{}_\mu - m\delta^{ij})\psi^j &= i\sqrt{2}\bar\psi^i_+\gamma^0 D^{ij}{}_+\psi_+{}^j + \bar\psi^i_+(i\gamma^\perp D^{ij}{}_\perp - m\delta^{ij})\psi_-{}^j \\ &+ \bar\psi^i_-\left[i\sqrt{2}\gamma^0 D^{ij}{}_-\psi_-{}^j + (i\gamma^\perp D^{ij}{}_\perp - m\delta^{ij})\psi_+{}^j\right] \qquad (2)\end{aligned}$$

This shows that the minus components $\psi_-{}^j$ are in fact nondynamical (Lagrange multiplier) fields without kinetic terms. The variation of the action with respect to $\bar\psi^j{}_-$ and $\psi^j{}_-$ leads to the following gauge-covariant constraint equation

$$i\sqrt{2}\, D^{ij}{}_-\psi_-{}^j = -(i\gamma^0\gamma^\perp D^{ij}{}_\perp - m\gamma^0\delta^{ij})\psi_+{}^j, \qquad (3)$$

and its conjugate. The $\psi^j{}_-$ components may thus be eliminated in favor of the dynamical components ψ^j_+

$$\psi_-{}^j(x) = \frac{i}{\sqrt{2}}\left[U^{-1}(x)\frac{1}{\partial_-}U(x)\right]^{jk}(i\gamma^0\gamma^\perp D^{kl}{}_\perp - m\gamma^0\delta^{kl})\psi_+{}^l(x). \qquad (4)$$

Here, for a fixed τ, $U(x)$ is an $N_c \times N_c$ gauge matrix in the fundamental representation of $SU(N_c)$ and it satisfies

$$\partial_- U(x) = -ig\, U(x)\, A_-(x) \qquad (5)$$

where $U(x)$ is functional of $A_-(x)$ only and $A_- = A^a{}_-\lambda^a/2$. It has the formal solution

$$U(x^-, x^\perp) = U(x_0^-, x^\perp)\,\widetilde{\mathcal{P}}\, exp\left\{-ig\int_{x^-{}_0}^{x^-} dy^- A_-(y^-, x^\perp)\right\} \qquad (6)$$

where $\widetilde{\mathcal{P}}$ indicates the anti-path-ordering along the longitudinal direction x^-. U has a series expansion in the powers of the coupling constant. In the l.c. gauge with A_- set to zero, we find from (3)

$$i\sqrt{2}\,\psi_-{}^j(x) = -\frac{1}{\partial_-}\,(i\gamma^0\gamma^\perp D^{kl}{}_\perp - m\gamma^0\delta^{kl})\;\psi_+{}^l(x). \tag{7}$$

The free field propagator for the dynamical component ψ_+ is determined from the quadratic terms in the Lagrangian density:

$$i\sqrt{2}\psi_+{}^\dagger\partial_+\psi_+ + \psi_+{}^\dagger(i\gamma^0\gamma^\perp\partial_\perp - m\gamma^0)\psi_- \tag{8}$$

contained in (2) where the color index is suppressed. Here we have also the free field constraint equation $2i\partial_-\psi_- = (i\gamma^\perp\partial_\perp + m)\gamma^+\psi_+$ which determines the dependent field ψ_-. The equation of motion for the independent component ψ_+ is nonlocal in the longitudinal direction

$$\left[4\partial_+ + (m + i\gamma^\perp\partial_\perp)\gamma^-\frac{1}{\partial_-}(m + i\gamma^\perp\partial_\perp)\gamma^+\right]\psi_+ = 0. \tag{9}$$

The free field Hamiltonian formulation can be constructed by following the Dirac procedure [22, 23]. The constraint equation arises now as a second class constraint on the canonical phase space. The Dirac bracket which takes care of these constraints is easily constructed. The effective free LF Hamiltonian is found to be $\mathcal{H}^{lf} = -\bar{\psi}_+(i\gamma^\perp\partial_\perp - m)\psi_-$, and the canonical quantization performed by the correspondence of the Dirac brackets with the (anti-)commutators leads to the following nonvanishing *local* anti-commutation relation

$$\{\psi_+(\tau, x^-, x^\perp), \psi_+{}^\dagger(\tau, y^-, y^\perp)\} = \frac{1}{\sqrt{2}}\Lambda^+\delta(x^- - y^-)\delta^2(x^\perp - y^\perp). \tag{10}$$

on the LF in contrast to the non-local LF-commutator found, for example, for the scalar field or gauge field (Section 4). Eq. (10) was proposed earlier in Ref. [13]. The equation of motion (9) for ψ_+ is recovered as an Heisenberg equation of motion if we employ (10).

The propagator in momentum space may be derived by going over to the Fourier transform of $\psi(x)$ over the complete set of linearly independent plane wave solutions of the free Dirac equation, say, for the longitudinal momentum $p^+ > 0$. Such a set is spanned by $u^{(r)}(p)e^{-ip\cdot x}$ together with $v^{(r)}(p)e^{ip\cdot x}$ where $u^{(r)}(p)$ and

$$v^{(r)}(p) \equiv u^{(r)}(p)_c = C\gamma^{0T}u^{(r)}(p)^*$$

are linearly independent solutions of the free Dirac equation in momentum space: $(\gamma^\mu p_\mu - m)u(p) = 0$ and $p\cdot x = (p^-x^+ + p^+x^- - p^\perp x^\perp)$.

A useful form [31] of the solution for the four-spinors in the context of LF quantization is

$$u^{(r)}(p) = \frac{1}{(\sqrt{2}p^+ m)^{\frac{1}{2}}} \left[\sqrt{2}p^+\Lambda^+ + (m + \gamma^\perp p_\perp)\Lambda^-\right] \widetilde{u}^{(r)} \tag{11}$$

where the constant spinors $\widetilde{u}^{(r)}$ satisfy $\gamma^0\widetilde{u}^{(r)} = \widetilde{u}^{(r)}$ and $\Sigma_3\widetilde{u}^{(r)} = r\widetilde{u}^{(r)}$ with $\Sigma_3 = i\gamma^1\gamma^2$ and $r = \pm$. The normalization and the completeness relations are:

$$\bar{u}^{(r)}(p)u^{(s)}(p) = \delta_{rs} = -\bar{v}^{(r)}(p)v^{(s)}(p)$$

$$\sum_{r=\pm} u^{(r)}(p)\bar{u}^{(r)}(p) = \frac{(\not{p} + m)}{2m}$$

$$\sum_{r=\pm} v^{(r)}(p)\bar{v}^{(r)}(p) = \frac{(\not{p} - m)}{2m}$$

and C is the charge conjugation matrix [20]. The Λ^+ projection of (11) is by construction very simple, ${u^{(r)}}_+(p) = (\sqrt{2}p^+/m)^{\frac{1}{2}}(\Lambda^+\widetilde{u}^{(r)})$. The ${u^{(r)}}_+(p)$ are eigenstates of Σ_3 as well, while the $\widetilde{u}^{(r)}$ correspond to rest frame spinors when $\sqrt{2}p^\pm = m$.

The Fourier transform expansion may be written as

$$\psi(x) = \frac{1}{\sqrt{(2\pi)^3}} \sum_{r=\pm} \int d^2p^\perp dp^+\, \theta(p^+)\sqrt{\frac{m}{p^+}} \left[b^{(r)}(p)u^{(r)}(p)e^{-ip\cdot x} + d^{\dagger(r)}(p)v^{(r)}(p)e^{ip\cdot x}\right] \tag{12}$$

The presence of the factor $\theta(p^+)$ in (12) follows also from the considerations of the *covariant phase space* (or *LIPS*) *factor* which is found relevant [32] in the context of the analysis of the physical processes. For $\psi_+ \equiv \Lambda^+\psi$ we find

$$\psi_+(x) = \frac{(\sqrt{2})^{\frac{1}{2}}}{\sqrt{(2\pi)^3}} \sum_{r=\pm} \int d^2p^\perp dp^+\theta(p^+) \left[b^{(r)}(p)\widetilde{u}_+^{(r)}e^{-ip\cdot x} + {d^\dagger}^{(r)}(p)\widetilde{v}_+^{(r)}e^{ip\cdot x}\right], \tag{13}$$

where $\widetilde{u}$ and $\widetilde{v}$ are constant spinors, and the integrations are to be taken from $-\infty$ to ∞ not only for $p^\perp$ but also over p^+ which is very convenient when using generalized functions. It can be verified that the anti-commutation relations (10) are satisfied if the creation and the annihilation operators are assumed to satisfy the canonical anti-commutation relations, with the nonvanishing ones given by: $\{b^{(r)}(p), {b^\dagger}^{(s)}(p')\} = \{d^{(r)}(p), {d^\dagger}^{(s)}(p')\} = \delta_{rs}\delta(p^+ - p'^+)\delta^2(p^\perp - p'^\perp)$.

The free propagator for the independent component ψ_+ is easily derived using (13)

$$\begin{aligned}\left\langle 0\right| T(\psi_{+A}(x)\psi^\dagger_{+B}(0))\left|0\right\rangle &= \left\langle 0\right| \left[\theta(\tau)\psi_{+A}(x)\psi^\dagger_{+B}(0) - \theta(-\tau)\psi^\dagger_{+B}(0)\psi_{+A}(x)\right]\left|0\right\rangle \\ &= \frac{1}{\sqrt{2}}\frac{\Lambda^+_{AB}}{(2\pi)^3}\int d^2q^\perp dq^+\theta(q^+)\left[\theta(\tau)e^{-iqx} - \theta(-\tau)e^{iqx}\right]\end{aligned} \tag{14}$$

where $A, B = 1 \cdots 4$ label the spinor components. The only relevant differences, compared with the case of the scalar field, are, apart from the appearance of the projection operator, the absence of the factor $(1/2q^+)$ in the integrand of (14) and the negative sign of the second term in the fermionic case. The two effects, however, compensate, when one factors out the exponential, giving rise to the factor $[\theta(q^+) + \theta(-q^+)]$ which may be interpreted as unity in the distribution theory sense, parallel to what one finds in the derivation of the scalar field propagator on the LF. The straightforward use of the integral representation $2\pi i\,\theta(\tau)\,e^{-ip\tau} = \int d\lambda\, e^{(-i\lambda\tau)}/(p-\lambda-i\epsilon)$ of $\theta(\pm\tau)$, together with the standard manipulations in the second term to factor out the exponential, lead to

$$\Big\langle 0|T(\psi^i{}_+(x)\psi^{\dagger j}{}_+(0))|0\Big\rangle = \frac{i\delta^{ij}}{(2\pi)^4}\int d^4q\,\frac{\sqrt{2}q^+\,\Lambda^+}{(q^2-m^2+i\epsilon)}\,e^{-iq.x}, \tag{15}$$

where for convenience we have renamed the dummy integration variable λ as q^- and $d^4q = d^2q^\perp dq^+ dq^-$ with all integrations ranging from $-\infty$ to ∞. We have restored the color index as well. The propagator (15) is causal and contains no instantaneous term [13]. The integrand factor may also be expressed as $\approx\ [\Lambda^+(\not{q}+m)\Lambda^-/(q^2-m^2+i\epsilon)]\,\gamma^0$. One can verify that the propagator satisfies the equation for the Green's function corresponding to the equation of motion of ψ_+, Eq. (9).

4 Gauge Field Propagator in l.c. Gauge

The relevant quadratic terms in the Lagrangian density which determine the free gauge field propagators are

$$\frac{1}{2}\left[F^a{}_{+-}F^a{}_{+-} + 2F^a{}_{+\perp}F^a{}_{-\perp} - F^a{}_{12}F^a{}_{12}\right] + B^aA^a{}_- + \bar{c}^a\partial_- c^a. \tag{16}$$

We observe that in the *front form* framework, the fields $A^a{}_+$ as well as B^a have no kinetic terms, and they enter in the action as auxiliary multiplier fields. Also, since the ghost fields decouple, it is sufficient to study the free abelian gauge theory with the following action

$$\int d^2x^\perp dx^-\left\{\frac{1}{2}\Big[(F_{+-})^2 - (F_{12})^2 + 2F_{+\perp}F_{-\perp}\Big] + BA_-\right\} \tag{17}$$

where $F_{\mu\nu}$ stands for $(\partial_\mu A_\nu - \partial_\nu A_\mu)$ in the present section. The gauge field equations of motion are $\Box A_\mu = \partial_\mu(\partial\cdot A) - B\delta_\mu{}^-$, $A_- = 0$, $\mu = -, +, \perp$ and $\perp = 1, 2$, and as a consequence $\partial_- B = 0$. The canonical momenta following from (17) are $\pi^+ = 0$, $\pi_B = 0$, $\pi^\perp = F_{-\perp}$, and $\pi^- = F_{+-} = (\partial_+ A_- - \partial_- A_+)$ which indicates that we are dealing with a constrained dynamical system. The Dirac procedure will be followed in order to construct the self-consistent Hamiltonian theory which is required for performing canonical quantization. It also serves as a convenient framework to discuss

the Lorentz covariance properties of the theory. The canonical Hamiltonian density is

$$\mathcal{H}_c = \frac{1}{2}(\pi^-)^2 + \frac{1}{2}(F_{12})^2 - A_+(\partial_-\pi^- + \partial_\perp\pi^\perp) - BA_- \tag{18}$$

The primary constraints following from (17) are $\pi^+ \approx 0$, $\pi_B \approx 0$ and $\eta^\perp \equiv \pi^\perp - \partial_- A_\perp + \partial_\perp A_- \approx 0$, where $\approx$ stands for the *weak equality* relation. We now require the persistency in τ of these constraints employing the preliminary Hamiltonian, which is obtained by adding to the canonical Hamiltonian the primary constraints multiplied by the undetermined Lagrange multiplier fields u_+, $u_\perp$, and u_B. In order to obtain the Hamilton's equations of motion, we assume initially the standard Poisson brackets for all the dynamical variables present in (18).

We are then led to the following secondary constraints

$$\begin{aligned} \Phi \equiv \partial_-\pi^- + \partial_\perp\pi^\perp &\approx 0, \\ A_- &\approx 0 \end{aligned} \tag{19}$$

which are already present in (18) multiplied by Lagrange multiplier fields. Requiring also the persistency of Φ and A_- leads to another secondary constraint

$$\Psi \equiv \pi^- + \partial_- A_+ \approx 0. \tag{20}$$

The procedure stops at this stage, and no more constraints are seen to arise since further repetition leads to equations which would merely determine the multiplier fields.

Let us now analyze the nature of the phase space constraints. In spite of the gauge-fixing term introduced in the initial Lagrangian, there still remains on the canonical LF phase space a first class constraint $\pi_B \approx 0$. An inspection of the equations of motion shows that we may add to the set of constraints found above an additional external constraint $B \approx 0$. This would make the whole set of constraints in the theory second class. Dirac brackets satisfy the property such that we can set the above set of constraints as *strong* equality relations inside them. The equal-τ Dirac bracket $\{f(x), g(y)\}_D$ which carries this property is straightforward to construct. Hamilton's equations now employ the Dirac brackets rather than the Poisson ones. The phase space constraints $\pi^+ = 0$, $\eta^\perp = 0$, $A_- = 0$, $\Phi = 0$, $\Psi = 0$, $\pi_B = 0$, and $B = 0$ thus effectively eliminate all the canonical momenta from the theory. The surviving dynamical variables are $A_\perp$ while A_+ is a dependent variable which satisfies $\partial_-(\partial_- A_+ - \partial_\perp A_\perp) = 0$. The *reduced* Hamiltonian is found to be

$${H_0}^{LF} = \frac{1}{2}\int d^2x^\perp dx^- \left[(\partial_- A_+)^2 + \frac{1}{2}F_{\perp\perp'}F^{\perp\perp'}\right], \tag{21}$$

where we have retained the dependent variable A_+ for convenience.

The canonical quantization of the theory at equal-τ is performed *via* the correspondence $i\{f(x), g(y)\}_D \to [f(x), g(y)]$ where the latter indicates the commutators among the corresponding field operators. The equal-LF-time commutators of the transverse components of the gauge field are found to be

$$\left[A_\perp(\tau, x^-, x^\perp), A_{\perp'}(\tau, y^-, y^\perp)\right] = i\delta_{\perp\perp'} K(x, y)$$

where $K(x, y) = -(1/4)\epsilon(x^- - y^-)\delta^2(x^\perp - y^\perp)$. The commutators are nonlocal in the longitudinal coordinate but there is no violation [9] of the microcausality principle on the LF. At equal LF-time, $(x - y)^2 = -(x^\perp - y^\perp)^2 < 0$, is nonvanishing for $x^\perp \neq y^\perp$ but $\delta^2(x^\perp - y^\perp)$ vanishes for such spacelike separation. The commutators of the transverse components of the gauge fields are physical, having the same form as the commutators of scalar fields in the *front form* theory.

The Heisenberg equations of motion employing (21) lead to the Lagrange equations for the independent fields which assures us of the self-consistency [22] of the *front form* Hamiltonian theory in the l.c. gauge. We also find that the commutators of A_+ are identical to the ones obtained by substituting A_+ by $(\partial_\perp/\partial_-)A_\perp$. This is a consequence of the definition of the Dirac bracket itself and manipulations on it with the partial derivatives. Hence in the l.c. gauge on the LF we obtain the Lorentz condition $\partial \cdot A = 0$ as an operator equation as well.

We now turn our attention to the momentum space description. The LF commutators of the gauge field may be realized in momentum space by the following Fourier transform

$$A_\perp(x) = \frac{1}{\sqrt{(2\pi)^3}} \int d^2k^\perp dk^+ \frac{\theta(k^+)}{\sqrt{2k^+}} \left[a_\perp(\tau, k^+, k^\perp)e^{-i\bar{k}\cdot x} + a^\dagger_\perp(\tau, k^+, k^\perp)e^{i\bar{k}\cdot x}\right] \quad (22)$$

where $\bar{k} \cdot x = k^+x^- - k^\perp x^\perp$ and $a_\perp$, $a^\dagger_\perp$ are operators which satisfy the equal-τ canonical commutation relations with the nonvanishing ones given by $[a_\perp(\tau, k^+, k^\perp), a^\dagger{}_{\perp'}(\tau, k'^+, k'^\perp)] = \delta_{\perp\perp'}\, \delta^3(k - k')$ where $\delta^3(k - k') \equiv \delta(k^+ - k'^+)\, \delta^2(k^\perp - k^{\perp'})$. The Heisenberg equation of motion for $A_\perp(x)$ then leads to $a_\perp(\tau, k^+, k^\perp) = a_\perp(k^+, k^\perp)$ $exp(-ik^-x^+)$ where k^- is defined through the dispersion relation, $2k^-k^+ = k^\perp k^\perp$. The operators $a_\perp(k^+, k^\perp)$ and $a^\dagger{}_\perp(k^+, k^\perp)$ are thus associated with the *massless* gauge field quanta. The Fourier transform (22) may then be rewritten as

$$A_\perp(x) = \frac{1}{\sqrt{(2\pi)^3}} \int d^2k^\perp dk^+ \frac{\theta(k^+)}{\sqrt{2k^+}} \left[a_\perp(k^+, k^\perp)e^{-ik\cdot x} + a^\dagger_\perp(k^+, k^\perp)e^{ik\cdot x}\right] \quad (23)$$

where $k \cdot x = (k^-x^+ + k^+x^- + k_\perp x^\perp)$ and $k^\mu k_\mu = 0$. The Fourier transform (23) is of the typical form of the *front form* theory where the bosonic fields satisfy nonlocal LF commutation relation; it does not carry in it any explicit information on the mass

of the field. For example, the massive scalar field quantized on the LF also has a similar Fourier transform, but the dispersion relation is that of a massive field. The commutators of A_+ are realized if we write for its Fourier transform

$$A_+(x) = \frac{1}{\sqrt{(2\pi)^3}} \int d^2k^\perp dk^+ \, \frac{\theta(k^+)}{\sqrt{2k^+}} \left[a_+(k^+, k^\perp) e^{-ik\cdot x} + a_+^\dagger(k^+, k^\perp) e^{ik\cdot x} \right] \tag{24}$$

where $a_+(k)$ is determined from $[\, k^+ a_+(k) + k^\perp a_\perp(k) \,] = 0$.

Knowing the Fourier transforms of the field operators, the free propagators in momentum space may be derived from the definition of the propagator of the quantized fields. We find

$$\begin{aligned} \left\langle 0 \right| T(A^a{}_\perp(x) A^b{}_\perp(0)) \left| 0 \right\rangle &= \left\langle 0 \right| \left[\theta(\tau) A^a{}_\perp(x) A^b{}_\perp(0) + \theta(-\tau) A^b{}_\perp(0) A^a{}_\perp(x) \right] \left| 0 \right\rangle \\ &= \frac{i\delta^{ab}}{(2\pi)^4} \int d^4k \; e^{-ik\cdot x} \; \frac{-g_{\perp\perp'}}{k^2 + i\epsilon} \end{aligned} \tag{25}$$

where we have restored the gauge index a. In view of (24) we may write the gauge field propagator in the l.c. gauge in the following convenient form

$$\left\langle 0 \right| T(A^a{}_\mu(x) A^b{}_\nu(0)) \left| 0 \right\rangle = \frac{i\delta^{ab}}{(2\pi)^4} \int d^4k \; e^{-ik\cdot x} \; \frac{D_{\mu\nu}(k)}{k^2 + i\epsilon} \tag{26}$$

where we have defined

$$D_{\mu\nu}(k) = D_{\nu\mu}(k) = -g_{\mu\nu} + \frac{n_\mu k_\nu + n_\nu k_\mu}{(n\cdot k)} - \frac{k^2}{(n\cdot k)^2} \, n_\mu n_\nu . \tag{27}$$

Here n_μ is a null four-vector, gauge direction, whose components are chosen to be $n_\mu = \delta_\mu{}^+$, $n^\mu = \delta^\mu{}_-$. We note that

$$\begin{aligned} D_{\mu\lambda}(k) D^\lambda{}_\nu(k) = D_{\mu\perp}(k) D^\perp{}_\nu(k) &= -D_{\mu\nu}(k), \\ k^\mu D_{\mu\nu}(k) = 0, \qquad & n^\mu D_{\mu\nu}(k) \equiv D_{-\nu}(k) = 0, \\ D_{\lambda\mu}(q) \, D^{\mu\nu}(k) \, D_{\nu\rho}(q') &= -D_{\lambda\mu}(q) D^{\mu\rho}(q'). \end{aligned} \tag{28}$$

The property that the gauge field propagator $i\, D_{\mu\nu}(k)/(k^2 + i\epsilon)$ is transverse not only to the gauge direction n_μ but also to k_μ, *i.e.*, it is doubly transverse, leads to appreciable simplifications in the computations in QCD as is illustrated below. In a sense our gauge propagator corresponds to the form used in Landau gauge, but here it is derived in the context of the noncovariant l.c. gauge. As usual with the non-covariant gauges, the propagator contains a non-covariant piece added to the covariant (Feynman gauge) propagator. It differs from the propagators derived [17, 18, 26] in equal-time quantized l.c. gauge QCD. The form (26) of the propagator reminds us of

the rules, in the context of the old-fashioned perturbation theory, laid down in Ref. [16] a long time ago, in the context of LF quantization. We will comment in Section 6 on the problem of handling the singularity near $(n \cdot k) \approx 0$ present in the propagator.

We can introduce the operators $b_{(\perp)}$ and $b^{\dagger}{}_{(\perp)}$, $(\perp) = (1), (2)$ representing the two independent states of transverse polarizations of a massless photon. They are assumed to obey the standard canonical commutation relations $[b_{(\perp)}(k^+, k^\perp), b^{\dagger}{}_{(\perp')}(k'^+, k'^\perp)]$ $= \delta_{(\perp)(\perp')}\ \delta^3(k - k')$. We write $a^\perp(k) = \sum_{(\perp')} E^\perp{}_{(\perp')}(k)\ b_{(\perp')}(k)$ where the $E_{(\perp)}{}^\mu(k)$ indicate the two independent polarization four-vectors. A convenient set may be chosen to be

$$E_{(\perp)}{}^\mu(k) = E^{(\perp)\mu}(k) = -\,D^\mu_\perp(k) \tag{29}$$

which have the property

$$\sum_{(\perp)=1,2} E^{(\perp)}{}_\mu(k) E^{(\perp)}{}_\nu(k) = D_{\mu\nu}(k), \qquad\qquad g^{\mu\nu} E^{(\perp)}{}_\mu(k) E^{(\perp')}{}_\nu(k) = g^{\perp\perp'} \tag{30}$$

$$k^\mu E^{(\perp)}_\mu(k) = 0, \qquad\qquad n^\mu E^{(\perp)}_\mu \equiv E^{(\perp)}_- = 0 \tag{31}$$

The Fourier transform of the gauge field may then be expressed in the standard form

$$A^{\mu a}(x) = \frac{1}{\sqrt{(2\pi)^3}} \int d^2k^\perp dk^+ \frac{\theta(k^+)}{\sqrt{2k^+}} \sum_{(\perp)} E_{(\perp)}{}^\mu(k) \left[b^a_{(\perp)}(k^+, k^\perp) e^{-ik\cdot x} + b^{\dagger a}_{(\perp)}(k^+, k^\perp) e^{ik\cdot x} \right] \tag{32}$$

where the l.c. gauge $A^a_- = 0$, along with the Lorentz condition, is already incorporated in it. The momentum space expressions of LF energy and momentum confirm the interpretation of $b_{(\perp)}$ and $b^{\dagger}{}_{(\perp)}$ as the Fock space operators of annihilation and creation of massless transverse gauge field quanta. Only the physical transverse degrees of freedom appear in the gauge field expansion.

5 The QCD Hamiltonian in l.c. Gauge

The Dyson-Wick perturbation theory expansion in the interaction representation requires that we separate the full Hamiltonian into the free theory component, discussed in Sections 3 and 4, and an interaction coupling-constant-dependent piece.

The equations of motion following from (1) result in

$$2i\partial_- \psi^i{}_- = 2i\partial_- \widetilde{\psi}^i_- + g\gamma^\perp A_\perp{}^a (t^a)^{ij} \gamma^+ \psi^j{}_+ \tag{33}$$

and

$$2i\partial_+ \psi^i{}_+ = (i\gamma^\perp \partial_\perp + m)\gamma^- \psi^i{}_- + g\gamma^\perp A_\perp{}^a (t^a)^{ij} \gamma^- \psi^j{}_- - 2g A_+{}^a (t^a)^{ij} \psi^j{}_+, \tag{34}$$

along with

$$\partial_-(\partial_- A_+{}^a - \partial_- \widetilde{A}^a_+) = -g f_{abc} A_\perp{}^b \partial_- A^{\perp c} + g\bar{\psi}^i \gamma^+ (t^a)^{ij} \psi^j \tag{35}$$

where we define [2] $\widetilde{A}^a_+$ and $\widetilde{\psi}^i_-$ by $\partial_- \widetilde{A}^a_+ = \partial_\perp A_\perp{}^a$ and $2i\partial_- \widetilde{\psi}^i_- = (i\gamma^\perp \partial_\perp + m)\gamma^+ \psi^i{}_+$ respectively. The combination $(\psi^i{}_+ + \widetilde{\psi}^i_-)$, when $g = 0$, satisfies the free Dirac equation. Hence we can separate straightforwardly the free and the interaction parts of the Lagrangian; the former coinciding with ones employed in Sections 3 and 4.

The interaction Hamiltonian in the l.c. gauge, $A^a_- = 0$, then follows: [2]

$$\begin{aligned}
\mathcal{H}_{int} = -\mathcal{L}_{int} \;=\; & -g\, \bar{\psi}^i \gamma^\mu\, (t^a)^{ij}\, \psi^j\, A^a_\mu \\
& + \frac{g}{2}\, f^{abc}\, (\partial_\mu A^a{}_\nu - \partial_\nu A^a{}_\mu) A^{b\mu} A^{c\nu} \\
& + \frac{g^2}{4}\, f^{abc} f^{ade} A_{b\mu} A^{d\mu} A_{c\nu} A^{e\nu} \\
& - \frac{g^2}{2}\, \bar{\psi}^i \gamma^+\, \gamma^\mu\, A^a_\mu\, (t^a)^{ij}\, \frac{1}{i\partial_-}\, \gamma^\nu\, A^b_\nu\, (t^b)^{jk}\, \psi^k \\
& - \frac{g^2}{2}\, (\frac{1}{i\partial_-}\, j^+_a)\, (\frac{1}{i\partial_-}\, j^+_a)
\end{aligned} \tag{36}$$

where

$$j^+_a = \bar{\psi}^i \gamma^+ (t_a)^{ij} \psi^j + f_{abc} (\partial_- A_{b\mu}) A^{c\mu} \tag{37}$$

Here ψ^i and A_μ obey free field equations of motion and their Fourier transforms are given in (12) and (32). A sum over distinct quark and lepton flavors (in QED), not written explicitly, is understood in (37) and (36).

The perturbation theory expansion in the interaction representation, where we time-order with respect to the LF time τ, can now be built following the Dyson-Wick [20] procedure. There are no ghost interaction terms to consider. The instantaneous interaction contributions (the last two terms in (36)) can be dealt with systematically. Such terms are also required in abelian QED theory, obtained by suppressing in the above interaction the additional terms of nonabelian theory. For example, the tree level *seagull* term dominates the classical Thomson formulae for the scattering at the vanishingly small photon energies. The instantaneous counter terms also serve to restore the manifest Lorentz invariance, which was broken by the use of noncovariant l.c. gauge and the noncovariant propagator. The information on the l.c. gauge is encoded in the remarkable properties of the gauge field propagator in the LF framework. The vertices in momentum space, following from the Fourier transforms given in Sections 3 and 4, and required for the illustrations below are summarized in the Appendix.

6 Illustrations

6.1 Electron-Muon Scattering

The contributions to the matrix element from the mediation of the gauge field is

$$-e^2\,[\bar{u}_e(p_1')\gamma^\mu u_e(p_1)\;\bar{u}_\mu(p_2')\gamma^\nu u_\mu(p_2)]\;\;\frac{i\,D_{\mu\nu}(q)}{q^2+i\epsilon}. \tag{38}$$

where $q=-p_1'+p_1=p_2'-p_2$. Using the mass-shell conditions for the external lines it reduces to

$$-i\,e^2\left[\bar{u}_e(p_1')\gamma^\mu u_e(p_1)\;\bar{u}_\mu(p_2')\gamma^\nu u_\mu(p_2)\;\;\frac{-g_{\mu\nu}}{q^2+i\epsilon}-\frac{1}{q^{+2}}\bar{u}_e(p_1')\gamma^+ u_e(p_1)\;\bar{u}_\mu(p_2')\gamma^+ u_\mu(p_2)\right]. \tag{39}$$

The second term here originates from the noncovariant terms in the gauge propagator. It is easily shown to be compensated by the instantaneous contribution to the matrix element deriving from the corresponding last term in (36), of abelian QED theory. The familiar covariant expression for the matrix element is then recovered.

6.2 The β-Function in Yang-Mills Theory

In this section we will illustrate the renormalization procedure in LF-quantized l.c. gauge QCD by an explicit computation of the one-loop effective running coupling constant, for simplicity, in the pure nonabelian Yang-Mills theory. As we shall show, all noncovariant terms drop out, so that the renormalization factors are scalars. The renormalization of the theory then becomes straightforward. Gross and Wilczek [33] and Politzer [34] computed the β-function in QCD from the gluonic vertex in the conventional theory. The corresponding LF computation becomes simpler because the gauge propagator in l.c. gauge is transverse with respect to both k^μ and the gauge direction n^μ, and ghost fields are absent.

Gluon Self-energy corrections:

The propagator modification is given by

$$\delta_{ab}\frac{iD_{\lambda\delta}(q)}{q^2}+\delta_{aa'}\frac{iD_{\lambda\mu}(q)}{q^2}\,\Pi^{\mu\nu}_{a'b'}(q)\,\delta_{b'b}\frac{iD_{\nu\delta}(q)}{q^2}+\cdots \tag{40}$$

The contribution to the gluon polarization tensor $\Pi^{\mu\nu}_{ab}(q)$ coming solely from the three-gluon interaction is (Fig. 1a)

$$\Pi^{\mu\nu}_{ab}(q)\;\;=\;\;\int\frac{d^dk}{(2\pi)^d}\,\frac{1}{2}\,(-g_d)^2\,f_{adc}\,f_{bcd}\,F^{\mu\alpha\beta}(-q,-k,k+q)$$

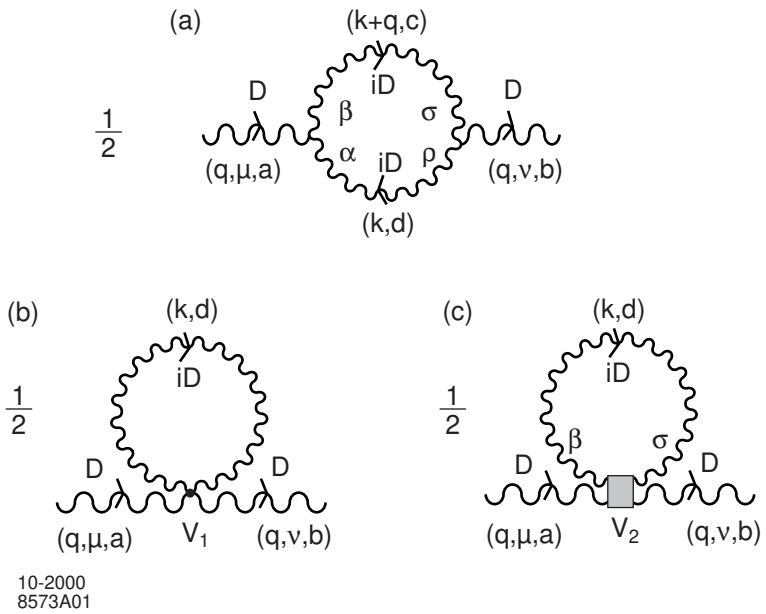

Figure 1: Yang-Mills self-energy to one loop. (a) Gluon self-energy diagram $\Pi^{\mu\nu}_{ab}(q)$; (b) tadpole diagram containing vertex V_1, vanishing in dimensional regularization; (c) non-vanishing tadpole diagram containing vertex V_2.

$$\begin{aligned} & F^{\nu\sigma\rho}(q,-k-q,k)\,\frac{iD_{\beta\sigma}(k+q)}{(k+q)^2}\,\frac{iD_{\alpha\rho}(k)}{k^2} \\ = \; & \frac{1}{2}\,g^2\,\delta_{ab}\,C_A\,\Pi^{\mu\nu}(q). \end{aligned} \tag{41}$$

Here associated to the three outgoing momenta, p_1^λ, p_2^μ, p_3^ν, satisfying $(p_1+p_2+p_3)^\mu = 0$, we define

$$F_{\lambda\mu\nu}(p_1,p_2,p_3) = (p_1-p_2)_\nu g_{\lambda\mu}+(p_2-p_3)_\lambda g_{\mu\nu}+(p_3-p_1)_\mu g_{\nu\lambda} = -F_{\lambda\nu\mu}(p_1,p_3,p_2), \tag{42}$$

use $f_{adc}f_{bcd} = -C_A\delta_{ab}$ and write

$$\Pi^{\mu\nu}(q) = \mu^{4-d}\int \frac{d^dk}{(2\pi)^d}\frac{1}{[k^2+i0]\,[(k+q)^2+i0]}\;I^{\mu\nu}(q,k), \tag{43}$$

with

$$\begin{aligned} I^{\mu\nu}(q,k) \;=\; & [-(2k+q)^\mu g^{\alpha\beta}+(k-q)^\beta g^{\mu\alpha}+(2q+k)^\alpha g^{\mu\beta}]D_{\alpha\rho}(k) \\ & [-(2k+q)^\nu g^{\rho\sigma}+(k-q)^\sigma g^{\nu\rho}+(2q+k)^\rho g^{\nu\sigma}]D_{\sigma\beta}(k+q). \end{aligned} \tag{44}$$

The dimensionless coupling is indicated by g while $g_d = (\mu)^{\frac{4-d}{2}}g$ and μ indicates the mass parameter associated with the dimensional regularization which we will be adopting.

We first note that every internal gluon line carries a factor $D_{\mu\nu}$, and the polarization vector of an external gluon is $E^{\mu}_{(\perp)} = -D^{\mu}_{\perp}$. The object of interest relevant in the renormalization of the theory under consideration is clearly the combination $D_{\lambda\mu}(q)\,\Pi^{\mu\nu}_{ab}(q)\,D_{\nu\delta}(q)$. We may therefore use the transversity properties of $D^{\mu\nu}(q)$ to simplify the original expression and consider instead the following reduced expression for $I^{\mu\nu}$ in the integrand

$$\begin{aligned} I^{\mu\nu}(q,k) &= [-(2k+q)^{\mu}g^{\alpha\beta} - 2q^{\beta}g^{\mu\alpha} + 2q^{\alpha}g^{\mu\beta}]D_{\alpha\rho}(k) \\ &\quad [-(2k+q)^{\nu}g^{\rho\sigma} - 2q^{\sigma}g^{\nu\rho} + 2q^{\rho}g^{\nu\sigma}]D_{\sigma\beta}(k+q). \end{aligned} \tag{45}$$

Explicitly

$$\begin{aligned} I^{++} &= 2(2k+q)^{+}(2k+q)^{+} \\ I^{+\perp} &= I^{\perp+} = 2(2k+q)^{+}(2k+q)^{\perp} + 2(2k+q)^{+}\left[\frac{1}{(k+q)^{+}} - \frac{1}{k^{+}}\right]S_{\perp} \\ I^{\perp\perp'} &= 2\,(2k+q)^{\perp}(2k+q)^{\perp'} \\ &\quad -2\,[(2k+q)_{\perp}S_{\perp'} + (2k+q)_{\perp'}S_{\perp}]\,\left[\frac{1}{(k+q)^{+}} - \frac{1}{k^{+}}\right] \\ &\quad -4\,g^{\perp\perp'}\,S_{\perp}S_{\perp}\left[\frac{1}{(k+q)^{+2}} + \frac{1}{k^{+2}}\right] \\ &\quad -8\,S_{\perp}S_{\perp'}\,\frac{1}{k^{+}\,(k+q)^{+}} \end{aligned} \tag{46}$$

where

$$\begin{aligned} S_{\mu} \equiv S_{\mu}(k,q) &= (k_{\mu}q^{+} - q_{\mu}k^{+}) \\ &= -S_{\mu}(q,k) = S_{\mu}(k+aq,q) = S_{\mu}(k,q+bk) \end{aligned} \tag{47}$$

Here the properties given in (28), like $D^{\perp}_{\rho}(k) = [-\delta^{\perp}_{\rho} + (k^{\perp}/k^{+})\delta^{+}_{\rho}]$, $D^{\mu\nu}(k)D_{\mu\nu}(k) = 2$, $D_{\lambda\mu}(q)D^{\mu}{}_{\perp}(k) = -D_{\lambda\perp}(q)$ were used to simplify the expressions.

In the renormalization procedure we need to isolate the divergent terms in the matrix element. In order to compute the integrals we will employ the dimensional regularization together with the Mandelstam [35] and Leibbrandt [36] prescription in the l.c. gauge, which was derived [17] also in the context of equal-time canonical quantization, to treat the *spurious* singularities occurring in the noncovariant part of the gauge field propagator. We remark that the LF coordinates are the natural ones to use in *front form* theory. In two dimensional massless theory on the LF, for example, the causal prescription for the $k^2 \approx 0$ singularity in $1/k^2 \equiv 1/(2k^{+}k^{-})$ is naturally associated with the choice given by the causal ML prescription for the $1/k^{+}$ singularity. It is hence suggested, since we do continuation in d, that in the LF framework the dimensional regularization plus ML prescription is mathematically

sound procedure apart from being very convenient. The ML prescription is often written as

$$\frac{1}{q\cdot n} = \lim_{\epsilon\to 0}\frac{(q\cdot n^*)}{(q\cdot n)(q\cdot n^*)+i\epsilon} \tag{48}$$

where $\epsilon \to 0+$ and the light-like four-vector n^*_μ represents the dual of n_μ with the components given by, $n^*_\mu = \delta^-_\mu$. We recall that such a pair of null vectors, n_μ and n^*_μ, arise quite naturally in the LF framework, for example, in the LF quantized QCD in covariant gauge [24], when we define[§] the linearly independent set of four gauge field polarization vectors. Unlike the *principal value* prescription, the n^*_μ- prescription is consistent with both Wick rotation and power counting [18, 17].

The divergent part of $\Pi^{\mu\nu}$ may be computed straightforwardly employing the available list of integrals [18, 17]. We find

$$\begin{aligned} div\,\Pi^{++} &= \frac{2}{3}\,q^{+2}\,I^{div} \\ div\,\Pi^{+\perp} &= -\frac{10}{3}\,q^+q^\perp\,I^{div} \\ div\,\Pi^{\perp\perp'} &= 2\left[\frac{11}{3}(q^2g^{\perp\perp'}-q^\perp q^{\perp'})-8\,q^+q^-g^{\perp\perp'}\right]\,I^{div} \end{aligned} \tag{49}$$

Here $(2\pi)^4 I^{div} = 2i\pi^2/(4-d) \to i\pi^2(2/\epsilon)$, with $d=(4-\epsilon)$, $\epsilon\to 0+$, is the pole term in the divergent integral

$$(2\pi)^4\,\mu^{4-d}\int\frac{d^dk}{(2\pi)^d}\,\frac{1}{k^2(k-q)^2} = i\pi^2\,\left[N_\epsilon - ln\frac{-q^2}{\mu^2}+\cdots\right]+o(\epsilon) \tag{50}$$

where

$$N_\epsilon = \left[\frac{2}{\epsilon}-\gamma_E+ln(4\pi)\right]$$

For brevity we indicate sometimes only the pole term I^{div}. The expressions of Π^{++}, $\Pi^{+\perp}$ agree with the corresponding expressions computed in Ref. [18], in the conventional l.c. gauge QCD, where a different gluon propagator is used. The expression for $\Pi^{\perp\perp'}$ is found to be different. As a consequence in the LF quantized theory we find an additional (noncovariant) term in

$$D_{\lambda\mu}(q)\,\Pi^{\mu\nu}_{ab}(q)\,D_{\nu\delta}(q) = \frac{g^2}{16\pi^2}C_A\delta_{ab}\,(-\frac{11}{3}q^2+8q^+q^-)\;i\,\left[N_\epsilon - ln\frac{-q^2}{\mu^2}+\cdots\right]\,D_{\lambda\delta}(q) \tag{51}$$

[§] $e^{(+)} = (1,\vec{k}/k^0)/\sqrt{2}$, $e^{(-)} = (1,-\vec{k}/k^0)/\sqrt{2}$, $e^{(1)} = (0,\vec{\epsilon}(k;1))$, $e^{(2)} = (0,\vec{\epsilon}(k;2))$ where $\vec{\epsilon}(k;1)$, $\vec{\epsilon}(k;2)$ and $\vec{k}/|\vec{k}|$ constitute the usual orthonormal set of three-vectors. In l.c. gauge $n\cdot n^* = 1$, $n\cdot n = n^*\cdot n^* = 0$ and associated to any four-vector q_μ we may define the four vectors $q_{||}$ and $q_{(\perp)}$ by $q_{||_\mu}\,(n\cdot n^*) = (n^*\cdot q\,n_\mu + n\cdot q\,n^*_\mu)$ and $q_{(\perp)_\mu} = q_\mu - q_{||_\mu}$.

where we have written also explicitly the finite $ln(-q^2/\mu^2)$ part accompanying the pole term, which will be required below for the computation of the running gauge coupling constant. It is worth remarking that, in view of the properties of $D_{\mu\nu}$, the computation of (51) does not require the evaluation of components other than those given in (49). We note also that

$$q_\nu I^{\mu\nu} = 2\,(q^2 + 2k.q)\ \left[(2k+q)^\mu + (q^\beta g^{\mu\alpha} - q^\alpha g^{\mu\beta})\right]\ D^\rho{}_\beta(k+q) D_{\rho\alpha}(k). \tag{52}$$

The corresponding divergent part is shown to be

$$div\ q_\nu\ \Pi^{\nu\mu}(q) = -8\, q^- q^{+2}\, D^\mu{}_+(q)\ I^{div}. \tag{53}$$

which allows us to compute $div\ \Pi^{-+}(q) = \ (\frac{10}{3}q^2 - \frac{22}{3}q^+q^-)\ I^{div}$ by setting $\mu = +$.

The result (51) obtained here is different from the earlier l.c. gauge computations [18] in the conventional framework. However, the noncovariant piece, $8q^+q^-$, in (51) is compensated by the contributions coming from the *tadpole* graphs.

There are two *tadpole* graphs (Figs. 1b and 1c) to be considered. The one associated with the four-gluon coupling V_1 gives a vanishing result, but the contribution coming from the instantaneous interaction $V_2(p_1, p_2, p_3, p_4)$ is found to be nonvanishing in dimensional regularization due to the momentum dependence of the vertex itself. The divergent part of the matrix element is easily reduced to

$$\begin{aligned} & \mu^{4-d} \int \frac{d^dk}{(2\pi)^d}\, \frac{1}{2}(ig^2) C_A \delta_{ab} \left[-(\frac{k^+ - q^+}{k^+ + q^+})^2 - (\frac{k^+ + q^+}{k^+ - q^+})^2 \right] \frac{iD^{\mu\nu}(k)}{k^2} D_{\lambda\mu}(q) D_{\nu\delta}(q) \\ = \ & \frac{g^2}{16\pi^2}\, C_A\, \delta_{ab}\, (-8q^+q^-)\ i\ \left[N_\epsilon - ln\frac{-q^2}{\mu^2} + \cdots \right]\ D_{\lambda\delta}(q) \end{aligned} \tag{54}$$

Here we made use of the useful identity $D_{\lambda\mu}(q)\, D^{\mu\nu}(k)\, D_{\nu\delta}(q) = D_{\nu\delta}(q)$ to arrive at the second line. The net divergent part of the gluon self-energy correction is covariant and found to be

$$D_{\lambda\mu}(q)\, \Pi^{\mu\nu}_{ab}(q)\, D_{\nu\delta}(q) = \frac{g^2}{16\pi^2} C_A \delta_{ab}\, (-\frac{11}{3}q^2)\ i\ \left[N_\epsilon - ln\frac{-q^2}{\mu^2} + \cdots \right]\ D_{\lambda\delta}(q). \tag{55}$$

which is ($\propto q^2$). The vanishing gluon mass hence remains unaltered due to the one-loop gluon self-energy correction.

The multiplicative renormalization scalar factor Z_3 which corrects the gluon propagator is defined by

$$Z_3\ \delta_{ab} \frac{iD_{\lambda\delta}(q)}{q^2} = \delta_{ab} \frac{iD_{\lambda\delta}(q)}{q^2} + \delta_{aa'} \frac{iD_{\lambda\mu}(q)}{q^2}\, \Pi^{\mu\nu}_{a'b'}(q)\, \delta_{b'b} \frac{iD_{\nu\delta}(q)}{q^2} + \cdots \tag{56}$$

and we find

$$Z_3 = 1 + \frac{g^2}{16\pi^2}\, C_A\, \frac{11}{3}\ \left[N_\epsilon - ln\frac{-q^2}{\mu^2} + \cdots \right] + \cdots \tag{57}$$

Vertex corrections:

In pure Yang-Mills theory the gluon vertex corrections to one-loop arise from the three-gluon interaction alone, the triangle diagram, (Fig. 2a) and from the two types of *swordfish* graphs (Figs. 2b and 2c) in which one of the two vertices carries a four-gluon interaction, which may be of type V_1 or type V_2, while the other one contains a three-gluon interaction. The complete vertex to order g^3 is written as

$$-gf_{abc}\mathcal{F}_{\lambda\mu\nu} = -gf_{abc}\,[F_{\lambda\mu\nu} + \Delta_{\lambda\mu\nu}](p_1,p_2,p_3) = -gf_{abc}\,F_{\lambda\mu\nu}(p_1,p_2,p_3)\,(1+\bar{\Delta})\ . \quad (58)$$

Consider first the triangle diagram. Since, as remarked before, each gluon line carries with it a factor of D we will simplify the expressions right from the start making use also of the presence of the factor $D_{\lambda'\lambda}(p_1)D_{\mu'\mu}(p_2)D_{\nu'\nu}(p_3)$, coming from the external gluon lines. The matrix element for the one loop correction to order g^2 is written as

$$(-g_d^2)\,(-\frac{1}{2}C_A\,f^{abc})\,(i)^3\ T^{\lambda\mu\nu}(p_1,p_2,p_3) \quad (59)$$

where we have use $f_{aa'b'}f_{bb'c'}f_{cc'a'} = (C_A/2)f_{abc}$. A factor of i comes from each of the gluon propagators and the expression for $T^{\lambda\mu\nu}(p_1,p_2,p_3)$ is given by (Fig. 2a)

$$\int\frac{d^dq}{(2\pi)^d}\qquad \frac{D_{\alpha\alpha'}(q)D_{\beta\beta'}(k)D_{\gamma\gamma'}(l)}{(q^2+i\epsilon)(k^2+i\epsilon)(l^2+i\epsilon)}[(p_1-q)^\beta g^{\alpha\lambda} + (q-k)^\lambda g^{\alpha\beta} + (k-p_1)^\alpha g^{\lambda\beta}]$$
$$[(-q-p_2)^\gamma g^{\alpha'\mu} + (p_2-l)^{\alpha'} g^{\gamma\mu} + (l+q)^\mu g^{\alpha'\gamma}]$$
$$[(-k+l)^\nu g^{\beta'\gamma'} + (-l-p_3)^{\beta'} g^{\nu\gamma'} + (p_3+k)^{\gamma'} g^{\beta'\nu}] \quad (60)$$

where $k = -(q+p_1)$, $l = (q-p_2)$, $p_1+p_2+p_3 = 0$ and the $D's$ associated with the external gluon lines are understood.

Proceeding as before we may consider instead the following reduced expression

$$8\int\frac{d^dq}{(2\pi)^d}\qquad \frac{D_{\alpha\alpha'}(q)D_{\beta\beta'}(k)D_{\gamma\gamma'}(l)}{(q^2+i\epsilon)(k^2+i\epsilon)(l^2+i\epsilon)}[p_1^\beta g^{\alpha\lambda} + q^\lambda g^{\alpha\beta} - p_1^\alpha g^{\lambda\beta}]$$
$$[-p_2^\gamma g^{\alpha'\mu} + p_2^{\alpha'} g^{\gamma\mu} + q^\mu g^{\alpha'\gamma}][-k^\nu g^{\beta'\gamma'} - p_3^{\beta'} g^{\nu\gamma'} + p_3^{\gamma'} g^{\beta'\nu}] \quad (61)$$

The divergent terms in $T^{\lambda\mu\nu}$ are then easily identified and may be rewritten as follows

$$8\int\frac{d^dq}{(2\pi)^d}\frac{1}{q^2k^2l^2}\qquad \Big[2q^\lambda q^\mu k^\nu + p_3^\alpha q^\lambda q^\mu\{D_\alpha^\nu(l) - D_\alpha^\nu(k)\}$$
$$-k^\nu q^\lambda p_2^\alpha\{D_\alpha^\mu(q) - D_\alpha^\mu(l)\} - k^\nu q^\mu p_1^\alpha\{D_\alpha^\lambda(k) - D_\alpha^\lambda(q)\}\Big]\ . \quad (62)$$

The contribution to the divergent part comes solely from the first term, since the divergent terms coming from the noncovariant terms vanish. The divergent part of the

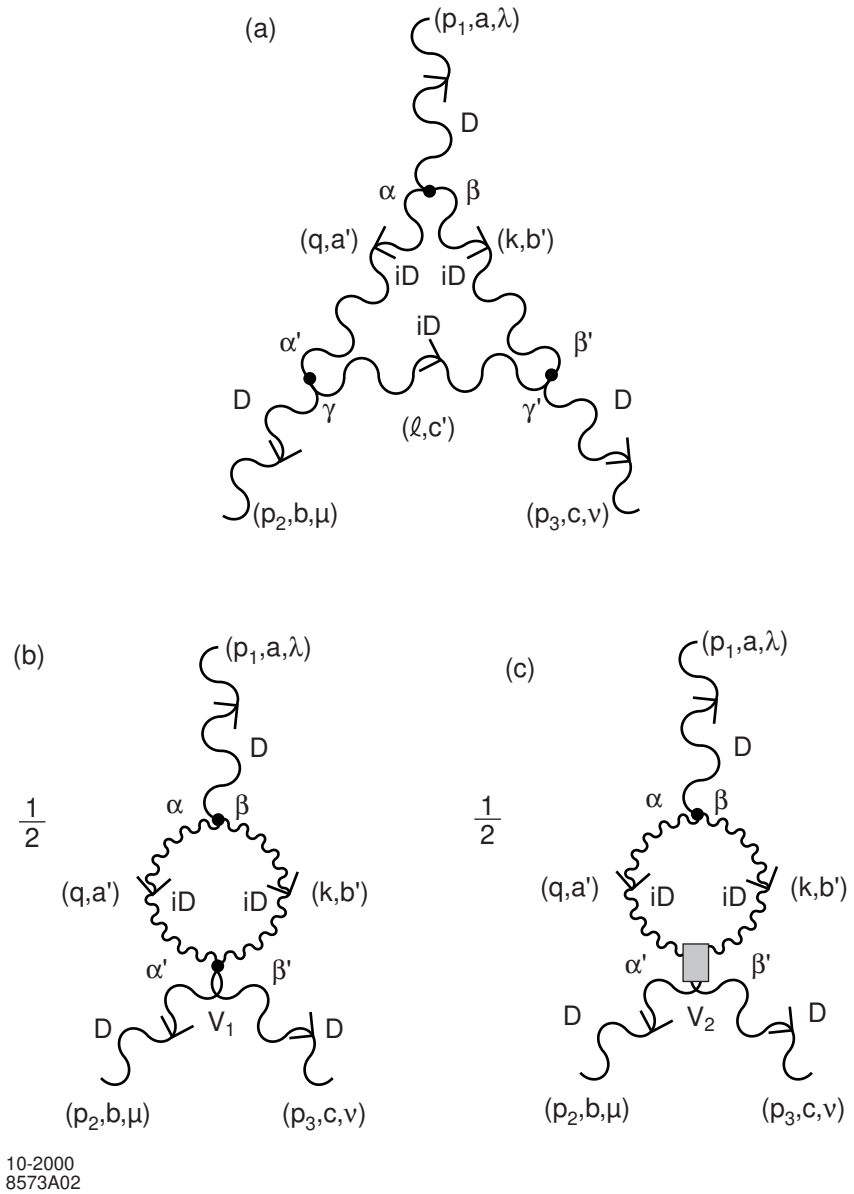

Figure 2: Three-gluon vertex diagrams. (a) Triangle diagram; (b) swordfish diagram containing vertex V_1; the other two diagrams are obtained by cyclic permutations of the external line indices; (c) swordfish diagrams containing vertex V_2; the other two diagrams are obtained by cyclic permutations of the external line indices.

one-loop correction from the triangle diagram has the form $-gf_{abc}\,F_{\lambda\mu\nu}(p_1,p_2,p_3)\,\bar{\Delta}_1$ with

$$\bar{\Delta}_1 = \frac{g^2}{16\pi^2} C_A\,(\frac{1}{3})\,\left[N_\epsilon - ln\frac{Q^2}{\mu^2} + \cdots\right] \tag{63}$$

where $Q^2 = -(p_2+p_3)^2$. It is covariant and different from the one found [17, 18] in equal-time l.c. gauge framework where a different propagator is used, and the corresponding expressions contain noncovariant pieces.

The total contribution from each type of *swordfish* diagram comes from three similar diagrams, the two others being constructed from the first one by cyclic permutations of the set of labels of the three external gluon lines. The net divergent contribution following from the two types of diagrams is given by $-gf_{abc}\,F_{\lambda\mu\nu}(p_1,p_2,p_3)\,\bar{\Delta}_2$ with

$$\bar{\Delta}_2 = \frac{g^2}{16\pi^2} C_A\,(-4)\,\left[N_\epsilon - ln\frac{Q^2}{\mu^2} + \cdots\right] \tag{64}$$

The noncovariant terms cancel out leading to the covariant result $\bar{\Delta}_2$ (see below).

The gluon vertex renormalization scalar factor Z_1 is defined by, $\bar{\Delta} = (\bar{\Delta}_1 + \bar{\Delta}_2)$,

$$-gf_{abc}F_{\lambda\mu\nu}(p_1,p_2,p_3)\;\frac{1}{Z_1} = -gf_{abc}F_{\lambda\mu\nu}(p_1,p_2,p_3)\;(1\,+\,\bar{\Delta}) \tag{65}$$

We find

$$\frac{1}{Z_1} = 1 - \frac{g^2}{16\pi^2}\,C_A\,(\frac{11}{3})\,\left[N_\epsilon - ln\frac{Q^2}{\mu^2} + \cdots\right] + \cdots \tag{66}$$

Since the factors Z_1 and Z_3 are found to be scalars the renormalization procedure is the same as in the covariant gauge theory. The gauge coupling constant renormalization scalar factor Z_g, defined by $Z_g = Z_1/(Z_3)^{3/2}$, is found to be

$$Z_g = \frac{Z_1}{(Z_3)^{3/2}} = 1 - \frac{g^2}{16\pi^2}\,C_A\,(\frac{11}{6})\,\left[N_\epsilon - ln\frac{-q^2}{\mu^2} + \cdots\right] + \cdots \tag{67}$$

Here we assume $Q^2 = -(p_1+p_2)^2 = -q^2 > 0$ at the vertex to do the evaluation of $Z_g(Q^2)$. It agrees with the result found [33, 34] in QCD, in the conventional *instant form* framework, to one-loop order when the quark fields are ignored. We note the equality $Z_1 = Z_3$ so that $Z_g = (Z_3)^{-\frac{1}{2}}$ in our framework.

The effective (or renormalized) gauge coupling is defined by $g_s(Q^2) = Z_g^{-1}\,g_o$, where we write g_o in place of g in the discussion above, may be expressed as a power series in the bare coupling constant g_o. In one loop order we find

$$\alpha_s(Q^2) = \alpha_o\,\left[1 + \alpha_o\,B(Q^2) + \cdots\right] \tag{68}$$

where $\alpha_o = g_o^2/(4\pi)$, $\alpha_s = g_s^2/(4\pi)$ and

$$\alpha_o\,B(Q^2) = \frac{\alpha_o}{4\pi}\,\beta_o\,\left[\frac{2}{\epsilon} - ln\frac{Q^2}{4\pi\mu^2} + \cdots\right] \tag{69}$$

with $\beta_o = 11\, C_A/3 > 0$. It diverges for $\epsilon = (4-d) \to 0$. However, the regularized expression $\alpha_o\,[B(Q^2) - B(M^2)] = -\alpha_o\,[\beta_o/(4\pi)]\,\ln(Q^2/M^2)$, with a convenient choice of $M^2 > 0$, is finite when $\epsilon \to 0$ and is independent of the parameter μ. We then arrive at the following expression for the running coupling constant to one-loop order

$$\alpha_s(Q^2) - \alpha_s(M^2) = -\frac{{\alpha_o}^2}{4\pi}\,\beta_o\,\ln\frac{Q^2}{M^2} + \cdots \tag{70}$$

For $Q^2 > M^2 > 0$ we find that $\alpha_s(Q^2) < \alpha_s(M^2)$, the signal of asymptotic freedom in nonabelian Yang Mills gauge theory. More specifically, the β function

$$\beta(\alpha_s) \equiv M^2\,\frac{\partial \alpha_s(M^2)}{\partial M^2} = \frac{\partial \alpha_s}{\partial (ln M^2)} = -\frac{\beta_o}{4\pi}\,{\alpha_o}^2 + \cdots \approx -\frac{\beta_o}{4\pi}\,\alpha_s^2 + \cdots \tag{71}$$

has the characteristic negative form of nonabelian Yang Mills gauge theory at small coupling.

A sketch of the computation involved in the *swordfish* diagrams is presented here. The matrix element simplifies considerably due to the nice properties of the $D's$ leading to the reduced form

$$\begin{aligned}&\frac{1}{2}\,(-ig_d^2)\,\frac{1}{2}C_A\,f_{abc}\int\frac{d^dq}{(2\pi)^d}\frac{1}{q^2k^2}iD_{\alpha\alpha'}(q)\,iD_{\beta,\beta'}(k)\\&2\left[p_1^\beta g^{\alpha\lambda} - p_1^\alpha g^{\beta\lambda} + q^\lambda g^{\alpha\beta}\right]\left[Ag^{\alpha'\mu}g^{\beta'\nu} + Bg^{\alpha'\nu}g^{\beta'\mu} + Cg^{\alpha'\beta'}g^{\mu\nu}\right]\end{aligned} \tag{72}$$

where

$$A = 3 - \frac{(q+p_2)^+(p_3+k)^+}{(q-p_2)^{+\,2}},\quad B = -3 + \frac{(k+p_2)^+(p_3+q)^+}{(q-p_3)^{+\,2}},\quad C = 2\frac{(k-q)^+(p_3-p_2)^+}{(q+k)^{+\,2}} \tag{73}$$

with a multiplication by the factor $D_{\lambda'\lambda}(p_1)D_{\mu'\mu}(p_2)D_{\nu'\nu}(p_3)$ being understood. The integrand may be simplified further and recast as

$$\begin{aligned}&p^\rho\Big[-g^{\widetilde{\lambda}\widetilde{\mu}}\,(\,AD_\rho^{\widetilde{\nu}}(k) - BD_\rho^{\widetilde{\nu}}(q)\,) + g^{\widetilde{\lambda}\widetilde{\nu}}\,(\,AD_\rho^{\widetilde{\mu}}(q) - BD_\rho^{\widetilde{\mu}}(k)\,)\\&+Cg^{\mu\nu}\,(\,AD_\rho^{\widetilde{\lambda}}(q) - D_\rho^{\widetilde{\lambda}}(k)\,)\Big] + q^\lambda g^{\widetilde{\mu}\widetilde{\nu}}\,(A+B) + 2C\,q^\lambda\,g^{\mu\nu}\end{aligned} \tag{74}$$

where $\widetilde{\mu} \equiv \perp = 1,2$ only. The divergent terms then can be picked up straightforwardly. Some of them drop out if we use also $p_1^\lambda D_{\lambda'\lambda}(p_1) = 0$ and $D_{-\mu} = 0$. When we add to this result the two expressions obtained from the present one by the cyclic permutations of 3-tuples (p_1,λ,a), (p_2,μ,b), and (p_3,ν,c), the noncovariant pieces drop out, leading to the covariant expression for $\bar{\Delta}_2$ given above.

The divergent terms arising from each type of the *swordfish* graphs contain covariant as well as noncovariant pieces. The contribution corresponding to the usual

four-gluon vertex V_1 in our framework agrees with the one found in the earlier l.c. gauge computations [17, 18], in the conventional framework. It thus gives us a consistency check of our calculations, since the computation here is insensitive to the $n_\mu n_\nu/k^{+2}$ term in our gauge propagator. The doubly transverse gauge propagator of the LF quantized theory greatly simplifies the computation: in compensation for the few extra interaction vertices, there are no ghost interactions.

6.3 Gluon Self-Energy corrections from Quark loop

For each flavor f of quark the gluon self-energy arising from a quark loop is given by

$$\Pi^{(F)\mu\nu}_{aa'} = (-1)\,(ig)^2 \sum_{ij} t^a_{ij} t^{a'}_{ji}\ \mu^{4-d}\,(i)^2 \int \frac{d^dk}{(2\pi)^d}\ \frac{Tr(\not k + m_f)\gamma^\mu(\not k + \not q + m_f)\gamma^\nu}{[k^2 - m_f{}^2]\,[(k+q)^2 - m_f{}^2]}. \tag{75}$$

The expression is understood o be multiplied by the factors $D_{\lambda\mu}(q)D_{\nu\delta}(q)$ which allows us to simplify the numerator from the start. The divergent part is computed to be

$$\begin{aligned} div\ D_{\lambda\mu}(q)\,\Pi^{(F)\mu\nu}_{aa'}\,D_{\nu\delta}(q) \ &= \ \frac{g^2}{16\pi^2}\,\delta_{aa'}\ T_F\ (\frac{4}{3}\,q^2)\ D_{\lambda\delta}(q) \\ & \quad i\left[N_\epsilon - 6\int_0^1 dx\,x(1-x)\,ln\,\frac{m_f^2 - x(1-x)\,q^2}{\mu^2}\right] \end{aligned} \tag{76}$$

where $\sum_{ij} t^a_{ij} t^{a'}_{ji} = T_F\delta_{aa'}$. The contributions from $\Pi^{(F)-\nu}_{aa'}$ or $\Pi^{(F)\mu-}_{aa'}$ are automatically suppressed in view of $D_{\mu-} = 0$, as they should, since $A_- = 0$ in the l.c. gauge. We have used also

$$D_{\lambda\mu}(q)\ (q^2 g^{\mu\nu} - q^\mu q^\nu)\ D_{\nu\delta}(q) = -q^2 D_{\lambda\delta}(q). \tag{77}$$

For the total number n_f of massless quarks we find

$$div\ D_{\lambda\mu}(q)\,\Pi^{(F)\mu\nu}_{aa'}\,D_{\nu\delta}(q) = \frac{g^2}{16\pi^2}\,\delta_{aa'}\,T_F\,(\frac{4}{3}\,n_f\,q^2)\,i\left[N_\epsilon - ln\frac{-q^2}{\mu^2} + \cdots\right]\ D_{\lambda\delta}(q). \tag{78}$$

The divergent contribution to gluon self-energy becomes

$$\begin{aligned} div\ D_{\lambda\mu}(q)\ \left[\Pi^{\mu\nu}_{aa'} + \Pi^{(F)\mu\nu}_{aa'}\right]\ D_{\nu\delta}(q) \ &= \ \frac{g^2}{16\pi^2}\,\delta_{aa'}\,(q^2)\,\left(-\frac{11}{3}C_A + \frac{4}{3}\,n_f\,T_f\right) \\ & \quad i\left[N_\epsilon - ln\frac{-q^2}{\mu^2} + \cdots\right]\ D_{\lambda\delta}(q). \end{aligned} \tag{79}$$

Hence

$$Z_3 = 1 + \frac{g^2}{16\pi^2}\,\left(\frac{11}{3}C_A - \frac{4}{3}\,n_f\,T_F\right)\,\left[N_\epsilon - ln\frac{-q^2}{\mu^2} + \cdots\right] \tag{80}$$

and

$$Z_3{}^{-\frac{1}{2}} = 1 - \frac{g^2}{16\pi^2}\left(\frac{11}{6}C_A - \frac{2}{3}n_f\,T_F\right)\left[N_\epsilon - ln\frac{-q^2}{\mu^2} + \cdots\right] \tag{81}$$

in our ghost free l.c. gauge LF quantized QCD framework. We also did not perform any renormalization of the gauge parameter, required in the conventional covariant gauge theory framework. A complete discussion of QCD in our framework will be given elsewhere [37].

7 Conclusions

The canonical quantization of l.c. gauge QCD in the *front form* theory has been derived employing the Dirac procedure to construct a self-consistent LF Hamiltonian theory. The formulation begins with the gauge-fixed BRS invariant quantum action, but the final result is ghost-free. The gauge field satisfies the Lorentz gauge condition as an operator equation as well in the light-cone gauge. The interaction Hamiltonian is obtained in a simple form by retaining the dependent components A_+ and ψ_-. Its form closely resembles the interaction Hamiltonian of covariant theory, except for the instantaneous interactions which are analogous to the Coulomb interactions in transverse gauge. The Dyson-Wick perturbation theory expansion based on equal-LF-time ordering has been constructed in a manner which allows one to perform high order computations in straightforward fashion.

In our framework of l.c. gauge formulation of gauge theory, gluons have only physical degrees of freedom, and no ghost fields need to be considered. Thus unitarity relations such as the optical theorem are manifest within each Feynman diagram, rather than as a consequence of cancelations over sets of diagrams [38]. This allows one to construct effective charges analogous to the Gell Mann-Low running coupling of QED based on the structure of self-energy diagrams using the pinch technique[39]. Since the absorptive part of these contributions are based on physical cross sections, one can define a physical and analytic renormalization scheme for QCD.

The propagator of the dynamical ψ_+ part of the free fermionic propagator in the *front form* theory is shown to be causal and not to contain instantaneous terms. The propagator of the massless gauge field is found to be doubly transverse both with respect to the four-momentum and the four-vector n_μ of gauge direction. The physical transverse polarization vectors may be conveniently identified as $E^\mu_{(\perp)}(k) \equiv -D^\mu_\perp(k)$ so that each gluon line, external or internal, carries with it a factor of $D_{\mu\nu}$. The remarkable properties of these factors give rise to much simplification already at the start of computations. It is worth stressing that the propagator obtained here follows from the straightforward application of the well-tested standard Dirac method for constructing Hamiltonian formulation for constrained dynamical systems. It differs from the ones found in the literature in the context of equal-time l.c. gauge QCD. In

substance, it corresponds to the rules of light-front-time-ordered perturbation theory on the LF [16]. The singularities in the noncovariant piece of the gauge field propagator can be handled using dimensional regularization which on the LF leads naturally to the causal (and seemingly mathematically sound) l.c. gauge ML prescription for $1/k^+$. The power-counting rules in l.c. gauge then become similar [40, 17, 41] to those found in covariant gauge theory.

Electron-muon scattering has been considered in order to illustrate the relevance of the instantaneous terms in the interaction Hamiltonian. They restore the manifest Lorentz covariance of the matrix element which was broken by the noncovariant gauge and the noncovariant terms in the gauge propagator. The same is found true in nonabelian gauge theory.

Despite the noncovariant gauge and broken manifest covariance of QCD in l.c. gauge, the renormalization factors associated with the gluonic vertex are found to be scalars. The renormalization procedure is thus similar to that of conventional covariant gauge theory. The additional interaction terms found are in a sense the appropriate counter terms which arise naturally in the canonical quantization in the LF framework presented. Higher order computations are required in order to check for any other terms.

The instantaneous interactions, in fact, are interesting by themselves. For example, the semi-classical limit of Thomson scattering is revealed at the tree level in the l.c. or covariant gauge [24] on the LF. This is relevant since a systematic procedure to obtain the semi-classical limit seems to be lacking in the *front form* theory. The LF framework is useful also for obtaining the non-relativistic limit of a relativistic field theory, for example, in the context of chiral perturbation theory. This is an alternative to the conventional framework where a functional integral technique and the Foldy-Wouthuysen transformation is employed.

We have demonstrated, in our framework, the equality $Z_1 = Z_3$ and $Z_g = (Z_3)^{-\frac{1}{2}}$ at one loop in Yang-Mills theory, which possibly would be true in higher orders as well. In view of the Slavnov-Taylor identities in QCD [42], it is then expected, for example, that in the renormalization of the qqg vertex the corrections to the vertex are compensated by the ones arising from the quark field renormalization, such that the coupling constant renormalization arises only from the gluonic self-energy. It is straightforward to show it in QED, and its complete discussion including that in QCD will be given elsewhere [37]. We have computed the renormalization of the effective gauge coupling defined from the three gluon vertex in l.c. gauge at one-loop order. The β function of the nonabelian Yang-Mills gauge theory is shown to be negative for vanishingly small coupling in the lowest order perturbation theory in agreement with conventional treatments [33, 34]. Computations in our ghost-free framework require comparable effort as calculations in covariant gauge because of the remarkable simplifications arising due to the doubly transverse l.c. gauge propagator. Higher-

loop computations should be possible in our formalism by making advantageous use of the techniques [40] which have recently been developed to handle multi-loop integrals involving noncovariant integrands.

The instantaneous interaction terms generated by l.c. gauge in the LF seem to have been missed in the analysis of *instant form* noncovariant gauge QCD based on functional integral quantization. Even with the appropriate counter terms found by imposing [17, 18, 26] the constraint of covariance or by requiring an extended BRS symmetry, the gauge propagators employed in that framework do not possess the very useful properties carried by the one in our LF framework.

It is worth remarking that we have made an *ad hoc* choice of only one (of the family) of the characteristic LF hyperplanes, $x^+ = const.$, in order to quantize the theory. The conclusions reached here and in the earlier works [11, 24] confirm the conjecture [11] concerning the irrelevance in the quantized theory of the fact that the hyperplanes $x^\pm = 0$ constitute characteristic surfaces of hyperbolic partial differential equation. The Hamiltonian version can clearly be implemented in DLCQ [25] which has been shown [43] to have a continuum limit. There is no loss of causality in DLCQ when the infinite volume limit is properly handled [44]. We also note that nonperturbative computations are often done on the LF in the closely related (l.c.) gauge $\partial_- A_- \approx 0$, such as to demonstrate [12] the existence of the *condensate* or θ-vacua in the Schwinger model.

Appendix

The Feynman Rules

The Dyson-Wick perturbation theory expansion on the LF can be realized in momentum space by employing the Fourier transform expressions of the fields and the propagators discussed in the text (Sections 2-5). Many of the rules of the Feynman diagrams, for example, the symmetry factor 1/2 for gluon loop, a minus sign associated with fermionic loops etc., are the same as those found in the conventional covariant framework [45]. There are some differences: for example, the external quark line now carries a factor $\theta(p^+)\sqrt{m/p^+}$ while the external gluon line a factor $\theta(q^+)/\sqrt{2q^+}$ or that the Lorentz invariant phase space factor is now as given in [32]. The external gluon line carries the polarization vector $E^{\mu(\perp)}(q) = E^\mu_{(\perp)} = -D^\mu_\perp(q)$. Its properties and the sum over the two polarization states are given Section 4. The notation and the corresponding discussion connected with the quark field is given in Section 3. The momentum space vertices, used in Section 6, associated with the interaction Hamiltonian (36) are given below:

Quark propagator:

$$i\delta_{ij}S(p), \quad \text{with} \qquad S(p) \equiv \frac{\not p + m}{p^2 - m^2 + i\epsilon}\,, \qquad \epsilon > 0,$$

where p_μ is the quark 4-momentum, i and j are color indices.
Gluon propagator:

$$i\delta^{ab}\frac{D_{\mu\nu}(q)}{q^2+i\epsilon}, \quad \text{with} \quad D_{\mu\nu}(q)=\left(-g_{\mu\nu}+\frac{n_\mu q_\nu+q_\mu n_\nu}{n\cdot q}-\frac{q^2}{(n\cdot q)^2}n_\mu n_\nu\right),$$

where q_μ is the gluon 4-momentum and n_μ is the gauge direction. We choose $n_\mu \equiv \delta_\mu^+$ and $n_\mu^* \equiv \delta_\mu^-$, the dual of n_μ.

Quark-quark-gluon vertex factor:

$$ig\,\gamma^\mu t^a$$

3-gluon vertex factor:

$$-g\,f_{abc}\,F_{\lambda\mu\nu}(p_1,p_2,p_3)$$

Here (p_1,λ,a), (p_2,μ,b), and (p_3,ν,c), label the three gluons at the vertex with outgoing momenta p_1^λ, p_2^μ and p_3^ν with the associated gauge indices a, b and c respectively. Also $F_{\lambda\mu\nu}(p_1,p_2,p_3)=(p_1-p_2)_\nu g_{\lambda\mu}+(p_2-p_3)_\lambda g_{\mu\nu}+(p_3-p_1)_\mu g_{\nu\lambda}=-F_{\lambda\nu\mu}(p_1,p_3,p_2)$ and $p_1+p_2+p_3=0$.

4-gluon vertex factors:

There are two types of 4-gluon vertices in this framework. The $V_1{}^{abcd}_{\alpha\mu\nu\beta}$ is the usual momentum-independent four-gluon vertex of the covariant gauge theory, while $V_2{}^{abcd}_{\alpha\mu\nu\beta}(p_1,p_2,p_3,p_4)$ is a new vertex generated by l.c. gauge LF quantized QCD, with the momentum dependence as well, coming from the last term in the interaction Hamiltonian (36):

$$\begin{aligned} V_1{}^{abcd}_{\alpha\mu\nu\beta} &= -ig^2\Big[f^{eab}f^{ecd}(g^{\alpha\nu}g^{\beta\mu}-g^{\alpha\beta}g^{\mu\nu}) \\ &\quad +f^{eac}f^{ebd}(g^{\alpha\mu}g^{\beta\nu}-g^{\alpha\beta}g^{\mu\nu}) \\ &\quad +f^{ead}f^{ecb}(g^{\alpha\nu}g^{\beta\mu}-g^{\alpha\mu}g^{\beta\nu})\Big] \end{aligned}$$

and

$$\begin{aligned} V_2{}^{abcd}_{\alpha\mu\nu\beta}(p_1,p_2,p_3,p_4) &= +ig^2\left[f^{eab}f^{ecd}g^{\alpha\mu}g^{\beta\nu}\,\frac{(p_1-p_2)^+(p_3-p_4)^+}{(p_1+p_2)^{+2}}\right. \\ &\quad +f^{eac}f^{ebd}g^{\alpha\nu}g^{\beta\mu}\,\frac{(p_1-p_3)^+(p_2-p_4)^+}{(p_1+p_3)^{+2}} \\ &\quad \left.+f^{ead}f^{ecb}g^{\alpha\beta}g^{\mu\nu}\,\frac{(p_1-p_4)^+(p_3-p_2)^+}{(p_1+p_4)^{+2}}\right] \end{aligned}$$

Here (p_1,α,a), (p_2,μ,b), (p_3,ν,c), and (p_4,β,d) indicate the four outgoing gluons at the vertex with $p_1+p_2+p_3+p_4=0$.

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