SLAC-PUB-8601 [1]
hep-ph/0009292
October 2000

Large Two-Loop Contributions to $g-2$ from a Generic Pseudoscalar Boson

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Abstract

We calculate the dominant contributions to the the muon $g-2$ at the two-loop level due to a pseudoscalar boson that may exist in any exotic Higgs sector in most extensions of the standard model. The leading effect comes from diagrams of the Barr-Zee type. A sufficiently light pseudoscalar Higgs boson can give rise to contribution as large as the electroweak contribution which is measurable in the next round of $g-2$ experiment. The coming improved data on muon $g-2$ can put the best limit on the possible existence of a light pseudoscalar boson in physics beyond the standard model.

PACS numbers: 14.80.Cp, 14.60.Ef, 13.40.Em

[1] Work supported by the Department of Energy, Contract DE-AC03-76SF00515

Precision measurement of the anomalous magnetic moment of the muon, $a_\mu \equiv \frac{1}{2}(g_\mu - 2)$, can provide not only a sensitive test of quantum loop effects in the electroweak standard model (SM), but also a probe to potential "new physics". The experimental average in 1998 Particle Data Book gives [1] $a_\mu^{\rm exp} = 11659230(84) \times 10^{-10}(\pm 7.2$ ppm). Recent measurements by E821 experiment at Brookhaven gives [2] $a_\mu^{\rm exp} = 11659250(150) \times 10^{-10}(\pm 13$ ppm) (1997 data) and $a_\mu^{\rm exp} = 11659191(59) \times 10^{-10}$ (1998 data). Combining with previous data, this can be translated[4] into

$$a_\mu^{\rm exp} = 11659210(46) \times 10^{-10}(\pm 3.9 \text{ ppm}) \ . \tag{1}$$

The E821 experiment is expected[3] to reduce the error soon by more than a factor of 10 to ± 0.35 ppm[4] with one month of dedicated running. With subsequent longer dedicated runs it could statistically approach the anticipated systematic uncertainty of about $\pm(10-20) \times 10^{-11}$ [5].

The contributions to a_μ are traditionally divided into

$$a_\mu = a_\mu^{\rm QED} + a_\mu^{\rm Hadronic} + a_\mu^{\rm EW} + \Delta a_\mu, \tag{2}$$

representing QED, hadronic, electroweak and the exotic (beyond the standard model) contributions respectively. The QED loop contributions have been computed to very high order [8]

$$\begin{aligned} a_\mu^{\rm QED} &= \frac{\alpha}{2\pi} \\ &+0.765857381(51)\left(\frac{\alpha}{\pi}\right)^2 + 24.050531(40)\left(\frac{\alpha}{\pi}\right)^3 \\ &+126.02(42)\left(\frac{\alpha}{\pi}\right)^4 + 930(170)\left(\frac{\alpha}{\pi}\right)^5 \end{aligned} \tag{3}$$

The most precise fine structure constant $\alpha = 1/137.03599944(57)$ can be obtained by inverting the similar formula for the electron $g_e - 2$ from the data.[6]. This gives

$$a_\mu^{\rm QED} = 116584706(2) \times 10^{-11} \tag{4}$$

Much more precise than the expected experimental reach. The hadronic contribution, by considering of the hadronic vacuum polarization diagram, is $a_\mu^{\rm Hadronic} = 6771(77) \times 10^{-11}$ [7], the SM electroweak contribution up to two-loop level gives $a_\mu^{\rm EW} = 151(4) \times 10^{-11}$[8, 9]

using $\sin^2\theta_W = 0.224$ and $M_H = 250$ GeV. (In comparison, the one-loop SM electroweak contribution is 195×10^{-11}). The total value of the standard model is:

$$a_\mu(\mathrm{SM}) = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{Hadronic}} + a_\mu^{\mathrm{EW}} = 116591628(77) \times 10^{-11} (\pm 0.66 \text{ ppm}). \tag{5}$$

The biggest theoretical uncertainty still comes from the strong interaction, however, it is still smaller than the experimental uncertainty. The hadronic uncertainty can be reduced further by measuring the hadronic photon polarization effect directly, and there are many experiments which intend to achieve this goal.

Compared with the latest experimental value, the two are still consistent, however one can tell that the experimental value is biased toward the high side of the standard model prediction. Given that $a_\mu^{\mathrm{Hadronic}}$ and $a_\mu^{\mathrm{EW,one-loop}}$ are both positive, one can conclude that the current data already probe these contributions. Note that $a_\mu^{\mathrm{EW,two-loop}}$ is negative. Naively one can extract from the SM prediction and data that Δa_μ between $(+100.9$ — $-6.5) \times 10^{-10}$ is still allowed. It will be very interesting to see if there is disagreement if the experimental data is reduced by a factor of 10 as expected.

Even without the recent experimental improvement, $g-2$ data has already provided non-trivial constraints[13] on physics beyond the standard model. For example, the one-loop results of the minimal supersymmetric standard model (MSSM), by considering smuon-smuon-neutralino and chargino-chargino-sneutrino loops, is well known[10, 11]. Its contribution to anomalous magnetic moment will depend on the masses of supersymmetric particles and $\tan\beta$.

In theories beyond the standard model (BSM), it is generic that there are many additional scalar or pseudoscalar bosons. In particular, some of the pseudoscalar bosons can potentially be light because of its pseudo-Goldstone nature, accidentally or otherwise. However, in collider search, it is well known that it is much harder to search for or constrain the pseudoscalar neutral boson than the scalar neutral or charged one. Therefore it is particularly interesting to see if one can constrain or discover such particle using low energy precision experiments. In this paper we wish to report that if the theory BSM has a light enough pseudoscalar boson, its two-loop contribution to muon $g-2$ can be as large as the one-loop electroweak effect. As a result the muon $g-2$ can provide a very strong probe on a very large class of theories beyond the standard model.

The one-loop contributions to $g-2$ from a scalar or pseudoscalar boson was calculated

many times in the past[12]. Besides two powers of m_μ that is required by the kinematic and definition, these one-loop contributions are further suppressed by another two powers of m_μ/M_a. However, the result is enhenced by a $\ln m_\mu/M_a$ enhencement due to the diagram in which the photon is emitted by the internal muon. Therefore for light enough Higgs mass, some limit can be derived from $g-2$ data just based on one-loop result. Nevertheless, as we shall see later, the two loop contribution is typically larger than the one-loop one by a factor of 2–10 for Higgs mass from 10 – 100 GeV. In addition, the one-loop and two-loop contributions have different signs for both scalar and pseudoscalar contributions. Therefore the one-loop contributions actually partially cancel the larger two-loop contributions.

The two-loop contribution of a scalar boson has been calculated in Ref.[8, 9] in the context of the standard model. The contribution of any scalar boson beyond the standard model can in principle be extracted from that calculation and we shall not dwell on this here except to note that the scalar boson gives negative contribution while the pseudoscalar gives positive contribution to Δa_μ. Also, we have parameterized our input Lagrangian as model independent as possible in order to make our gauge invariant result widely applicable to a large class of models.

For Higgs mass larger than roughly $3GeV$, the dominant Higgs related contribution to lepton anomalous magnetic moment is through the two-loop Barr-Zee type diagram[14], as in Fig. 1. Compared with the one-loop graph, the Yukawa coupling of the heavy fermion in the first loop together with the mass insertion of the heavy fermion will give rise to $(m_f/m_\ell)^2$ enhancement which can overcome the extra loop suppression factor of $\alpha/16\pi^2$ and two extra photonic couplings. The gauge boson could be photon or Z^0. The Z^0 contribution is typically smaller by two orders of magnitude. It is included in this manuscript just for completeness. Note that unless CP violation occurs in the Higgs potential, the pseudoscalar boson does not make gauge boson loop contribution to $g-2$.

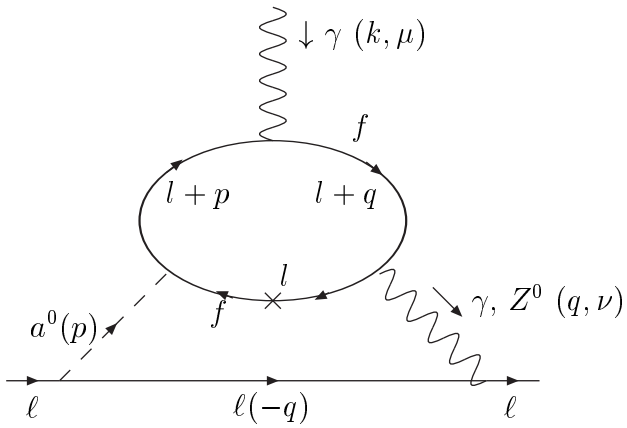

Fig. 1 The dominant two-loop graph involving a pseudoscalar boson that contributes to $g_\ell - 2$. The cross location denotes a possible mass insertion.

The form of the gauge invariant vertex function $\Gamma^{\mu\nu}$ of a pseudoscalar boson a^0 of momentum (p) turning into two photons $(-k,\mu)$, (q,ν) due to the internal fermion or gauge boson loop is

$$\Gamma^{\mu\nu} = P(q^2)\epsilon^{\mu\nu\alpha\beta}p_\alpha q_\beta \ . \tag{6}$$

In general, the heavy fermion generation dominates in the loop. The Yukawa coupling is parameterized in a model independent expression,

$$\mathcal{L} = i\frac{gA_f m_f}{2M_W}\bar{f}\gamma_5 f a^0, \tag{7}$$

Integrating the fermion loop momentum, we obtain the form factor

$$P(q^2) = N_c^f \frac{gA_f e^2 q_f^2 m_f^2}{8\pi^2 M_W}\int_0^1 \frac{dz}{m_f^2 - z(1-z)q^2}. \tag{8}$$

where m_f and q_f are the mass and the charge of the internal fermion in the loop. The color trace gives $N_c^b = N_c^t = 3$, $N_c^\tau = 1$. The above vertex is further connected to the lepton propagator to produce anomalous magnetic dipole moment $a_\ell^{\gamma a^0}$ for the lepton ℓ,

$$a_\ell^{\gamma a^0} = \frac{\alpha^2}{8\pi^2\sin^2\theta_W}\frac{m_\ell^2 A_\ell}{M_W^2}\sum_{f=t,b,\tau} N_c^f q_f^2 A_f \frac{m_f^2}{M_a^2}\mathcal{F}\left(\frac{m_f^2}{M_a^2}\right) \ , \tag{9}$$

$$\mathcal{F}(x) = \int_0^1 \frac{\ln\frac{x}{z(1-z)}dz}{x - z(1-z)} \ . \tag{10}$$

$\mathcal{F}(1) = \frac{4}{\sqrt{3}}\mathrm{Cl}_2(\frac{\pi}{3})$, with the Clausen's function $\mathrm{Cl}_2(\theta) = -\int_0^\theta \ln\left(2\sin\frac{\theta}{2}\right)d\theta$. As $x \gg 1$, $x\mathcal{F}(x)$ has the asymptotic form $2 + \ln x$. On the other extreme limit $x \ll 1$, $\mathcal{F}(x)$ approaches to $\frac{\pi^2}{3} + \ln^2 x$.

For the graph with the inner photon replaced by Z^0 boson, its contribution to a_μ can be calculated in a similar fashion,

$$a_\ell^{Z^0 a^0} = \frac{\alpha^2 m_\ell^2 A_\ell g_V^\ell}{8\pi^2 \sin^4\theta_W \cos^4\theta_W M_Z^2} \sum_{f=t,b,\tau} \frac{N_c^f A_f q_f g_V^f m_f^2}{M_Z^2 - M_a^2} \left[\mathcal{F}\left(\frac{m_f^2}{M_Z^2}\right) - \mathcal{F}\left(\frac{m_f^2}{M_a^2}\right) \right] , \quad (11)$$

with $g_V^f = \frac{1}{2}T_3(f_L) - q_f \sin^2\theta_W$. Note that, for both pseudoscalar and scalar boson contribution, only the vector coupling of Z^0 to heavy fermion contributes to the effective vertex due to Furry theorem. Numerically, this Z^0 mediated contribution turns out to be about two order of magnitude smaller than that of the photon mediated one. One suppression factor comes from the massive Z^0 propagator and the other one comes from the smallness of the leptonic vector coupling of Z^0 boson, which is proportional to $(-\frac{1}{4} + \sin^2\theta_W) \sim -0.02$.

Taking the pattern of Yukawa couplings in MSSM as an example, we set A_f as $\cot\beta$ ($\tan\beta$) for the u (or d)-type fermion. The contributions due to the top quark t , the bottom quark b and the tau lepton τ as well as the total are displayed in Fig. 2 for both $\tan\beta = 30$ and 50. In this MSSM pattern the t contribution is insensitive to $\tan\beta$ and both the b and the τ contributions, which are roughly the same order of magnitude, dominate over that of the top quark for a large $\tan\beta$ and a light pseudoscalar mass M_a. For $M_a \lesssim 15$ GeV, the τ contribution is larger than the b-quark contribution. The total two-loop photonic contribution from the pseudoscalar boson, $a_\mu^{\gamma a^0}$, can be as large as 10^{-8} for a large $\tan\beta$ when $M_a \leq 10$ GeV as shown in Fig. 2. For example, for $M_a = 10$ GeV and $\tan\beta = 50$, $a_\mu^{\gamma a^0} = 1.2 \times 10^{-8}$, which is above the upper limit allowed by the current experiment bound. Generically, for $M_a \sim 80$–100 GeV, $\tan\beta \sim 50$, a_μ ranges in $(7-9)\times 10^{-10}$ which is close to the electroweak contribution. Note that the pseudoscalar contribution has the same sign as the hadronic or electroweak contributions.

To derive constraint from the data one must combine the one- and two-loop contributions. The well known one-loop contribution[12] due to the pseudoscalar a^0 is

$$a_\ell^a(\text{1-loop}) = -\frac{{m_\ell}^2}{8\pi^2 {M_a}^2} \left(\frac{g A_\ell m_\ell}{2M_W}\right)^2 H\left(\frac{m_\ell^2}{M_a^2}\right) , \quad \text{with } H(y) = \int_0^1 \frac{x^3 dx}{1 - x + x^2 y} . \quad (12)$$

For small y, $H(y) \to -\ln y - \frac{11}{6} > 0$. Note that the one-loop contribution is always negative in contrast to the two-loop contribution. In Fig. 2, we draw the absolute value of the one-loop contribution for easy comparison. For small M_a such as 10 GeV, the

one-loop contribution can be as large as half of the two loop contribution and cancelling in $g_\mu - 2$. Complete cancellation occurs around $3GeV$ for a large $\tan\beta$. However the one-loop effect becomes smaller for larger M_a due to its additional suppression factor of $(m_\mu^2/M_a^2)\ln(M_a^2/m_\mu^2)$ and basically negligible for $M_a > 50GeV$.

To compare our result with the recent data, we note that, in the framework of the standard model, roughly an uncertainty of Δa_μ between $(+100.9 \text{ — } -6.5) \times 10^{-10}$ can still be accommodated by the data. The upper and the lower bounds are illustrated in the two lightly shaded regions in Fig. 2. Note that for M_a lighter than roughly $3GeV$, the negative one-loop contribution dominates and the total pseudoscalar Higgs contribution becomes negative. In Fig. 2, we draw the absolute value of the total in this case. As already emphasized by previous one-loop analysis, the region with M_a lighter than roughly $2.9GeV$ is already ruled out by the current $g-2$ experiment. The E821 experiment is expected[4] to announce its new result with error reduced by more than a factor of 10 very soon. It is hard to predict the consequence of this improved data since even the central value may be shifted. However, as a reference point, we plot the line $\Delta a_\mu \leq 10^{-9}$ in Fig. 2 as a potential consequence assuming the central value remains the same.

In CP conserving MSSM, there is already a theoretical lower bound[15] on $M_a \geq 60$ GeV. However, when CP violation is included in the analysis[16], or in more general supersymmetric models or in general two or more Higgs doublet models [17], very little can be said about the potentially light pseudoscalar Higgs boson. The model independent nature of our calculation makes it possible to derive relatively strong limit on the pseudoscalar boson sector in any theory beyond the standard model using the hard earned data on muon $g-2$. If future data reduce the uncertainty of $g_\mu - 2$ in the way that we expect, the pseudoscalar boson of less than 75 GeV for $\tan\beta = 50$ can be easily ruled out except in a very narrow region of cancellation (around $3GeV$).

Note that in general multi-Higgs doublet models, the $\tan\beta$ factor in our analysis may be supplemented by additional factor of mixing matrix elements. In addition, in any specific model, there may be additional two-loop contributions, such as the ones involving the physical charged Higgs boson or the neutral scalar boson. We assume that these contributions do not accidentally cancel each other. Given that the experimental limit on the masses of the charged Higgs boson as well as the neutral scalar boson are already quite

high, it is very unlikely they will cancel the contribution of a relatively light pseudoscalar boson.

In conclusion, in this letter we report a set of analytic formulas for the two-loop contributions of a generic pseudoscalar boson to lepton anomalous magnetic moment. Such pseudoscalar bosons may exist in any theory beyond the standard model and they are typically harder to constrain using collider experimental data. In this paper, we show that strong constraint on such sector can be derived from the precision data on muon anomalous magnetic moment from the going and future experiments. We hope our work add importance and urgency to these low energy precision experiments.

Acknowledgments This work was supported in parts by National Science Council of R.O.C., by U.S. Department of Energy (Grant No. DE-FG02-84ER40173). We wish to thank A. Pilaftsis, M. Krawczyk, T. Rizzo and Bogdan Grzadkowski for discussions and DC wishes to thank SLAC Theory Group for hospitality.

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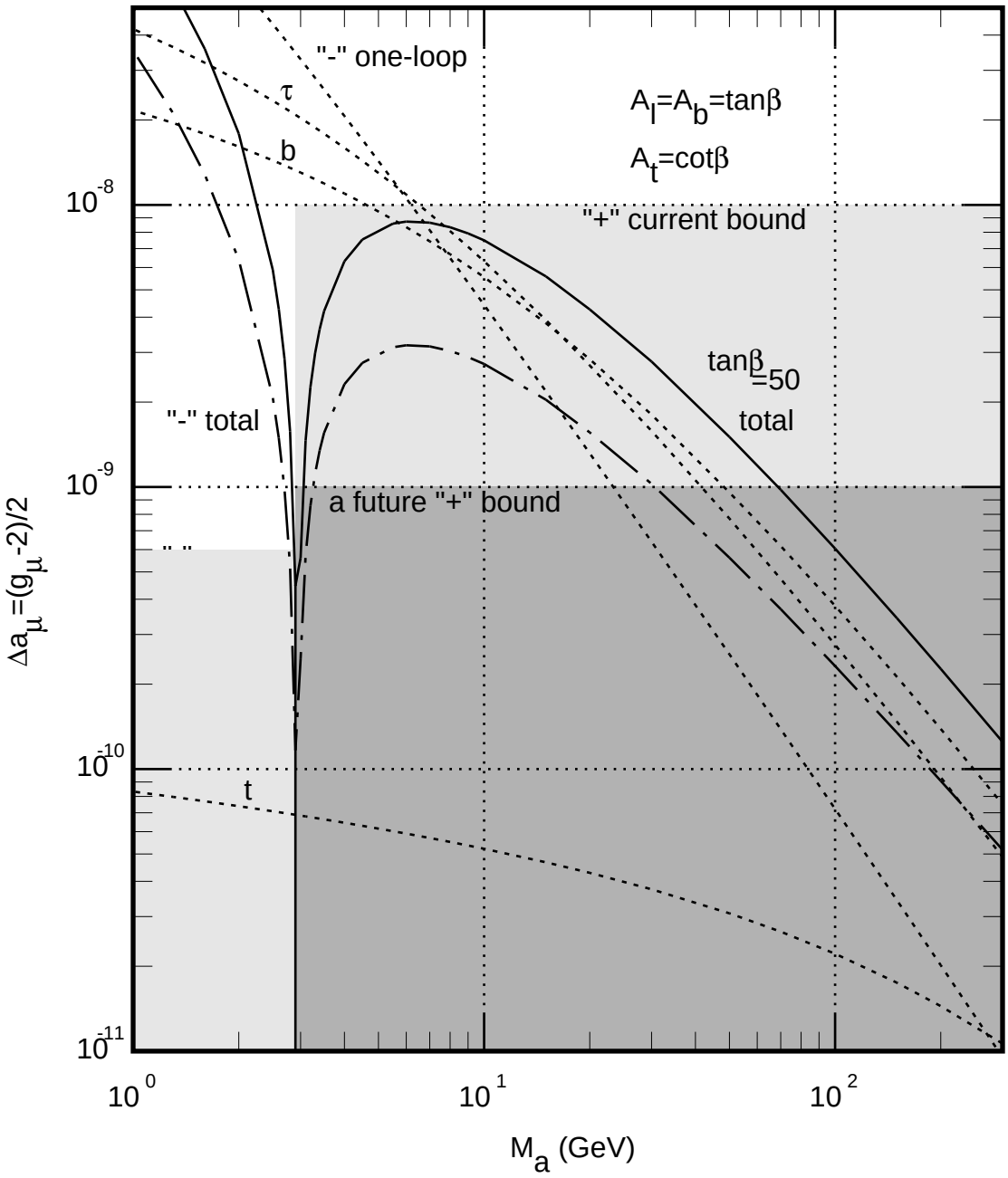

Fig. 2: Two-loop Contributions from the inner t,b, τ loops to the muon $g_\mu - 2$ due to the pseudoscalar a^0 versus M_a at $\tan\beta = 50$, as well as the negative contribution from the one-loop diagram. The sum in solid (Dashed-dotted) curve shows cancellation at low M_a mass for $\tan\beta = 50$ (30). The shaded areas are the current bound and the expected bound from future data.