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LIGHT-FRONT QCD IN LIGHT-CONE GAUGE*

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Abstract

The light-front (LF) quantization [1] of QCD in light-cone (l.c.) gauge is discussed. The Dirac method is employed to construct the LF Hamiltonian and theory quantized canonically. The Dyson-Wick perturbation theory expansion based on LF-time ordering is constructed. The framework incorporates in it simultaneously the Lorentz gauge condition as an operator equation as well. The propagator of the dynamical ψ_+ part of the free fermionic propagator is shown to be causal while the gauge field propagator is found to be transverse. The interaction Hamiltonian is re-expressed in the form closely resembling the one in covariant theory, except for additional instantaneous interactions, which can be treated systematically. Some explicit computations in QCD are given.

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1 Introduction

The quantization of relativistic field theory at fixed light-front time $\tau = (t - z/c)/\sqrt{2}$, was proposed by Dirac [2] half a century ago. It has found important applications [3, 4, 5, 6] in gauge theory and string theory. The light-front (LF) quantization of QCD in its Hamiltonian form provides an alternative approach to lattice gauge theory for the computation of nonperturbative quantities. We discuss here [7] the LF quantization of QCD gauge field theory in l.c. gauge employing the Dyson-Wick S-matrix expansion based on LF-time-ordered products. The case of covariant gauge has been discussed in our earlier work [8].

2 QCD action in light-cone gauge

The LF coordinates are defined as $x^\mu = (x^+ = x_- = (x^0 + x^3)/\sqrt{2}, x^- = x_+ = (x^0 - x^3)/\sqrt{2}, x^\perp)$, where $x^\perp = (x^1, x^2) = (-x_1, -x_2)$ are the transverse coordinates and $\mu = -, +, 1, 2$. The coordinate $x^+ \equiv \tau$ will be taken as the LF time, while x^- is the longitudinal spatial coordinate.

The quantum action of QCD in l.c. gauge is described in the standard notation by

$$\mathcal{L}_{QCD} = -\frac{1}{4} F^{a\mu\nu} F^a{}_{\mu\nu} + B^a A^a{}_- + \bar{c}^a \mathcal{D}^{ab}_- c^b + \bar{\psi}^i (i\gamma^\mu D^{ij}{}_\mu - m\delta^{ij})\psi^j. \tag{1}$$

Here $\bar{c}^a, c^a$ are anticommuting ghost fields and auxiliary fields $B^a(x)$ are introduced in the *linear* gauge-fixing term. The action is invariant under BRS symmetry transformations. Since B^a carries canonical dimension three no quadratic terms in them are permitted.

3 Spinor field propagator on the LF

The quark field term in LF coordinates reads

$$\begin{aligned} \bar{\psi}^i (i\gamma^\mu D^{ij}{}_\mu - m\delta^{ij})\psi^j &= i\sqrt{2}\bar{\psi}^i_+ \gamma^0 D^{ij}{}_+ \psi_+{}^j + \bar{\psi}^i_+ (i\gamma^\perp D^{ij}{}_\perp - m\delta^{ij})\psi_-{}^j \\ &+ \bar{\psi}^i_- \left[i\sqrt{2}\gamma^0 D^{ij}{}_- \psi_-{}^j + (i\gamma^\perp D^{ij}{}_\perp - m\delta^{ij})\psi_+{}^j \right] \end{aligned} \tag{2}$$

where [8] $\psi_\pm = \Lambda^\pm \psi$. This shows that the minus components $\psi_-{}^j$ are in fact nondynamical fields without kinetic terms. Their equations of motion in l.c. gauge lead to

the constraint equations

$$i\sqrt{2}\,\psi_-{}^j(x) = -\frac{1}{\partial_-}\,\left(i\gamma^0\gamma^\perp D^{kl}{}_\perp - m\gamma^0\delta^{kl}\right)\,\psi_+{}^l(x). \tag{3}$$

The free field propagator of ψ_+ is determined from the quadratic terms (suppressing the color index) $i\sqrt{2}\psi_+{}^\dagger\partial_+\psi_+ + \psi_+{}^\dagger(i\gamma^0\gamma^\perp\partial_\perp - m\gamma^0)\psi_-$ where $2i\partial_-\psi_- = (i\gamma^\perp\partial_\perp + m)\gamma^+\psi_+$. The equation of motion for the independent component ψ_+ is nonlocal in the longitudinal direction. In the quantized theory we find the following nonvanishing *local* anticommutator $\{\psi_+(\tau, x^-, x^\perp), \psi_+{}^\dagger(\tau, y^-, y^\perp)\} = \frac{1}{\sqrt{2}}\Lambda^+\delta(x^- - y^-)\delta^2(x^\perp - y^\perp)$. They may be realized in momentum space through the following Fourier transform [8]

$$\psi(x) = \frac{1}{\sqrt{(2\pi)^3}}\sum_{r=\pm}\int d^2p^\perp dp^+ \quad \theta(p^+) \quad \sqrt{\frac{m}{p^+}}\left[b^{(r)}(p)u^{(r)}(p)e^{-ip\cdot x}\right.$$
$$\left. + \quad d^{\dagger(r)}(p)v^{(r)}(p)e^{ip\cdot x}\right] \tag{4}$$

where

$$u^{(r)}(p) = \frac{1}{(\sqrt{2}p^+m)^{\frac{1}{2}}}\left[\sqrt{2}p^+\Lambda^+ + (m + \gamma^\perp p_\perp)\Lambda^-\right]\tilde{u}^{(r)} \tag{5}$$

and the nonvanishing anticommutation relations are given by: $\{b^{(r)}(p), b^{\dagger(s)}(p')\} = \{d^{(r)}(p), d^{\dagger(s)}(p')\} = \delta_{rs}\delta(p^+ - p'^+)\delta^2(p^\perp - p'^\perp)$.

The free propagator then follows to be [8]

$$< 0|T(\psi^i{}_+(x)\psi^{\dagger j}{}_+(0))|0 >= \frac{i\delta^{ij}}{(2\pi)^4}\int d^4q\,\frac{\sqrt{2}q^+\,\Lambda^+}{(q^2 - m^2 + i\epsilon)}\,e^{-iq.x}. \tag{6}$$

It is causal and contains no instantaneous term.

4 Gauge field propagator in l.c. gauge

In the l.c. gauge the ghost fields decouple and it is sufficient to study the free abelian gauge theory with the action

$$\int d^2x^\perp dx^-\left\{\frac{1}{2}\left[(F_{+-})^2 - (F_{12})^2 + 2F_{+\perp}F_{-\perp}\right] + BA_-\right\} \tag{7}$$

where $F_{\mu\nu} = (\partial_\mu A_\nu - \partial_\nu A_\mu)$. Following the Dirac procedure we show that the phase space constraints remove all the canonical momenta from the theory. The surviving variables are $A_\perp$ and A_+. The latter, however, is a dependent variable satisfying

$\partial_-(\partial_- A_+ - \partial_\perp A_\perp) = 0$. The construction of the Dirac bracket shows that in the l.c. gauge on the LF we simultaneously obtain the Lorentz condition $\partial \cdot A = 0$ as an operator equation as well. The *reduced* Hamiltonian is found to be ${H_0}^{LF} = \frac{1}{2}\int d^2x^\perp dx^- \left[(\partial_- A_+)^2 + \frac{1}{2}F_{\perp\perp'}F^{\perp\perp'}\right]$

The equal-τ commutators are $[A_\perp(x), A_\perp(y)] = i\delta_{\perp\perp'}K(x,y)$ where $K(x,y) = -(1/4)\epsilon(x^- - y^-)\delta^2(x^\perp - y^\perp)$. They are *nonlocal* in the longitudinal coordinate but there is no violation of the microcausality principle on the LF. They may be realized in momentum space by the following Fourier transform [7]

$$\begin{aligned} A^\mu(x) &= \frac{1}{\sqrt{(2\pi)^3}}\int d^2k^\perp dk^+ \frac{\theta(k^+)}{\sqrt{2k^+}} \\ &\quad \sum_{(\perp)} E_{(\perp)}{}^\mu(k)\left[b_{(\perp)}(k^+,k^\perp)e^{-ik\cdot x} + b^\dagger_{(\perp)}(k^+,k^\perp)e^{ik\cdot x}\right]. \end{aligned} \tag{8}$$

where k^- is shown [7] to be defined through the dispersion relation, $2k^-k^+ = k^\perp k^\perp$ corresponding to a *massless* photon. Here the nonvanishing commutators are given by $[b_{(\perp)}(k), b^\dagger{}_{(\perp')}(k')] = \delta_{(\perp)(\perp')}\,\delta^3(k-k')$. The free gluon propagator is hence found to be [7]

$$\begin{aligned} <0|\,T(A^a{}_\mu(x)A^b{}_\nu(0))\,|0> &= \frac{i\delta^{ab}}{(2\pi)^4}\int d^4k\; e^{-ik\cdot x}\;\frac{D_{\mu\nu}(k)}{k^2+i\epsilon}, \\ D_{\mu\nu}(k) &= D_{\nu\mu}(k) = -g_{\mu\nu} + \frac{n_\mu k_\nu + n_\nu k_\mu}{(n\cdot k)} - \frac{k^2}{(n\cdot k)^2}\,n_\mu n_\nu \end{aligned} \tag{9}$$

with $n_\mu = \delta_\mu{}^+$, $E_{(\perp)}{}^\mu(k) = E^{(\perp)\mu}(k) = -D^\mu_\perp(k)$, $k^\mu D_{\mu\nu}(k) = 0$, $n^\mu D_{\mu\nu}(k) = 0$.

5 QCD Hamiltonian in l.c. gauge

The interaction Hamiltonian in the l.c. gauge, $A^a_- = 0$, may be rewritten [7] as

$$\begin{aligned} \mathcal{H}_{int} &= +\frac{g}{2}f^{abc}(\partial_\mu A^a{}_\nu - \partial_\nu A^a{}_\mu)A^{b\mu}A^{c\nu} + \frac{g^2}{4}f^{abc}f^{ade}A_{b\mu}A^{d\mu}A_{c\nu}A^{e\nu} \\ &\quad -g\,\bar\psi^i\gamma^\mu A_\mu{}^{ij}\psi^j - \frac{g^2}{2}\,\bar\psi^i\gamma^+\,(\gamma^{\perp'}A_{\perp'})^{ij}\,\frac{1}{i\partial_-}\,(\gamma^\perp A_\perp)^{jk}\,\psi^k \\ &\quad -\frac{g^2}{2}\,j^+_a\,\frac{1}{(\partial_-)^2}\,j^+_a \end{aligned} \tag{10}$$

where $j^+_a = \bar\psi^i\gamma^+(t_a)^{ij}\psi^j + f_{abc}(\partial_- A_{b\mu})A^{c\mu}$ and a sum over distinct flavours, not written explicitly, is to be understood.

The fact that gluons have only physical degrees of freedom in l.c. gauge may provide an analysis of coupling renormalization similar to that of the pinch technique, which is currently being discussed [9] in order to obtain a shorter expansion and scheme for QCD. In addition, the couplings of gluons in the l.c. gauge provides a simple procedure for the factorization of soft and hard gluonic corrections in high momentum transfer inclusive and exclusive reactions.

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