Matter effects in the $D^0 - \overline{D^0}$ system

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We discuss the impact of matter effects in the $D^0 - \overline{D^0}$ system. We show that such effects could, in principle, be measured, but that they cannot be used to probe the mass difference x_D or the lifetime difference y_D. This occurs because the mixing effects and the matter effects decouple at short times. We also comment briefly on the B systems.

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I. INTRODUCTION

The $D^0 - \overline{D^0}$ system is of great current interest. Until recently, the best probes on the mixing resulted only in upper bounds on $x_D^2 + y_D^2$ of order 10^{-2} [1]. This changed with the recent results from CLEO [2] and FOCUS [3]. The current situation is beautifully summarized in reference [4]: there is evidence for $y_D \sim 10^{-2}$ and for a large strong phase, corresponding to large SU(3) breaking effects. The problem with the strong, final-state interaction phases is that they are not known. They are fit in the same experiments that look for y_D. It would be interesting if matter effects could be used to shed some light on this issue. The idea arises from the kaon system, where the phases due to matter effects are measured experimentally. Thus, we would be dealing with a *known* CP-even phase.

There is one important difference between the kaon and D systems. For a beam of neutral kaons in vacuum, the depletion from the beam is controlled by two exponentials, $\exp(-\Gamma_S\, t)$ and $\exp(-\Gamma_L\, t)$, corresponding to the short-lived (K_S) and long-lived (K_L) components. Here $|\Delta\Gamma| \sim 2\Gamma$ and the two exponential fallouts are clearly separated. As a result, we may wait for the K_S component to decay away and use the matter effects on K_L to regenerate K_S; a phenomenon known as 'regeneration'. In the D (and the B_d) system the situation is very different: $|\Delta\Gamma| \ll \Gamma$ and the leading behavior in vacuum is given by $\exp(-\Gamma\, t)$, with the hyperbolic sine and cosine of $\Delta\Gamma\, t$ acting as small perturbations. In this system, the depletion from the beam is controlled by Γ. Because we cannot resolve the two exponentials, the classical 'regeneration' experiments cannot be carried out and a new analysis is required. This is the question we address here.

In section II we review the general features of propagation of neutral mesons systems in matter. In section III we use an extrapolation from known kaon results to argue that these effects might, in principle, be sought. Unfortunately, we will also show that, even if visible, the matter effects cannot be used to probe x_D or y_D. This is shown to be a generic feature of the 'small-time' approximation in the time-evolution. We draw our conclusions in section IV, where we remark briefly on the B_d and B_s systems.

II. PROPAGATION IN MATTER

Let us consider a generic neutral meson system $P^0 - \overline{P^0}$. Assuming CPT-conservation in vacuum, this system is characterized by two complex eigenvalues, $\mu_a = m_a - i\Gamma_a/2$ and $\mu_b = m_b - i\Gamma_b/2$, and by a mixing parameter, q/p. It is convenient to define the average and the difference of the eigenvalues,

$$\mu = \frac{\mu_a + \mu_b}{2} = m - \frac{i}{2}\Gamma,$$
$$\Delta\mu = \mu_a - \mu_b = \Delta m - \frac{i}{2}\Delta\Gamma. \tag{1}$$

These quantities describe the system in vacuum and its time evolution is well known [5].

The new effects resulting from the interaction with matter can be described by the elastic forward scattering amplitudes of P^0 and $\overline{P^0}$, which we denote by f and $\bar{f}$, respectively. These enter the effective Hamiltonian through

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$$\chi = -\frac{2\pi N}{m} f, \quad \text{and} \quad \bar{\chi} = -\frac{2\pi N}{m} \bar{f}, \tag{2}$$

where N is the density of scattering centers in the medium. As before, we define the average and the difference of these parameters as

$$\begin{aligned} \chi_{\rm av} &= \frac{\chi + \bar{\chi}}{2}, \\ \Delta\chi = \chi - \bar{\chi} &= -\frac{2\pi N}{m} \Delta f, \end{aligned} \tag{3}$$

with $\Delta f = f - \bar{f}$.

In the presence of matter, the time evolution of meson states with a given initial flavor may be written as [6–9],

$$\begin{aligned} e^{i(\mu+\chi_{\rm av})t}\, |P^0(t)\rangle &= \left[\cos(\Omega\, t/2) - i\frac{\Delta\chi}{\Omega}\sin(\Omega\, t/2)\right] |P^0\rangle + \frac{q}{p}\left[-i\frac{\Delta\mu}{\Omega}\sin(\Omega\, t/2)\right] |\overline{P^0}\rangle, \\ e^{i(\mu+\chi_{\rm av})t}\, |\overline{P^0}(t)\rangle &= \frac{p}{q}\left[-i\frac{\Delta\mu}{\Omega}\sin(\Omega\, t/2)\right] |P^0\rangle + \left[\cos(\Omega\, t/2) + i\frac{\Delta\chi}{\Omega}\sin(\Omega\, t/2)\right] |\overline{P^0}\rangle, \end{aligned} \tag{4}$$

where $\Omega = \sqrt{(\Delta\mu)^2 + (\Delta\chi)^2}$ [10]. The evolution in vacuum is reproduced by setting $\chi_{\rm av}$ and $\Delta\chi$ to zero.

As mentioned in the introduction, in the D (and in the B_d) system the depletion from the beam in vacuum is controlled by Γ. This *one* exponential is then modulated by trigonometric functions of $\Delta\mu\, t = \frac{\Delta\mu}{\Gamma}\tau$, where $\tau = \Gamma\, t$ is the time measured in lifetime units. Therefore, one introduces

$$\frac{\Delta\mu}{\Gamma} = x - iy. \tag{5}$$

In most cases of interest the matter effects satisfy $\Gamma > |{\rm Im}\chi_{\rm av}|$ and $|\Delta\mu| > |\Delta\chi|$, and the depletion is still chiefly determined by Γ. To describe the trigonometric functions in matter we need Eq. (5) and a new complex parameter. This may be choosen to be $\Delta\chi/\Gamma$, which determines the depletion (statistics available) by the time the effects of $\Delta\chi$ become important. However, since we know the current experimental reach on $\Delta\mu$, it is easiest to use it to access the experimental reach on $\Delta\chi$, through[1]

$$2r = \frac{\Delta\chi}{\Delta\mu}. \tag{6}$$

Our figure of merit will be $|\Delta\chi/\Delta\mu|$.

Henceforth we will refer to those effects due to $\Delta\mu$ as 'mixing effects', and to those effects due to $\Delta\chi$ as 'matter effects'. The fundamental question is: when do the matter effects and the mixing effects couple to each other? The answer is easily found by looking at Eqs. (4). They can couple in two ways. The first and simplest way occurs when the production, propagation and decay occur all in the same medium and the decay is flavor tagging. In that case, the matter and mixing effects couple mainly through Ω. The second way occurs when both flavors can decay into the same final state, or when the system crosses several media. For example, for events occuring completely in the medium, with the decays going into CP-eigenstates. Or, when the production and the initial part of the evolution occur in matter, with the final part of the evolution and its decay occuring in vacuum. In these cases, besides Ω, there can also be effects proportional to $\Delta\mu\,\Delta\chi$, which arise from the interference between the coefficients of the sine terms multiplying $|P^0\rangle$ and of the sine terms multiplying $|\overline{P^0}\rangle$.

III. MATTER EFFECTS IN THE $D^0 - \overline{D^0}$ SYSTEM

A. Interplay between mixing and matter effects

We will now show that, even when visible, the matter effects cannot be used to probe x_D or y_D. The main reason is that the complex trigonometric functions are controled by $(x - iy)\tau$ and, with current technology, one can only

[1] Although it is conventional to use this factor of two in the definition of r, it seems to confuse rather than enlighten the issue.

follow τ up to 5 or 10, corresponding to 5 to 10 average lifetimes in the D proper frame. This is mostly due to the statistics of perfectly reconstructed events. It is obvious that, given unlimmited event samples, one could follow the full decay curve including times of order $1/y_D$. We mention this because the 'no-go theorem' we are about to present hinges on the experimental constraint that one can only follow the decay curves for time $\tau \ll 1/y_D$. Under these circumstances, we can expand the trigonometric functions on the right-hand side of Eqs. (4) as

$$\cos(\Omega\, t/2) \pm i\frac{\Delta\chi}{\Omega}\sin(\Omega\, t/2) \approx 1 \pm i\frac{\Delta\chi}{2}\, t - \frac{(\Delta\mu)^2 + (\Delta\chi)^2}{8}\, t^2 \mp i\Delta\chi\, \frac{(\Delta\mu)^2 + (\Delta\chi)^2}{48}\, t^3 + \bigcirc(t^4), \tag{7}$$

and

$$-\, i\frac{\Delta\mu}{\Omega}\sin(\Omega\, t/2) \approx -i\frac{\Delta\mu}{2}\, t + i\Delta\mu\, \frac{(\Delta\mu)^2 + (\Delta\chi)^2}{48}\, t^3 + \bigcirc(t^5), \tag{8}$$

for the right-sign and wrong-sign transitions, respectively.

The old results on semileptonic wrong-sign decays [11] measure the magnitude squared of Eq. (8), given to leading order by

$$\frac{|\Delta\mu|^2}{4\Gamma^2}\,\tau^2 = \frac{x^2 + y^2}{4}\tau^2. \tag{9}$$

Let us now consider a final state f to which both P^0 and $\overline{P^0}$ can decay. We denote the decay amplitudes by A_f and $\bar{A}_f$, respectively. Such a decay has a term linear in t arising from the interference between Eqs. (7) and (8),

$$\mathrm{Im}\left[\Delta\mu\,\frac{q}{p}\bar{A}_f\, A_f^*\right]\, t. \tag{10}$$

CLEO [2] has looked at $f = K^+\pi^-$ which is a Cabibbo-allowed decay for $\overline{D^0}$ and a Doubly-Cabibbo-suppressed decay for D^0. FOCUS [3] has looked at the CP-eigenstate $f = K^+K^-$.

Could one look at matter effects with semileptonic decays? In principle yes. The time evolution of the right-sign semileptonic decays is proportional to the magnitude squared of Eq. (7), which has a term linear in t,

$$\mp\,\mathrm{Im}\left(\frac{\Delta\chi}{\Gamma}\right)\,\tau. \tag{11}$$

Even if $|\Delta\chi|$ is an order of magnitude smaller than $|\Delta\mu|$, this term should still be easier to detect than the quadratic term in Eq. (9). This idea will be explained in detail below.

For the moment we wish to return to our fundamental question: can matter effects be used to give a new handle on $\Delta\mu$? The answer is no! First consider right-sign semileptonic decays and magnitude square the right-hand side of Eq. (7). We see that only at order τ^3 do we find effects proportional to $\Delta\mu\,\Delta\chi$. Even at order τ^2, the effects decouple as $\mathrm{Re}(\Delta\mu)^2$ and $\mathrm{Re}(\Delta\chi)^2$. Next consider wrong-sign semileptonic decays and magnitude square the right-hand side of Eq. (8). This case is even worse, since the effects proportional to $\Delta\mu\,\Delta\chi$ only arise at order τ^4. Now consider again a final state f to which both D^0 and $\overline{D^0}$ can decay. In the presence of matter Eq. (10) gets changed into

$$\mathrm{Im}\left[\Delta\mu\,\frac{q}{p}\bar{A}_f\, A_f^*\left(t + i\frac{\Delta\chi^*}{2}t^2\right)\right]. \tag{12}$$

In this case there is an effect proportional to $\Delta\mu\,\Delta\chi^*$ at order t^2. But, if $|\Delta\chi|$ is an order of magnitude smaller than $|\Delta\mu|$, then this effect will be an order of magnitude smaller than that in Eq. (9). Notice that, in particular, there is no effect proportional to Ωt.

In this discussion we have implicitly considered only events which occur totally in matter. However, the matter-vacuum transition can only promote the same type of interference effects we have already discussed [12].

B. Observing matter effects

We have already argued that matter effects cannot be used as a handle on x_D and y_D. We have yet to prove that they are observable at all, even in principle. Let us consider the decay $D^0 \to K^- l^+ \nu_l$. Using Eqs. (4) and (7) we find

$$\Gamma\left[D^0(t) \to K^- l^+ \nu_l\right] \approx e^{-\Gamma t}\, e^{2\mathrm{Im}\,\chi_{\mathrm{av}}\, t}\, |A_o|^2\, (1 + \mathrm{Im}\,\Delta\chi\; t)\,, \tag{13}$$

where we have denoted $A_o = A(D^0 \to K^- l^+ \nu_l)$.[2] We now need to estimate the linear term.

However, since the elastic forward scattering amplitudes for D^0 and $\bar{D}^0$ (f and $\bar{f}$) are not known, we have to estimate them somehow. We recall that the matter effects in the kaon system arise because K^0 interacts quasi-elastically with the nucleons, while $\overline{K^0}$ suffers inelastic interactions, such as

$$\begin{aligned} \overline{K^0} \ + \ (p^+, n) \ &\to \pi^0 + \ (\Sigma^+, \Sigma^0) \\ s\bar{d} \ + \ (uud, udd) &\to d\bar{d} \ + \ (uus, uds). \end{aligned} \tag{14}$$

The counterpart in the D system is

$$\begin{aligned} D^0 \ + \ (p^+, n) \ &\to \pi^0 + \ (\Sigma_c^+, \Sigma_c^0) \\ c\bar{u} \ + \ (uud, udd) &\to u\bar{u} \ + \ (udc, ddc). \end{aligned} \tag{15}$$

The kinematics are different because $m(D^0) \approx 1865\text{MeV} \gg m(K^0) \approx 498\text{MeV}$ and $m(\Sigma_c^+) \approx 2454\text{MeV} \gg m(\Sigma^+) \approx 1189\text{MeV}$. However, on the one hand we will be interested in the high momentum range (larger than 25GeV), where the difference becomes less important. And, on the other hand, we are not interested in exact results but, rather, wish to learn whether the matter effects are in principle observable. Therefore, we will simply scale the results from the kaon system to the D system.

We will use an empirical scaling law determined by Gsponer and collaborators from their measurements in C, Al, Cu, Sn, and Pb for kaon momenta between 30 to 150 GeV. They found that [13,14]

$$|\Delta f| = 1.13\text{fm} \left(\frac{A}{\text{g mol}^{-1}}\right)^{0.758} \left(\frac{p_K}{\text{GeV}c^{-1}}\right)^{0.386}, \tag{16}$$

where A the atomic number of the material. This result exhibits a power law momentum dependence in accordance with Regge theory [15], which also predicts that $\arg \Delta f$ should be constant and given by $-(1+0.386)\pi/2 = -0.693\pi$ [14]. The power-law approximation is fairly good down to a few GeV/c momentum, where low-energy resonances set in [16]. We neglect these effects since we will be concentrating on the high momentum range. We will also need the imaginary part of $f + \bar{f}$. Again we rely on Gsponer and collaborators, who found [18,19]

$$\text{Im}(f + \bar{f}) = 1.895\text{fm} \left(\frac{A}{\text{g mol}^{-1}}\right)^{0.840} \left(\frac{p_K}{\text{GeV}c^{-1}}\right) \tag{17}$$

in the kaon sector.

We start by noticing that the density of scattering centers is a medium is given by

$$N = \frac{N_A \rho}{A}, \tag{18}$$

where N_A is Avogadro's number, ρ is the density of the material, and A is its atomic number. Using Eqs. (3) and (16), we obtain

$$\begin{aligned} \frac{\Delta\chi_D}{\Gamma_D} &= \frac{1}{m_D \Gamma_D} \left(-\frac{2\pi N_A \rho}{A}\right) \Delta f \\ &= -5.6 \times 10^{-5} e^{i0.307\pi} \left(\frac{A}{\text{g mol}^{-1}}\right)^{-0.242} \left(\frac{\rho}{\text{g cm}^{-3}}\right)^{-0.242} \left(\frac{p_D}{\text{GeV}c^{-1}}\right)^{0.386}. \end{aligned} \tag{19}$$

This result should be compared with that obtained for the neutral kaons, where -5.6×10^{-5} is substituted by 0.09. The sign difference arises from the fact that the inelastic interactions occur for D^0, not $\overline{D^0}$; for example, Eq. (15) involves D^0, while Eq. (14) involves $\overline{K^0}$. Since we are assuming that the forward scattering amplitudes coincide, the difference in magnitudes arises from

[2] To illustrate this effect, we have simplified the discussion by considering a 'thought-experiment' with the events taking place (production, evolution, and decay) inside the material. For these purposes, a muon in the final state would be most readily observed.

$$\frac{|\Delta\chi_D|/\Gamma_D}{|\Delta\chi_K|/\Gamma_K} = \frac{m_K\Gamma_K}{m_D\Gamma_D} = \frac{498\text{MeV}}{1865\text{MeV}}\frac{0.415\times10^{-12}s}{2\times0.893\times10^{-10}s} = 6.2\times10^{-4}. \tag{20}$$

The results for carbon and tungsten are listed in table I for a variety of D momenta chosen within a range accessible to FOCUS [17]. Tungsten is a preferable material since it leads to values which are roughly 4.4 times larger than those obtained in carbon. For tungsten and momenta above $100\text{GeV}c^{-1}$ the effects are of order 0.2%. However, the current experiments already probe $\Delta\mu_D/\Gamma_D$ of order 1%. Therefore, in principle, the linear term in Eq. (13) would be detected with an experiment of this type.

Of course, we still need to show that the absorption term $\exp(2\text{Im}\,\chi_{\text{av}}\,t)$ does not affect greatly the number of events. Using Eq. (17), we find

$$2\frac{\text{Im}\,\chi_{\text{av}\ D}}{\Gamma_D} = -9.4\times10^{-5}\left(\frac{A}{\text{g mol}^{-1}}\right)^{-0.160}\left(\frac{\rho}{\text{g cm}^{-3}}\right)\left(\frac{p_D}{\text{GeV}c^{-1}}\right). \tag{21}$$

For tungsten and a momentum of $100\text{GeV}c^{-1}$, the ratio becomes 0.08. Looking back at the right-hand side of Eq. (13), this result means that the absorption exponential, $\exp(2\text{Im}\,\chi_{\text{av}}\,t)$, does not compete with the decay exponential, $\exp(-\Gamma t)$.

Although we have been detailed about our estimates, our objective is not to propose a specific experiment, but rather to emphasize that one is not, in principle, out of reach. Therefore, we will not address the obvious list of questions: i) How does one identify the primary and secondary vertices?; ii) How does one instrument the material?; iii) What is the ideal momentum range?; etc. We stress that the results presented here are based on the expectation that we can extrapolate from the kaon to the D system. They should be taken as estimates and not as reliable predictions. In particular, the presence of resonances could enhance or suppress these estimates.

IV. CONCLUSIONS

The situation in the B_d system is similar to the one in the D system in that $\Delta\Gamma \ll \Gamma$. Using the same naive high-momenta, scaling law used in Eq. (20), we obtain

$$\frac{|\Delta\chi_B|/\Gamma_B}{|\Delta\chi_K|/\Gamma_K} = \frac{m_K\Gamma_K}{m_B\Gamma_B} = 8.2\times10^{-4}. \tag{22}$$

However, in the B_d system $|\Delta\mu| \sim x_d \sim 0.7$, meaning that $|\Delta\chi|$ cannot compete with Γ nor with $|\Delta\mu|$.

The situation in the B_s is very different. From the point of view of the lifetime difference it is actually closer in spirit to the kaon system [20]. Indeed, in the B_s system $|\Delta\Gamma|/\Gamma \sim 0.15$ [21], and one may talk about two exponentials. However, B_s contains no quarks of the first family and, hence, it would seem inappropriate to estimate the matter effects as we have done above.

In conclusion, we have analyzed the possibility of detecting matter effects in the $D^0 - \overline{D^0}$ system. We conclude that these effects are, in principle, accessible. Unfortunately, when the time dependence is followed only for a few lifetimes, the matter effects and the mixing effects decouple. As a result, matter effects cannot be used to provide a new handle on x_D or y_D.

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$\frac{p_D}{\text{GeV}/c^{-1}}$	$100\left\lvert\frac{\Delta\chi_D}{\Gamma_D}\right\rvert_{\text{in carbon}}$	$100\left\lvert\frac{\Delta\chi_D}{\Gamma_D}\right\rvert_{\text{in tungsten}}$
25	0.024	0.106
50	0.032	0.138
100	0.042	0.182
150	0.048	0.212
200	0.054	0.236
250	0.058	0.258

TABLE I. Values of $\Delta\chi_D/\Gamma_D$ for carbon and tungsten, as a function of the D momentum.