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Basis-independent analysis of the sneutrino sector in R-parity violating supersymmetry

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Abstract

In R-parity-violating supersymmetric models (with a conserved baryon number), there are no quantum numbers that distinguish the lepton-doublet and down-type Higgs supermultiplets. As a result, the R-parity-violating parameters depend on the basis choice for these superfields, although physical observables are independent of the choice of basis. This paper presents a basis-independent computation of the sneutrino/antisneutrino squared-mass splitting in terms of basis-independent quantities. Techniques are developed for an arbitrary number of sneutrino generations; specific results are provided for the one, two and three generation cases.

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I. INTRODUCTION

In low-energy supersymmetric extensions of the Standard Model, lepton and baryon number conservation are not automatically respected by the most general set of renormalizable interactions [1]. Nevertheless, experimental observations imply that lepton number violating effects, if they exist, must be rather small. Moreover, baryon number violation, if present, must be consistent with the observed stability of the proton. If one wants to enforce lepton and baryon number conservation, it is sufficient to impose one extra discrete symmetry. In the minimal supersymmetric extension of the Standard Model (MSSM), a multiplicative symmetry called R-parity is introduced [2], such that the R quantum number of an MSSM field of spin S, baryon number B and lepton number L is given by $(-1)^{[3(B-L)+2S]}$. By introducing B–L conservation modulo 2, one eliminates all dimension-four lepton number and baryon number-violating interactions.

The observation of neutrino mixing effects in solar and atmospheric [3] neutrinos suggest that lepton-number is not an exact global symmetry of the low-energy theory. One can develop a supersymmetric model of neutrino masses that generalizes the see-saw mechanism while maintaining R-parity as a good symmetry [4,5] (where lepton number is violated by two units). In this paper, we consider the alternative possibility that neutrino masses and mixing arise in a theory of R-parity violation, in which lepton number is violated by one unit [6]. In the most general R-parity-violating (RPV) model, both B and L are violated. However, it is difficult to relax both lepton and baryon number conservation in the low-energy theory without generating a proton decay rate many orders of magnitude above the present bounds. It is possible to enforce baryon number conservation, while allowing for lepton number violating interactions by imposing a discrete baryon $\mathbf{Z_3}$ symmetry on the low-energy theory [7], in place of the standard $\mathbf{Z_2}$ R-parity. Henceforth, we consider R-parity-violating low-energy supersymmetry with an unbroken discrete baryon $\mathbf{Z_3}$ symmetry. This model exhibits lepton-number-violating phenomena such as neutrino masses, sneutrino/antisneutrino mixing, and lepton-number violating decays.

In RPV low-energy supersymmetry, there is no quantum number that distinguishes the lepton supermultiplets $\hat{L}_m$ and the down-type Higgs supermultiplet $\hat{H}_D$ (m is a generation label that runs from 1 to $n_g = 3$). Each supermultiplet transforms as a $Y = -1$ weak doublet under the electroweak gauge group. It is therefore convenient to denote $\hat{L}_0 \equiv \hat{H}_D$ and unify the four supermultiplets by one symbol $\hat{L}_\alpha$ ($\alpha = 0, 1, \ldots, n_g$). Then, the relevant terms in the (renormalizable) superpotential are

$$W = \epsilon_{ij}\left[-\mu_\alpha \hat{L}^i_\alpha \hat{H}^j_U + \tfrac{1}{2}\lambda_{\alpha\beta m}\hat{L}^i_\alpha \hat{L}^j_\beta \hat{E}_m + \lambda'_{\alpha n m}\hat{L}^i_\alpha \hat{Q}^j_n \hat{D}_m\right] , \tag{1.1}$$

where $\hat{H}_U$ is the up-type Higgs supermultiplet, the $\hat{Q}_n$ are doublet quark supermultiplets, the $\hat{D}_m$ are singlet down-type quark supermultiplets and the $\hat{E}_m$ are the singlet charged

lepton supermultiplets. Note that μ_α and $\lambda'_{\alpha nm}$ are vectors and $\lambda_{\alpha\beta m}$ is an antisymmetric matrix in the generalized lepton flavor space.

Next, the soft-supersymmetry-breaking terms are also generalized in similar way. The relevant terms are

$$V_{\rm soft} = (M^2_{\widetilde{L}})_{\alpha\beta}\,\widetilde{L}^{i*}_\alpha \widetilde{L}^i_\beta - (\epsilon_{ij} b_\alpha \widetilde{L}^i_\alpha H^j_U + \text{h.c.}) + \epsilon_{ij}[\tfrac{1}{2} a_{\alpha\beta m}\widetilde{L}^i_\alpha \widetilde{L}^j_\beta \widetilde{E}_m + a'_{\alpha nm}\widetilde{L}^i_\alpha \widetilde{Q}^j_n \widetilde{D}_m + \text{h.c.}]\,, \tag{1.2}$$

where the fields appearing in eq. (1.2) are the scalar partners of the superfields that appear in eq. (1.1). Here, b_α and $a'_{\alpha nm}$ are vectors, $a_{\alpha\beta m}$ is an antisymmetric matrix and $(M^2_{\widetilde{L}})_{\alpha\beta}$ is a Hermitian matrix in the generalized lepton flavor space.

When the scalar potential is minimized (see Section II), one finds a vacuum expectation value for the neutral scalar fields: $\langle \tilde{L}_\alpha \rangle = v_\alpha/\sqrt{2}$ and $\langle H_U \rangle = v_u/\sqrt{2}$. To make contact with the usual notation of the MSSM, we define the length of the vector v_α by $v_d \equiv (v_\alpha v_\alpha)^{1/2}$ and $\tan\beta \equiv v_u/v_d$. The mass of the W boson constrains the value $v^2 \equiv v_u^2 + v_d^2 = (246\ \text{GeV})^2$.

So far, there is no distinction between the neutral Higgs bosons and neutral sleptons. Nevertheless, we know that RPV-interactions, if present, must be small. It is tempting to choose a particular convention corresponding to a specific choice of basis in the generalized lepton flavor space. For example, one can choose to *define* the down-type Higgs multiplet such that $\langle H_D \rangle \equiv \langle \tilde{L}_0 \rangle = v_d/\sqrt{2}$ and $\langle \tilde{L}_m \rangle = 0$. This means that we let the dynamics (which determines the direction of the vacuum expectation value in the generalized lepton flavor space) choose the definition of the down-type Higgs field. In this basis, all the RPV-parameters are well defined and must be small to satisfy phenomenological constraints.

Nevertheless, the above convention is only one possible basis choice. Other conventions are equally sensible. For example, one could choose a second basis where $\mu_m = 0$ and a third basis where $b_m = 0$. In each case, the corresponding RPV-parameters are small. But comparing results obtained in different bases requires some care. Moreover, it is often desirable to study the evolution of couplings from some high (unification) scale to the low-energy (electroweak) scale. The renormalization group equations for the RPV-parameters is not basis preserving. That is, a particular basis choice at the high energy scale will lead to some complicated effective basis choice at the low-energy scale.

The problems described above can be ameliorated by avoiding basis-specific definitions of parameters. The challenge of such an approach is to determine a set of basis-independent RPV parameters, in the spirit of the Jarlskog invariant which characterizes the strength of CP-violation in the Standard Model [8]. Such an approach has been applied to RPV models in the past, where neutrino masses [9–13], early universe physics [14] and the Higgs sector [15] were studied. It is instructive to examine the neutrino spectrum of the RPV model. At tree level, one neutrino become massive due to the RPV mixing of the neutrinos and the neutralinos. The other $n_g - 1$ neutrinos remain massless at tree-level, although they can

acquire smaller radiative masses at one-loop. To first order in the small RPV-parameter the basis independent formula can be written in the following form [10,16]:

$$m_\nu = \frac{m_Z^2 \mu M_{\tilde{\gamma}} \cos^2\beta}{m_Z^2 M_{\tilde{\gamma}} \sin 2\beta - M_1 M_2 \mu} |\hat{v} \times \hat{\mu}|^2 \,, \tag{1.3}$$

where $M_{\tilde{\gamma}} \equiv \cos^2\theta_W M_1 + \sin^2\theta_W M_2$ depends on gaugino mass parameters M_1 and M_2. In eq. (1.3), $\hat{v}$ and $\hat{\mu}$ are unit vectors in the v_α and μ_α directions, respectively. It is convenient to introduce the notation of the cross-product of two vectors. Although the cross-product technically exists only in three-dimensions, the dot product of two cross-products can be expressed as a product of dot-products

$$(a \times b) \cdot (c \times d) = (a \cdot c)(b \cdot d) - (a \cdot d)(b \cdot c) \,, \tag{1.4}$$

which exists in any number of dimensions. This notation is useful, since any expression that involves the cross-product of two vectors vanishes if the corresponding vectors are parallel. This provides a nice geometrical characterization of the small RPV-parameters of the model. For example, $|\hat{v} \times \hat{\mu}|^2 = \sin^2\xi$ where ξ is the angle between $\hat{v}$ and $\hat{\mu}$. Thus, in eq. (1.3), $|\hat{v} \times \hat{\mu}|^2$ is the small RPV-parameter, while the prefactor can be computed in the R-parity-conserving (RPC) limit of the model.

In this paper, we focus on a basis-independent description of the RPV-parameters that govern the sneutrino spectrum. Due to the lepton-number violation of the model, $\Delta L = 2$ interactions are generated which gives rise to sneutrino/antisneutrino mixing [5,17–21,16,22,13]. In this case, the sneutrino ($\tilde{\nu}$) and antisneutrino ($\bar{\tilde{\nu}}$), which are eigenstates of lepton number, are no longer mass eigenstates. The mass eigenstates are superpositions of $\tilde{\nu}$ and $\bar{\tilde{\nu}}$, and sneutrino mixing effects can lead to a phenomenology analogous to that of K–$\overline{K}$ and B–$\overline{B}$ mixing [5]. The mass splitting between the two sneutrino mass eigenstates is related to the magnitude of lepton number violation, which is typically characterized by the size of neutrino masses [5,18]. As a result, the sneutrino/antisneutrino mass splitting is expected generally to be very small. Yet, it can be detected in many cases, if one is able to observe the lepton number oscillation [5].

In contrast to the neutrino sector (where only one neutrino mass eigenstate acquires a tree-level mass), in general *all* sneutrinos/antisneutrino pairs are split in mass at tree level. For simplicity, we consider the case of a CP-conserving scalar sector. In the RPC limit, the CP-even scalar sector consists of two Higgs scalars (h^0 and H^0, with $m_{h^0} < m_{H^0}$) and n_g generations of CP-even sneutrinos $(\tilde{\nu}_+)_m$, while the CP-odd scalar sector consists of the Higgs scalar, A^0, the Goldstone boson (which is absorbed by the Z), and n_g generations of CP-odd sneutrinos $(\tilde{\nu}_-)_m$. Here, we have implicitly chosen a flavor basis in which the sneutrinos are mass eigenstates. Moreover, the $(\tilde{\nu}_\pm)_m$ are mass degenerate (separately for each m), so that the standard practice is to define eigenstates of lepton number: $\tilde{\nu}_m \equiv [(\tilde{\nu}_+)_m + i(\tilde{\nu}_-)_m]/\sqrt{2}$ and $\overline{\tilde{\nu}}_m \equiv \tilde{\nu}_m^*$. When R-parity is violated, the sneutrinos in each CP-sector mix with the

corresponding Higgs scalars, and the mass degeneracy of $(\tilde{\nu}_+)_m$ and $(\tilde{\nu}_-)_m$ is broken. In ref. [16] we computed the mass-splitting in a special basis where $v_m = 0$ and the matrix $(M^2_{\tilde{\nu}\tilde{\nu}^*})_{ij}$ [which is the 3×3 block sub-matrix of $(M^2_{\tilde{\nu}\tilde{\nu}^*})_{\alpha\beta}$ defined in eq. (2.2)] is diagonal. In this basis, we identified the b_m as the relevant small RPV-parameters. To leading order in b_m^2,

$$(\Delta m^2_{\tilde{\nu}})_m = \frac{-4\, b_m^2\, m_Z^2\, m^2_{\tilde{\nu}_m}\, \sin^2\beta}{(m_H^2 - m^2_{\tilde{\nu}_m})(m_h^2 - m^2_{\tilde{\nu}_m})(m_A^2 - m^2_{\tilde{\nu}_m})}\,, \tag{1.5}$$

where $(\Delta m^2_{\tilde{\nu}})_m \equiv (m^2_{\tilde{\nu}_+})_m - (m^2_{\tilde{\nu}_-})_m$. As in the neutrino case described above, we may evaluate the prefactor that multiplies b_m^2 in the RPC limit. In deriving eq. (1.5), it was assumed that all RPC Higgs and sneutrino masses are all distinct. If degeneracies exist, the above formula must be modified.

The goal of this paper is to reanalyze the sneutrino mass spectrum in a basis-independent formalism. We identify the small RPV-parameters that govern the sneutrino/antisneutrino mass splittings. Our technique will also allow us to generalize the analysis to treat the case of scalar mass degeneracies. In Section II, we derive a convenient form for the CP-even and CP-odd scalar squared-mass matrices. We compute the sneutrino/antisneutrino squared-mass difference in the case of one sneutrino flavor in Section III. In Section IV, we generalize to an arbitrary number of generations, and exhibit explicit formulae for the two and three generation cases. The latter results assume that in the RPC limit, there are no degeneracies among different sneutrino flavors. The degenerate case is treated in Section V. A discussion of our results and conclusions are presented in Section VI. Details of our computations are provided in six appendices.

II. MINIMUM CONDITION AND BASIC EQUATIONS

We begin our analysis by collecting the relevant formulae given in ref. [16]. We assume that the scalar sector is CP-conserving,[a] which implies that the scalar fields can be defined such that $M^2_{\tilde{L}}$ is a real symmetric matrix and b_α and μ_α are real. The vacuum expectation value $\langle L_\alpha \rangle \equiv v_\alpha/\sqrt{2}$ is determined by minimizing the scalar potential. With the assumption of CP-conservation, one can separate out the scalar potential for the CP-even and CP-odd sector, $V = V_{\rm even} + V_{\rm odd}$. Then, v_α is determined by minimizing $V_{\rm even}$, and the resulting condition is given by:

[a] In the MSSM, the Higgs sector is automatically CP-conserving at tree-level, since all phases can be removed by suitable redefinitions of the fields. In the RPV model, new phases enter through $(M^2_{\tilde{L}})_{\alpha\beta}$, b_α and μ_α, which cannot all be simultaneously removed in the general case.

$$(M^2_{\tilde{\nu}\tilde{\nu}^*})_{\alpha\beta} v_\beta = v_u b_\alpha \,, \tag{2.1}$$

where

$$(M^2_{\tilde{\nu}\tilde{\nu}^*})_{\alpha\beta} \equiv (M^2_{\tilde{L}})_{\alpha\beta} + \mu_\alpha \mu_\beta - \tfrac{1}{8}(g^2 + g'^2)(v_u^2 - v_d^2)\delta_{\alpha\beta} \,. \tag{2.2}$$

Note that eq. (2.1) determines both the size of v_α and its direction. In a perturbative treatment of RPV terms, we can use the RPC value for the squared magnitude $v_d^2 \equiv \sum_\alpha v_\alpha v_\alpha$, and then use eq. (2.1) to determine the direction of v_α in the generalized lepton flavor space.

We next separate the scalar squared-mass matrices into CP-odd and CP-even blocks. In the H_U–$\widetilde{L}_\alpha$ basis, the CP-odd squared-mass matrix is given by

$$M^2_{\rm odd} = \begin{pmatrix} b_\rho v_\rho / v_u & b_\beta \\ b_\alpha & (M^2_{\tilde{\nu}\tilde{\nu}^*})_{\alpha\beta} \end{pmatrix} \,, \tag{2.3}$$

while the CP-even squared-mass matrix is given by

$$M^2_{\rm even} = \begin{pmatrix} \frac{1}{4}(g^2 + g'^2) v_u^2 + b_\rho v_\rho / v_u & -\frac{1}{4}(g^2 + g'^2) v_u v_\beta - b_\beta \\ -\frac{1}{4}(g^2 + g'^2) v_u v_\alpha - b_\alpha & \frac{1}{4}(g^2 + g'^2) v_\alpha v_\beta + (M^2_{\tilde{\nu}\tilde{\nu}^*})_{\alpha\beta} \end{pmatrix} \,, \tag{2.4}$$

where $(M^2_{\tilde{\nu}\tilde{\nu}^*})_{\alpha\beta}$ is defined in eq. (2.2).

To compute the squared-mass differences of the corresponding CP-even and CP-odd sneutrinos, we must diagonalize both of the above matrices. We wish to employ a perturbative procedure by identifying the small RPV-parameters, without resorting to a specific choice of basis. Our strategy is to recast the two scalar squared-mass matrices in a more convenient form.

First, consider the CP-odd squared mass matrix [eq. (2.3)]. Note that the vector $(-v_u, v_\beta)$ is an eigenvector of $M^2_{\rm odd}$ with zero eigenvalue; this is the Goldstone boson that is absorbed by the Z. We can remove the Goldstone boson by introducing the following orthogonal $(n_g + 2) \times (n_g + 2)$ matrix

$$U_o = \begin{pmatrix} -v_u/v & v_d/v & 0 \\ v_\beta/v & v_u v_\beta/(v_d v) & X_{\beta i} \end{pmatrix} \,, \tag{2.5}$$

where $v \equiv (v_u^2 + v_d^2)^{1/2}$. Note that the index i runs from 1 to n_g; thus $X_{\alpha i}$ is an $(n_g + 1) \times n_g$ matrix. The orthogonality of U_o implies that each column of U_o is a real unit vector and different columns are orthogonal. In addition, the set $\{v_\beta/v, X_{\beta i}\}$ forms an orthonormal set of vectors in an $(n_g + 1)$-dimensional vector space. It follows that:

$$v_\alpha X_{\alpha i} = 0 \,, \tag{2.6}$$

$$X_{\alpha i} X_{\alpha j} = \delta_{ij} \,, \tag{2.7}$$

$$X_{\alpha i} X_{\beta i} = \delta_{\alpha\beta} - \frac{v_\alpha v_\beta}{v_d^2} \,. \tag{2.8}$$

In our computations, no explicit realization of the $X_{i\alpha}$ will be required. A simple computation yields:

$$U_o^T M^2_{\rm odd} U_o = \begin{pmatrix} 0 & 0_\beta \\ 0_\alpha & (\widetilde{M}^2_{\rm odd})_{\alpha\beta} \end{pmatrix} , \tag{2.9}$$

where 0_β [0_α] is a row [column] matrix of zeros and[b]

$$\widetilde{M}^2_{\rm odd} = \begin{pmatrix} v^2(v\cdot b)/(v_u v_d^2) & v b_\beta X_{\beta i}/v_d \\ v X_{j\alpha} b_\alpha/v_d & X_{j\alpha}(M^2_{\tilde\nu\tilde\nu^*})_{\alpha\beta} X_{\beta i} \end{pmatrix} , \tag{2.10}$$

where $v\cdot b \equiv v_\alpha b_\alpha$. The eigenstates of $\widetilde{M}_{\rm odd}$ correspond to the CP-odd Higgs boson A^0 and n_g generations of CP-odd sneutrinos.

It turns out that it is also convenient to rotate the CP-even squared-mass matrix, but by a slightly different orthogonal transformation. In the limit of $m_Z = 0$, the CP-even squared-mass matrix also possesses a Goldstone boson, which we can explicitly isolate. Comparing the CP-odd and CP-even cases, we see that when $g = g' = 0$ the two matrices are related by $b_\alpha \to -b_\alpha$ and $v_u \to -v_u$. Thus, if we introduce $U_e \equiv U_o(v_u \to -v_u)$ and define $\widetilde{M}^2_{\rm even} \equiv U_e^T M^2_{\rm even} U_e$, then

$$\widetilde{M}^2_{\rm even} = \begin{pmatrix} m_Z^2\cos^2 2\beta & -m_Z^2\cos 2\beta\sin 2\beta & 0 \\ -m_Z^2\cos 2\beta\sin 2\beta & m_Z^2\sin^2 2\beta + v^2(v\cdot b)/(v_u v_d^2) & -v b_\beta X_{\beta i}/v_d \\ 0 & -v X_{j\alpha} b_\alpha/v_d & X_{j\alpha}(M^2_{\tilde\nu\tilde\nu^*})_{\alpha\beta} X_{\beta i} \end{pmatrix} , \tag{2.11}$$

where we used $m_Z^2 = \frac{1}{4}(g^2+g'^2)v^2$ and $\tan\beta \equiv v_u/v_d$. The eigenstates of $\widetilde{M}^2_{\rm even}$ correspond to the CP-even Higgs bosons h^0 and H^0 and n_g generations of CP-even sneutrinos.

In the RPC limit, one can choose a basis in which $v_m = b_m = X_{0m} = 0$. It follows that the sneutrino and Higgs mass matrices decouple and one recovers the known RPC result. A basis-independent characterization of the RPC limit in the scalar (sneutrino/Higgs) sector is the condition that the vectors b_β and v_β are aligned.[c] Equivalently, by using the transformed mass matrices given above, it is clear that the quantities

$$B_i \equiv \frac{v b_\beta X_{\beta i}}{v_d} \tag{2.12}$$

can be identified as n_g basis-independent parameters that vanish in the RPC limit, and thus provide good candidates for the small quantities that can be used in a perturbative

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[b]We define $X_{i\alpha}$ to be the transpose of the matrix $X_{\alpha i}$. When no ambiguity arises, we will not explicitly exhibit the transpose symbol (superscript T).

[c]For example, if one chooses the basis where $v_m = 0$, then by eq. (2.6), $X_{0m} = 0$. In this basis, the b_m are the small RPV-parameters.

expansion. If b_β and v_β are aligned, then eq. (2.6) implies that the $B_i = 0$ and we are back to the RPC limit.

Although the B_i provide a basis-independent set of small RPV-parameters, the explicit dependence on $X_{\alpha i}$ is inconvenient. Clearly, it is preferable to re-express the B_i directly in terms of the original model parameters. In the following sections, we will exhibit this procedure in the one-generation case and generalize it to the multi-generation case. To this end, it is convenient to introduce a set of new RPV-parameters. In the case of $n_g \leq 3$, only one new vector is required for our final results:

$$c_\alpha \equiv \frac{(M^2_{\tilde{\nu}\tilde{\nu}^*})_{\alpha\beta} b_\beta}{b^2} \,. \tag{2.13}$$

For $n_g > 3$, further vectors are required. It is convenient to introduce a series of vectors

$$c^{(n+1)}_\alpha \equiv (M^2_{\tilde{\nu}\tilde{\nu}^*})_{\alpha\beta} c^{(n)}_\beta \,, \tag{2.14}$$

where $c^{(1)}_\alpha \equiv c_\alpha$. Clearly, the maximal number of linearly independent vectors (along with b and v) is $n = n_g - 1$, although not all of these will appear in our final results.

To simplify the presentation we introduce the following shorthand for the elements of the transformed CP-even and CP-odd squared-mass matrices:

$$\widetilde{M}^2_{\rm odd} = \begin{pmatrix} A & B_i \\ B_j & C_{ij} \end{pmatrix} \,, \tag{2.15}$$

$$\widetilde{M}^2_{\rm even} = \begin{pmatrix} D & E & 0 \\ E & F + A & -B_i \\ 0 & -B_j & C_{ij} \end{pmatrix} \,. \tag{2.16}$$

Our strategy is to employ a first-order perturbative analysis to diagonalize these matrices, taking the small parameters to be the B_i [eq. (2.12)]. The result of such a calculation will be basis-independent expressions for the sneutrino/antisneutrino squared-mass splittings. We first illustrate the method for one sneutrino generation, and then generalize it to the multi-generational case.

III. THE CASE OF ONE GENERATION

In the one generation case we can drop the Roman indices from X, B and C, namely we define $X_\alpha \equiv X_{\alpha 1}$, $B \equiv B_1$ and $C \equiv C_{11}$. To zeroth order in B, the CP-even and CP-odd sneutrino squared-masses are equal to C. The corrections can be calculated perturbatively.

The eigenvalue equation for the CP-odd squared-mass matrix, $\det(\widetilde{M}^2_{\rm odd} - \lambda I) = 0$ reads

$$(A - \lambda)(C - \lambda) - B^2 = 0 \,. \tag{3.1}$$

For B small, $\lambda = C + \mathcal{O}(B^2)$, so we can take $\lambda - C = aB^2$ and solve for a. Thus, to first order in B^2, the squared mass of the CP-odd sneutrino is

$$m^2_{\rm odd} = C - \frac{B^2}{A - C}\,. \tag{3.2}$$

A similar analysis for the CP-even squared-mass matrix yields the following result for the squared-mass of the CP-even sneutrino:

$$m^2_{\rm even} = C - \frac{B^2(D - C)}{(F + A - C)(D - C) - E^2}\,. \tag{3.3}$$

The squared-mass splitting, $\Delta m^2_{\tilde\nu} = m^2_{\rm even} - m^2_{\rm odd}$, is given by

$$\Delta m^2_{\tilde\nu} = \frac{-FCB^2}{(A - C)[(F + A - C)(D - C) - E^2]}\,, \tag{3.4}$$

where we have used the fact that $FD = E^2$. Note that the only small parameter in eq. (3.4) is B^2. We can therefore use the RPC values for the prefactor that multiplies B^2. For example, as noted above, $C = m^2_{\tilde\nu}$ is the RPC sneutrino squared-mass.

Although $B = vb_\alpha X_\alpha/v_d^2$ is expressed in a basis-independent manner, the explicit dependence on X_α is inconvenient. Clearly, it is preferable to re-express B directly in terms of the original model parameters. To this end, note that the orthogonality condition [eq. (2.6)] implies that X_α and v_α are orthogonal, and thus a dot product of any vector with X is equivalent to a cross product with v. Using eq. (A2) for B^2 and the RPC values for the other parameters of eq. (3.4), we end up with

$$\Delta m^2_{\tilde\nu} = \frac{-4\,b^2 m_Z^2\, m^2_{\tilde\nu}\,\sin^2\beta}{(m^2_{H^0} - m^2_{\tilde\nu})(m^2_{h^0} - m^2_{\tilde\nu})(m^2_{A^0} - m^2_{\tilde\nu})}|\hat v \times \hat b|^2\,, \tag{3.5}$$

where $\hat b$ is a unit vector in the b_α direction and the square of the cross-product is formally defined according to eq. (1.4). It is easy to check [see Appendix B] that in the special basis where $v_1 = 0$, the basis-independent result above [eq. (3.5)] reduces to the basis-dependent result quoted in eq. (1.5).

The basis-independent result obtain in eq. (3.5) is still not in optimal form, since it depends on v, which is a derived quantity that requires one to determine the minimum of the scalar potential [eq. (2.1)]. However, we can employ the vector c_α [defined in eq. (2.13)] to our advantage by noting that in the $n_g = 1$ case [see eq. (C6)],

$$|b \times c|^2 = m^4_{\tilde\nu}|\hat v \times \hat b|^2\,. \tag{3.6}$$

Consequently, we can express the sneutrino squared-mass splitting in the one-generation case directly in terms of fundamental parameters of the RPV-Lagrangian in a completely basis-independent form.

IV. THE CASE OF AN ARBITRARY NUMBER OF GENERATIONS

In this section, we obtain results for an arbitrary number of generations. We then explicitly exhibit the corresponding results for $n_g = 2$ and 3 generations. The eigenvalue equation for the CP-odd scalar squared-mass matrix is

$$(A - \lambda)\det(C - \lambda I) + Y^{(N)}(\lambda) = 0\,, \tag{4.1}$$

where I is the $N \times N$ unit matrix, $N \equiv n_g$, and

$$Y^{(N)}(\lambda) \equiv B_i \mathrm{cof}\left[(\widetilde{M}^2_{\rm odd} - \lambda I)_{0i}\right]\,, \tag{4.2}$$

where the sum over the repeated index i is assumed implicitly. As usual, the cofactor is defined as $\mathrm{cof}[A_{ij}] = (-1)^{i+j}\det\tilde{A}(i,j)$ where $\tilde{A}(i,j)$ is the matrix A whose ith row and jth column are removed. In the special case of a one-dimensional matrix, we can define $\mathrm{cof}[A_{11}] = 1$.

Let $\lambda_m^{(0)}$ $(m = 1, 2, \cdots, N)$ be the roots of eq. (4.1) to zeroth order in the B_i, namely $\det(C - \lambda_m^{(0)} I) = 0$. For small B_i, we insert $\lambda_m = \lambda_m^{(0)} + (\delta\lambda_m)_{\rm odd}$ into eq. (4.1). Working to the lowest non-trivial order in the B_i, we make use of eq. (D2) to obtain

$$(\delta\lambda_m)_{\rm odd} = \frac{Y^{(N)}(\lambda_m)}{(A - \lambda_m)\det'(C - \lambda_m I)}\,, \tag{4.3}$$

where $\det' A$ is the product of all the non-zero eigenvalues of A. In this analysis, we assume that there are no degenerate eigenvalues (the degenerate case will be considered in Section V); hence

$$\det'(C - \lambda_m I) = \prod_{i \neq m}(\lambda_i - \lambda_m)\,. \tag{4.4}$$

Note that we do not distinguish between $\lambda_m^{(0)}$ and λ_m in eq. (4.3). Since the $B_i B_j$ are the small parameters, any distinction between the two estimates for λ_m would yield a result that is higher order in the product of the B_i.

By a similar technique, we may solve the eigenvalue equation for the CP-even scalar squared-mass matrix. Noting that $\lambda_m^{(0)}$ is the same in both the CP-odd and CP-even squared-mass computations, we can write $\lambda_m = \lambda_m^{(0)} + (\delta\lambda_m)_{\rm even}$. The end result is

$$(\delta\lambda_m)_{\rm even} = \frac{Y^{(N)}(\lambda_m)(D - \lambda_m)}{[(D - \lambda_m)(F + A - \lambda_m) - E^2]\det'(C - \lambda_m I)}\,. \tag{4.5}$$

We may evaluate the denominator of the above expression in the RPC limit (where $B^2 = 0$). In this limit,

$$\begin{aligned}(D - \lambda_m)(F + A - \lambda_m) - E^2 &= (m_h^2 - \lambda_m)(m_H^2 - \lambda_m)\,,\\ A - \lambda_m &= m_A^2 - \lambda_m\,.\end{aligned} \tag{4.6}$$

The squared-mass difference of the mth sneutrino/antisneutrino pair is denoted by $\Delta m^2_{\tilde{\nu}_m} = (m^2_{\rm even})_m - (m^2_{\rm odd})_m$. Plugging in the results of eqs. (4.3) and (4.5), we obtain

$$\Delta m^2_{\tilde{\nu}_m} = \frac{Y^{(N)}(\lambda_m)}{\det'(C-\lambda_m I)}\left[\frac{D-\lambda_m}{(m^2_h-\lambda_m)(m^2_H-\lambda_m)} - \frac{1}{(m^2_A-\lambda_m)}\right] . \tag{4.7}$$

We may further simplify this result by employing RPC values for any expression that multiplies a term of order B_iB_j. In particular, we can make use of the well known tree-level MSSM Higgs results: $m^2_{h^0}+m^2_{H^0} = m^2_{A^0}+m^2_Z$ and $m^2_{H^0}m^2_{h^0} = m^2_{A^0}m^2_Z\cos^2 2\beta$. Moreover, we may take $\lambda_m = m^2_{\tilde{\nu}_m}$. The end result is:

$$\Delta m^2_{\tilde{\nu}_m} = \frac{m^2_{\tilde{\nu}_m} m^2_Z \sin^2 2\beta\, Y^{(N)}(m^2_{\tilde{\nu}_m})}{(m^2_A-m^2_{\tilde{\nu}_m})(m^2_h-m^2_{\tilde{\nu}_m})(m^2_H-m^2_{\tilde{\nu}_m})\prod_{i\neq m}(m^2_{\tilde{\nu}_i}-m^2_{\tilde{\nu}_m})} . \tag{4.8}$$

The small RPV parameters that govern the above expression has been completely isolated into $Y^{(N)}$. One additional consequence of this result is a simple sum rule that holds for an appropriately weighted sum of sneutrino squared-mass differences. The sum rule and its derivation is given in Appendix E.

We next derive a method for computing $Y^{(N)}(\lambda)$. First, we evaluate $Y^{(N)}$ for $\lambda = 0$. From eq. (4.2), it is straightforward to evaluate $Y^{(N)}(0) = B_i{\rm cof}\left[(\widetilde{M}^2_{\rm odd})_{0i}\right]$. Using eq. (2.15),

$$Y^{(N)}(0) = -B_iB_j{\rm cof}\left[C_{ij}\right] . \tag{4.9}$$

Note that for $N=1$, $Y^{(1)}(\lambda) = -B^2$, independent of the value of λ. One can extend eq. (4.9) for arbitrary λ. We have found the following recursion relation:

$$Y^{(N)}(\lambda) = -B_iB_j{\rm cof}\left[C_{ij}\right] - \lambda Y^{(N-1)}(\lambda) , \tag{4.10}$$

with $Y^{(1)}(\lambda) = -B^2$. The proper use of this equation requires some care. One must first express $Y^{(N-1)}$ covariantly in terms of the $(N-1)$-dimensional vector B_i and $(N-1)\times(N-1)$ dimensional matrix C_{ij}. Then, the term $Y^{(N-1)}$ that appears in eq. (4.10) is given by precisely the same expression [obtained in the $(N-1)$-dimensional case], but with B_i and C_{ij} now N-dimensional objects. For example,

$$Y^{(2)}(\lambda) = Y^{(2)}(0) + \lambda B^2 , \tag{4.11}$$

but in eq. (4.11), $B^2 = \sum_{i=1}^N B_iB_i$, with $N=2$.

The solution to the recursion relation [eq. (4.10)] is

$$Y^{(N)}(\lambda) = \sum_{k=0}^{N-1}(-1)^k\lambda^k Y^{(N-k)}(0) . \tag{4.12}$$

Again, we emphasize that the $Y^{(N-k)}$ are first obtained by an $(N-k)$-dimensional computation. Once these terms are expressed covariantly in terms of B_i and C_{ij}, the resulting expressions for $Y^{(N-k)}$ may be used in eq. (4.12), with B_i and C_{ij} promoted to full N-dimensional objects. Thus, for each extra generation, we need only calculate one new invariant, $Y^{(N)}(0)$.

We illustrate the general formulae above for the cases of $N = 1$, 2 and 3 generations. The case of $N = 1$ is trivial. Here $Y^{(1)}(0) = -B^2$. Using eq. (A2), we quickly recover the result of eq. (3.5). For the case of $N = 2$, we use eqs. (4.9) and (4.12) to obtain:

$$Y^{(2)}(\lambda) = B^2\left[\lambda - \mathrm{Tr}(C)\right] + B_i B_j C_{ij}\,. \tag{4.13}$$

Using the results of Appendix A we can express $Y^{(2)}$ directly in terms of the model parameters:

$$Y^{(2)}(\lambda) = \frac{1}{v_d^2\cos^2\beta}\left\{|v\times b|^2\left[\lambda - \mathrm{Tr}(M^2_{\tilde{\nu}\tilde{\nu}^*})\right] + b^2(v\times b)\cdot(v\times c)\right\}\,. \tag{4.14}$$

The final result for the sneutrino squared-mass splittings in the two generation case is

$$\Delta m^2_{\tilde{\nu}_m} = \frac{4m^2_{\tilde{\nu}_m} m_Z^2\tan^2\beta\left\{|v\times b|^2\left[m^2_{\tilde{\nu}_m} - \mathrm{Tr}(M^2_{\tilde{\nu}\tilde{\nu}^*})\right] + b^2(v\times b)\cdot(v\times c)\right\}}{v^2(m^2_{\tilde{\nu}_n} - m^2_{\tilde{\nu}_m})(m_A^2 - m^2_{\tilde{\nu}_m})(m_h^2 - m^2_{\tilde{\nu}_m})(m_H^2 - m^2_{\tilde{\nu}_m})}\,, \tag{4.15}$$

where $n \neq m$ and we have put $\lambda_m = m^2_{\tilde{\nu}_m}$. We may evaluate $\mathrm{Tr}(M^2_{\tilde{\nu}\tilde{\nu}^*})$ in the RPC limit:

$$\mathrm{Tr}(M^2_{\tilde{\nu}\tilde{\nu}^*}) = |b|\tan\beta + m^2_{\tilde{\nu}_1} + m^2_{\tilde{\nu}_2}\,, \tag{4.16}$$

where $|b| \equiv (b_\alpha b_\alpha)^{1/2}$. In Appendix B, we verify that eqs. (1.5) and (4.15) agree in the special basis.

As in the previous section, we note that eq. (4.15) depends on the derived quantity v. At the expense of a somewhat more complex result, we can re-express eq. (4.15) in terms of the vectors b, c, and a new vector $c^{(2)}$ introduced in eq. (2.14), as shown in Appendix C.

For $N = 3$ generations, the new invariant that arises is again obtained from eq. (4.9):

$$Y^{(3)}(0) = \tfrac{1}{2}B^2\left[\mathrm{Tr}(C^2) - [\mathrm{Tr}(C)]^2\right] + B_iB_jC_{ij}\,\mathrm{Tr}(C) - B_iB_jC_{ik}C_{kj}\,. \tag{4.17}$$

Following the procedure outlined above, eqs. (4.10) and (4.13) yield:

$$Y^{(3)}(\lambda) = Y^{(3)}(0) - \lambda Y^{(2)}(\lambda)\,, \tag{4.18}$$

where $Y^{(2)}(\lambda)$ is given by eq. (4.14), with v, b and c promoted to three-dimensional vectors and $M^2_{\tilde{\nu}\tilde{\nu}^*}$ promoted to a 3×3 matrix. An explicit evaluation of $Y^{(3)}(0)$ is given in eq. (A11). Inserting the corresponding results into eq. (4.8), we end up with the sneutrino squared-mass splittings in the three generation case:

$$\begin{aligned}\Delta m^2_{\tilde{\nu}_m} = {} & \frac{-4m^2_{\tilde{\nu}_m} m_Z^2\tan^2\beta}{v^2(m^2_{\tilde{\nu}_n} - m^2_{\tilde{\nu}_m})(m^2_{\tilde{\nu}_k} - m^2_{\tilde{\nu}_m})(m_A^2 - m^2_{\tilde{\nu}_m})(m_h^2 - m^2_{\tilde{\nu}_m})(m_H^2 - m^2_{\tilde{\nu}_m})} \\ & \times\Bigg\{|v\times b|^2\left[m^4_{\tilde{\nu}_m} - m^2_{\tilde{\nu}_m}\mathrm{Tr}(M^2_{\tilde{\nu}\tilde{\nu}^*}) - \tfrac{1}{2}[\mathrm{Tr}(M^4_{\tilde{\nu}\tilde{\nu}^*}) - [\mathrm{Tr}(M^2_{\tilde{\nu}\tilde{\nu}^*})]^2\right] \\ & \quad + b^2(v\times b)\cdot(v\times c)\left[m^2_{\tilde{\nu}_m} - b\cdot c - \mathrm{Tr}(M^2_{\tilde{\nu}\tilde{\nu}^*})\right] + b^2v_d^2|b\times c|^2\Bigg\}\,,\end{aligned} \tag{4.19}$$

where $n \neq k \neq m$. The traces in the RPC limit are given by:

$$\mathrm{Tr}(M^2_{\tilde{\nu}\tilde{\nu}^*}) = |b| \tan\beta + \sum_{k=1}^{3} m^2_{\tilde{\nu}_k} \,,$$
$$\mathrm{Tr}(M^4_{\tilde{\nu}\tilde{\nu}^*}) = b^2 \tan^2\beta + \sum_{k=1}^{3} m^4_{\tilde{\nu}_k} \,. \tag{4.20}$$

Again, we can check that in the special basis [see Appendix B], eqs. (1.5) and (4.19) agree. The extension of these results to four and more generations is straightforward.

V. THE DEGENERATE CASE

So far we have assumed that the sneutrinos are non-degenerate. We expect this assumption to hold in any realistic model, since the sneutrino/antisneutrino mass splittings are of order the neutrino masses. Thus in order for the result of the previous section not to hold, the flavor degeneracy has to be very good, namely the mass splitting between different sneutrino flavors should be much smaller than the neutrino mass. In any realistic model, we do not expect such a high degree of degeneracy. Even in models where supersymmetry breaking is flavor blind, a mass-splitting between sneutrino flavors will be generated via renormalization group (RG) evolution (from the scale of primordial supersymmetry breaking to the electroweak scale) that is proportional to the corresponding charged lepton masses. Even a very small amount of running is sufficient to generate a mass splitting that is many orders of magnitude larger than the neutrino mass.

Nevertheless, as a mathematical exercise and for completeness, we generalize the results of the previous sections to the case of degenerate sneutrinos. First we give a basis-dependent argument that explains how one can obtain the sneutrino squared-mass splittings in the degenerate case from the results already obtained in the non-degenerate case without any additional calculation. A basis-independent proof is relegated to Appendix F.

Consider a case with n_f sneutrinos, of which n_d sneutrinos are degenerate in mass (where $2 \leq n_d \leq n_f$) in the RPC limit. Consider the n_d degenerate sneutrinos and their corresponding antisneutrinos. Of these, $n_d - 1$ sneutrino/antisneutrino pairs remain degenerate when RPV effects are included, while one pair is spilt in mass.[d] In total, $n_f - n_d + 1$ sneutrino/antisneutrino pairs are split in mass. The corresponding squared-mass differences are then given by eq. (4.8) for the $(n_f - n_d + 1)$-generation case, but with all vectors and tensors appearing in the formula promoted to n_f dimensions. The proof of this assertion is as follows. For the case of n_d degenerate sneutrinos, the matrix C [that appears in eqs. (2.15) and (2.16)] has n_d degenerate eigenvalues. Thus, we are free to make arbitrary rotations

[d] This corrects a misstatement made at the end of Section III in ref. [16].

within the n_d dimensional subspace corresponding to the degenerate states. By a suitable rotation, we can choose of basis in which only one of the B_i within the degenerate subspace is non-zero. In this basis the CP-odd and the CP-even squared-mass matrices [eqs. (2.15) and (2.16)] separate into (n_d-1) and (n_f-n_d+1)-dimensional blocks. Clearly, the sneutrino eigenvalues in the corresponding (n_d-1)-dimensional blocks are not affected by the presence of RPV terms, while the $(n_f - n_d + 1)$-dimensional block can be treated by the methods of Section IV.

Further generalizations, where more than one set of sneutrinos are each separately degenerate, can also be studied. The procedure for computing the resulting sneutrino squared-mass differences is now clear, so we shall not elaborate further.

VI. DISCUSSION

This paper provides formulae for the sneutrino/antisneutrino squared-mass differences at tree-level in terms of basis-independent R-parity-violating (RPV) quantities. In contrast to the neutrino sector, where only one tree-level neutrino mass is generated by RPV-effects, we expect that *all* sneutrino/antisneutrino squared-mass differences are generated at tree-level with roughly the same order of magnitude. The sneutrino/antisneutrino mass difference is expected to be of the same order of magnitude as the (tree-level) neutrino mass. However, these quantities could be significantly different, as they depend on independent RPV-parameters. One can also analyze the case of degenerate masses for different sneutrino flavors; although this case can only arise as a result of a high degree of fine-tuning of low-energy parameters. The pattern of sneutrino/antisneutrino squared-mass differences would provide some insight into the fundamental origin of lepton flavor at a very high energy scale.

The sneutrino/antisneutrino squared-mass splittings can be explored either directly by observing sneutrino oscillation [5], or indirectly via its effects on other lepton number violating processes, such as neutrinoless double beta decay [17] and neutrino masses [22,13]. Moreover, the effects of tree-level sneutrino/antisneutrino squared-mass splittings on neutrino masses are expected to be significant. The neutrino spectrum is determined by the relative size of the different RPV couplings that control three sources of neutrino masses: (i) the tree-level mass, (ii) the sneutrino induced one-loop masses, and (iii) the trilinear RPV induced one-loop masses [16]. Since only one neutrino acquires a tree-level mass, the other two mechanisms are responsible for the masses of the other two neutrinos. In the literature, only the trilinear RPV-induced one-loop masses have been considered in most studies. In refs. [22] and [13], it is argued that the sneutrino-induced one-loop contributions to the neutrino masses are generically dominant, since the trilinear RPV-induced one-loop masses are additionally suppressed by a factor proportional to the Yukawa coupling squared.

The results of our basis-independent formalism are useful for comparing the two radiative neutrino mass generation mechanisms. In particular, in models in which a theory

of flavor determines the structure of the soft-supersymmetry-breaking parameters at some high energy scale, RG-evolution provides the connection between the observed low-energy spectrum and the high-energy values of the fundamental parameters of the theory [10,23]. Basis-dependent quantities are not renormalization-group invariant; hence the RG-evolution of basis-independent quantities can significantly simplify the analysis. For example, the direction of the vacuum expectation value of the generalized slepton/Higgs scalar field is dynamically generated at each energy scale. Since the model parameters generically depend on the scale, the direction of the vacuum expectation value in the generalized lepton flavor space is scale dependent. Clearly, in the basis-independent approach, such complications are avoided. This will be the subject of a subsequent paper.

A few possible directions for future research are worth noting. First, recall that in this paper, we assumed that CP was conserved in the scalar sector. If CP is violated, the required analysis is more complicated. Instead of diagonalizing separately CP-even and CP-odd squared-mass matrices, one must diagonalize a single squared-mass matrix in which the formerly CP-even and CP-odd states can mix. Then, one must identify the two sneutrino mass eigenstates (in the limit of small RPV couplings). It should be possible to extend the techniques developed in this paper to address this more general case. Second, in exploring the phenomenology of sneutrino interactions (production cross-sections and decay), one can generally assume that RPV-couplings are irrelevant except in the decay of the lightest sneutrino state. In that case, new RPV-couplings enter, in particular the corresponding λ and λ' parameters given in eq. (1.1). In the spirit of this paper, one should also develop a basis-independent formalism to describe the RPV sneutrino decay. We hope to return to some of these issues in a future work.

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APPENDIX A: EVALUATION OF $Y^{(N)}(0)$ FOR $N = 1, 2$ AND 3

Using eq. (4.9) for $Y^{(N)}(0)$, we provide below the explicit computation for the cases of $N = 1, 2$ and 3. The computation makes use of the definitions of B_i and C_{ij}:

$$B_i \equiv \frac{b_\beta X_{\beta i}}{\cos\beta}\,,$$
$$C_{ij} \equiv X_{\alpha j} M^2_{\alpha\beta} X_{\beta i}\,, \tag{A1}$$

where $M^2 \equiv M^2_{\tilde{\nu}\tilde{\nu}^*}$, and the properties of the $X_{\alpha i}$ given in eqs. (2.6)–(2.8).

The case of $N = 1$ is very simple.

$$Y^{(1)}(0) = -B^2 = \frac{-1}{\cos^2\beta} b_\alpha X_{\alpha i} b_\beta X_{\beta i} = \frac{-1}{\cos^2\beta}\left[b^2 - \frac{(b\cdot v)^2}{v_d^2}\right] = \frac{-b^2}{\cos^2\beta}|\hat{v}\times\hat{b}|^2\,, \tag{A2}$$

where the product of cross-products, defined in eq. (1.4), can be used in any number of dimensions.

For the case of $N = 2$, we compute

$$\begin{aligned} Y^{(2)}(0) &= B_i B_j C_{ij} - B^2 \mathrm{Tr}(C) \\ &= \frac{1}{\cos^2\beta}\left[X_{\alpha i} M^2_{\alpha\beta} X_{\beta j} b_\mu X_{\mu i} b_\nu X_{\nu j} - X_{\alpha i} M^2_{\alpha\beta} X_{\beta i} b_\mu X_{\mu j} b_\nu X_{\nu j}\right] \\ &= \frac{1}{v_d^2\cos^2\beta}\left\{b^2 v_d^2 (c\cdot b) - v_u b^2 (b\cdot v) - [b^2 v_d^2 - (b\cdot v)^2]\,\mathrm{Tr}(M^2)\right\} \\ &= \frac{1}{v_d^2\cos^2\beta}\left\{b^2 (v\times b)\cdot(c\times v) - |v\times b|^2\,\mathrm{Tr}(M^2)\right\}\,. \end{aligned} \tag{A3}$$

Note that the resulting expression has simplified considerably after introducing the vector c [defined in eq. (2.13)].

The case of $N = 3$ is more involved.

$$Y^{(3)}(0) = \tfrac{1}{2}B^2\left[\mathrm{Tr}(C^2) - [\mathrm{Tr}(C)]^2\right] + B_i B_j C_{ij}\,\mathrm{Tr}(C) - B_i B_j C_{ik} C_{kj}\,. \tag{A4}$$

We calculate separately the two terms above. First, B^2 is obtained from eq. (A2) and

$$\begin{aligned} \mathrm{Tr}(C^2) - [\mathrm{Tr}(C)]^2 &= \left[X_{\alpha i} M^2_{\alpha\beta} X_{\beta j} X_{\mu j} M^2_{\mu\nu} X_{\nu i} - X_{\alpha i} M^2_{\alpha\beta} X_{\beta i} X_{\mu j} M^2_{\mu\nu} X_{\nu j}\right] \\ &= \mathrm{Tr}(M^4) - [\mathrm{Tr}(M^2)]^2 - \frac{2v_u}{v_d^2}\left[v_u b^2 - (b\cdot v)\,\mathrm{Tr}(M^2)\right]\,. \end{aligned} \tag{A5}$$

Next, we evaluate

$$\begin{aligned} &[B_i B_j C_{ij}\,\mathrm{Tr}(C) - B_i B_j C_{ij}^2]\cos^2\beta \\ &\quad= X_{\alpha i} M^2_{\alpha\beta} X_{\beta i} X_{\mu j} M^2_{\mu\nu} X_{\nu k} b_\sigma X_{\sigma k} b_\rho X_{\rho j} - X_{\alpha i} M^2_{\alpha\beta} X_{\beta j} X_{\mu j} M^2_{\mu\nu} X_{\nu k} b_\sigma X_{\sigma k} b_\rho X_{\rho i} \\ &\quad= M^2_{\mu\nu} b_\mu b_\nu\,\mathrm{Tr}(M^2) - M^2_{\alpha\mu} M^2_{\mu\nu} b_\nu b_\alpha - \frac{v_u}{v_d^2}\left[b^2 (b\cdot v)\,\mathrm{Tr}(M^2) - (b\cdot v) M^2_{\mu\nu} b_\nu b_\alpha\right] \\ &\qquad + \frac{v_u}{v_d^2}\left[v_u b^4 - b^2 (b\cdot v)\,\mathrm{Tr}(M^2)\right] + \frac{v_u}{v_d^4}\left[(b\cdot v)^3\,\mathrm{Tr}(M^2) - v_u b^2 (b\cdot v)^2\right]\,. \end{aligned} \tag{A6}$$

The above result can be simplified further. First, the last two terms can be combined by noting that

$$v_u b^4 - b^2(b\cdot v)\,\mathrm{Tr}(M^2) + \frac{1}{v_d^2}\left[(b\cdot v)^3\,\mathrm{Tr}(M^2) - v_u b^2 (b\cdot v)^2\right] = \frac{|v\times b|^2}{v_d^2}\left[v_u b^2 - (b\cdot v)\,\mathrm{Tr}(M^2)\right]. \tag{A7}$$

This term will end up canceling a similar term in eq. (A5).

At this point, it is convenient to re-express some of the terms of eq. (A6) in terms of the vector c. First, we observe that

$$\begin{aligned} v_d^2 M^2_{\mu\nu} b_\mu b_\nu\,\mathrm{Tr}(M^2) - v_u b^2 (b\cdot v)\,\mathrm{Tr}(M^2) &= b^2\left[(b\cdot c)v_d^2 - (b\cdot v)(c\cdot v)\right]\,\mathrm{Tr}(M^2) \\ &= b^2 (v\times c)\cdot(v\times b)\,\mathrm{Tr}(M^2)\,. \end{aligned} \tag{A8}$$

In deriving the above result, we noted that $M^2_{\mu\nu} b_\nu v_\nu = v_u b^2 = b^2(v\cdot c)$ [using eqs. (2.1) and (2.13) and the fact that $M^2_{\tilde\nu\tilde\nu^*}$ is a symmetric matrix], which implies that

$$v_u = v\cdot c\,. \tag{A9}$$

Second,

$$\begin{aligned} v_d^2 M^2_{\alpha\mu} M^2_{\mu\nu} b_\nu b_\alpha - v_u (b\cdot v) M^2_{\mu\nu} b_\nu b_\nu &= b^4 c^2 v_d^2 - v_u b^2 (b\cdot v)(b\cdot c) \\ &= b^2\left[v_d^2 |b\times c|^2 + (b\cdot c)(v\times b)\cdot(v\times c)\right]\,. \end{aligned} \tag{A10}$$

Collecting all of the above results, the final expression is quite compact:

$$\begin{aligned} Y^{(3)}(0) = \frac{1}{v_d^2\cos^2\beta}\Big\{&\tfrac{1}{2}|v\times b|^2\left[\mathrm{Tr}(M^4) - [\mathrm{Tr}(M^2)]^2\right] + b^2 v_d^2 |b\times c|^2 \\ &+ b^2 (v\times c)\cdot(v\times b)\left[\mathrm{Tr}(M^2) - (b\cdot c)\right]\Big\}\,. \end{aligned} \tag{A11}$$

APPENDIX B: SNEUTRINO SQUARED-MASS SPLITTING FORMULAE IN THE SPECIAL BASIS

We define the special basis in which $v_m = 0$ (*i.e*, the neutral scalar vacuum expectation values determines the definition of the down-type Higgs field) and the matrix $(M^2_{\tilde\nu\tilde\nu^*})_{ij}$ [which is the 3×3 block sub-matrix of $(M^2_{\tilde\nu\tilde\nu^*})_{\alpha\beta}$ defined in eq. (2.2)] is diagonal. As in Appendix A we define $M^2 \equiv M^2_{\tilde\nu\tilde\nu^*}$.

In the special basis, one can use eqs. (2.1), (2.13) and (A9) to obtain the following relations:

$$c_0 = \tan\beta\,, \qquad M^2_{00} = b_0 c_0\,, \qquad M^2_{0i} = b_i\tan\beta\,, \qquad c_i = \frac{b_i(M^2_{00} + M^2_{ii})}{b^2}\,. \tag{B1}$$

Using these results, it follows that

$$|v \times b|^2 = v_d^2 \sum_i b_i^2 \,, \tag{B2}$$

$$b^2 (v \times b) \cdot (v \times c) = v_d^2 \left[M_{00}^2 \sum_i b_i^2 + \sum_i M_{ii}^2 b_i^2 \right] . \tag{B3}$$

We will also need a similar expression for $|b \times c|^2$. First, we note that the last relation of eq. (B1) implies

$$c^2 = c_0^2 + \frac{1}{b^4} \sum_i b_i^2 (M_{00}^2 + M_{ii}^2)^2 \,, \qquad b \cdot c = M_{00}^2 + \frac{1}{b^2} \sum_i b_i^2 (M_{00}^2 + M_{ii}^2) \,. \tag{B4}$$

These results can be used to obtain:

$$|b \times c|^2 = b^2 c^2 - (b \cdot c)^2 = \frac{1}{b^2} \sum_i (b_i M_{ii}^2)^2 + \mathcal{O}(b_i^4) \,. \tag{B5}$$

We now turn to the specific cases. For the case of one generation, the basis-independent result is given in eq. (3.5). In the special basis, eq. (B2) yields

$$b^2 |\hat{v} \times \hat{b}|^2 = b_1^2 . \tag{B6}$$

Inserting this result into eq. (3.5), we immediately obtain eq. (1.5).

In the two generation case, the basis-independent result is given in eq. (4.15). In the special basis, eqs. (B2) and (B3) yield

$$b^2 (v \times b) \cdot (v \times c) = v_d^2 \left[M_{00}^2 (b_1^2 + b_2^2) + M_{11}^2 b_1^2 + M_{22}^2 b_2^2 \right] \,, \tag{B7}$$

$$[M_{ii}^2 - \mathrm{Tr}(M^2)] |v \times b|^2 = -v_d^2 \left(M_{00}^2 + M_{jj}^2 \right) (b_1^2 + b_2^2) \,, \tag{B8}$$

for the two cases of $i = 1$, $j = 2$ and $i = 2$, $j = 1$, respectively. Adding the above two equations, one finds

$$[M_{ii}^2 - \mathrm{Tr}(M^2)] |v \times b|^2 + b^2 (v \times b) \cdot (v \times c) = (M_{ii}^2 - M_{jj}^2) b_i^2 v_d^2 \,. \tag{B9}$$

Working to leading order in the RPV-parameters b_i^2, we may set the diagonal elements of M^2 to their RPC values, $M_{ii}^2 = m_{\tilde{\nu}_i}^2$. Plugging the result into eq. (4.15), one again recovers eq. (1.5).

In the three generation case, the basis-independent result is given in eq. (4.19). Again, it is sufficient to work to leading order in the b_i^2. Then, one finds that in the special basis,

$$\begin{aligned} [M_{11}^2]^2 - M_{11}^2 \mathrm{Tr}(M^2) - \tfrac{1}{2}[\mathrm{Tr}(M^4) - [\mathrm{Tr}(M^2)]^2] &= M_{00}^2 \left[M_{22}^2 + M_{33}^2 \right] + M_{22}^2 M_{33}^2 + \mathcal{O}(b_i^2) \,, \\ M_{11}^2 - b \cdot c - \mathrm{Tr}(M^2) &= -M_{22}^2 - M_{33}^2 + \mathcal{O}(b_i^2) \,. \end{aligned} \tag{B10}$$

Using these results and those of eqs. (B2), (B3) and (B5), we end up with

$$\left\{[M_{11}^2]^2 - M_{11}^2 \mathrm{Tr}(M^2) - \tfrac{1}{2}[\mathrm{Tr}(M^4) - [\mathrm{Tr}(M^2)]^2\right\} |v \times b|^2 + b^2 v_d^2 |b \times c|^2$$
$$+ b^2 (v \times b) \cdot (v \times c) \left[M_{11}^2 - b \cdot c - \mathrm{Tr}(M^2)\right] = v_d^2\, b_1^2\, (M_{11}^2 - M_{22}^2)(M_{11}^2 - M_{33}^2)\,.$$

Two additional equations can be generated by permuting the indices 1,2 and 3. Finally, setting $M_{ii}^2 = m_{\tilde{\nu}_i}^2$ and plugging the result into eq. (4.19), one confirms eq. (1.5) for the third time.

APPENDIX C: HOW TO ELIMINATE v IN FAVOR OF OTHER VECTORS

Our final expressions for the sneutrino squared mass differences depend on basis-independent products of vectors, v, b, $c, \ldots$, and traces of powers of $M_{\tilde{\nu}\tilde{\nu}^*}^2$. However, the vector v is not a fundamental parameter of the model, but a derived parameter which arises as a solution to eq. (2.1). With some manipulation, it is possible to eliminate v in favor of the other vectors (which correspond more directly to the fundamental supersymmetric model parameters, namely b and a series of vectors obtained by multiplying b some number of times by $M_{\tilde{\nu}\tilde{\nu}^*}^2$). In this appendix, we illustrate the procedure in the case of the one and two generation models.

In the one generation model, $M_{\tilde{\nu}\tilde{\nu}^*}^2$ is a 2×2 matrix. Consider an arbitrary 2×2 matrix A and its characteristic equation $\det(A - \lambda I) = 0$. Since any matrix satisfies its own characteristic equation, we obtain[e]

$$A^2 - A\mathrm{Tr}(A) + \det(A) = 0\,, \tag{C1}$$

which after multiplication by A^{-1} yields

$$A^{-1} = \frac{\mathrm{Tr}(A) - A}{\det(A)}\,. \tag{C2}$$

Using eq. (C2) we can express $|v \times b|^2$ in terms of $|b \times c|^2$. Let $A \equiv M_{\tilde{\nu}\tilde{\nu}^*}^2$, and use eqs. (2.1) and (2.13) to obtain

$$v = v_u A^{-1} b = \frac{v_u\,[b\mathrm{Tr}(A) - b^2 c]}{\det(A)}\,, \tag{C3}$$

We now substitute eq. (C3) for v in $|v \times b|^2 \equiv b^2 v_d^2 - (v \cdot b)^2$.

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[e] We henceforth suppress the obvious factors of the identity matrix I.

$$b^2 v_d^2 = \frac{b^4 v_u^2}{[\det(A)]^2} \left\{ [\mathrm{Tr}(A)]^2 - 2(b\cdot c)\mathrm{Tr}(A) + b^2 c^2 \right\} ,$$

$$(v\cdot b)^2 = \frac{b^4 v_u^2}{[\det(A)]^2} \left\{ [\mathrm{Tr}(A)]^2 - 2(b\cdot c)\mathrm{Tr}(A) + (b\cdot c)^2 \right\} . \tag{C4}$$

Subtracting these two equations, we end up with

$$|v\times b|^2 = \frac{b^4 v_u^2}{[\det(A)]^2} |b\times c|^2 . \tag{C5}$$

Since $|b\times c|^2$ is the small RPV parameter, we may evaluate $\det(A)$ in the RPC limit. Using eq. (2.1) in the RPC limit, $[\det(A)]^2 = m_{\tilde\nu}^4\, b^2 \tan^2\beta$. The end result is

$$|b\times c|^2 = m_{\tilde\nu}^4 |\hat v\times \hat b|^2 . \tag{C6}$$

In the two generation model, $M^2_{\tilde\nu\tilde\nu^*}$ is a 3×3 matrix. The procedure again employs the characteristic equation. For an arbitrary 3×3 matrix,

$$A^{-1} = \frac{A^2 - A\mathrm{Tr}(A) + \mathrm{Sym}_2 A}{\det(A)} , \tag{C7}$$

where[f]

$$\mathrm{Sym}_2(A) \equiv \sum_{i<j} \lambda_i\lambda_j \ = \tfrac{1}{2}\left\{ [\mathrm{Tr}(A)]^2 - \mathrm{Tr}(A^2) \right\} , \tag{C8}$$

where λ_j are the eigenvalues of A. We can again solve for v following the method used in the one-generation case:

$$v = v_u A^{-1} b = \frac{v_u\,[b^2 d - b^2 c\,\mathrm{Tr}(A) + b\,\mathrm{Sym}_2(A)]}{\det(A)} , \tag{C9}$$

where we have defined $d \equiv c^{(2)} = Ac$. After some algebra, we obtain[g]

$$|v\times b|^2 = \frac{b^4 v_u^2}{[\det(A)]^2} \left[|b\times d|^2 + |b\times c|^2 [\mathrm{Tr}(A)]^2 - 2(b\times c)(b\times d)\mathrm{Tr}(A) \right] , \tag{C10}$$

and

$$(v\times b)\cdot(v\times c) = \frac{b^2 v_u^2}{[\det(A)]^2} \Big[(d\times b)\cdot(d\times c)[b^2 - \mathrm{Sym}_2(A)] \\ + b^2 (c\times d)\cdot(c\times b)\mathrm{Tr}(A) + |b\times c|^2\,\mathrm{Tr}(A)\,\mathrm{Sym}_2(A) \Big] . \tag{C11}$$

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[f] To prove eq. (C8), simply note that $\mathrm{Tr}(A^2) = \sum_k \lambda_k^2$.

[g] Observe that the result for $|v\times b|$ depends on the number of generations, *i.e.* the dimension of the matrix A [compare eqs. (C5) and (C10)].

It is easy to evaluate the three invariants in the RPC limit:[h]

$$\begin{aligned}
\mathrm{Tr}(A) &= |b|\tan\beta + m_1^2 + m_2^2\,, \\
\det(A) &= m_1 m_2 |b|\tan\beta\,, \\
\mathrm{Sym}_2(A) &= m_1^2 m_2^2 + |b|\tan\beta(m_1^2+m_2)^2\,,
\end{aligned} \tag{C12}$$

where $m_i^2 \equiv m_{\tilde{\nu}_i}^2$.

Inserting the above results into eq. (4.15) yields the desired result. Further algebraic manipulations of the resulting expression do not lead to a particularly simple result.

APPENDIX D: EVALUATION OF $\det[A-(\lambda_m-\epsilon)I]$

Consider a general $N\times N$ matrix A, with eigenvalues λ_k. Then,

$$\det(A-\lambda I) = \prod_k^N (\lambda_k - \lambda)\,, \tag{D1}$$

where the product is taken over all N eigenvalues (some of which might be degenerate). First, suppose that there are no degenerate eigenvalues. If λ_m is one of the eigenvalues and $\epsilon \ll 1$, then

$$\begin{aligned}
\det[A-(\lambda_m-\epsilon)I] &= \epsilon \prod_{i\neq m}(\lambda_i-\lambda_m) + \mathcal{O}(\epsilon^2) \\
&= \epsilon\,\det{}'(A-\lambda_m I) + \mathcal{O}(\epsilon^2)\,,
\end{aligned} \tag{D2}$$

where $\det' M$ is the product of all the non-zero eigenvalues of M [see eq. (4.4)].

The case of degenerate eigenvalues is easily handled. We can still use eq. (4.4) if it is understood that all terms in which λ_i is equal to the degenerate eigenvalue are omitted from the product. If λ_d is an eigenvalue which is n_d-fold degenerate, then eq. (D2) is generalized to

$$\det[A-(\lambda_d-\epsilon)I] = \epsilon^{n_d}\det{}'(A-\lambda_d I) + \mathcal{O}(\epsilon^{n_d+1})\,. \tag{D3}$$

In Section IV and Appendix F, we have employed these results with $\epsilon = -\delta\lambda_m$ and $\epsilon = -\delta\lambda_d$, respectively.

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[h] Since $v\cdot c = v_u$, it follows that in the RPC limit, $|c| = \tan\beta$.

APPENDIX E: SNEUTRINO SQUARED-MASS SPLITTING SUM RULES

In the case of N sneutrino generations, one can calculate the corresponding sneutrino squared-mass splittings. In the case of non-degenerate tree-level sneutrino masses, the squared-mass splittings were obtained in eq. (4.8). In the case of degenerate masses, one employs the modified results according to the discussion given in Section V and Appendix F. We then find the following interesting sum rule:

$$\sum_{m=1}^{N} \frac{v^2(m_A^2 - m_{\tilde{\nu}_m}^2)(m_h^2 - m_{\tilde{\nu}_m}^2)(m_H^2 - m_{\tilde{\nu}_m}^2)}{4m_Z^2 \tan^2\beta} \frac{\Delta m_{\tilde{\nu}_m}^2}{m_{\tilde{\nu}_m}^2} = -|v \times b|^2 \,. \tag{E1}$$

We shall prove this result for the non-degenerate case. Using eqs. (4.8) and (A2), we see that eq. (E1) is equivalent to the following result:

$$\sum_{m=1}^{N} \frac{Y^{(N)}(\lambda_m)}{\prod_{i\neq m}(\lambda_i - \lambda_m)} = Y^{(1)}(0) \,. \tag{E2}$$

To prove eq. (E2), we insert the expansion for $Y^{(N)}(\lambda_m)$ [eq. (4.12)] into eq. (E2), and make use the following identity:

$$S_{N,k} \equiv \sum_{m=1}^{N} \frac{\lambda_m^k}{\prod_{i\neq m}(\lambda_m - \lambda_i)} = \begin{cases} 0\,, & k = 0, 1, \ldots, N-2\,, \\ 1\,, & k = N-1\,, \end{cases} \tag{E3}$$

where all the λ_m are assumed to be distinct.[i]

Eq. (E3) is established as follows. Let $f(x) = (x-\lambda_1)(x-\lambda_2)\cdots(x-\lambda_N)$, where the λ_m are distinct. Consider the resolution of $x^{k+1}/f(x)$ into partial fractions (where k is an integer such that $0 \leq k \leq N-2$):

$$\frac{x^{k+1}}{f(x)} = \sum_{m=1}^{N} \frac{A_m}{x - \lambda_m} \,. \tag{E4}$$

Combining denominators, it follows that:

$$x^{k+1} = \sum_{m=1}^{N} A_m \prod_{i\neq m} (x - \lambda_i) \,. \tag{E5}$$

The right hand side of eq. (E5) is a polynomial of degree $N-1$ or less. Since this must be an identity for all x, we can solve for each coefficient A_m separately by setting $x = \lambda_m$:

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[i]Note that in eq. (E3), the sign of the factors $\lambda_m - \lambda_i$ is reversed compared to eq. (E2). Thus, an extra factor of $(-1)^{N-1}$ is generated which cancels with the corresponding sign in front of $Y^{(1)}$ in eq. (4.12).

$$A_m = \frac{\lambda_m^{k+1}}{\prod_{i\neq m}(\lambda_m - \lambda_i)} \,. \tag{E6}$$

Inserting this result into eq. (E4) and setting $x = 0$ yields eq. (E3) for the case of $0 \leq k \leq N-2$. The case of $k = 0$ where one of the λ_m vanishes must be treated separately, although it is easy to show that the end result is unchanged. Thus, $S_{N,k} = 0$ for $0 \leq k \leq N-2$.

To derive eq. (E3) in the case of $k = N-1$, we set $k = N-2$ in eq. (E5). On the right hand side of eq. (E5), we note that the term proportional to x^{N-1} arises simply by setting the $\lambda_i = 0$. It follows that $\sum_{m=1}^{N} A_m = 1$ (for $k = N-2$) which is precisely equivalent to $S_{N,N-1} = 1$ and the proof is complete.

Finally, we note a useful recursion relation satisfied by the $S_{N,k}$. Multiply the mth term of eq. (E3) by $(\lambda_m - \lambda_{N+1})/(\lambda_m - \lambda_{N+1})$. One immediately deduces that relation:

$$S_{N,k} = S_{N+1,k+1} - \lambda_{N+1} S_{N+1,k} \,. \tag{E7}$$

The boundary conditions for the recursion relation are: $S_{N,0} = 0$ for $N \geq 2$ (which is a consequence of the proof given above), and $S_{1,0} = 1$.[j] It follows that $S_{N,k} = 0$ for $1 \leq k \leq N-2$. Choosing $k = N-1$ in eq. (E7) then yields $S_{N+1,N} = S_{N,N-1}$; it follows that $S_{N+1,N} = 1$ for all $N \geq 1$. Finally, it is easy to increase k further. For example, eq. (E7) implies that $S_{N+1,N+1} = S_{N,N} + \lambda_{N+1}$. It follows that $S_{N,N} = \sum_{i=1}^{N} \lambda_i$. And so on.

The degenerate case can be treated as a limiting case of the non-degenerate results obtained above. In the final analysis, we find that eq. (E1) applies in general, and serves as a useful check of our results.

APPENDIX F: BASIS-INDEPENDENT TREATMENT OF THE DEGENERATE CASE

The degenerate case for n_f flavors was treated in Section V. If n_d sneutrino/antisneutrino pairs are degenerate in mass in the RPC limit, then when RPV-effects are incorporated, one finds that $n_d - 1$ pairs remain degenerate, while $n_f - n_d + 1$ sneutrino/antisneutrino pairs are split in mass. The squared-mass splittings of the latter can be obtained from the corresponding formulae of the non-degenerate $n_f - n_d + 1$ flavor case.

In this appendix, we briefly sketch the required steps of a proof that generalizes the basis-independent results of Section IV. Consider first the squared-mass matrix of the CP-odd scalars [eq. (2.15)]. The characteristic equation, eq. (4.1), is still valid in the case of

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[j] The condition $S_{1,0} = 1$ formally defines the sum [eq. (E3)] in the case of $N = 1$. Alternatively, one can check by explicit evaluation that $S_{2,1} = 1$. Thus, we see that the assigned definition of $S_{1,0}$ is consistent. Note that one can similarly define $S_{1,k} = \lambda_1^k$.

degenerate sneutrinos. First we consider the quantity $Y^{(N)}(\lambda)$ [eq. (4.2)]. Suppose that the matrix C which appears in $\widetilde{M}^2_{\rm odd}$ has an eigenvalue λ_d that is n_d-fold degenerate (with the remaining eigenvalues of C distinct). We assert that the following formula holds:

$$Y^{(N)}(\lambda) = (\lambda_d - \lambda)^{n_d-1} Y^{(N-n_d+1)}_{\rm deg}(\lambda)\,, \tag{F1}$$

where $Y^{(N-n_d+1)}_{\rm deg}(\lambda)$ is obtained as follows. First, one evaluates $Y^{(N-n_d+1)}(\lambda)$ as in Section IV, and expresses the result covariantly in terms of the vector B_i and the matrix C_{ij}. Next, these quantities are reinterpreted as N-dimensional objects. Finally, all traces that appear in the result are replaced by:

$$\mathrm{Tr}'\, C^n \equiv \mathrm{Tr}\, C^n - (n_d - 1)\lambda_d^n\,. \tag{F2}$$

Consider first the effect of the RPV terms on the non-degenerate sneutrinos. Then the analysis of Section IV can be used, and we obtain eq. (4.8) for the squared-mass splitting of sneutrino/antisneutrino pairs. If we now insert eq. (F1) for $Y^{(N)}(m^2_{\tilde\nu_m})$, we see that we obtain a new formula which has the same form as eq. (4.8), with the following modifications: (i) $Y^{(N)}(m^2_{\tilde\nu_m})$ is replaced by $Y^{(N-n_d+1)}_{\rm deg}(m^2_{\tilde\nu_m})$; and (ii) the product that appears in the denominator of eq. (4.8) is modified to $\prod'_{i\neq m}(m^2_{\tilde\nu_i} - m^2_{\tilde\nu_m})$, where the prime indicates that degenerate squared-masses appear only once in the product. To obtain a covariant expression for $Y^{(N-n_d+1)}_{\rm deg}(m^2_{\tilde\nu_m})$, we first obtain the expression of $Y^{(N-n_d+1)}(m^2_{\tilde\nu_m})$ in terms of the various vectors (v, b, c, ...) and traces of powers of M^2 using the results of Section IV and Appendix A. The resulting expression can then be used for $Y^{(N-n_d+1)}_{\rm deg}(m^2_{\tilde\nu_m})$ by replacing $\mathrm{Tr}\, M^{2n}$ with $\mathrm{Tr}'\, M^{2n}$ [the latter is defined by replacing C with M^2 in eq. (F2)] and interpreting all the vectors and matrices as N-dimensional objects.

Finally, consider the effect of the RPV terms on the n_d degenerate sneutrinos. Now, we must return to eq. (4.1) and insert $\lambda = \lambda_d^{(0)} + (\delta\lambda_d)_{\rm odd}$. Working to the lowest non-trivial order in the B_i, we make use of eqs. (F1) and (D3) to obtain

$$[(\delta\lambda_d)_{\rm odd}]^{n_d} = \frac{Y^{(N-n_d+1)}_{\rm deg}(\lambda_d)\,[(\delta\lambda_d)_{\rm odd}]^{n_d-1}}{(A-\lambda_d)\,\det'(C-\lambda_d I)}\,. \tag{F3}$$

The solution to this equation has $n_d - 1$ degenerate solution, $(\delta\lambda_d)_{\rm odd} = 0$, and one non-degenerate solution for $(\delta\lambda_d)_{\rm odd}$ which has the same form as eq. (4.3) for the $(N - n_d + 1)$-dimensional problem. As described above, we can make use of the relevant covariant expressions obtained in Section IV and Appendix A by replacing Tr with Tr$'$ and interpreting all the vectors and matrices as N-dimensional objects. The end result is that $n_d - 1$ sneutrino/antisneutrino pairs remain degenerate, while one of the original degenerate pairs is split according to the $N - n_d + 1$-dimensional version of eq. (4.8) [with all vectors and tensors promoted to N-dimensional objects]. One can also check that the sum rule obtained in Appendix E for sneutrino squared-mass differences (appropriately weighted) applies even when there are degenerate sneutrino masses.

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