A novel Final Focus design for future linear colliders.

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Abstract

The length, complexity and cost of the present Final Focus designs for linear colliders grows very quickly with the beam energy. In this letter, a novel final focus system is presented and compared with the one proposed for NLC [1]. This new design is simpler, shorter and cheaper, with comparable bandwidth, tolerances and tunability. Moreover, the length scales slower than linearly with energy allowing for a more flexible design which is applicable over a much larger energy range.

1 INTRODUCTION

The main task of a linear collider final focus system (FFS) is to focus the beams to the small sizes required at the interaction point (IP). To achieve this, the FFS forms a large and almost parallel beam at the entrance to the final doublet (FD), which contains two or more strong quadrupole lenses. For the nominal energy, the beam size at the IP is then determined by $\sigma = \sqrt{\varepsilon\,\beta^*}$ where ε is the beam emittance and β^* is the betatron function at the IP (typically about 1–0.1 mm). However, for a beam with an energy spread σ_E (typically 0.1–1%), the beam size is diluted by the chromaticity of these strong lenses. The chromaticity scales as L^*/β^*, where L^* (typically 2–4 m) is the distance from the interaction point to the final doublet, and thus the chromatic dilution of the beam size $\sigma_E L^*/\beta^*$ is very large. The design of a final focus system is therefore driven primarily by the necessity of compensating the chromaticity of the final doublet.

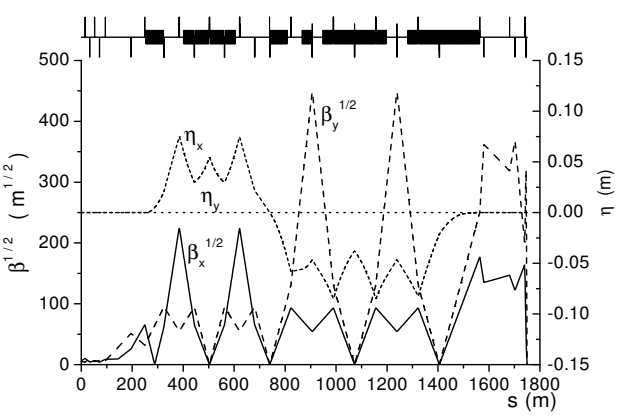

Figure 1: Optics of the traditional Final Focus for the Next Linear Collider showing horizontal and vertical betatron (β) and dispersion (η) functions. Focusing and defocusing quadrupoles are indicated as up and down bars on the magnet plot above the optics, while the bends are centered.

In a "traditional" final focus system (SLC [2], FFTB [3] or the new linear collider designs) the chromaticity is compensated in dedicated chromatic correction sections (CCX and CCY) by sextupoles placed in high dispersion and high beta regions. The geometric aberrations generated by the sextupoles are canceled by using them in pairs with an identity transformation between them. As an example, the design of the NLC Final Focus System [1] with $L^* = 2$ m, $\beta_x^* = 10$ mm and $\beta_y^* = 0.12$ mm is shown in Fig.1. The advantage of the traditional FFS is its separated optics with strictly defined functions and straightforward cancellation of geometrical aberrations. This makes such a system relatively simple for design or analysis.

The major disadvantage is that the chromaticity of the FD is not locally compensated. As a direct consequence there are intrinsic limitations on the bandwidth of the system due to the unavoidable breakdown of the proper phase relations between the sextupoles and the FD for different energies. This precludes the perfect cancellation of the chromatic aberrations. Moreover, the system is very sensitive to any disturbance of the beam energy in between the sources of chromaticity, whether due to longitudinal wakefields or synchrotron radiation. In particular, the bends in the system have to be long and weak to minimize the additional energy spread generated. In addition, the phase slippage of the off-momentum particles drastically limits the dynamic aperture of the system. Therefore very long and problematic collimation sections are required in order to eliminate these particles that would otherwise hit the FD and/or generate background in the detector. The collimation section optics itself also becomes a source of aberrations since very large beta and dispersion functions are required. As a result of all these limitations, the length of the beam delivery system becomes a significant fraction of the length of the entire accelerator, and scaling to higher energies is difficult.

2 "IDEAL" FINAL FOCUS SYSTEM

Taking into account the disadvantages of the traditional approach, one can formulate the requirements for a more "ideal" final focus:

1. The chromaticity should be corrected as locally as possible.
2. The number of bends should be minimized.
3. The dynamic aperture or, equivalently, the preservation of the linear optics should be as large as possible.
4. The system should be as simple as possible.
5. The system should be optimized for flat beams.

It is very straightforward, starting from the IP, to build such a system:

1. A Final Doublet is required to provide the focusing.
2. The FD generates chromaticity, so two sextupoles interleaved with these quadrupoles and a bend upstream to generate dispersion across the FD will locally cancel the chromaticity.
3. The sexupoles generate geometric aberrations, so two more sextupoles in phase with them and upstream of the bend are required.
4. In general four more quadrupoles are needed upstream to match the incoming beta function.

A schematic of such a FF is shown in Fig.2.

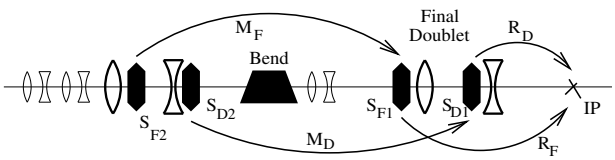

Figure 2: Optical layout of the new final focus.

The second order aberrations are cancelled when the following conditions are satisfied. The x-pair and y-pair of sextupoles must be separated by transfer matrices M_F and M_D :

$$M_F = \begin{vmatrix} F & 0 & 0 & 0 \\ F_{21} & 1/F & 0 & 0 \\ 0 & 0 & F & 0 \\ 0 & 0 & F_{43} & 1/F \end{vmatrix} ; \; M_D = \begin{vmatrix} D & 0 & 0 & 0 \\ D_{21} & 1/D & 0 & 0 \\ 0 & 0 & D & 0 \\ 0 & 0 & D_{43} & 1/D \end{vmatrix}$$

where all nonzero parameters are arbitrary. The sextupole integrated strengths K_S are determined by the equations:

$$K_{SF2} = -F^3 K_{SF1} \quad ; \quad K_{SD2} = -D^3 K_{SD1}$$
$$K_{SF1} = \frac{\xi_{x1} + \xi_{x2}}{R^3_{F12}\eta'} \quad ; \quad K_{SD1} = \frac{\xi_y}{R^3_{D34}\eta'}$$
$$\xi_{x1} = \xi_{x2} \quad ; \quad \xi_x = \frac{d^2x}{dx' dE/E}$$

x and x' are the beam coordinates at the IP, ξ_{x1} is the horizontal chromaticity of the system upstream of the bend, ξ_{x2} is the chromaticity downstream, ξ_y is the vertical chromaticity. R_F, R_D are the transfer matrices defined in Fig.2. The angular dispersion at the IP, η', is necessarily nonzero in the new design, but can be small enough that it does not significantly increase the beam divergence. Part of the horizontal chromaticity must be generated upstream of the bend in order to cancel the second order dispersion of the system.

The third order geometric aberrations generated by the sextupoles are:

$$U_{1222} = K_{SD} K_{SF} R^2_{D12} R^2_{F12} \psi_{12}$$
$$U_{3444} = K_{SD} K_{SF} R^2_{D34} R^2_{F34} \psi_{12}$$
$$U_{1244} = U_{3224} = -\frac{1}{2} K_{SD} K_{SF} [\psi_{12} R^2_{D34} R^2_{F12}$$
$$+ \psi_{12} R^2_{D12} R^2_{F34} - 4\psi_{34} R_{D12} R_{D34} R_{F12} R_{F34}]$$

where ψ_{12} and ψ_{34} are the elements of the transfer matrix between S_{F1} and S_{D1}. The term U_{3444} is small if the last quadrupole is defocusing. U_{1222} is negligible for typical parameters of flat beams. U_{1244} and U_{3224} can be made to vanish. Similar constraints hold for third order chromo-geometric aberrations. All these constraints can be satisfied with the simple system described above. A system with the same demagnification as the NLC FF and comparable optical performance can be constructed in a length of about 300 m.

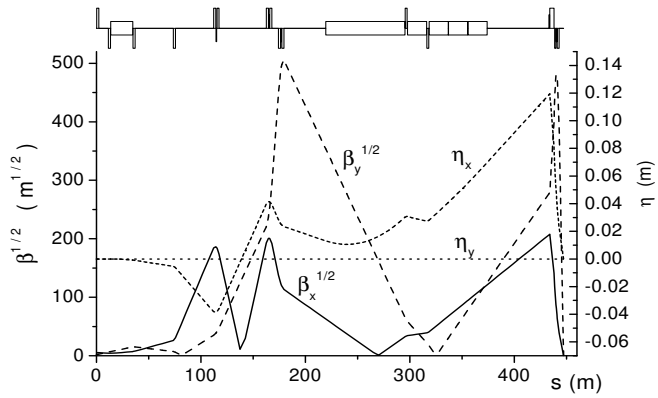

Figure 3: Optics of the new NLC Final Focus System showning horizontal and vertical betatron (β) and dispersion (η) functions similarly to Fig.1.

3 PERFORMANCE OF THE NEW FF

The new FF system has potentially much better performance than the traditional design. The "minimal" optics concept can be further improved by adding more elements to minimize residual aberrations. An additional bend upstream of the second sextupole pair can decrease the chromaticity through the system. An additional sextupole upstream and in phase with the last one further reduces aberrations in the horizontal plane. The proposed optics is shown in Fig.3. The flat beam parameters for which the system has been optimized are given in Table 1. It should be noted that this new system has an $L^* = 4$ m, which is twice the original value. This allows the use of large bore superconducting quadrupoles and simplifies the design of the detector. Although the chromaticity is doubled due to the larger L^*, the performance of the system is still comparable or better than for the original NLC FF design.

Table 1: Beam parameters

Beam energy, GeV	500
Normalized emittances $\gamma\varepsilon_x$ /$\gamma\varepsilon_y$ (μm)	4 / 0.06
Beta-functions β_x / β_y at IP (mm)	9.5 / 0.12
Beam sizes σ_x / σ_y at IP (nm)	197 / 2.7
Beam divergence θ_x / θ_y at IP (μrad)	21/23
Energy spread σ_E (10^{-3})	3
Dispersion' η'_x at IP (10^{-3})	5.4

Figs.4,5,6 compare the bandwidth of the NLC FF and the new design. Figs.4,5 show the IP beamsize as a function of energy and the luminosity reduction as a function of energy spread. Fig.6 shows the bandwidth in terms of the beamsize at the final doublet. The bandwidth is derived

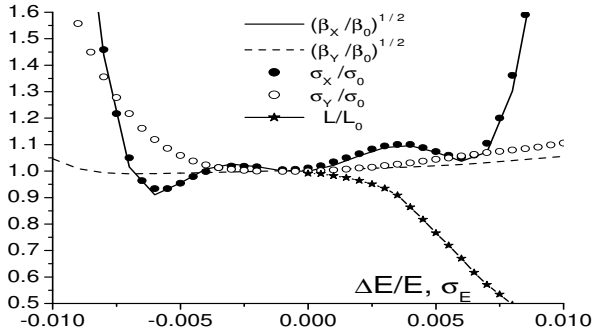

Figure 4: IP bandwidth of the traditional NLC Final Focus. Normalized betatron functions and normalized luminosity equivalent beam size (determined by tracking) versus energy offset $\Delta E/E$, and normalized luminosity versus rms energy spread σ_E.

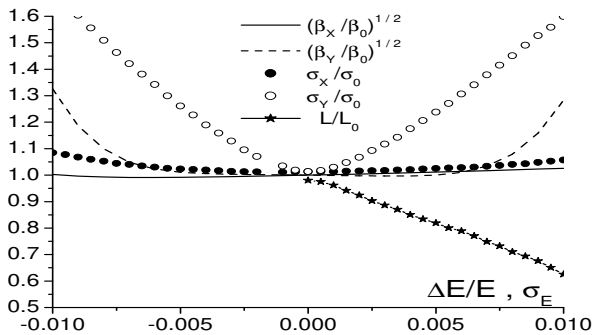

Figure 5: IP bandwidth of the New NLC Final Focus with definitions as in Fig.4.

from the variation of the beta function and the beam sizes as they actually contribute to luminosity, which is determined by tracking. The beam size bandwidth is narrower than the beta function bandwidth because of higher order cross-plane chromatic aberrations. While the IP bandwidth for these two systems is comparable, the FD bandwidth is much wider for the new FF.

Fig.7 shows the particle distribution at the face of the final doublet for the traditional FF and for the new FF. Particles of the incoming beam are placed on a surface of an ellipsoid with dimensions $N_\sigma(x, x', y, y', E) = (800, 8, 4000, 40, 20)$ times larger than the nominal parameters. The beam is very distorted in the traditional FF while the nonlinear terms are still negligible for the new FF. The nonzero dispersion across the FD in the new system has little affect on the dynamic aperture. In addition, it should be noted that the design aperture of the NLC final doublet is about $r_a = 10$ mm while for the new FF with twice longer L^* this aperture can be as large as $r_a = 40$ mm. Therefore the collimation requirements for the new FF may be relaxed by a factor of at least one hundred in the IP phase, and by a factor of at least 3 for the FD phase and energy without increasing particle losses at the FD.

Due to the shorter length of the system, there would also be less regeneration of the beam halo in the final focus itself

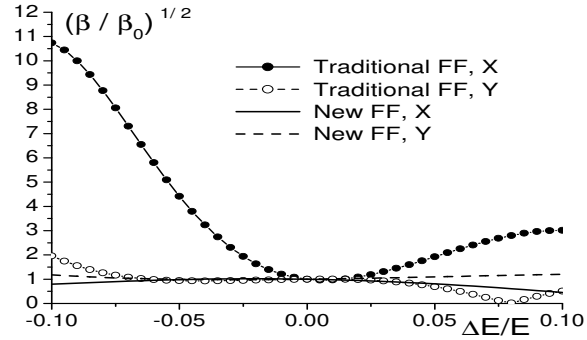

Figure 6: FD bandwidth of the Traditional and New NLC Final Focus. Normalized betatron functions at the final doublet versus energy offset $\Delta E/E$.

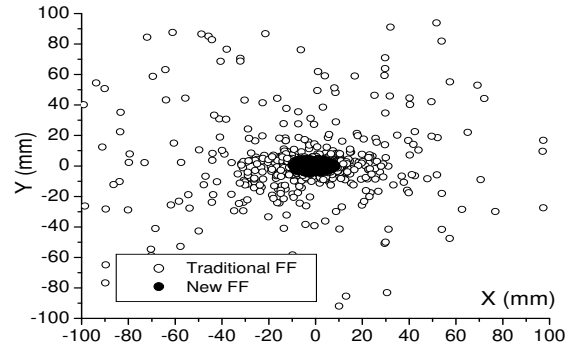

Figure 7: Beam at the entrance of the final doublet for the traditional NLC FF and for the new FF, as determined by particle tracking. Particles of the incoming beam are placed on a surface of an ellipsoid with dimensions $N_\sigma(x, x', y, y', E) = (800, 8, 4000, 40, 20)$ times larger than the nominal beam sizes.

from beam-gas scattering, reducing an additional source of background.

4 SCALING WITH EMITTANCE AND ENERGY

To maintain optimal performance of the system with larger incoming beam emittances, the bend field must increase like $B_0 \propto \sqrt{\varepsilon}$, as is seen from Fig.8. The increased field is necessary to hold constant the contribution of high order aberrations to the IP beam size, as well as the contribution of the IP angular dispersion $\eta'_{\rm IP}$ to the beam divergence.

The dependence of the luminosity on beam energy is shown in Fig.9. Clearly a fixed length final focus has a wide range of energies where it could operate, especially if the bend field is rescaled.

The scaling to higher energies is much easier with the new design. For a wide range of parameters, the IP spot size dilution is dominated by the energy spread created by synchrotron radiation in the bends. This scales like

$$\frac{\Delta\sigma_y}{\sigma_y} \propto \frac{\gamma^5}{L^2}\eta_B'^3 \propto (\gamma\varepsilon_y)^{3/2}\left(\frac{\eta'_B L}{\eta'_{\rm IP}}\right)^3\left(\frac{\eta'^2_{\rm IP}}{\varepsilon_y}\right)^{3/2}\frac{\gamma^{7/2}}{L^5}$$

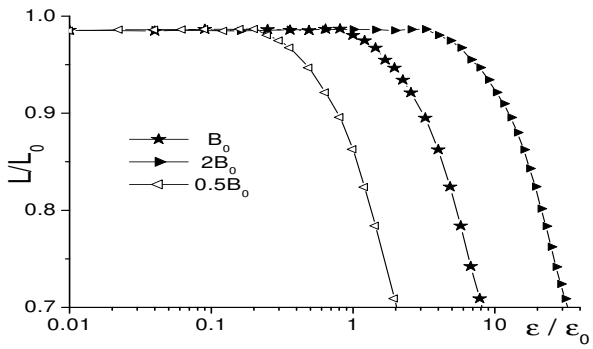

Figure 8: Luminosity versus emittance for the new Final Focus for the nominal, twice larger and twice lower field in the bends. Tracking without synchrotron radiation.

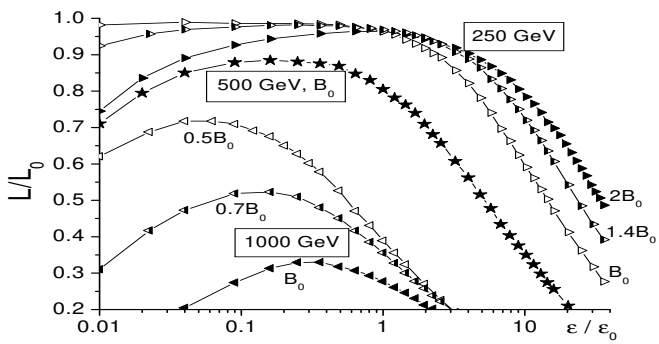

Figure 9: Luminosity versus emittance for different field in the bends and beam energy for the new Final Focus. Tracking includes synchrotron radiation in bends and quadrupoles.

where η'_B is the angular dispersion produced by the bends where the bend length is assumed to be proportional to the total length of the system L. The terms in the parenthesis are constant if the IP angular dispersion is proportional to the beam divergence and if we conservatively assume that the normalized emittance will be the same at higher energies. In this case the length of the system scales with energy as

$$L \propto \gamma^{7/10}.$$

If, however, the achievable normalized emittance scales approximately inversely with energy, as is assumed in [4], then the scaling becomes

$$L \propto \gamma^{2/5}.$$

In this case, with the new design, the FF for a 3 TeV center of mass energy collider could be only about 700 m long.

The beam also emits synchrotron radiation in the quadrupoles which becomes more of a problem at higher energies. This can be reduced in the new design because the larger bandwidth allows the FD quadrupoles to be lengthened to minimize the synchrotron radiation generated in them.

5 CONCLUSION

We have developed a new Final Focus system that has better properties than the systems so far considered and built. It is much shorter, providing a significant cost reduction for the collider. The system has similar bandwidth and several orders of magnitudes larger dynamic aperture. This reduces the backgrounds and relaxes the design of the collimation section. It is also compatible with an L^* which is twice as long as that in the traditional NLC FF design, which simplifies engineering of the Interaction Point area. Finally, its favorable scaling with beam energy makes it attractive for multi-TeV colliders.

We believe that further improvements of the performance of the system are possible.

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6 REFERENCES

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