SLAC-PUB-8434
April 2000
FISIST/5-2000/CFIF

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Presented at 9th Lomonosov Conference on Elementary Particle Physics,
9/20/1999—9/26/1999, Moscow, Russia

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Work supported by Department of Energy contract DE–AC03–76SF00515.

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Mixing and CP violation with Quasidegenerate Majorana Neutrinos

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Abstract

We study the exactly degenerate case for three lefthanded Majorana neutrinos building a general parametrization for the leptonic mixing matrix characterized by two angles and one CP violating phase and identify a weak-basis invariant relevant, in this case, for CP violation. After lifting the degeneracy, this parametrization accommodates the present data on atmospheric and solar neutrinos, as well as neutrinoless double beta decay. Some of the leptonic mixing ansätze suggested in the literature correspond to special cases of this parametrization.

To appear in "Particle Physics on the Boundary of the Millenium", Proceedings of the 9th Lomonosov Conference on Elementary Particle Physics, ed. by Alexander Studenikin published by Interregional Centre for Advanced Studies, Moscow, Russia, 2000

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[2] On sabbatical leave, visiting SLAC - Stanford Linear Accelerator Center, partially supported by Department of Energy, contract DE-AC03-76SF00515, FLAD - Fundação Luso Americana para o Desenvolvimento and GTAE - Grupo Teórico de Altas Energias.
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1 Introduction

In the Standard Model (SM) neutrinos are massless and there is no mixing in the leptonic sector. Currently there are indications for neutrino oscillations in solar [1], atmospheric [2] and accelerator [3] experiments with the strongest evidence coming from the Super-Kamiokande atmospheric neutrino data [2]. The LSND experiment is the only one that has not been confirmed independently. Neutrino oscillations are a clear sign of physics beyond the SM requiring the existence of massive neutrinos. The interpretation of the solar and atmospheric anomalies can be done in the framework of three lefthanded neutrinos, without additional sterile neutrinos. In our work [4], we have considered the case of three highly degenerate Majorana neutrinos and we analised in detail leptonic mixing and CP violation in the limit of exactly degenerate masses identifying, in this limit, a weak-basis invariant which controls the strength of CP violation. We showed that a two angle parametrization suggested by the exact degeneracy limit can fit all the present atmospheric and solar neutrino data in agreement with the experimental bound imposed by neutrinoless double beta decay. Our two angle parametrization has the interesting property of reproducing, for specific choices of the angles, several of the mixing schemes proposed in the literature such as bimaximal mixing [5], democratic mixing [6] as well as the scheme suggested by Georgi and Glashow [7].

2 The limit of exact degeneracy

The experimental constraints on squared neutrino mass differences coming from solar and atmospheric experiments together with the assumption that neutrino masses might be of the order of $1eV$ lead to highly degenerate masses [8].

The terms of the Lagrangean relevant for our discussion are

$$\mathcal{L}_{\rm mass} \;=\; -\,(\nu_{L_\alpha})^T\; C^{-1}\; m_{\alpha\beta} \quad \nu_{L_\beta} \;+\; \text{h.c.} \tag{1}$$

and

$$\mathcal{L}_{\rm W} \;=\; \frac{g}{2} \quad (\;\overline{e},\quad \overline{\mu},\quad \overline{\tau}\;)_L\,\gamma_\mu \quad U \quad \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L W^\mu \quad + \text{ h.c.} \tag{2}$$

$\mathcal{L}_{\rm mass}$ is a generic Majorana mass term for the three lefthanded neutrinos where $m = (m_{\alpha\beta})$ is a 3×3 complex symmetric mass matrix, and ν_{L_α} denote weak eigenstates. In principle the matrix m could be an effective Majorana mass matrix within a framework with three left-handed and three right-handed neutrinos. We shall work in the weak-basis (WB) where the charged lepton mass matrix is diagonal, real and positive. The neutrino mass matrix can be diagonalized by the transformation

$$U^T \cdot m \cdot U \;=\; \text{diag}(m_{\nu_1},\; m_{\nu_2},\; m_{\nu_3}) \tag{3}$$

so that the weak eigenstates ν_{L_α}, are related to the mass eigenstates, ν_{L_i}, by $\nu_{L_\alpha} = U_{\alpha i}\ \nu_{L_i}$ and the fields in the charged current given by Eq.(2) are already physical fields.

In general the neutrino diagonalization matrix U considered above can be parametrized by three angles and three phases that are CP violating. It is well known that in the case of Dirac neutrinos only three angles and one phase are required to parametrize U and that in the limit of exact degeneracy U can be rotated away into the unit matrix through a redefinition of the neutrino fields. In this respect Majorana neutrinos are fundamentally different and only if the theory is CP invariant and the three degenerate neutrinos have the same CP parity, can the matrix U be rotated away.

In the limit of exact degeneracy with μ the common neutrino mass we shall denote the mixing matrix by $U_\circ$ and define a dimensionless mass matrix by $Z_\circ$ given by $Z_\circ = m/\mu$. From Eq(3) we obtain

$$Z_\circ \ = U_\circ^\star \cdot U_\circ^\dagger \tag{4}$$

so that $Z_\circ$ is a unitary symmetric matrix and can be written without loss of generality as

$$Z_\circ \ = \ K^* \cdot \begin{pmatrix} c_\theta & s_\theta c_\phi & s_\theta s_\phi \\ s_\theta c_\phi & w_1 & w_2 \\ s_\theta s_\phi & w_2 & w_3 \end{pmatrix} \cdot K^* \tag{5}$$

where c and s stand for cosine and sine respectively, the w_i may be complex entries and K is a diagonal unitary matrix . After a WB transformation under which $Z_\circ \rightarrow \ K \cdot Z_\circ \cdot K$, $Z_\circ$ transforms into

$$Z_\circ \ = \ \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\phi & s_\phi \\ 0 & s_\phi & -c_\phi \end{pmatrix} \cdot \begin{pmatrix} c_\theta & s_\theta & 0 \\ s_\theta & z_{22} & z_{23} \\ 0 & z_{23} & z_{33} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\phi & s_\phi \\ 0 & s_\phi & -c_\phi \end{pmatrix} \tag{6}$$

Unitarity of $Z_\circ$, requires that either s_θ or z_{23} vanish. The case $s_\theta = 0$ leads to CP invariance. Assuming $s_\theta \neq 0$ the most general form for $Z_\circ$ is given by:

$$Z_\circ \ = \ \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\phi & s_\phi \\ 0 & s_\phi & -c_\phi \end{pmatrix} \cdot \begin{pmatrix} c_\theta & s_\theta & 0 \\ s_\theta & -c_\theta & 0 \\ 0 & 0 & e^{i\alpha} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\phi & s_\phi \\ 0 & s_\phi & -c_\phi \end{pmatrix} \tag{7}$$

In the cases $\alpha = 0, \pi$ there is, once again, CP conservation. It can be readily verified that this parametrization does not include the trivial case where CP is a good symmetry and all neutrinos have the same CP parity [9], corresponding to the eigenvalues $(1, 1, 1)$. In fact in the case of CP consevation $Z_\circ$ is diagonalized by an orthogonal transformation leaving invariant both $\mathrm{Tr}(Z_\circ)$ and $\det(Z_\circ)$ and there is no choice of parameters in Eq.(7) leading to a trace and determinant corresponding to this particular case. On the other hand the set of eigenvalues $(1, -1, 1)$ and $(1, -1, -1)$ corresponding to one neutrino with opposite CP parity to the other two can be obtained for $\alpha = 0$ and $\alpha = \pi$, respectively. The diagonalization of the matrix $Z_\circ$ through the transformation of Eq.(3), together

with the requirement of positive diagonal elements requires a $U_\circ$ matrix given by

$$U_\circ \;=\; \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\phi & s_\phi \\ 0 & s_\phi & -c_\phi \end{pmatrix} \cdot \begin{pmatrix} \cos(\frac{\theta}{2}) & \sin(\frac{\theta}{2}) & 0 \\ \sin(\frac{\theta}{2}) & -\cos(\frac{\theta}{2}) & 0 \\ 0 & 0 & e^{-i\alpha/2} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & i & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (8)$$

$U_\circ$ is not an orthogonal matrix even in the CP conserving limit, i.e., for $\alpha = 0$, π, as a result it cannot be rotated away through a redefinition of the neutrino fields.

From this discussion we see that in the case of three degenerate Majorana neutrinos the parametrization of U requires two angles and one phase and there may be CP violation [10].It can be shown that a necessary and sufficient condition for CP invariance, in the degenerate limit, is:

$$G \equiv \; \mathrm{Tr}\left[\; (m \cdot h \cdot m^\star) \;,\; h^\star\right]^3 \;=\; 0 \quad (9)$$

where $h = m_\ell \cdot m_\ell^\dagger$, and m_ℓ denotes the charged lepton mass matrix. G is a WB invariant and can be written as

$$\begin{array}{l} G = 6i \;\Delta_m \;\mathrm{Im}[(Z_\circ)_{11}(Z_\circ)_{22}(Z_\circ)^\star_{12}(Z_\circ)^\star_{21}] \\ = \frac{3i}{2} \;\Delta_m \;\cos(\theta)\sin^2(\theta)\;\sin^2(2\phi)\;\sin(\alpha) \end{array} \quad (10)$$

where $\Delta_m = \mu^6\;(m_\tau^2 - m_\mu^2\;)^2(m_\tau^2 - m_e^2\;)^2(m_\mu^2 - m_e^2\;)^2$ is a multiplicative factor which contains the different masses of the charged leptons and the common neutrino mass μ. In Refs.[10] [11] various examples of CP-odd WB-invariants were constructed, but all of those invariants automatically vanish in the limit of exact degeneracy whilst the invariant of Eq.(9) only vanishes if CP is conserved. It was shown in Ref. [4] that the imposition of maximal CP violation leads to a structure of the Majorana neutrino mass of the type that one obtains in the framework of universal strength for Yukawa couplings [12].

3 Lifting the degeneracy. Phenomenological implications

In the limit of exact degeneracy, the leptonic mixing matrix $U_\circ$ is param- etrized by two angles θ, ϕ and one phase α and is defined only up to an arbitrary orthogonal transformation $U_\circ \to U_\circ \cdot O$. The physically interesting case corresponds to quasidegenerate neutrinos. Let us assume that the degeneracy is lifted through a small perturbation:

$$Z = Z_\circ + \varepsilon\; Q \quad (11)$$

where ε is a small parameter and Q is a symmetric complex matrix of order one. It was shown in Ref. [4] that in the presence of a small perturbation around the degeneracy limit, the mixing matrix becomes, to leading order, $U_\circ \cdot O$, where O is no longer arbitrary, being the orthogonal matrix which diagonalizes the symmetric real matrix A, defined by $A = \mathrm{Re}(U_\circ^T \cdot Q \cdot U_\circ)$. As a result, for

quasidegenerate neutrinos, to leading order, only one CP violating phase appears in the leptonic mixing, namely the phase α present in $U_\circ$ and the question of whether the two angle parametrization given by Eq.(8) can accommodate the present experimental data on atmospheric and solar neutrinos, as well as the constraints on double beta decay immediately arises. Of course this corresponds to the case where the matrix $U_\circ^T \cdot Q \cdot U_\circ$ is already a real and diagonal matrix lifting the degeneracy as required experimentally.

The constraints arising from neutrinoless double beta decay put an upper bound on $< m >$, an average neutrino mass, given in standard notation by

$$< m > = \sum_i U_{ei}^2 \; m_{\nu_i} \; = \; m_{ee}^\star \tag{12}$$

where the U_{ei} denote the elements of the first row of the mixing matrix U, and m_{ee} is the $(1,1)$ element of the mass matrix m. At present, the strongest bound is $| < m > | = |m_{ee}| < 0.2 \; eV$ [13]. In the limit of exact degeneracy, we have $m_{ee} = \mu \; \cos(\theta)$, where we have used the parametrization of Eq.(8). If we fix $\mu = 1 \; eV$, then neutrino masses are equal to a precision sufficient to neglect their differences, and the experimental bound on m_{ee} immediately translates into a single bound on the parameter θ, namely $|\cos(\theta)| < 0.2$.

In our framework, without sterile neutrinos, the atmospheric neutrino data supports the existence of oscillations of atmospheric neutrinos to tau neutrinos with a large mixing angle satisfying the bound $\sin^2(2\theta_{\rm atm}) > 0.82$, and the neutrino mass square difference in the range $5 \times 10^{-4} \; eV^2 < \Delta m^2_{\rm atm} < 6 \times 10^{-3} \; eV^2$. This interpretation is further supported by recent data from the CHOOZ reactor netrino experiment [14] leading to the upper bound $| \; U_{e3} \; | \leq (0.22 - 0.14)$.

In the context of three left-handed neutrinos, the probability for a neutrino ν_α to oscillate to other neutrinos is

$$1 - P(\nu_\alpha \to \nu_\alpha) = 4 \sum_{i<j} U_{\alpha i} U_{\alpha i}^\star U_{\alpha j}^\star U_{\alpha j} \quad \sin^2 \left[\frac{\Delta m^2_{ji}}{4} \frac{L}{E} \right] \tag{13}$$

where $\Delta m^2_{ji} = |m_j^2 - m_i^2|$, E is the neutrino energy and L denotes the distance travelled by the neutrino between the source and the detector. Since in the range L/E that is relevant for atmospheric neutrinos the term in $\sin^2[(\Delta m^2_{21}/4)(L/E)]$ can be disregarded, we may identify $\sin^2(2\theta_{\rm atm})$ with $4(U_{21} U_{21}^\star U_{23}^\star U_{23} + U_{22} U_{22}^\star U_{23}^\star U_{23})$. In the framework of our two-angle parametrization of Eq.(8), the above combination of matrix elements has a simple form and one obtains $\sin^2(2\theta_{\rm atm}) = \sin^2(2\phi)$, i.e., $\theta_{\rm atm}$ can be identified with the angle ϕ and thus the atmospheric neutrino data leads to the constraint $\sin^2(2\phi) > 0.82$.

The discrepancy between the observed and the calculated [15] solar neutrino fluxes also requires neutrino oscillations, although at this stage various schemes are still possible, namely within the framework of the MSW mechanism [16] there is a small angle solution $\sin^2(2\theta_{\rm sol}) \approx 7 \times 10^{-3}$ with $\Delta m^2_{\rm sol} \approx 6 \times 10^{-6} \; eV^2$, and a large angle solution $\sin^2(2\theta_{\rm sol}) \sim 0.65 - 0.97$ with $\Delta m^2_{\rm sol} \sim (2 - 20) \times 10^{-5} \; eV^2$. Another solution could be vacuum oscillations with $\sin^2(2\theta_{\rm sol}) \approx 0.9$

and $\Delta m^2_{\rm sol} \approx 10^{-10}\ eV^2$. Since in our two-angle parametrization one has $U_{13} = 0$ we obtain $\sin^2(2\theta_{\rm sol}) = 4U_{11}U^\star_{11}U^\star_{12}U_{12}$ leading to $\sin^2(2\theta_{\rm sol}) = \sin^2(\theta)$, i.e., in our parametrization $2\theta_{\rm sol} = \theta$.

We see that each of the experiments considered above independently constrains a single parameter. Also, it is clear from Eq.(12), that with small solar neutrino mixing the bound from double beta decay would not be satisfied for quasidegenerate neutrinos with masses of the order of 1 eV.

Finally we show that some of the neutrino mixing schemes proposed in the literature correspond to specific cases of the two-angle parametrization suggested by Eq.(8).

(a) *Bimaximal Mixing* [5]: In this scheme the lines of the neutrino mixing matrix have the following structure:

$$L_1 = \left(\tfrac{1}{\sqrt{2}}, \tfrac{-1}{\sqrt{2}}, 0\right); \qquad L_2 = \left(\tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{\sqrt{2}}\right); \qquad L_3 = \left(\tfrac{-1}{2}, \tfrac{-1}{2}, \tfrac{1}{\sqrt{2}}\right) \tag{14}$$

This pattern of neutrino mixing is obtained within the two-angle parame- trization for the following values of θ, ϕ and α:

$$\alpha = 0\ ; \quad \cos(\theta/2) = -\sin(\theta/2) = -\cos(\phi) = \sin(\phi) = \frac{1}{\sqrt{2}} \tag{15}$$

(b) *Democratic Mixing* [6]: In this case the neutrino mixing matrix has, to a very good approximation, the following form:

$$L_1 = \left(\tfrac{1}{\sqrt{2}}, \tfrac{-1}{\sqrt{2}}, 0\right); \qquad L_2 = \left(\tfrac{1}{\sqrt{6}}, \tfrac{1}{\sqrt{6}}, \tfrac{-2}{\sqrt{6}}\right); \qquad L_3 = \left(\tfrac{1}{\sqrt{3}}, \tfrac{1}{\sqrt{3}}, \tfrac{1}{\sqrt{3}}\right) \tag{16}$$

Within the two-angle parametrization, one obtains the democratic mixing for the following values of the parameters:

$$\alpha = 0\ ; \quad \cos(\theta/2) = -\sin(\theta/2) = \frac{1}{\sqrt{2}}\ ; \quad \cos(\phi) = \frac{1}{\sqrt{2}}\,\sin(\phi) = \frac{-1}{\sqrt{3}} \tag{17}$$

In the above analysis, we have not paid attention to the factors "i" appearing in our two-angle parametrization of Eq.(8). As we have previously emphasized, these factors of "i" have to do with the fact that in the construction of the two-angle parametrization, we have implicitly assumed that in the limit of CP invariance (i.e. $\sin(\alpha) \to 0$), one of the Majorana neutrinos has relative CP parity opposite to the other two. The factors of "i" do not play any rôle in the analysis of atmospheric and solar neutrino data, but are crucial in the analysis of double beta decay.

(c) *Georgi-Glashow mass matrix* [7]: Using an analysis of the present neutrino data Georgi and Glashow have suggested the following approximate form for the Majorana neutrino mass matrix

$$\begin{aligned} &(m)_{1i} = \mu\left(0, \tfrac{1}{\sqrt{2}}, \tfrac{1}{\sqrt{2}}\right); \quad (m)_{2i} = \mu\left(\tfrac{1}{\sqrt{2}}, \tfrac{1}{2}, \tfrac{-1}{2}\right); \\ &(m)_{3i} = \mu\left(\tfrac{1}{\sqrt{2}}, \tfrac{-1}{2}, \tfrac{1}{2}\right) \end{aligned} \tag{18}$$

From Eq.(7) it follows that this neutrino mass matrix is obtained, within the two-angle parametrization for the following values of its parameters,

$$\alpha = 0; \quad \sin(\theta) = 1; \quad \cos(\phi) = \sin(\phi) = \frac{1}{\sqrt{2}} \tag{19}$$

Acknowledgments

M. N. R is grateful to the Organizers of the the 9th Lomonosov Conference on Elementary Particle Physics for the warm hospitality. M. N. R. received financial support from FCT - Fundação de Ciência e Tecnologia through GTAE - Grupo Teórico de Altas Energias and CFIF - Centro de Física das Interacções Fundamentais, to participate at the Conference.

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