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Design and implementation of IIR algorithms for control of longitudinal coupled-bunch instabilities *

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Abstract

The recent installation of third-harmonic RF cavities at the Advanced Light Source has raised instability growth rates, and also caused tune shifts (coherent and incoherent) of more than an octave over the required range of beam currents and energies. The larger growth rates and tune shifts have rendered control by the original bandpass FIR feedback algorithms unreliable. In this paper we describe an implementation of an IIR feedback algorithm offering more flexible response tailoring. A cascade of up to 6 second-order IIR sections (12 poles and 12 zeros) was implemented in the DSPs of the longitudinal feedback system. Filter design has been formulated as an optimization problem and solved using constrained optimization methods. These IIR filters provided 2.4 times the control bandwidth as compared to the original FIR designs. Here we demonstrate the performance of the designed filters using transient diagnostic measurements from ALS and DAΦNE.

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Abstract. The recent installation of third-harmonic RF cavities at the Advanced Light Source has raised instability growth rates, and also caused tune shifts (coherent and incoherent) of more than an octave over the required range of beam currents and energies. The larger growth rates and tune shifts have rendered control by the original bandpass FIR feedback algorithms unreliable. In this paper we describe an implementation of an IIR feedback algorithm offering more flexible response tailoring. A cascade of up to 6 second-order IIR sections (12 poles and 12 zeros) was implemented in the DSPs of the longitudinal feedback system. Filter design has been formulated as an optimization problem and solved using constrained optimization methods. These IIR filters provided 2.4 times the control bandwidth as compared to the original FIR designs. Here we demonstrate the performance of the designed filters using transient diagnostic measurements from ALS and DAΦNE.

INTRODUCTION

A family of digital bunch-by-bunch longitudinal feedback systems has been built and installed at several accelerators [1]. These systems have been commissioned and used in routine operation at the ALS, DAΦNE, PEP-II, and BESSY-II.

An array of general-purpose digital signal processors (DSPs) is used for the feedback law computation. In the bunch-by-bunch processing formalism all DSPs run the same algorithm to compute the kick signals for every bunch in the ring.

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Recently, installation of third harmonic cavities at the ALS posed new requirements for the longitudinal feedback control. A search for ways to satisfy these constraints resulted in development of several new control algorithm design methods.

FEEDBACK FILTER DESIGN

The arrival-time error τ of a bunch is given by [2]

$$\ddot{\tau} + 2\lambda_{rad}\dot{\tau} + \omega_s^2\tau = \frac{\alpha e}{ET_o}v \tag{1}$$

where α is the momentum compaction factor, E/e is the nominal beam energy in Volts, T_o is the revolution period, and v is the total wake voltage seen by the bunch. Feedback system has to control the bunch motion which is a superposition of the unstable eigenmodes. For small-signal analysis of the bunch-by-bunch feedback (without saturation) it is sufficient to consider the mode with the highest growth-rate (λ). Then we can rewrite Eq. (1) as follows

$$\ddot{\tau} - 2\lambda\dot{\tau} + \omega_s^2\tau = \frac{\alpha e}{ET_o}v_{fb} \tag{2}$$

Using feedback law $v_{fb} = -G_{fb}\dot{\tau}$ results in a constant damping rate $\frac{\alpha e}{2ET_o}G_{fb}$ independent of ω_s. However in the physical implementation one has to contend with the output stage saturation effects as well as processing delay. A differentiator transfer function results in gain rising linearly with frequency leading to saturation of the feedback output on input noise. Additionally, the processing and sampling delays produce a non-negligible filter transfer function phase slope. For example, at the ALS such delays produce a phase slope of $4.6^o/kHz$. Thus, a differentiator with a ideal 90^o phase shift would have 54^o phase error at the synchrotron frequency of 11.8 kHz.

Limitations of the above algorithm can be easily solved for cases when mode to mode tune shift is small. Then it is possible to use a narrowband differentiator approximation.

Narrowband control algorithms

In the DSP-based longitudinal feedback systems described here a short (4 - 12 taps) FIR filter has been used as a narrowband differentiator. Such filter has to provide control of gain and phase of the response at the synchrotron frequency. In addition, DC rejection is needed to prevent unique synchronous phases of the bunches from using up output amplifier power. Similarly, some frequency selectivity is helpful so that output power is not spent on out-of-band noise.

Using samples of a sinusoid at the synchrotron frequency as filter coefficients satisfies the gain and phase adjustment requirements. Removing the mean of the

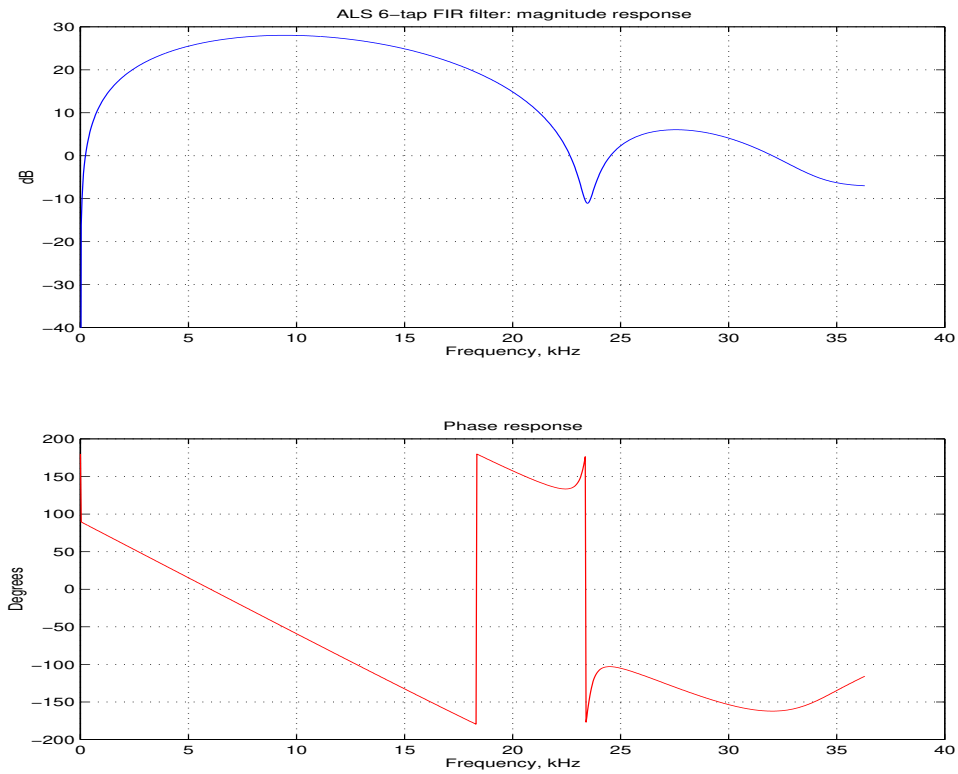

FIGURE 1. Frequency response of the ALS 6-tap FIR filter. Both sampling zero-order hold and processing delay are included.

coefficient vector provides necessary DC rejection. The resulting filter has a band-pass transfer function centered at ω_s. Fig. 1 shows impulse response and transfer function of the FIR filter in use at the ALS until 11/1999.

Control bandwidth widening

One approach to increasing the control bandwidth is to maintain the filter phase constant within the required range of the synchrotron frequencies. Such response tailoring required more flexibility than a 12-tap FIR filter could provide. An infinite impulse response (IIR) algorithm was examined as a way of compactly implementing complex transfer functions. Filter design has been formulated as frequency response approximation problem [3]. Desired phase and magnitude of the transfer function within the control passband were specified based on the response of the FIR filter at the nominal ω_s. A weighting function was used to differentiate the important features of the frequency response. The approximation problem has been solved using constrained optimization tools [4].

Fig. 2 illustrates frequency response of a 12 pole and 12 zero IIR filter for the ALS designed using the frequency response method. This filter has been used for

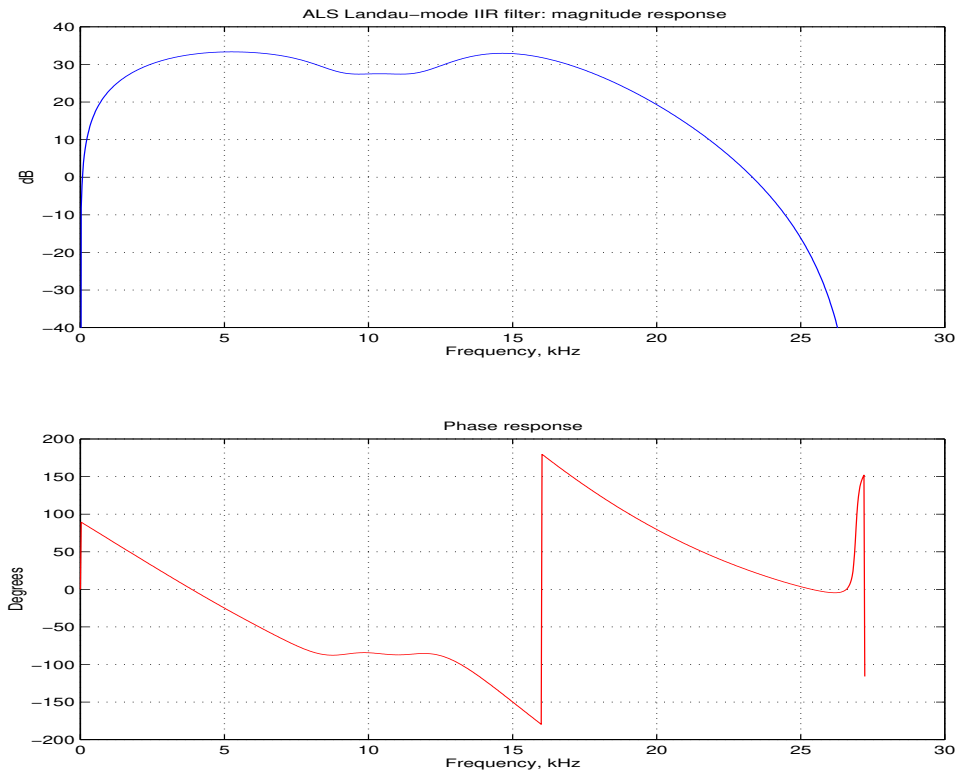

FIGURE 2. ALS IIR filter providing nearly constant phase response within the control bandwidth.

day-to-day operation at ALS and has been shown to provide necessary damping for the 7 kHz to 12.5 kHz range of synchrotron frequencies. A grow/damp [5] measurement shown in Fig. 3 illustrates performance of the filter near full design current.

Beam and feedback model

Unfortunately the frequency domain design method does not address system stability directly. In order to quantify damping performance as a function of ω_s we developed a simplified beam/feedback system model. Taking Laplace transform of Eq. 2 and converting time of arrival error τ to phase at RF frequency we get

$$H(s) = \frac{\Theta(s)}{V_{fb}(s)} = \frac{360\alpha f_{rf} f_{rev}}{(E/e)} \frac{1}{s^2 - 2\lambda s + \omega_s^2}, degrees/volt \qquad (3)$$

Using this transfer function for the beam model and combining it with the known discrete transfer function for the filter we obtain a closed-loop model. Fig. 4 shows the block diagram of such a model implemented in Simulink [6].

For each accelerator the closed-loop model is verified as follows. A number of grow/damp measurements is done varying beam and feedback conditions such as

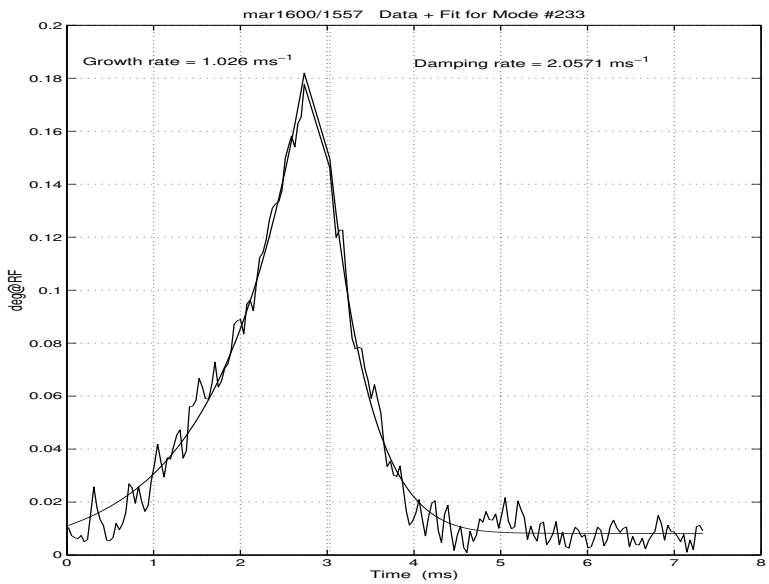

FIGURE 3. A grow/damp measurement with an IIR filter. The damping rate achieved shows that system could withstand a threefold growth rate increase.

fill pattern, current, FIR filter gain and phase, etc. Then the closed loop model is simulated under the same conditions. Fig. 5 compares the simulation results with the grow/damp measurements for DAΦNE e^+ ring.

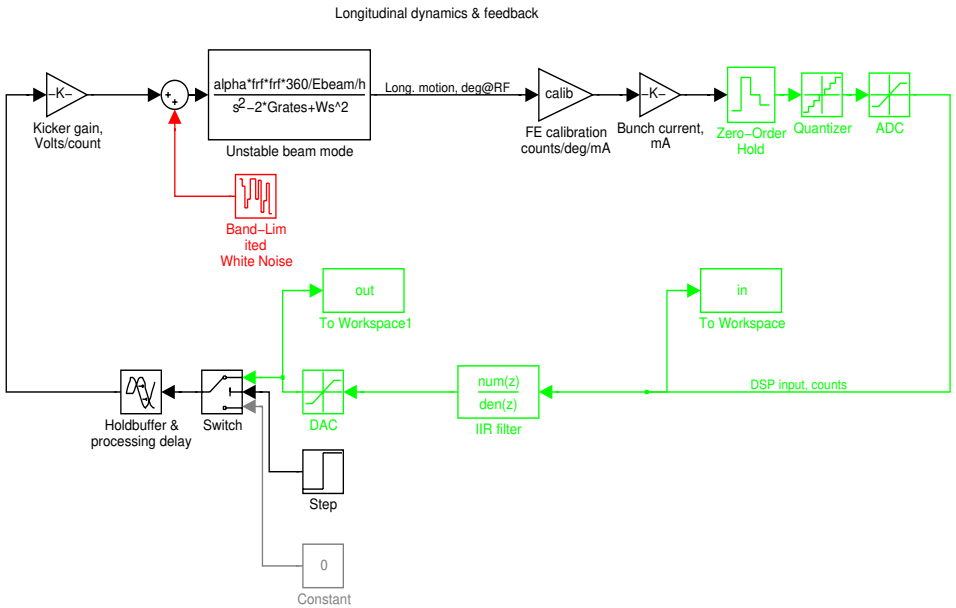

FIGURE 4. Closed-loop model of the beam and the feedback system. A switch after the DAC block is used to open the feedback loop and simulate a grow/damp measurement.

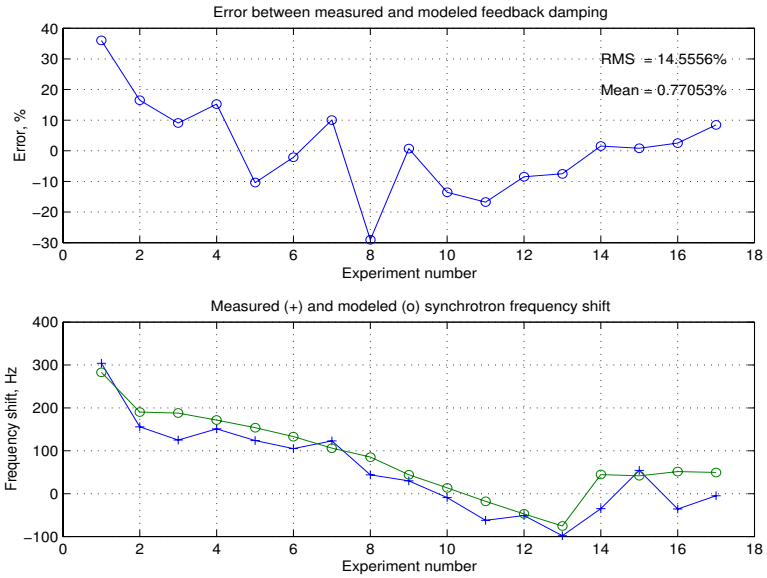

FIGURE 5. Closed-loop model verification for DAΦNE positron ring. In 17 measurements both feedback damping and feedback-induced synchrotron frequency shift agree well with simulation results. Measurements are taken at currents from 45 mA to 123 mA and feedback gains from 1.8 to 6.3.

Closed-loop filter design

Closed-loop model can be used to design new feedback filters. In this approach closed-loop poles of the system are evaluated within a range of synchrotron fre-

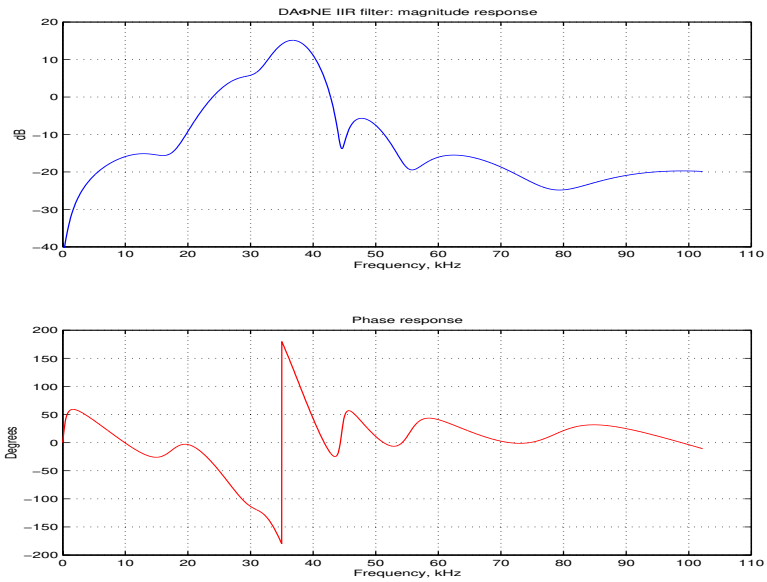

FIGURE 6. Transfer function of the DAΦNE IIR filter. Notice the narrowband structure of the filter response

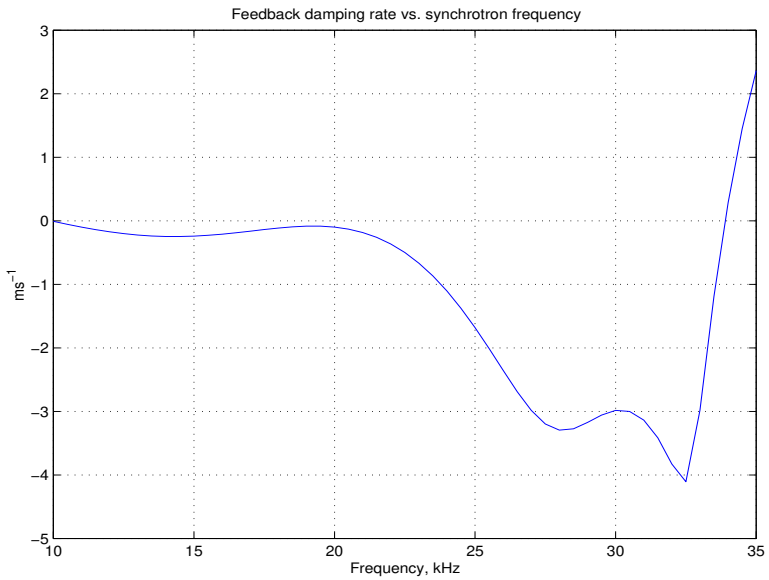

FIGURE 7. Simulated feedback damping rate for the DAΦNE IIR filter at 200 mA. In-band damping is twice the open-loop unstable mode growth rates of 1.5 ms^{-1} while in the 10 to 27 kHz band filter continues to provide some negative feedback.

quencies. These frequencies sample bands where feedback is expected to provide damping. Corresponding vectors of open-loop growth rates are used to shape damping requirements. Such shaping is needed to differentiate frequency ranges where large damping is needed from those where non-positive feedback is sufficient. For example, several machines (ALS, DAΦNE) exhibit large tune shift of mode 0 at high currents as RF cavity detuning changes to compensate for beam loading. At DAΦNE mode 0 frequency drops from 30 kHz at low currents to 14 kHz at 500 mA. At that frequency the standard FIR filter provides positive feedback and excites large motion of mode 0.

Using closed-loop design we require large damping around the synchrotron frequency (27-33 kHz). A second band of frequencies covers 10 kHz to 27 kHz range. In that range neutral feedback behavior is specified. Using the largest closed-loop pole as a parameter characterizing system dynamics, the filter design problem is posed as minimax constrained optimization. Fig. 6 illustrates the transfer function for the DAΦNE filter designed by the minimax method. In fig. 7 simulated feedback damping is graphed vs. frequency.

Implementation issues

IIR filtering is inherently more difficult in implementation due to the internal feedback. Care must be taken to avoid stability, quantization and saturation problems. In the longitudinal feedback system the IIR filter is realized as a cascade of 6 second-order sections. In order to guarantee stability of the filter and avoid

intra-section saturation filter poles are limited to a disc around the origin of radius 0.7 dictated by the dynamic range of the 16 bit implementation. Additionally filter poles are sorted by magnitude to avoid premature inter-section saturation. Pairing neighboring zeros and poles is helpful in further reducing saturation effects.

SUMMARY

Choice between IIR and FIR filtering is driven by the length of the impulse response needed. For FIR filter response length translates directly into the number of taps. The existing LFB environment allows FIR filters of 16 tap maximum length due to memory limitations. IIR filters shown here exhibit non-negligible impulse response of 40 - 50 samples. Thus using an IIR algorithm provides a computationally efficient implementation of more complex filters.

Among several IIR filter design approaches one based on a closed-loop model provides an objective measure of the filter performance. Since the frequency domain design method has no such a measure of acceptable phase and magnitude response deviations it produces more conservative designs than those possible with the closed-loop approach. However the frequency domain design provides an acceptable substitute when closed-loop model is unavailable.

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