SLAC-PUB-8337
February 2000
hep-th/0001206
SU-ITP-00/02
IASSNS-HEP-00/05

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Submitted to *Physical Review D*

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Work supported by Department of Energy contract DE–AC03–76SF00515.

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arXiv:hep-th/0001206 31 Jan 2000

Self-tuning flat domain walls in 5d gravity and string theory

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We present Poincare invariant domain wall ("3-brane") solutions to some 5-dimensional effective theories which can arise naturally in string theory. In particular, we find theories where Poincare invariant solutions exist for arbitrary values of the brane tension, for certain restricted forms of the bulk interactions. We describe examples in string theory where it would be natural for the quantum corrections to the tension of the brane (arising from quantum fluctuations of modes with support on the brane) to maintain the required form of the action. In such cases, the Poincare invariant solutions persist in the presence of these quantum corrections to the brane tension, so that no 4d cosmological constant is generated by these modes.

January 2000

1. Introduction

Some time ago, it was suggested that the cosmological constant problem may become soluble in models where our world is a topological defect in some higher dimensional spacetime [1]. Recently such models have come under renewed investigation. This has been motivated both by brane world scenarios (see for instance [2,3,4]) and by the suggestion of Randall and Sundrum [5] that the four-dimensional graviton might be a bound state of a 5d graviton to a 4d domain wall. At the same time, new ideas relating 4d renormalization group flows to 5d AdS gravity via the AdS/CFT correspondence [6] have inspired related approaches to explaining the near-vanishing of the 4d cosmological term [7,8]. These authors suggested (following [1]) that quantum corrections to the 4d cosmological constant could be cancelled by variations of fields in a five-dimensional bulk gravity solution. The results of this paper might be regarded as a concrete partial realization of this scenario, in the context of 5d dilaton gravity and string theory. A different AdS/CFT motivated approach to this problem appeared in [9].

In the thin wall approximation, we can represent a domain wall in 5d gravity by a delta function source with some coefficient $f(\phi)$ (where ϕ is a bulk scalar field, the dilaton), parametrizing the tension of the wall. Quantum fluctuations of the fields with support on the brane should correct $f(\phi)$. In this paper, we present a concrete example of a 5d dilaton gravity theory where one can find Poincare invariant domain wall solutions for *generic* $f(\phi)$. The constraint of finding a finite 4d Planck scale then restricts the sign of f and the value of $\frac{f'}{f}$ at the wall to lie in a range of order one. Thus fine-tuning is not required in order to avoid having the quantum fluctuations which correct $f(\phi)$ generate a 4d cosmological constant. One of the requirements we must impose is that the 5d cosmological constant Λ should vanish.[1] This would be natural in scenarios where the bulk is supersymmetric (though the brane need not be), or where quantum corrections to the bulk are small enough to neglect in a controlled expansion.

For suitable choices of $f(\phi)$, this example exhibits the precise dilaton couplings which naturally arise in string theory. There are two interesting and distinct contexts in which this happens. One is to consider $f(\phi)$ corresponding to tree-level dilaton coupling ($Ve^{-2\phi}$ in string frame, for some constant V). This form of the dilaton coupling is not restricted to tree-level *perturbative* string theory – it occurs for example on the worldvolumes of

[1] It is possible that an Einstein frame bulk cosmological term which is independent of ϕ will also allow for similar physics [10].

NS branes in string theory. There, the dynamics of the worldvolume degrees of freedom does not depend on the dilaton – the relevant coupling constant is dilaton independent. Therefore, quantum corrections to the brane tension due to dynamics of worldvolume fields would be expected to maintain the "tree-level" form of $f(\phi)$, while simply shifting the coefficient V of the (string frame) $e^{-2\phi}$. The other form of $f(\phi)$ natural in string theory involves a power series in e^{ϕ}. This type of coupling occurs when quantum corrections are controlled by the dilaton in string theory.

In either case, as long as we only consider quantum corrections which modify $f(\phi)$ but maintain the required form of the bulk 5d gravity action, this means that quantum corrections to the brane tension do not destabilize flat space; they do not generate a four-dimensional cosmological constant. We will argue that some of our examples should have a microscopic realization in string theory with this feature, at leading order in a controllable approximation scheme. It is perhaps appropriate to call this "self-tuning" of the cosmological constant because the 5d gravity theory and its matter fields respond in just the right way to shifts in the tension of the brane to maintain 4d Poincare invariance. Note that here, as in [5], there is a distinction between the brane tension and the 4d cosmological constant.

There are two aspects of the solutions we find which are not under satisfactory control. Firstly, the curvature in the brane solutions of interest has singularities at finite distance from the wall; the proper interpretation of these singularities will likely be crucial to understanding the mechanism of self-tuning from a four-dimensional perspective. We cut off the space at these singularities. The wavefunctions for the four-dimensional gravitons in our solutions vanish there. Secondly, the value of the dilaton ϕ diverges at some of the singularities; this implies that the theory is becoming strongly coupled there. However, the curvature and coupling can be kept arbitrarily weak at the core of the wall. Therefore, some aspects of the solutions are under control and we think the self-tuning mechanism can be concretely studied. We present some preliminary ideas about the microscopic nature of the singularities in §3.

A problem common to the system studied here and that of [5] is the possibility of instabilities, hidden in the thin wall sources, that are missed by the effective field theory analysis. Studying thick wall analogues of our solutions would probably shed light on this issue. We do not resolve this question here. But taking advantage of the stringy dilaton couplings possible in our set of self-tuned models, we present a plausibility argument for the existence of stringy realizations, a subject whose details we leave for future work [10].

Another issue involves solutions where the wall is not Poincare invariant. This could mean it is curved (for example, de Sitter or Anti de Sitter). However it could also mean that there is a nontrivial dilaton profile along the wall (one example being the linear dilaton solution in string theory, which arises when the tree-level cosmological constant is nonvanishing). This latter possibility is a priori as likely as others, given the presence of the massless dilaton in our solutions.

Our purpose in this paper is to argue that starting with a Poincare invariant wall, one can find systems where quantum corrections leave a Poincare invariant wall as a solution. However one could also imagine starting with non Poincare invariant wall solutions of the same 5d equations (and preliminary analysis suggests that such solutions do exist in the generic case, with finite 4d Planck scale). We are in the process of systematically analyzing the fine tuning of initial conditions that considering a classically Poincare invariant wall might entail [10].

The paper is organized as follows. In §2, we write down the 5d gravity + dilaton theories that we will be investigating. We solve the equations of motion to find Poincare invariant domain walls, both in the cases where the 5d Lagrangian has couplings which provide the self-tuning discussed above, and in more general cases. In §3, we describe several possible embeddings of our results into a more microscopic string theory context. We close with a discussion of promising directions for future thought in §4.

There have been many interesting recent papers which study domain walls in 5d dilaton gravity theories. We particularly found [11] and [12] useful, and further references may be found there.

This research was inspired by very interesting discussions with O. Aharony and T. Banks. While our work on Poincare invariant domain walls and self-tuning was in progress, we learned that very similar work was in progress by Arkani-Hamed, Dimopoulos, Kaloper and Sundrum [13]. In particular, before we had obtained the solutions in §2.3 and §2.4, R. Sundrum told us that they were finding singular solutions to the equations and were hoping the singularities would "explain" a breakdown of 4d effective field theory on the domain wall.

2. Poincare-invariant 4d Domain Wall Solutions

2.1. Basic Setup and Summary of Results

Let us consider the action

$$S = \int d^5x\sqrt{-G}\left[R - \frac{4}{3}(\nabla\phi)^2 - \Lambda e^{a\phi}\right] + \int d^4x\sqrt{-g}(-f(\phi)) \tag{2.1}$$

describing a scalar field ϕ and gravity living in five dimensions coupled to a thin four-dimensional domain wall. Let us set the position of the domain wall at $x_5 = 0$. Here we follow the notation of [5] so that the metric $g_{\mu\nu}$ along the four-dimensional slice at $x_5 = 0$ is given in terms of the five-dimensional metric G_{MN} by

$$\begin{aligned} &g_{\mu\nu} = \delta^M_\mu \delta^N_\nu G_{MN}(x_5 = 0) \\ &\mu, \nu = 1, \ldots, 4 \\ &M, N = 1, \ldots, 5 \end{aligned} \tag{2.2}$$

For concreteness, in much of our discussion we will make the choice

$$f(\phi) = Ve^{b\phi} \tag{2.3}$$

However, most of our considerations will *not* depend on this detailed choice of $f(\phi)$ (for reasons that will become clear). With this choice, (2.1) describes a family of theories parameterized by V, Λ, a, and b. If $a = 2b = 4/3$, the action (2.1) agrees with tree-level string theory where ϕ is identified with the dilaton. (That is, the 5d cosmological constant term and the 4d domain wall tension term both scale like $e^{-2\phi}$ in string frame.) In §3 we will discuss a very natural context in which this type of action arises in string theory, either with the specific form (2.3) or with more general $f(\phi)$.

In the rest of this section we will derive the field equations arising from this action and construct some interesting solutions of these equations. In particular, we will be interested in whether there are Poincare-invariant solutions for the metric of the four-dimensional slice at $x_5 = 0$ for generic values of these parameters (or more generally, for what subspaces of this parameter space there are Poincare-invariant solutions in four dimensions). We will also require that the geometry is such that the four-dimensional Planck scale is finite. Our main results can be summarized in three different cases as follows:

(I) For $\Lambda = 0$, $b \neq \pm\frac{4}{3}$ but otherwise arbitrary, and arbitrary magnitude of V we find a Poincare-invariant domain wall solution of the equations of motion. For $b = 2/3$, which is the value corresponding to a brane tension of order $e^{-2\phi}$ in string frame, the sign of V

must be positive in order to correspond to a solution with a finite four-dimensional Planck scale, but it is otherwise unconstrained. This suggests that for fixed scalar field coupling to the domain wall, quantum corrections to its tension V do not spoil Poincare invariance of the slice. In §3 we will review examples in string theory of situations where worldvolume degrees of freedom contribute quantum corrections to the $e^{-2\phi}$ term in a brane's tension. Our result implies that these quantum corrections do not need to be fine-tuned to zero to obtain a flat four-dimensional spacetime.

For a generic choice of $f(\phi)$ in (2.1) (including the type of power series expansion in e^{ϕ} that would arise in perturbative string theory), the same basic results hold true: We are able to find Poincare invariant solutions without fine-tuning f. Insisting on a finite 4d Planck scale gives a furthur constraint on f'/f at the wall, forcing it to lie in a range of order one.

Given a solution with one value of V and $\Lambda = 0$, a self-tuning mechanism is in fact clear from the Lagrangian (for $b \neq 0$). In (2.1) we see that if $\Lambda = 0$ (or $a = 0$), the only non-derivative coupling of the dilaton is to the brane tension term, where it appears in the combination $(-V)e^{b\phi}$. Clearly given a solution for one value of V, there will be a solution for any value of V obtained by absorbing shifts in V with shifts in ϕ. With more general $f(\phi)$, similar remarks hold: the dilaton zero mode appears only in f, and one can absorb shifts in V by shifting ϕ.

However, in the special case $b = 0$ (where $f(\phi)$ is just a constant), we will also find flat solutions for generic V. This implies that the freedom to vary the dilaton zero mode is not the only mechanism that ensures the existence of a flat solution for arbitrary V.

(II) For $\Lambda = 0$, $b = \pm 4/3$, we find a different Poincare-invariant solution (obtained by matching together two 5d bulk solutions in a different combination than that used in obtaining the solutions described in the preceding paragraph (I)). A solution is present for any value of V. This suggests that for fixed scalar field coupling to the domain wall, quantum corrections to its tension V do not spoil Poincare-invariance of the slice. Again the sign of V must be positive in order to have a finite four-dimensional Planck scale.

(III) We do not find a solution (nor do we show that none exists) for general Λ, V, a, and b (in concordance with the counting of parameters in [11]). However, for each Λ and V there is a choice of a and b for which we do find a Poincare invariant solution using a simple ansatz.

For $a = 0$, and general b, Λ, and V we are currently investigating the existence of self-tuning solutions. Their existence would be in accord with the fact that in this case, as in the cases with $\Lambda = 0$, the dilaton zero mode only appears in the tension of the wall. This means again that shifts in V can be absorbed by shifting ϕ, so if one finds a Poincare invariant solution for any V, one does not need to fine-tune V to solve the equations.

2.2. Equations of Motion

The equations of motion arising for the theory (2.1), with our simple choice for $f(\phi)$ given in (2.3), are as follows. Varying with respect to the dilaton gives:

$$\sqrt{-G}\left(\frac{8}{3}\nabla^2\phi - a\Lambda e^{a\phi}\right) - bV\delta(x_5)e^{b\phi}\sqrt{-g} = 0 \tag{2.4}$$

The Einstein equations for this theory are:

$$\begin{aligned}&\sqrt{-G}\left(R_{MN} - \frac{1}{2}G_{MN}R\right)\\ &-\frac{4}{3}\sqrt{-G}\left[\nabla_M\phi\nabla_N\phi - \frac{1}{2}G_{MN}(\nabla\phi)^2\right]\\ &+\frac{1}{2}\left[\Lambda e^{a\phi}\sqrt{-G}G_{MN} + \sqrt{-g}Vg_{\mu\nu}\delta^\mu_M\delta^\nu_N\delta(x_5)\right] = 0\end{aligned} \tag{2.5}$$

We are interested in whether there are solutions with Poincare-invariant four-dimensional physics. Therefore we look for solutions of (2.4) and (2.5) where the metric takes the form

$$ds^2 = e^{2A(x_5)}(-dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2) + dx_5^2 \tag{2.6}$$

With this ansatz for the metric, the equations become

$$\frac{8}{3}\phi'' + \frac{32}{3}A'\phi' - a\Lambda e^{a\phi} - bV\delta(x_5)e^{b\phi} = 0 \tag{2.7}$$

$$6(A')^2 - \frac{2}{3}(\phi')^2 + \frac{1}{2}\Lambda e^{a\phi} = 0 \tag{2.8}$$

$$3A'' + \frac{4}{3}(\phi')^2 + \frac{1}{2}e^{b\phi}V\delta(x_5) = 0 \tag{2.9}$$

where $'$ denotes differentiation with respect to x_5. The first one (2.7) is the dilaton equation of motion, the second (2.8) is the 55 component of Einstein's equations, and the last (2.9) comes from a linear combination (the difference) of the $\mu\nu$ component of Einstein's equation and the 55 component.

We will mostly consider the simple ansatz

$$A' = \alpha\phi'. \tag{2.10}$$

However for the case $a = 0$, $\Lambda \neq 0$ we will integrate the equations directly.

2.3. $\Lambda = 0$ Case

Let us first consider the system with $\Lambda = 0$. We will first study the bulk equations of motion (i.e. the equations of motion away from $x_5 = 0$) where the δ-function terms in (2.7) and (2.9) do not come in. Note that because the delta function terms do not enter, the bulk equations are independent of our choice of $f(\phi)$ in (2.1). We will then consider the conditions required to match two bulk solutions on either side of the domain wall of tension $Ve^{b\phi}$ at $x_5 = 0$. We will find two qualitatively different ways to do this, corresponding to results (I) and (II) quoted above. We will also find that for fairly generic $f(\phi)$, the same conclusions hold.

Bulk Equations: $\Lambda = 0$

Plugging the ansatz (2.10) into (2.8) (with $\Lambda = 0$) we find that

$$6\alpha^2(\phi')^2 = \frac{2}{3}(\phi')^2 \tag{2.11}$$

which is solved if we take

$$\alpha = \pm\frac{1}{3} \tag{2.12}$$

Plugging this ansatz into (2.7) we obtain

$$\frac{8}{3}(\phi'' + 4(\pm\frac{1}{3})(\phi')^2) = 0 \tag{2.13}$$

Plugging it into (2.9) we obtain

$$3(\pm\frac{1}{3})\phi'' + \frac{4}{3}(\phi')^2 = 0 \tag{2.14}$$

With either choice of sign for α, these two equations become identical in bulk. For $\alpha = \pm\frac{1}{3}$, we must solve

$$\phi'' \pm \frac{4}{3}(\phi')^2 = 0 \tag{2.15}$$

in bulk. This is solved by

$$\phi = \pm\frac{3}{4}\log|\frac{4}{3}x_5 + c| + d \tag{2.16}$$

where c and d are arbitrary integration constants.

Note that there is a singularity in this solution at

$$x_5 = -\frac{3}{4}c \tag{2.17}$$

Our solutions will involve regions of spacetime to one side of this singularity; we will assume that it can be taken to effectively cut off the space. At present we do not have much quantitative to say about the physical implications of this singularity. The results we derive here (summarized above) strongly motivate further exploring the effects of these singularities on the four-dimensional physics of our domain wall solutions.

At $x_5 = 0$ there is localized energy density leading to the δ-function terms in (2.7) and (2.9). We can solve these equations by introducing appropriate discontinuities in ϕ' at the wall (while insisting that ϕ itself is continuous). We will now do this for two illustrative cases (the first being the most physically interesting).

Solution (I):

Let us take the bulk solution with $\alpha = +\frac{1}{3}$ for $x_5 < 0$, and the one with $\alpha = -\frac{1}{3}$ for $x_5 > 0$. So we have

$$\phi(x_5) = \phi_1(x_5) = \frac{3}{4}\log|\frac{4}{3}x_5 + c_1| + d_1, \qquad x_5 < 0 \tag{2.18}$$

$$\phi(x_5) = \phi_2(x_5) = -\frac{3}{4}\log|\frac{4}{3}x_5 + c_2| + d_2, \qquad x_5 > 0 \tag{2.19}$$

where we have allowed for the possibility that the (so far) arbitrary integration constants can be different on the two sides of the domain wall.

Imposing continuity of ϕ at $x_5 = 0$ leads to the condition

$$\frac{3}{4}\log|c_1| + d_1 = -\frac{3}{4}\log|c_2| + d_2 \tag{2.20}$$

This equation determines the integration constant d_2 in terms of the others.

To solve (2.7) we then require

$$\frac{8}{3}(\phi_2'(0) - \phi_1'(0)) = bVe^{b\phi(0)} \tag{2.21}$$

while to solve (2.9) we need

$$3\Big(\alpha_2\phi_2'(0) - \alpha_1\phi_1'(0)\Big) = -\frac{1}{2}Ve^{b\phi(0)} \tag{2.22}$$

(where $\alpha_1 = +\frac{1}{3}$ and $\alpha_2 = -\frac{1}{3}$). These two matching conditions become

$$-\frac{8}{3}(\frac{1}{c_1} + \frac{1}{c_2}) = bVe^{bd_1}|c_1|^{\frac{3}{4}b} \tag{2.23}$$

and

$$\frac{1}{c_2} - \frac{1}{c_1} = -\frac{1}{2} V e^{b d_1} |c_1|^{\frac{3}{4} b} \tag{2.24}$$

Solving for the integration constants c_1 and c_2 we find

$$\frac{2}{c_2} = \left[-\frac{3b}{8} - \frac{1}{2} \right] V e^{b d_1} |c_1|^{\frac{3}{4} b} \tag{2.25}$$

$$\frac{2}{c_1} = \left[-\frac{3b}{8} + \frac{1}{2} \right] V e^{b d_1} |c_1|^{\frac{3}{4} b} \tag{2.26}$$

Note that as long as $b \neq \pm \frac{4}{3}$, we here find a solution for the integration constants c_1 and c_2 in terms of the parameters b and V which appear in the Lagrangian and the integration constant d_1. (As discussed above, the integration constant d_2 is then also determined).[2] In particular, for scalar coupling given by b, there is a Poincare-invariant four-dimensional domain wall for any value of the brane tension V; V does not need to be fine-tuned to find a solution. As is clear from the form of the 4d interaction in (2.1), one way to understand this is that the scalar field ϕ can absorb a shift in V since the only place that the ϕ zero mode appears in the Lagrangian is multiplying V. However since we can use these equations to solve for $c_{1,2}$ without fixing d_1, a more general story is at work; in particular, even for $b = 0$ we find solutions for arbitrary V.

A constraint on the sign of V arises, as we will now discuss, from the requirement that there be singularities (2.17) in the bulk solutions, effectively cutting off the x_5 direction at finite volume.

More General $f(\phi)$

If instead of (2.3) we include a more general choice of f in the action (2.1), the considerations above go through unaltered. The choice of f only enters in the matching conditions (2.21) and (2.22) at the domain wall. The modified equations become

$$\frac{8}{3}(\phi_2'(0) - \phi_1'(0)) = \frac{\partial f}{\partial \phi}(\phi(0)) \tag{2.27}$$

$$3\Big(\alpha_2 \phi_2'(0) - \alpha_1 \phi_1'(0)\Big) = -\frac{1}{2} f(\phi(0)) \tag{2.28}$$

In terms of the integration constants, these become:

[2] We will momentarily find a disjoint set of $\Lambda = 0$ domain wall solutions for which b will be forced to be $\pm 4/3$, so altogether there are solutions for any b.

$$-\frac{8}{3}(\frac{1}{c_1}+\frac{1}{c_2}) = \frac{\partial f}{\partial \phi}(\frac{3}{4}\log|c_1|+d_1) \tag{2.29}$$

$$\frac{1}{c_2}-\frac{1}{c_1} = -\frac{1}{2}f(\frac{3}{4}\log|c_1|+d_1) \tag{2.30}$$

Clearly for generic $f(\phi)$, one can solve these equations.

Obtaining a Finite 4d Planck Scale

Consider the solution (2.18) on the $x_5 < 0$ side. If $c_1 < 0$, then there is never a singularity. Let us consider the four-dimensional Planck scale. It is proportional to the integral [5]

$$\int dx_5 \; e^{2A(x_5)} \tag{2.31}$$

In the $x_5 < 0$ region, this goes like

$$\int dx_5 \sqrt{|\frac{4}{3}x_5 + c_1|} \tag{2.32}$$

If $c_1 < 0$, then there is no singularity, and this integral is evaluated from $x_5 = -\infty$ to $x_5 = 0$. It diverges. If $c_1 > 0$, then there is a singularity at (2.17). Cutting off the volume integral (2.32) there gives a finite result. Note that the ansatz (2.10) leaves an undetermined integration constant in A, so one can tune the actual value of the 4d Planck scale by shifting this constant.

In order to have a finite 4d Planck scale, we therefore impose that $c_1 > 0$. This requires $V(\frac{1}{2}-\frac{3b}{8}) > 0$. For the value $b = 2/3$, natural in string theory (as we will discuss in §3), this requires $V > 0$. With this constraint, there is similarly a singularity on the $x_5 > 0$ side which cuts off the volume on that side.

These conditions extend easily to conditions on $f(\phi)$ in the more general case. We find

$$\begin{aligned} -\frac{3}{8}\frac{\partial f}{\partial \phi}(\phi(0)) - \frac{1}{2}f(\phi(0)) < 0 \\ -\frac{3}{8}\frac{\partial f}{\partial \phi}(\phi(0)) + \frac{1}{2}f(\phi(0)) > 0 \end{aligned} \tag{2.33}$$

This means that $f(\phi)$ must be positive at the wall (corresponding to a positive tension brane), and that

$$-\frac{4}{3} < \frac{f'}{f} < \frac{4}{3} \tag{2.34}$$

So although f does not need to be fine-tuned to achieve a solution of the sort we require, it needs to be such that f'/f is in the range (2.34).

Let us discuss some of the physics at the singularity. Following [5,11], we can compute the x_5-dependence of the four-dimensional graviton wavefunction. Expanding the metric about our solution (taking $g_{\mu\nu} = e^{2A}\eta_{\mu\nu} + h_{\mu\nu}$), we find

$$h_{\mu\nu} \propto \sqrt{|\frac{4}{3}x_5 + c|} \tag{2.35}$$

At a singularity, where $|\frac{4}{3}x_5 + c|$ vanishes, this wavefunction also vanishes. Without understanding the physics of the singularity, we cannot determine yet whether it significantly affects the interactions of the four-dimensional modes.

It is also of interest to consider the behavior of the scalar ϕ at the singularities. In string theory this determines the string coupling. In our solution (I), we see that

$$\begin{aligned} x_5 &\to -\frac{3}{4}c_1 \Rightarrow \phi \to -\infty \\ x_5 &\to -\frac{3}{4}c_2 \Rightarrow \phi \to \infty \end{aligned} \tag{2.36}$$

So in string theory, the curvature singularity on the $x_5 < 0$ side is weakly coupled, while that on the $x_5 > 0$ side is strongly coupled. It may be possible to realize these geometries in a context where supersymmetry is broken by the brane, so that the bulk is supersymmetric. In such a case the stability of the high curvature and/or strong-coupling regions may be easier to ensure. In any case we believe that the results of this section motivate further analysis of these singular regions, which we leave for future work.

Putting everything together, we have found the solution described in case (I) above. It should be clear that since $f(\phi)$ only appears in (2.1) multiplying the delta function "thin wall" source term, we can always use the choice (2.3) in writing matching conditions at the wall for concreteness. To understand what would happen with a more general f, one simply replaces $Ve^{b\phi(0)}$ with $f(\phi(0))$ and $bVe^{b\phi(0)}$ with $\frac{\partial f}{\partial \phi}(\phi(0))$ in the matching equations. We will not explicitly say this in each case, but it makes the generalization to arbitrary f immediate.

Solution (II):

A second type of solution with $\Lambda = 0$ is obtained by taking α to have the same sign on both sides of the domain wall. So we have

$$\phi(x_5) = \phi_1(x_5) = \pm\frac{3}{4}\log|\frac{4}{3}x_5 + c_1| + d_1, \qquad x_5 < 0 \tag{2.37}$$

$$\phi(x_5) = \phi_2(x_5) = \pm\frac{3}{4}\log|\frac{4}{3}x_5 + c_2| + d_2, \qquad x_5 > 0 \tag{2.38}$$

The matching conditions then require $b = \mp\frac{4}{3}$ for consistency of (2.7) and (2.9) (in the case with more generic $f(\phi)$, this generalizes to the condition $\frac{\partial f}{\partial \phi}(\phi(0)) = \mp\frac{4}{3}f(\phi(0))$). This is not a value of b that appears from a dilaton coupling in perturbative string theory. It is still interesting, however, as a gravitational low-energy effective field theory where V does not have to be fine-tuned in order to preserve four-dimensional Poincare invariance. We find a solution to the matching conditions with

$$\begin{aligned} c_1 &= c, \qquad x_5 > 0 \\ c_2 &= -c, \qquad x_5 < 0 \\ d_1 &= d_2 = d \\ e^{\mp\frac{4}{3}d} &= \frac{4}{V}\frac{c}{|c|} \end{aligned} \tag{2.39}$$

for some arbitrary constant c, and any V. This gives the results summarized in case (II) above. The value $b = \mp 4/3$, which is required here, was excluded from the solutions (I) derived in the last section.

As long as we choose c such that there are singularities on both sides of the domain wall, we again get finite 4d Planck scale. As we can see from (2.37) and (2.38), having singularities on either side of the origin requires c to be positive. Then we see from (2.39) that we can find a solution for arbitrary positive brane tension V.

Let us discuss the physics of the singularities in this case. As in solutions (I), the graviton wavefunction decays to zero at the singularity like $(x - x_{sing})^{\frac{1}{2}}$. For $b = -4/3$, $\phi \to -\infty$ at the singularities on both sides, while for $b = \frac{4}{3}$, $\phi \to \infty$ at the singularities on both sides.

Putting solutions (I) and (II) together, we see that in the $\Lambda = 0$ case one can find a Poincare invariant solution with finite 4d Planck scale for any positive tension V and any choice of b in (2.1). As we have seen, this in fact remains true with (2.3) replaced by a more general dilaton dependent brane tension $f(\phi)$.

Two-Brane Solutions

One can also obtain solutions describing a pair of domain walls localized in a compact fifth dimension. In case (I), one can show that such solutions always involve singularities. In case (II), there are solutions which avoid singularities while maintaining the finiteness

of the four-dimensional Planck scale. They however involve extra moduli (the size of the compactified fifth dimension) which may be stabilized by for example the mechanism of [14]. The singularity is avoided in these cases by placing a second domain wall between $x_5 = 0$ and the would-be singularity at $\frac{4}{3}x_5 + c = 0$. This allows us in particular to find solutions for which ϕ is bounded everywhere, so that the coupling does not get too strong. This is a straightforward generalization of what we have already done and we will not elaborate on it here.

2.4. $\Lambda \neq 0$ *(Solution III)*

More generally we can consider the entire Lagrangian (2.1) with parameters Λ, V, a, b. In this case, plugging in the ansatz (2.10) to equations (2.7)–(2.9), we find a bulk solution

$$\begin{aligned} \phi &= -\frac{2}{a}\log(\frac{a(\mp\sqrt{B})}{2}x_5 + d) \\ B &= \frac{\Lambda}{\frac{4}{3} - 12\alpha^2} \\ \alpha &= -\frac{8}{9a} \end{aligned} \tag{2.40}$$

We find a domain wall solution by taking one sign in the argument of the logarithm in (2.40) for $x_5 < 0$ and the opposite sign in the argument of the logarithm for $x_5 > 0$. Say for instance that $a > 0$. Then we could take the $-$ sign for $x > 0$ and the $+$ sign for $x < 0$, and find a solution which terminates at singularities on both sides if we choose $d > 0$.

The matching conditions then require

$$V = -12\alpha\sqrt{B} \tag{2.41}$$

and

$$b = -\frac{4}{9\alpha} \tag{2.42}$$

So we see that here V must be fine-tuned to the Λ-dependent value given in (2.41). This is similar to the situation in [5], where one fine-tune is required to set the four-dimensional cosmological constant to zero. Like in our solutions in §2.1, there is one undetermined parameter in the Lagrangian. But here it is a complicated combination of Λ and V (namely, $\frac{V}{\sqrt{\Lambda}}$), and we do not have an immediate interpretation of variations of this parameter as arising from nontrivial quantum corrections from a sector of the theory.

The fact, apparent from equations (2.40) and (2.42), that $b = a/2$ in this solution makes its embedding in string theory natural, as we will explain in the next section.

$\Lambda \neq 0$, $a = 0$

In this case, the bulk equations of motion become (in terms of $h \equiv \phi'$ and $g \equiv A'$)

$$\begin{aligned} & h' + 4hg = 0 \\ & 6g^2 - \frac{2}{3}h^2 + \frac{1}{2}\Lambda = 0 \\ & 3g' + \frac{4}{3}h^2 = 0 \end{aligned} \tag{2.43}$$

We can solve the second equation for g in terms of h, and then integrate the first equation to obtain $h(x_5)$. For $g \neq 0$ the third equation is then automatically satisfied. We will not need detailed properties of the solution, so we will not include it here. The solutions are more complicated than those of §2.3. We are currently exploring under what conditions one can solve the matching equations to obtain a wall with singularities cutting off the x_5 direction on both sides [10]. If such walls exist, they will also exhibit the self-tuning phenomenon of §2.3, since the dilaton zero mode can absorb shifts in V and doesn't appear elsewhere in the action.

3. Toward a String Theory Realization

3.1. $\Lambda = 0$ *Cases*

Taking $\Lambda = 0$ is natural in string theory, since the tree-level vacuum energy in generic critical closed string compactifications (supersymmetric or not) vanishes. One would expect bulk quantum corrections to correct Λ in a power series in $g_s = e^{\phi}$. However, the analysis of §2.3 may still be of interest if the bulk corrections to Λ are small enough. This can happen for instance if the supersymmetry breaking is localized in a small neighborhood of the wall and the x_5 interval is much larger, or more generally if the supersymmetry breaking scale in bulk is small enough.

General $f(\phi)$

The examples we have found in §2 which "self-tune" the 4d cosmological constant to zero have $\Lambda = 0$ with a broad range of choices for $f(\phi)$. We interpret this as meaning that quantum corrections to the brane tension, which would change the form of f, do not destabilize the flat brane solution. The generality of the dilaton coupling $f(\phi)$ suggests

that our results should apply to a wide variety of string theory backgrounds involving domain walls. We now turn to a discussion of some of the features of particular cases.

D-branes

In string theory, one would naively expect codimension one D-branes (perhaps wrapping a piece of some compact manifold) to have $f(\phi)$ given by a power series of the form

$$f(\phi) = e^{\frac{5}{3}\phi} \sum_{n=0}^{\infty} c_n e^{n\phi} \tag{3.1}$$

The c_0 term represents the tree-level D-brane tension (which goes like $\frac{1}{g_s}$ in string frame). The higher order terms in (3.1) represent quantum corrections from the Yang-Mills theory on the brane, which has coupling $g_{YM}^2 = e^{\phi}$.

If one looks for solutions of the equations which arise with the choice (2.3) for $f(\phi)$ with positive V and $b = 5/3$ (the tree level D-brane theory), then there are no solutions with finite 4d Planck scale. The constraints of §2.3 cannot be solved to give a single wall with singularities on both sides cutting off the length in the x_5 direction. However, including quantum corrections to the D-brane theory to get a more generic f as in (3.1), there is a constraint on the magnitude of $\frac{\partial f}{\partial \phi}(\phi(0))$ divided by $f(\phi(0))$ which can be obeyed. Therefore, one concludes that for our mechanism to be at work with D-brane domain walls, the dilaton ϕ must be stabilized away from weak coupling – the loop corrections to (3.1) must be important.

The Case $f(\phi) = Ve^{\frac{2}{3}\phi}$ and NS Branes

Another simple way to get models which could come out of string theory is to set $b = 2/3$ in (2.3), so

$$f(\phi) = Ve^{\frac{2}{3}\phi} \tag{3.2}$$

Then (2.1) becomes precisely the Einstein frame action that one would get from a "3-brane" in string theory with a string frame source term proportional to $e^{-2\phi}$. In this case, ϕ can also naturally be identified with the string theory dilaton. This choice of b is possible in solutions of the sort summarized in result (I) in §2.1.

However, after identifying ϕ with the string theory dilaton, if we really want to make this specific choice for $f(\phi)$ we would also like to find branes where it is natural to expect that quantum corrections to the brane tension (e.g. from gauge and matter fields living on the brane) would shift V, but not change the overall ϕ dependence of the source term.

This can only happen if the string coupling $g_s = e^{\phi}$ is *not* the field-theoretic coupling parameter for the dynamical degrees of freedom on the brane.

Many examples where this happens are known in string theory. For example, the NS fivebranes of type IIB and heterotic string theory have gauge fields on their worldvolume whose Yang-Mills coupling does not depend on g_s [15,16,17]. This can roughly be understood from the fact that the dilaton grows to infinity down the throat of the solution, and its value in the asymptotic flat region away from this throat is irrelevant to the coupling of the modes on the brane. Upon compactification, this leads to gauge couplings depending on sizes of cycles in the compactification manifold (in units of α') [16,18]. For instance, in [18] gauge groups which arise "non-perturbatively" in singular heterotic compactifications (at less supersymmetric generalizations of the small instanton singularity [15]) are discussed. There, the 4d gauge couplings on a heterotic NS fivebrane wrapped on a two-cycle go like

$$g_{YM}^2 \sim \frac{\alpha'}{R^2} \tag{3.3}$$

Here R is the scale of this 2-cycle in the compactification manifold. In [18], this was used to interpret string sigma model worldsheet instanton effects, which go like $e^{-\frac{R^2}{\alpha'}}$, in terms of nonperturbative effects in the brane gauge group, which go like $e^{-\frac{8\pi^2}{g_{YM}^2}}$. So this is a concrete example in which nontrivial dilaton-independent quantum corrections to the effective action on the brane arise. One can imagine analogous examples involving supersymmetry breaking. In such cases, perturbative shifts in the brane tension due to brane worldvolume gauge dynamics would be a series in $\frac{\alpha'}{R^2}$ and not $g_s = e^{\phi}$.

In particular, one can generalize such examples to cases where the branes are domain walls in 5d spacetime (instead of space-filling in 4d spacetime as in the examples just discussed), but where again the brane gauge coupling is not the string coupling. Quantum corrections to the brane tension in the brane gauge theory then naturally contribute shifts

$$e^{\frac{2}{3}\phi} V \to e^{\frac{2}{3}\phi}(V + \delta V) \tag{3.4}$$

to the (Einstein frame) $b = 2/3$ source term in (2.1), without changing its dilaton dependence.

Most of our discussion here has focused on the case where ϕ is identified with the string theory dilaton. However, in general it is possible that some other string theory modulus could play the role of ϕ in our solutions, perhaps for more general values of b.

Resemblance to Orientifolds

In our analysis of the equations, we find solutions describing a 4d gravity theory with zero cosmological constant if we consider singular solutions and cut off the fifth dimension at these singularities. The simplest versions of compactifications involving branes in string theory also include defects in the compactification which absorb the charge of the branes and cancel their contribution to the cosmological constant in four dimensions, at least at tree level. Examples of these defects include orientifolds (in the context of D-brane worlds), S-duals of orientifolds (in the context of NS brane worlds), and Horava-Witten "ends of the world" (in the context of the strongly coupled heterotic string).

Our most interesting solutions involve two different behaviors on the two sides of the domain wall. On one side the dilaton goes to strong coupling while on the other side it goes to weak coupling at the singularity. This effect has also been seen in brane-orientifold systems [19].

It would be very interesting to understand whether the singularities we find can be identified with orientifold-like defects, as these similarities might suggest. Then their role (if any) in absorbing quantum corrections to the 4d cosmological constant could be related to the effective negative tension of these defects. However, various aspects of our dilaton gravity solutions are not familiar from brane-orientifold systems. In particular, the existence of solutions with curved 4d geometry on the same footing as our flat solutions does not occur in typical perturbative string compactifications. In any case, note that (as explained in §3.1) our mechanism does not occur in the case of weakly coupled D-branes and orientifolds.

3.2. $\Lambda \neq 0$ *Cases*

Some of the $\Lambda \neq 0$ cases discussed in §2.4 could also arise in string theory. As discussed in [20,21] one can find closed string backgrounds with nonzero tree level cosmological constant $\Lambda < 0$ by considering subcritical strings. In this case, the cosmological term would have dilaton dependence consistent with $a = 4/3$ in bulk. Using equations (2.40) and (2.42), this implies $b = 2/3$, which is the expected scaling for a tree-level brane tension in the thin-wall approximation as well.

One would naively expect to obtain vacua with such negative bulk cosmological constants out of tachyon condensation in closed string theory [20,21]. It is then natural to consider these domain walls (in the $a = 4/3, b = 2/3$ case) as the thin wall approximation

to "fat" domain walls which could be formed by tachyon field configurations which interpolate between different minima of a closed string tachyon potential. In the context of the Randall-Sundrum scenario, such "fat" walls were studied for example in [11,22,23].

It would be interesting to find cases where the $\Lambda \neq 0$, $a = 0$ solutions arise from a more microscopic theory. However, it is clear that the dilaton dependence of (2.1) is then not consistent with interpreting ϕ as the string theory dilaton. Perhaps one could find a situation where ϕ can be identified with some other string theoretic modulus, and Λ can be interpreted as the bulk cosmological constant after other moduli are fixed.

4. Discussion

The concrete results of §2 motivate many interesting questions, which we have only begun to explore. Answering these questions will be important for understanding the four-dimensional physics of our solutions.

The most serious question has to do with the nature of the singularities. There are many singularities in string theory which have sensible physical resolutions, either due to the finite string tension or due to quantum effects. Most that have been studied (like flops [24] and conifolds [25]) involve systems with some supersymmetry, though some (like orbifolds [26]) can be understood even without supersymmetry. We do not yet know the proper interpretation of our singularities, though as discussed in §3 there are intriguing similarities to orientifold physics in our system. After finding the solutions, we cut off the volume integral determining the four-dimensional Planck scale at the singularities. It is important to determine whether this is a legitimate operation.

It is desirable (and probably necessary in order to address the question in the preceding paragraph) to embed our solutions microscopically into M theory. As discussed in §3, some of our solutions appear very natural from the point of view of string theory, where the scalar ϕ can be identified with the dilaton. It would be interesting to consider the analogous couplings of string-theoretic moduli scalars other than the dilaton. Perhaps there are other geometrical moduli which couple with different values of a and b in (2.3) than the dilaton does.

It is also important to understand the effects of quantum corrections to quantities other than $f(\phi)$ in our Lagrangian. In particular, corrections to Λ and corrections involving different powers of e^{ϕ} in the bulk (coming from loops of bulk gravity modes) will change the nature of the equations. It will be interesting to understand the details of curved 4d

domain wall solutions to the corrected equations [27,11,10]. More specifically, it will be of interest to determine the curvature scale of the 4d slice, in terms of the various choices of phenomenologically natural values for the Planck scale. Since the observed value of the cosmological constant is nonzero according to studies of the mass density, cosmic microwave background spectral distribution, and supernova events [28], such corrected solutions might be of physical interest.

Perhaps the most intriguing physical question is what happens from the point of view of four-dimensional effective field theory (if such a description in fact exists). Understanding the singularity in the 5d background is probably required to answer this question. One possibility (suggested by the presence of the singularity and by the self-tuning of the 4d cosmological constant discovered here) is that four-dimensional effective field theory breaks down in this background, at least as far as contributions to the 4d cosmological constant are concerned. In [5] and analogous examples, there is a continuum of bulk modes which could plausibly lead to a breakdown of 4d effective field theory in certain computations. In our theories, cutting off the 5d theory at the singularities leaves finite proper distance in the x_5 direction. This makes it unclear how such a continuum could arise (in the absence of novel physics at the singularities, which could include "throats" of the sort that commonly arise in brane solutions). So in this system, any breakdown of 4d effective field theory is more mysterious.

Acknowledgements

We are indebted to O. Aharony and T. Banks for interesting discussions which motivated us to investigate this subject. We would also like to thank R. Sundrum for many helpful discussions about closely related topics; we understand that Arkani-Hamed, Dimopoulos, Kaloper and Sundrum have uncovered very similar results [13]. We thank H. Verlinde for interesting discussions, and in particular for several helpful comments about the potential generality of these results. In addition we are grateful to M. Dine, N. Kaloper, S. Shenker, M. Shmakova, L. Susskind and E. Verlinde for stimulating discussions. We would like to acknowledge the kind hospitality of the School of Natural Sciences at the Institute for Advanced Study during the early stages of this work. S.K. is supported in part by the Ambrose Monell Foundation and a Sloan Foundation Fellowship, M.S. is supported in part by an NSF Graduate Research Fellowship, and E.S. is supported in part by a DOE OJI Award and a Sloan Foundation Fellowship. S.K. and E.S. are supported in part by the DOE under contract DE-AC03-76SF00515.

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