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Open string instantons and superpotentials

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We study the F-terms in $\mathcal{N} = 1$ supersymmetric, $d = 4$ gauge theories arising from $D(p+3)$-branes wrapping supersymmetric p-cycles in a Calabi-Yau threefold. If p is even the spectrum and superpotential for a single brane are determined by purely classical ($\alpha' \to 0$) considerations. If $p = 3$, superpotentials for massless modes are forbidden to all orders in α' and may only be generated by open string instantons. For this latter case we find that such instanton effects are generically present. Mirror symmetry relates even and odd p and thus perturbative and nonperturbative superpotentials; we provide a preliminary discussion of a class of examples of such mirror pairs.

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1. Introduction

The study of D-branes in Calabi-Yau threefolds is of both formal and phenomenological interest. As philosophical tools, Calabi-Yau threefolds provide a natural arena for studying nonperturbative stringy geometry. D-branes are excellent probes of this geometry as they are sensitive to structure well below the string scale (at weak string coupling) [1,2]. Furthermore a deep understanding of mirror symmetry requires understanding its action on D-branes [3,4,5].

The phenomenological interest is served by studying space-filling D-branes in type I or type II string theories, in configurations preserving $\mathcal{N} = 1$ SUSY in four dimensions.[1] These configurations fall into two classes [6,7]: 6-branes in type IIA wrapping special Lagrangian cycles of the threefold, and odd-p-branes in type I or type IIB wrapping even-dimensional cycles. The latter configurations (up to orientifolds) can be written as coherent sheaves on the threefold [3,4,8] (or its mirror) and so involve the same type of data as heterotic compactifications [9,10]. The D-brane limit allows one to study gauge field data when it is intrinsically stringy (much as Landau-Ginzburg compactifications allow one to study intrinsically stringy aspects of geometric data), via open string techniques.

In closed-string compactifications on Calabi-Yau threefolds, worldsheet instanton effects are the most well-understood source of truly stringy physics. They drastically modify the geometry at short distances: in addition they lead to interesting physical effects such as the generation of nontrivial superpotentials in heterotic compactifications [11,12].[2] In this work we will study the effects of *open*-string instantons on D-brane physics, in particular on the superpotential. For branes wrapping even-dimensional cycles, we will find that the superpotential can be determined from classical geometry; for branes wrapping special Lagrangian cycles it is generated entirely by nonperturbative worldsheet effects.

[1] The standard caveat is in force here: in order to satisfy Gauss' law for the various RR charges, one should either consider branes wrapping cycles in non-compact Calabi-Yau spaces, or consider configurations containing both branes and orientifolds. For our purposes the former assumption will suffice, but many of our considerations could also be applied to lower dimensional, non space-filling branes wrapping the same cycles.

[2] In fact, it has been proved in [13] that the most easily realized heterotic (0,2) models, those realized as gauged linear sigma models [14,15], are not destabilized by worldsheet instantons. As we will see in the following, it should be easier to find examples of D-brane models which exhibit disc instanton generated superpotentials.

This has interesting implications for mirror symmetry in the type II compactifications. To begin with, if a given mirror pair of cycles has massless deformations with a nontrivial superpotential, then the classical moduli spaces will not match under the mirror map. A holomorphic 2-cycle with its infinitesimal holomorphic deformations obstructed at some nontrivial order (so that a 5-brane wrapped around it will have massless chiral fields with a superpotential) will have as its mirror a 3-cycle which has flat directions to all orders in α'. On the other hand, if we start with a special Lagrangian cycle and find that worldsheet instantons destabilize or make nonsupersymmetric the D-branes wrapping them, the mirror will respectively either not exist or will be some classically nonsupersymmetric configuration. This is reminiscent of a common feature of dualities of $\mathcal{N} = 1$ gauge theories, where superpotentials generated by nonperturbative dynamics are dual to tree-level superpotentials [16]. However, it is a relatively novel situation for $\mathcal{N} = 1$ dualities of string vacua, where normally instanton effects map to instanton effects, as in heterotic/F-theory duality [17]. Here mirror symmetry should provide a powerful tool for summing open string instantons, as it does for closed string worldsheet instantons [18]. Note that the nonperturbative superpotentials we are discussing here are not explicable in terms of gauge dynamics involving the (perturbative) D-brane gauge theory; we would expect in some circumstances the instanton effects will be related to gauge instantons of a $non-perturbative$ D-brane gauge theory that arises at singular points in the brane moduli space. This is analogous to the fact that in heterotic (0,2) models, worldsheet instantons can sometimes be related to gauge instantons of nonperturbative gauge groups arising from singular compactifications [19].

In this paper we will begin to investigate disc instanton effects by asking whether nonperturbative superpotentials are generically generated by worldsheet (disc) instantons. We will find a story similar to that of the heterotic string [12]: when the open string instantons are isolated, nonvanishing (locally) runaway potentials may be generated.[3]

The plan of our paper is as follows: in §2 we will review the construction of $\mathcal{N} = 1$ four-dimensional theories via space-filling D-branes wrapped on Calabi-Yau threefolds in type II string theory. After discussing the results determined by classical geometry, we will discuss some constraints from string theory. First, at tree level the superpotentials are

[3] However, in contrast to the heterotic string story, the classical moduli spaces of the brane configurations we will study are naturally compact. Hence, the superpotentials we find will have minima which are not "at infinity" in field space.

computable via topological open string theory, and the hypermultiplets of the background closed-string theory decouple [20]. Furthermore, we will find that in the case of D6-branes wrapping special Lagrangian three-cycles, superpotentials are forbidden not only classically but to all orders in α', due to a Peccei-Quinn symmetry. Superpotentials in these cases can only be generated by topologically nontrivial disc instanton effects. In §3 we will discuss the generic superpotential terms that are allowed in the presence of an isolated holomorphic disc. Finally, in §4 we discuss promising directions for future work. Further results in explicit examples will appear in a companion paper [21].

There is a close relation between the ideas discussed in this paper and earlier work of Witten [22] and Vafa [23]. As this work was being completed, we were also informed of the related work [24] by mathematicians.

2. Classical geometry of D-branes on CY_3

We begin with D-brane configurations preserving four supercharges, to lowest order in g_s and α', in type II string theory on $M \times \mathbb{R}^4$ where M is a Calabi-Yau threefold. We will assume that the D-branes fill all of spacetime so that we realize an $\mathcal{N} = 1$, $d = 4$ gauge theory.

The internal configurations preserving four supersymmetries fall into two classes [6,7]: "A-type" branes wrapping special Lagrangian submanifolds of the threefold, and "B-type" branes which wrap holomorphic cycles of the Calabi-Yau. (The latter may have nontrivial holomorphic gauge bundles living on them as well, corresponding to bound states with lower-dimensional branes. We will for the most part ignore this possibility.) In the present discussion these exist in the type IIA and IIB theories respectively. We will discuss the associated gauge theories of each class in turn.

2.1. A-type branes

Spectrum

In the large-volume, large-complex-structure limit, supersymmetric A-type branes wrap special Lagrangian submanifolds. Let such a manifold Σ be described by a map

$$f : \Sigma \to M \ .$$

Recall that special Lagrangian submanifolds Σ are defined by the properties that:

$$\begin{aligned} \dim_{\mathbb{R}} \Sigma &= \frac{1}{2} \dim_{\mathbb{R}} M \\ f^*\omega &= 0 \\ f^*(\mathrm{Im}\, e^{i\theta}\Omega) &= 0 \end{aligned} \tag{2.1}$$

where ω is the Kähler form of M, Ω is the standard holomorphic $(3,0)$ form, and $e^{i\theta}$ is some phase.

N D6-branes wrapping a single supersymmetric cycle $\Sigma \subset M$ have a $U(N)$ vector multiplet arising from massless open string excitations polarized completely in $\mathbb{R}^4$. Massless open string excitations polarized in M form adjoint $U(N)$ chiral multiplets. We will focus on the case $N = 1$. To lowest order in α', the counting of massless chiral fields has been worked out. Σ lives in a family of deformations with dimension $b_1(\Sigma)$ [25] (*c.f.* also [26] for a clear discussion). More precisely, each basis vector in the tangent space to the space of deformations may be used to construct a nontrivial harmonic one-form on Σ, and vice-versa. Of course the space of such deformations (which has real dimension $b_1(\Sigma)$) cannot make up our set of chiral multiplets which are built from complex scalars: for example $b_1(\Sigma)$ need not be even. However, deformations of flat connections of the D6-brane gauge field on Σ also map one-to-one onto the space of harmonic one-forms on Σ, roughly because there is a Wilson line of the $U(1)$ gauge field around each 1-cycle. Thus for each element of the space $\mathcal{H}^1(\Sigma)$ of harmonic one-forms on Σ one has two real flat directions which may be described by a complex scalar [5]. In other words, we find $b_1(\Sigma)$ massless chiral multiplets, one for each non-trivial one-cycle or harmonic one-form on Σ.

Note that if we have branes wrapping several (mutually supersymmetric) 3-cycles, then we may get additional matter from any intersection points, in bifundamentals of the $U(1)$s of each cycle. A local example of this was discussed in [27]. In this work we will discuss branes wrapping single "primitive" 3-cycles: however, as explained in [27], interesting transitions to this more complicated case can occur as one varies background (closed string) hypermultiplets.

A natural choice of coordinates on the moduli space of the wrapped D6 brane is the following [23,28]. Let $\{\gamma_j\}$ be a basis for $\mathcal{H}_1(\Sigma)$. Choose minimal area discs D_j subject to the condition that

$$\partial D_j \;=\; \gamma_j \tag{2.2}$$

and let

$$w_j = \int_{D_j} \omega \ . \tag{2.3}$$

In other words, if there is a holomorphic disc in the relative homology class of D, then w_j will be the area. The w_j provide $b_1(\Sigma)$ real coordinates. They are complexified by $b_1(\Sigma)$ Wilson lines

$$a_j \ = \ \int_{\gamma_j} A \tag{2.4}$$

where A is the $U(1)$ gauge field on the wrapped brane. The coordinates in (2.3) and (2.4) are the real and imaginary parts of scalar components of the $b_1(\Sigma)$ chiral multiplets Φ_j on the brane:

$$\Phi_j = w_j + ia_j + \cdots \tag{2.5}$$

where $\cdots$ indicates higher components of the superfields.

Superpotentials and worldsheet instantons

As noted above, it follows from McLean's theorem [25] that at lowest order in α' the brane wrapping Σ has a moduli space of dimension $b_1(\Sigma)$. So far there may still be α' corrections which lift these flat directions. To lift moduli, one would need to generate either D-terms or F-terms in the low energy action; we will focus on the superpotential, since the chiral multiplets Φ_j are neutral (at least at generic points in the classical moduli space of the cycle) and do not appear in FI D-terms. At leading order in α', the superpotential $W(\Phi)$ identically vanishes. We now determine to what extent α' corrections of any sort are possible.

In fact, it turns out that there are no corrections to the open-string superpotential to any finite order in α': all contributions must come from nonperturbative corrections, arising from topologically nontrivial configurations. The arguments are almost identical to similar arguments for the heterotic string [11,12]. We will give two.

The first argument is a string theory argument. The (0)-picture vertex operator for a flat connection A at zero momentum on the D6-brane has the form:

$$V = \int_{\partial D} A_\mu(X) \partial_\alpha X^\mu d\sigma^\alpha \tag{2.6}$$

where X are coordinates on the brane and σ coordinates on the worldsheet D. Let A be polarized completely internally, so that it corresponds to a choice of Wilson lines around the elements of $H_1(\Sigma)$. If $X(\partial D)$ is a topologically trivial cycle on Σ, then A can be written as

an exact form $d\Lambda$. We see that V vanishes after an integration by parts. Thus topologically trivial disc amplitudes give no non-derivative couplings (such as superpotential terms) of the imaginary parts of chiral multiplets, to all orders in α'. Holomorphy thus requires the superpotential vanish to all finite orders in α'.

If the boundary maps to a topologically *non*-trivial cycle $\gamma_j \subset \Sigma$, this argument fails. Such discs are non-trivial elements of the relative homology class $H_2(M, \Sigma)$. These worldsheets will give terms weighted by the instanton action:

$$e^{-(w_j + ia_j)/\alpha'} \tag{2.7}$$

where w_j is the spacetime area of this disc as in (2.3), and a_j is its partner Wilson line (2.4). The α' dependence is decidedly non-perturbative. Note that to obtain the contribution (2.7) to the action, the map $X(\sigma)$ must be a *holomorphic* map from the disc to M with the desired boundary, and with the normal derivative to $X(\sigma)$ at the boundary taking values in the pullback of the normal bundle to Σ; this is a disc instanton. It follows from standard arguments that only such holomorphic maps have the correct zero mode count to contribute to a superpotential term in the spacetime theory.

The second argument is a spacetime argument. The space of $U(1)$ Wilson lines on a circle is the dual circle. Thus, any function of the chiral fields appearing in the effective action must be invariant under discrete shifts of their imaginary parts. Holomorphy then requires that the superpotential be a power series in $\exp(-[w_j + ia_j]/\alpha')$, which again gives a nonperturbative dependence on α'.

Examples of three-cycles

The best-known example arises in the Strominger-Yau-Zaslow formulation of mirror symmetry [5]: the claim is that any geometric Calabi-Yau with a geometric mirror can be written as a fibration of special Lagrangian T^3s. Mirror symmetry is fiberwise T-duality on these T^3s. D3-branes wrapping these fibers are mapped to D0-branes on the mirror. The T^3 has $b_1 = 3$ so all this is in accord with expectations: the mirror D0-brane and thus the wrapped D3-brane should have a 3-dimensional complex moduli space (which is the mirror threefold). Many examples of special Lagrangian three-cycles can be found as fixed loci of real structures. Some examples, which are homeomorphic to $\mathbb{RP}^3$, are contained in [6,20]. Note that these have a $\mathbb{Z}_2$ Wilson line degree of freedom as $\pi_1(\mathbb{RP}^3) = \mathbb{Z}_2$.

In addition, there has been some discussion of local and non-compact models. Ref. [28] contains some general discussion of noncompact supersymmetric three-cycles. A simple

example with an isolated disc instanton is the following. Take $z_{1,2,3}$ as coordinates on $\mathbb{C}^3$, and choose ω and Ω to be the obvious Kähler form and holomorphic three-form. Then the three-cycle Σ defined by

$$|z_1|^2 - t = |z_2|^2 = |z_3|^2 \tag{2.8}$$

$$Im(z_1 z_2 z_3) = 0, \quad Re(z_1 z_2 z_3) \geq 0 \tag{2.9}$$

with t positive is special Lagrangian, and diffeomorphic to $S^1 \times \mathbb{R}^2$. A generator γ of $H_1(\Sigma)$ is given by the concrete choice

$$\gamma : \{(t^{1/2} e^{i\theta}, 0, 0)\} \tag{2.10}$$

where θ runs from 0 to 2π. A holomorphic disc with boundary γ and area πt is given by

$$D_t : \{(z_1, 0, 0),\ |z_1|^2 \leq t)\} \tag{2.11}$$

2.2. B-type branes

In the large-volume, large-complex-structure limit, supersymmetric B-type branes wrap holomorphic cycles of M. One may also examine bound states which can be described as gauge bundles on the highest-dimensional branes (*c.f.* for example [20] for a discussion). For simplicity we will focus on branes wrapping primitive cycles, and in our examples we will discuss cases where $\mathcal{C}$ is a rational curve.

For N (space-filling) D-branes wrapping a given cycle $\mathcal{C} \subset M$ one again has a $U(N)$ vector multiplet arising from massless open strings polarized along the spacetime directions. Massless strings polarized along M give rise to adjoint chiral multiplets. Again we will focus on $N = 1$. For B-type branes wrapping $\mathcal{C}$ the infinitesimal supersymmetric deformations of the cycle are holomorphic sections of the normal bundle $\mathcal{N}_{\mathcal{C}}$. The number of such first-order deformations is therefore the dimension of the space of holomorphic sections, $H^0(\mathcal{C}, \mathcal{N}_{\mathcal{C}})$ (this is the cohomology group of the bundle $\mathcal{N}_{\mathcal{C}}$, not a relative cohomology group). These are the scalars in the massless chiral multiplets. There is no guarantee that these deformations do not have an obstruction at higher order.[4] Such obstructions, if they exist, correspond to elements of the group $H^1(\mathcal{C}, \mathcal{N}_{\mathcal{C}})$ [29]. More specifically, given an element of the cohomology group $H^0(\mathcal{C}, \mathcal{N}_{\mathcal{C}})$ one may try to construct a finite deformation

[4] For e.g. $\mathcal{C}$ a curve of genus $g \geq 1$, there are also $2g$ Wilson line degrees of freedom which parametrize the flat $U(1)$ bundles on $\mathcal{C}$. These pair up into g chiral multiplets and provide exactly flat directions. Similar comments apply if $\mathcal{C}$ is a four-cycle with $b_1(\mathcal{C}) \neq 0$.

by beginning with an infinitesimal deformation and constructing a finite deformation as a power series. H^1 measures the space of possible obstructions at each order in this series. Note that it may happen that although H^1 is nonvanishing, there is still a solution for this power series and thus a family of cycles. In the end, an obstruction should appear as a higher-order term in the superpotential for a brane wrapped around this cycle [20].

Furthermore, deforming the complex structure of M can also cause obstructions to (previously existing) deformations of $\mathcal{C}$. The basic statement is as follows (*c.f.* [30,31]). One may use the restriction map to $\mathcal{C}$ and the short exact sequence:

$$0 \to T_{\mathcal{C}} \to T_M|_{\mathcal{C}} \to \mathcal{N}_{\mathcal{C}} \to 0 \tag{2.12}$$

to write a map

$$r : H^1(M, T_M) \to H^1(\mathcal{C}, \mathcal{N}_{\mathcal{C}}) \ . \tag{2.13}$$

If we perturb the complex structure of M to first order by some element $\rho \in H^1(M, T_M)$, a deformation of $\mathcal{C}$ exists which preserves $\mathcal{C}$ as a holomorphic cycle if and only if $r(\rho) = 0$. Note that couplings of (open-string) chiral multiplets to (background closed-string) complex structure parameters in the superpotential are allowed and generic [20].

In the end, even counting these chiral multiplets is a harder problem on its face than for A-type branes, as the number of moduli depends not only on the intrinsic topology of the cycle but on the details of its embedding in M. (This is already apparent for rational curves in the quintic – *c.f.* [32,33].) Nonetheless, one may find a lot of specific examples for which computations are possible, especially for rational curves.

Some additional constraints exist as for A-type branes. First, the computation of the disc contribution to the superpotential can be reduced to a B-twisted open topological field theory calculation [20]. Again the Kähler parameters almost completely decouple from the superpotential: indeed, the computations of the dimension of $H^0(\mathcal{C}, \mathcal{N}_{\mathcal{C}})$ and of the obstruction depend completely on the complex structure. But in addition, all contributions to B-model computations come entirely from constant maps into the target space [34,22]. There are no worldsheet instanton corrections and tree level sigma model calculations will suffice: the superpotential can be deduced from classical geometry.

Examples of holomorphic curves and superpotentials

Many useful examples of holomorphic cycles exist in the literature. Several can be found or are referenced in [20]. We are particularly interested in two-cycles with nontrivial obstructed deformations.

The canonical example is simply a small resolution of the singular hypersurface in $\mathbb{C}^4$:

$$xy = z^2 - t^{2n} \tag{2.14}$$

(Such a small resolution is consistent with the Calabi-Yau condition as the space is non-compact.) If $n = 1$, then $H^0(\mathcal{C}, \mathcal{N}_{\mathcal{C}}) = 0$, and the curve is rigid. If $n > 1$, then $H^0(\mathcal{C}, \mathcal{N}_{\mathcal{C}})$ is one-dimensional — the normal bundle to this curve is $\mathcal{O}(0) \oplus \mathcal{O}(-2)$ — but there is an obstruction at nth order to deforming this curve [35]. This phenomenon can be described by a superpotential $W(\Phi) = \Phi^{n+1}$ [20].

It is easy to use $W(\Phi)$ to see the effect of a general deformation of complex structure on C. We perturb $W(\Phi)$ to

$$W_t(\Phi) = \Phi^{n+1} + tP(\Phi) + O(t^2), \tag{2.15}$$

where $P(\Phi)$ is an arbitrary polynomial in Φ subject only to the genericity condition $P'(0) \neq 0$. Solving $W_t'(\phi) = 0$, we get n solutions for the vev

$$\phi_k(t) = e^{2\pi ik/n} \left(-\frac{P'(0)}{n+1} \right)^{1/n} t^{1/n} + \ldots,$$

where the dots denote higher order terms in t. The geometric description of this perturbation of curves with normal bundle $\mathcal{O}(0) \oplus \mathcal{O}(-2)$ to n rigid rational curves was well known [36]. The geometric perturbation of contractible curves with normal bundle $\mathcal{O}(1) \oplus \mathcal{O}(-3)$ to rigid curves has recently been worked out in [37] and can be rephrased in terms of the perturbation of a superpotential if desired. The geometric description in the case of a general $\mathcal{O}(1) \oplus \mathcal{O}(-3)$ curve is not yet worked out, but the introduction of a superpotential can be expected to clarify the geometry.

Digression on holomorphic Chern-Simons theory

Another way to arrive at the superpotential in examples like (2.14), (2.15) is by studying a holomorphic analogue of the Chern-Simons action, discussed in [22,23].[5] In the

[5] We thank C. Vafa for pointing this out to us, and D. Diaconescu for related discussions.

following, we will suppress constants which enter harmlessly in our formulas. We think of the Calabi-Yau M as being obtained from the total space of the normal bundle $\mathcal{O}(0) \oplus \mathcal{O}(-2)$ by a modification of the complex structure. We choose holomorphic coordinates (z, z_0, z_1) on the normal bundle, with z being a coordinate on $\mathcal{C}$, z_0 being in $\mathcal{O}(0)$, and z_1 in $\mathcal{O}(-2)$. The curve $\mathcal{C}$ is identified with the zero section $z_0 = z_1 = 0$. The modification of complex structure is realized as usual by perturbing $\bar{\partial}$ by a tensor $A^i_{\bar{j}}$, i.e. $\bar{\partial}_{\bar{j}} \mapsto \bar{\partial}_{\bar{j}} + A^i_{\bar{j}}\partial_i$, where $A^i_{\bar{j}}$ is a TM valued $(0,1)$ form on M. We assume that the curve $\mathcal{C}$ remains holomorphic, and want to understand which deformations $z_i = \phi_i(z)$ $(i = 0, 1)$ remain holomorphic. The space of C^∞ deformations of $\mathcal{C}$ is identified with the space (ϕ_0, ϕ_1) of C^∞ sections of the normal bundle $\mathcal{N}_\mathcal{C}$. The relevant holomorphic Chern-Simons action is

$$\int_\mathcal{C} \left(\phi_0 \left(\bar{\partial} + A^i_{\bar{z}}\partial_i\right)\phi_1 - \phi_1 \left(\bar{\partial} + A^i_{\bar{z}}\partial_i\right)\phi_0\right). \tag{2.16}$$

Note that in (2.16) we only use the index $\bar{j} = \bar{z}$ in A. (2.16) expands as

$$\int_\mathcal{C} \phi_0\bar{\partial}\phi_1 + \phi_0 A^z_{\bar{z}}\partial\phi_1 + \phi_0 A^1_{\bar{z}} - \left(\phi_1\bar{\partial}\phi_0 + \phi_1 A^z_{\bar{z}}\partial\phi_0 + \phi_1 A^0_{\bar{z}}\right). \tag{2.17}$$

We make sense of this by respectively identifying ϕ_0, ϕ_1 with functions and $(1,0)$ forms on $\mathcal{C}$ (as would be expected in the twisted brane worldvolume theory [38]), while respectively identifying $A^0_{\bar{z}}, A^1_{\bar{z}}$ with $(0,1)$ and $(1,1)$ forms after pulling back to $\mathcal{C}$. Thus all the terms in (2.17) are $(1,1)$ forms on $\mathcal{C}$ and can be integrated.

The variations of (2.16) or (2.17) with respect to ϕ_0 and ϕ_1 give the conditions that the corresponding curve in X is holomorphic. In fact, the action of the topological theory on $\mathcal{C}$ actually becomes the superpotential in the four-dimensional $\mathcal{N} = 1$ theory arising from wrapping a D5 brane on $\mathcal{C}$. This is because the holomorphic Chern-Simons theory is the string field theory for the open string topological B-model [22], and therefore its action is the generating function of the topological correlation functions which give rise to superpotential terms in the physical theory.

To illustrate this fact, we now show that we can choose our tensor A so that (2.17) becomes $W(\Phi) = \Phi^{n+1}$. Since the obstructions to deforming $\mathcal{C}$ lie in $H^1(\mathcal{N}_\mathcal{C}) = H^1(\mathcal{O}(-2))$, we choose our A to have $A^1_{\bar{z}} = z_0^n dz \wedge d\bar{z}$ while the other $A^i_{\bar{z}}$ vanish (we can always choose such a gauge). Then the constant section $(\phi_0, \phi_1) = (t, 0)$ is holomorphic provided we put $t^n = 0$. So this A produces the required geometry.

The variation of (2.17) with respect to ϕ_1 shows that ϕ_0 is holomorphic. The variation of (2.17) with respect to ϕ_0 shows that $\bar{\partial}\phi_1$ is a multiple of ϕ_0^n. Substituting these back

into (2.17) (and performing the integral over the curve $\mathcal{C}$, which just produces a volume factor) gives a multiple of ϕ_0^{n+1}, as claimed. This proves that for any $\mathcal{O}(0) \oplus \mathcal{O}(-2)$ curve, the superpotential will be a polynomial of degree k for some k (or will vanish identically) – k is the only invariant of the complex structure in some neighborhood of the curve.

Another Example

Another example which we will use was detailed in Ref. [39] (see also sec. 9 of [31].) Here one has at a specific point in the complex structure moduli space an A_1 singularity fibered over a genus-g curve $\mathcal{S}$. At this point the collapsing cycles obviously form a family which is precisely $\mathcal{S}$. Deformations of the complex structure of M destroy this family, generically leaving $2g-2$ isolated curves. One may find $2g$ three-cycles by sweeping the collapsing curves over the one-cycles of $\mathcal{S}$, mapping $H_1(\mathcal{S})$ to $H_3(M)$.[6] This can be lifted to a map from $H^{(1,0)}(\mathcal{S})$ into $H^{(2,1)}(M) \simeq H^1(M, T_M)$ [39,31]. This gives g independent first-order deformations of complex structure.[7] We can use the map r (2.13) to project the relevant deformation onto $H^1(\mathcal{N})$ for each fiber of this collapsing surface. Now the spaces $H^1(\mathcal{N})$ are the fibers of a bundle over $\mathcal{S}$, and this bundle is identified with the canonical bundle of $\mathcal{S}$. So (2.13) gets included in the sequence of maps

$$H^{(1,0)}(\mathcal{S}) \to H^1(M, T_M) \to H^0(\mathcal{S}, K_{\mathcal{S}}). \tag{2.18}$$

This says two things. First of all, first order differentials on $\mathcal{S}$ lead to first order deformations of complex structure, realizing g deformations of complex structure. Second, upon perturbing by such a complex structure deformation, the only curves which survive the deformation are those which are located at zeros of the associated section of $K_{\mathcal{S}}$. Thus generally we will find a set of isolated curves with only massive chiral multiplets. However, at codimension one in the complex structure moduli space, zeros of the section of $K_{\mathcal{S}}$ will

[6] This is closely related to the formula for the superpotential in [40]. Fixing a point $s_0 \in \mathcal{S}$, then a path from s_0 to $s \in \mathcal{S}$ sweeps out a 3-chain in M, which can be integrated over a holomorphic 3-form, defining a function of s. If we define the potential this way in our context, there is a multiplicative ambiguity from the choice of holomorphic 3-form, reflected in the description in the main text by the choice of isomorphism $H^{(2,1)}(M) \simeq H^1(M, T_M)$.

[7] If M arises from Batyrev's construction of Calabi-Yau toric hypersurfaces by blowing up the curve $\mathcal{S}$ of A_1 singularities, these deformations of the complex structure of M are those which are not realizable by polynomial/toric deformations.

coincide and the resulting curve will have higher multiplicity: their deformations will be massless but obstructed at some higher, non-trivial order.

A superpotential which reflects this geometry be constructed as follows. Let the complex structure deformation be induced as above from an element $\omega \in H^{(1,0)}(S)$. At $\omega = 0$ the curve lives in a family which is precisely $\mathcal{S}$ so that a deformation of a curve at $z \in \mathcal{S}$ described by $\mathcal{N}$ can also be written as an element $\phi \in T_z^{(1,0)}\mathcal{S}$. One may then write the superpotential as:

$$W(\Phi;\omega) \;=\; \langle \omega, \Phi \rangle + \frac{1}{2!}\langle \partial\langle \omega, \Phi\rangle, \Phi\rangle + \frac{1}{3!}\langle \partial\langle \partial\langle \omega, \Phi\rangle, \Phi\rangle, \Phi\rangle + \cdots \tag{2.19}$$

Here Φ is the superfield associated to ϕ, ∂ is the the Dolbeault operator on $\mathcal{S}$ and $\langle , \rangle$ is the usual inner product between forms and vectors. It is understood that one is to evaluate the inner product at the point $p \in \mathcal{S}$ around which one is expanding, and convergence follows from the convergence of the power series representation of ω. For $\mathcal{S}$ of genus g, the expansion (2.19) can be truncated after $2g-1$ terms without changing the location and structure of the critical points. The closed string complex structure moduli act as parameters in the superpotential, through the choice of ω.

Let us explore the properties of (2.19) slightly more explicitly, to illustrate its features. Consider expanding (2.19) about some point on $\mathcal{S}$ where ω has an expansion in a local complex coordinate z

$$\omega \;\sim\; z^n dz. \tag{2.20}$$

We can represent the scalar field, which we are thinking of as a tangent vector to $\mathcal{S}$, as $\phi\frac{\partial}{\partial z}$ with ϕ complex. Then, expanding (2.19) around $z = 0$, we find

$$W(\Phi) \;\sim\; \Phi^{n+1}.$$

For $n = 0$ (i.e. around generic points on $\mathcal{S}$) there is no supersymmetric vacuum, while for $n > 0$ there are supersymmetric vacua. For $n = 1$, the vacuum is massive; for $n > 1$ there is a massless field, and the vacuum splits into $n-1$ massive vacua upon small perturbations of the complex structure of M (just as in the situation of (2.15)). For $\mathcal{S}$ of genus g, ω will generically have $2g-2$ isolated zeroes, giving rise to $2g-2$ massive supersymmetric vacua at generic points in the space of background closed string parameters. At various codimensions in the closed string moduli space, as one further specializes the multiplicities of the zeroes of ω, these $2g-2$ vacua merge in various combinations to yield theories with massless fields obstructed by higher order potentials.

A simpler way to write (2.19) locally on $\mathcal{S}$ is to write $\omega = df_\omega$ for a locally defined function on $\mathcal{S}$. Locally, such an f can be thought of as a function of ϕ. Then we simply have

$$W(\Phi, \omega) = f_\omega(\Phi).$$

While this formula is simpler in form than (2.19), it does not capture the global structure of the moduli space $\mathcal{S}$.

The above considerations are easily adapted to the more general situation considered in [39], where an A_N singularity is fibered over $\mathcal{S}$. If we denote the collapsing curve as $C_1 \cup \ldots \cup C_N$, then for each C_j we get g deformations of complex structure arising as in (2.18), yielding gN complex moduli. But we also have connected subsets $C_k \cup C_{k+1} \cup \ldots \cup C_{k+r}$ to which the above analysis applies. But since the first map in (2.18) depends linearly on the individual C_j, including these connected subsets does not give rise to any new complex structure deformations. So we get $N(N+1)/2$ superpotentials of the form (2.19) on $N(N+1)/2$ copies of $\mathcal{S}$, each of which depends on the gN complex moduli (only g of which appear in any one superpotential). Each of these superpotentials controls the obstructions to deforming curves of the form $C_k \cup C_{k+1} \cup \ldots \cup C_{k+r}$, and $2g-2$ curves of this type survive a generic deformation of complex structure.

3. Disc instantons

Type II string theory in the presence of a D-brane on a given special Lagrangian submanifold has the same net number of worldsheet (and spacetime) supersymmetries as a heterotic $(0,2)$ model; and as with heterotic (0,2) models, the nonrenormalization theorem for the spacetime superpotential is spoiled by worldsheet instanton effects. In light of results for $(0,2)$ models [12], it is fair to ask whether the generic D6-brane configuration is nonperturbatively stable.

We expect direct calculations of instanton effects to be difficult. But instantons in supersymmetric theories generate fermion zero modes which provide selection rules for CFT correlators. Using the rule of thumb that allowed terms are generic, we will see that three-cycles with an isolated disc instanton are destabilized nonperturbatively.[8] The argument is quite similar to that for heterotic $(0,2)$ models.

[8] Holomorphic discs ending on special Lagrangian cycles of Calabi-Yau threefolds are generically isolated [22].

The easiest way to count the zero modes for an isolated holomorphic disc is to begin with the amplitude for the sphere and get the disc by orbifolding with respect to a real involution, which will cut the number of zero modes in half. For an isolated sphere, the superconformal symmetry together with an index theorem shows that there are four holomorphic zero modes and four antiholomorphic zero modes [12,41], so we expect four fermion zero modes on the disc.

Consider a single D6-brane wrapping a special Lagrangian three-cycle Σ. The complex modulus $\phi = w + ia$ is associated with a cycle $\gamma \in H_1(\Sigma)$, using the notation and definitions of §2.1. Here we assume the isolated instanton corresponds to a disc D such that $\partial D = \gamma$ and D has minimal area. The most obvious, lowest-order term consistent with our perturbative nonrenormalization theorem is the exponential

$$W(\Phi) = e^{-\Phi/\alpha'} \tag{3.1}$$

where Φ is the superfield corresponding to ϕ. This will clearly destabilize the wrapped D6-brane, at least locally.

We will search for the superpotential (3.1) by examining small fluctuations Φ_j away from the above classical configuration $\Phi_0 = \phi$. Here j is an index in $H_1(\Sigma)$. The lowest-order terms directly computable via a CFT correlator will be those arising from the cubic term

$$\Phi_i \Phi_j \Phi_k e^{-\phi/\alpha'} \ .$$

We will focus on the term

$$S_{\text{cubic}} = C \int d^4x \phi_i \phi_j F_k \tag{3.2}$$

where F_k is the auxiliary field in Φ_k. Note that in the reduction to four dimensions, the operators above are arrived at by contour integrals in Σ, so C is proportional to a triple integral.

The vertex operators which enter in the calculation of (3.2) are easily presented in the covariant RNS formalism [42] (*c.f.* [20] for a general discussion of the CFT calculation of open-string superpotential terms). The (-1)-picture zero-momentum vertex operator for the scalar component ϕ_j is:

$$V_\phi^{(-1),j} = \theta_\mu^j(X) \psi^\mu e^{-\tilde{\phi}} \tag{3.3}$$

where $\tilde{\phi}$ is the bosonized superconformal ghost [42], θ^j_μ is the harmonic one-form (associated to γ_j) on the 3-cycle, and ψ^μ is a fermion with Dirichlet boundary conditions. The (0)-picture vertex operator for the auxiliary field is [43]:

$$V_F^{(0),j} = \Omega_{\rho\mu\nu}(X)\theta^j_\sigma g^{\sigma\rho}\psi^\mu\psi^\nu \ . \tag{3.4}$$

Here Ω is the the $(3,0)$ form, with the coordinates (but not the indices) restricted to Σ. Equation (3.4) is obtained by applying the unit spectral flow operator $\Omega_{\mu\nu\rho}\psi^\mu\psi^\nu\psi^\rho$ as in [43,10].

The three-point function

$$\langle V_\phi^{(0),i} V_\phi^{(-1),j} V_F^{(-1),k}\rangle \tag{3.5}$$

has the correct fermion and ghost number to be nonvanishing; in an instanton background, the four fermions in the vertex operators in (3.5) can soak up the relevant zero modes. Note that we are computing the integrand of the triple integral defining C in Eq. (3.2). Since holomorphic maps will preserve the order of marked points on the boundary, the ordering of (3.5) will be fixed for a given set of positions in this integrand.

This superpotential term can equivalently be computed as a correlator in the topological A-model open string theory [22,20]. Here one is computing the contribution to (3.5) (or more familiarly, a Yukawa coupling related to (3.5) by supersymmetry) in a sector where the map of the worldsheet to spacetime is a disc whose boundary $\gamma \subset \Sigma$ is topologically nontrivial. The path integral localizes onto the space of holomorphic maps, and the contribution is

$$\oint_\gamma \oint_\gamma \oint_\gamma dx_1 dx_2 dx_3 A_i(x_1)A_j(x_2)A_k(x_3) \tag{3.6}$$

(suppressed by the exponential of the area of the holomorphic disc), where the gauge fields A_i can be identified with the 1-forms θ^i in (3.3). Once again, for given positions in the integrand, the ordering of the vertex operators for $A_{i,j,k}$ must match the ordering of $x_{1,2,3}$ respectively.

The result is that the superpotential (3.1) is generic for an isolated instanton. With some interpretation added, this statement matches a calculation in ref. [22]. There it is shown that the string field theory for the topological open string A-model is equivalent to Chern-Simons theory on Σ with instanton corrections to the action. This instanton correction can be interpreted as precisely the superpotential we have calculated, as it generates the topological correlator we have discussed. Note that in [22], the dependence

on the area of the disc was added as a convergence factor, whereas in our discussion it is required by spacetime supersymmetry.

The topological string theory representation of the superpotential allows us to write the full worldsheet instanton contribution to the CFT correlator (3.5). First, note that while we have discussed A_i as a harmonic form, we can modify it by adding a BRST-trivial piece to give it support only in an arbitrarily small neighborhood around a two-cycle β_i which is Poincaré dual to γ_i. The result is as follows. Denote by $d^{\{n_a\}}_{\{m_l\}}(i,j,k)$ the number of holomorphic maps from a disc to M where the image $D \subset M$ has the following properties:[9]

i) $[\partial D] = \sum_l m_l \gamma_l$.

ii) The vertex operators $V^{i,j,k}$ are mapped in cyclic order to intersections of $\gamma = \partial D$ with $\beta_{i,j,k}$ respectively.[10]

iii) $D - \sum_l m_l D_l$, which is a closed two cycle in M, is in the homology class $\sum_a n_a K_a$.

Then, the three-point function receives a contribution

$$\begin{aligned}\langle V_F^i V_\phi^j V_\phi^k \rangle \sim \sum_{m_l, n_a \geq 0} & \left(\int_{\partial D} \theta^i\right) \left(\int_{\partial D} \theta^j\right) \left(\int_{\partial D} \theta^k\right) \times \\ & \times d^{\{n_a\}}_{\{m_l\}}(i,j,k) \prod_{l=1}^{b^1(\Sigma)} e^{-m_l(w_l + i a_l)/\alpha'} \prod_{a=1}^{h^{1,1}(M)} e^{-n_a t_a}\end{aligned} \tag{3.7}$$

from disc instantons, where t_a denotes the integral of the Kähler form over K_a (and for simplicity we are setting the closed string background B-field to zero). Although we have mostly used the language of the topological theory in deriving this result, it also holds for the three-point function in the physical theory.

The same kind of instanton sum also appears in [23], where the interpretation in terms of a superpotential for wrapped branes (and in particular the fact that these effects serve to obstruct the deformations of branes wrapped on special Lagrangian cycles) was not discussed.

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[9] As with "numbers" of rational curves in mirror symmetry, the correct notion of d when there are families of discs and/or including multiple covers would require much further discussion; we will be content here to be schematic. A proposal for the multiple cover contribution has recently been worked out by H. Ooguri and C. Vafa [44].

[10] One has to be careful if two vertex operators correspond to the same cycle. The support of A can be made arbitrarily small but finite. In this way nonzero contributions still generically come from the vertex operator insertions mapping to different points in γ.

Coupling to closed string background fields

It is clear from the form of the three-point functions (3.7) that the superpotential depends on the closed string background Kähler moduli, which enter through the worldsheet instanton action e^{-t_a}. The dependence of the disc instanton generated superpotential on Kähler moduli, and the fact that it does *not* depend on the background complex structure moduli in the IIA theory, is consistent with the nonrenormalization result of [20].

We can directly probe the dependence of the superpotential on closed string moduli by computing the CFT correlator

$$\langle V_K^{(-1,-1),a} V_F^{(0),j} \rangle \tag{3.8}$$

where $V_K^{(-1,-1),a}$ is the vertex operator for a closed string Kähler deformation. Again, the vertex operators in (3.8) can absorb the fermion zero modes which are present in an instanton background. In fact, the "mirror" couplings of open strings to background complex moduli in the superpotential generically exist at tree level in the B-model [20] – this is clear from the examples of §2.2, where a small perturbation of complex structure can obstruct families of holomorphic curves. The couplings (3.8) must then similarly exist, but due to Peccei-Quinn symmetries they should arise at the non-perturbative level in both the closed and open string worldsheet instanton expansions.

V_K^a represents an integral (1,1) form ω_a which could be used to perturb the Kähler form of M. We can choose ω_a to have support only infinitesimally near the four-cycle $L^a \subset M$ Poincare dual to K_a. Then, (3.8) has the expansion

$$\begin{aligned} \langle V_K^{(-1,-1),a} V_F^{(0),j} \rangle \sim & \sum_{m_l, n_b \geq 0} \left(\int_D \omega_a \right) \left(\int_{\partial D} \theta^j \right) \times \\ & \times d^{\{n_b\}}_{\{m_l\}}(a,j) \prod_{l=1}^{b^1(\Sigma)} e^{-m_l(w_l + i a_l)/\alpha'} \prod_{b=1}^{h^{1,1}(M)} e^{-n_b t_b} \end{aligned} \tag{3.9}$$

where $d^{\{n_b\}}_{\{m_l\}}(a,j)$ counts the number of holomorphic maps to discs $D \subset M$ which pass through L^a at the insertion point of V_K^a and β_j at the insertion point of V_F^j, and which in addition have $[\partial D] = \sum_l m_l \gamma_l$ and $[D - \sum_l m_l D_l] = \sum_b n_b K_b$.

4. Discussion

Space-filling D-branes wrapping supersymmetric cycles in Calabi-Yau manifolds provide one of the most natural classes of $\mathcal{N} = 1$ supersymmetric models in string theory, and are attractive as concrete realizations of "brane world" scenarios. In this paper, we have shown that the theories which arise from D6 branes wrapping supersymmetric three-cycles are in many ways analogous to heterotic (0,2) models. In particular, although they are supersymmetric to all orders in α', nonperturbative worldsheet effects can generate superpotentials and, perhaps, break supersymmetry.

These models differ from heterotic theories, however, in that mirror symmetry provides a dual description where the non-perturbative superpotential is computable at tree level in sigma model perturbation theory. This should be a powerful tool: most known dualities of $\mathcal{N} = 1$ models, like heterotic/F-theory duality, relate instanton computations to other instanton computations (with worldsheet instantons mapping to euclidean wrapped branes of various sorts [17]). The present situation is considerably rosier, and it will be very interesting to exploit this to sum up instantons in this class of $\mathcal{N} = 1$ string vacua.

The cases discussed in §2.2 (on the B-model side) should provide ideal test cases. In each case, one can realize (on a 5-brane wrapping a holomorphic curve) a theory with massless chiral fields, constrained by a higher order superpotential. The mirror D6 theory should provide us with an example of a brane wrapping a supersymmetric three-cycle Σ with $b_1(\Sigma) > 0$, but without a moduli space of the expected dimension. By the nonrenormalization theorem of §2.1, the moduli space on the A-model side must be lifted by a disc instanton generated superpotential. Work to explicitly construct the mirror cycles, and compute the relevant superpotentials, is under way [21]. Note that nonperturbative superpotentials which obstruct deformations of branes wrapped on three-cycles can resolve the puzzle for mirror symmetry raised by Thomas in [45]. On the other hand, we expect e.g. the supersymmetric T^3 used in [5] to derive mirror symmetry will survive instanton corrections. In the mirror picture this is obvious (since deformations of a point are unobstructed), and in the direct analysis presumably any holomorphic discs with boundary on the T^3 would come in families and cancel in their contribution to the superpotential.

In the regime where there are "small" holomorphic discs, new interesting phenomena should also occur. For instance, there are arguments in the mathematics literature that in some cases the classical moduli spaces of special Lagrangian three-cycles will be manifolds with boundary (see §5 of [28]). This cannot be the case for physical applications of the

sort we have discussed, involving wrapped branes in string theory. The moduli space (including Wilson line degrees of freedom) is that of a 4d $\mathcal{N} = 1$ supersymmetric D-brane field theory. Assuming supersymmetry isn't broken, the quantum moduli space of supersymmetric ground states must be a Kähler manifold; there is no known dynamics that can create boundaries at codimension one in the moduli space of 4d $\mathcal{N} = 1$ supersymmetric theories. The argument of [28] involves the fact that a holomorphic disc with boundary in the three-cycle is becoming very small; therefore, it is likely that some analogue of the phenomena discussed in [14] is occurring. Just as one can use theta angles to go around the boundaries of the classical Kähler cone and find intrinsically stringy Landau-Ginzburg phases of Calabi-Yau compactifications, it seems likely that one can use Wilson lines to go around the would-be boundary of moduli space discussed in [28] and find new, "quantum" supersymmetric three-cycles.

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