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The Paradox of Charmonium Production

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The Paradox of Charmonium Production

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Abstract

The CDF preliminary analysis on polarized charmonium production at moderate transverse momentum, $p_T \sim 4 - 20$ GeV, severely challenges the color octet model (COM), which predicts quarkonium to be transversely polarized with increasing p_T. Based on this data, we analyze the compatibility of the Tevatron and the photoproduction at HERA in the context of the COM. Due to the uncertainty on the extraction of non-relativistic QCD (NRQCD) matrix elements and a lack of complete next-to-leading order calculations, one cannot completely rule out the COM. Nonetheless, both collider experiments seem to push the input matrix elements to opposite directions, and the puzzle of quarkonium polarization remains unsolved.

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I. INTRODUCTION

The simplest mechanism based on perturbative QCD to explain quarkonium production, the color singlet model (CSM) [1], is not able to describe charmonium hadroproduction. This model underestimates J/ψ production, both in the central [2,3] and forward [4] rapidity regions. The data show that the bound state of heavy-quark pair is produced not only in the color singlet configuration, but there are additional states contributing to the final colorless vector meson. Based on the non-relativistic QCD model (NRQCD) [5], quarkonium production is understood as two-step phenomenon: $c\bar{c}$ pair production at perturbative level and the subsequent evolution to colorless vector meson through soft gluon emission at the non-perturbative domain. This argument is supported by the fact that the $c\bar{c}$ pair is produced at distance $1/m_Q$, m_Q standing for heavy quark mass, which is much smaller than $1/\Lambda_{QCD}$, the typical QCD scale for bound-state system. According to the color octet model (COM) formulation [6], a generic S-wave quarkonium state is described by the Fock state decomposition, schematically given by

$$|\psi_Q\rangle = \mathcal{O}(1)|Q\bar{Q}[^3S_1^{(1)}]\rangle + \mathcal{O}(v)|Q\bar{Q}[^3P_J^{(8)}]g\rangle + \mathcal{O}(v^2)|Q\bar{Q}[^1S_0^{(8)}]g\rangle + \mathcal{O}(v^2)|Q\bar{Q}[^3S_1^{(8)}]gg\rangle + ... \quad (1)$$

where v is a typical velocity of the heavy-quark pair. We use the usual spectroscopic notation $^{2S+1}L_J$, and the color state is indicated by (1) for singlet and (8) for the octet.

In the first approximation, the $Q\bar{Q}$ system is produced in a color singlet state, which already has the quantum numbers of the physical quarkonium. The octet contributions are suppressed by powers of v and α_s. The latter is due to the extra soft gluon radiation needed to produce the correct color and/or quantum numbers. In principle, the state $^3P_J^{(8)}$ can produce χ_J mesons or evolve nonperturbatively to a vector meson.

At the partonic level, the inclusive ψ (generically denoting the charmonium J/ψ and $\psi(2S)$) is given by

$$d\hat{\sigma}(a+b\to\psi+X) = \sum_n d\hat{\sigma}_n(a+b\to c\bar{c}[n]+X)\langle\mathcal{O}_n^\psi\rangle, \quad (2)$$

where $c\bar{c}[n]$ stands for the quark-pair in the generic state n. We denote $\hat{\sigma}_n$ as the cross section for the short distance c-pair production, which can be calculated perturbatively. The matrix elements of the transition $c\bar{c}[n]\to\psi$, $\langle\mathcal{O}_n^\psi\rangle$, cannot be calculated in the usual perturbation theory. Fortunately, they are assumed to be universal, and can be extracted from experiments.

Of course, in principle one could argue that the dominant long-distance matrix element should be $\langle^3S_1^{(1),\psi}\rangle$, the $c\bar{c}$ state already with the correct quantum number and color of vector meson. However, as already stated, from the Fermilab experiments the CSM itself cannot explain the transverse momentum p_T of the inclusive reaction $p\bar{p}\to\psi X$. The CSM differential cross section behaves like $d\sigma/dp_T \sim 1/p_T^6$, falling much faster than the data. The p_T dependence can be fixed combining the octet states $^3S_1^{(8)}$, $^1S_0$, ^3P_J, which is order of $m_c^3v_c^3$ according to NRQCD expansion. Particularly, the $^3S_1^{(8)}$ is fundamental for explaining a harder p_T spectrum. If nature favors the vector S-wave octet state to evolve to a vector meson, there is a strong consequence on quarkonium production. Because the $c\bar{c}$ bound-state

is originated from gluon jet, the COM predicts quarkonium to be transverse polarized on the limit $4m_c^2/p_T^2 \ll 1$ [7,8]. Indeed, it has been shown that in this limit, the charmonium fragmentation function $D_{g\to\psi}$ [9] could be recovered [6].

The ${}^3S_1^{(8)}$ plays the major role on the explanation of charmonium data at Tevatron, however the same is not true at HERA for $z > 0.2$, where $z = p_\psi \cdot p_p / p_\gamma \cdot p_p$. In this kinematic region, the COM predictions for photoproduction [10,11] are dominated by the states ${}^1S_0$ and ${}^3P_J$. Due to the universality, the values of these matrix elements extracted from CDF data should reproduce HERA data. However, the COM predicts an excess of events compared with HERA data [12,13] for $z \to 1$. This discrepancy could be explained by the higher-order QCD effects [14] or by the intrinsic transverse momentum of the partons [15]. Nonetheless, it seems these tentative solutions cannot reproduce quite well not only z, but other relevant kinematic distributions [16].

In the following, we make a quantitative study of the COM in the light of the experimental data and we show that is quite difficult to accommodate the J/ψ production at Tevatron and HERA, simultaneously. Even with the introduction of higher order QCD corrections, the COM will face another challenge: the interpretation of charmonium polarization. The preliminary CDF analysis [17] is pointing to unpolarized ψ production, contradicting the COM predictions.

Our strategy is at follows. We extract independently the non-perturbative matrix elements from both Tevatron and HERA data. After determining the solution that could satisfies both data, we show that actually it is incompatible to the polarization data.

II. HADRO AND PHOTOPRODUCTION IN THE COLOR OCTET MODEL

At the Tevatron, the inclusive ψ production cross section can be written as the usual form,

$$d\sigma^\lambda_{p\bar{p}\to\psi X}(s) = \int f_{a/p}(x_a) f_{b/\bar{p}}(x_b)\, \hat{\sigma}^\lambda_{ab\to\psi X}(\hat{s}), \tag{3}$$

where the $\hat{\sigma}$ is given by Eq. (2) and at perturbative level

$$\frac{d\hat{\sigma}^{(\lambda)}_{ab\to c\bar{c}X}[n]}{d\hat{t}} = A_{ab}[n] + B_{ab}[n][\epsilon(\lambda)\cdot k_a]^2 + C_{ab}[n][\epsilon(\lambda)\cdot k_b]^2 + D_{ab}[n][\epsilon(\lambda)\cdot k_a][\epsilon(\lambda)\cdot k_b], \tag{4}$$

where k_a and k_b are the momenta of the initial partons a and b and $\epsilon(\lambda)$ the polarization vector of ψ. The complete analytic expression can be found in [8,11]. The sum over λ yields the unpolarized cross section.

Since we want to detect a vector meson, the lowest order at the partonic level should be $2 \to 2$ process. At the Tevatron, the most important contribution comes from the subprocess

$$g\ g \to c\bar{c}[n]\ g, \tag{5}$$

although gq and $q\bar{q}$ bring some contribution, especially for high p_T.

At HERA, there are two types of mechanisms contributing to distinct regions of z. Direct ψ photoproduction, given by the partonic level subprocesses

$$\gamma^* \; g \to c\bar{c}[n] \; g \tag{6}$$
$$\gamma^* \; q(\bar{q}) \to c\bar{c}[n] \; q(\bar{q}), \tag{7}$$

which are important for $z > 0.2$, and by resolved photon mechanism through the partonic content of the γ^*, dominant for small z. All the relevant analytic expressions are listed in [11].

The complete $ep \to \psi X$ can be written as

$$d\sigma_{ep\to eJ/\psi X}(s) = \Gamma(Q^2, y) \; d\sigma_{\gamma^* p\to J/\psi X}(W^2), \tag{8}$$

where

$$\Gamma(Q^2, y) = \frac{\alpha}{2\pi y Q^2}[1 + (1-y^2)^2]. \tag{9}$$

The $d\sigma(W^2)$ can be related to the partonic cross section for the resolved photon process according to

$$d\sigma_{\gamma^* p\to J/\psi X}(W^2) = \int f_{i/\gamma}(x_i) f_{b/p}(x_b) \; d\hat{\sigma}_{ib\to\psi X}(\hat{s}). \tag{10}$$

For $i = \gamma$, $f_{i/\gamma}(x_i) = \delta(1 - x_i)$ reproduces the expression for direct production.

III. DISCUSSIONS AND CONCLUSIONS

We have performed a numerical calculation of the charmonium production cross section fixing $m_c = 1.5$ GeV and choosing the renormalization and factorization scale to be $\mu = \sqrt{p_{T\psi}^2 + m_\psi^2}$, where $m_\psi = 2m_c$. We make our analysis for three different parton distribution functions (PDF's) in the proton; MRS (R2) [18], CTEQ 4L [19], and GRV 94 LO [20]. For the resolved photons, we use GRV distribution function [21]. As the shape of $c\bar{c}[^1S_0^{(8)}]$ and $c\bar{c}[^3P_J^{(8)}]$ are almost identical, we use the usual combination $\langle {}^1S_0^{(8)}\rangle + \frac{k}{m_c^2}\langle {}^3P_J^{(8)}\rangle \equiv M_k$, fixing $k = 3$.

Evidently a careful analysis on μ, as well as m_c dependence would bring to better control of the theoretical uncertainties. Overall, even with an uncertainty with factor two, our conclusions still remain valid. Nevertheless, we consider a case with $m_c = 1.3$ GeV for a more complete check.

As we point out in the introduction, we extract the non-perturbative matrix elements independently, for both Tevatron and HERA. From the Fig. 1 we can see that the COM can accommodate quite well the CDF central ($|\eta_\psi| < 0.6$) direct J/ψ production data [2]. We should emphasize that the same set of matrix elements brings to an extraordinary agreement with the D0 forward ($2.5 < |\eta_\psi| < 3.7$) J/ψ production data [4]. However, this is not a surprise, once we only fit these free universal matrix elements without any constraint.

In the Fig. 2 (3) we show the z (y^*, the rapidity of J/ψ in the $\gamma^* p$ center-of-mass frame) distribution for J/ψ production at HERA, and once again, it seems COM can in principle fit well the H1 data [16].

In the Table I we collect the best set of the color octet NRQCD matrix elements that fit both data set independently for the three PDF's we are considering. For the color singlet

contribution, we have used $\langle {}^3S_1^{(1)} \rangle = 1.2$ GeV^3, following [6]. Since MRS (R2) is calculated at next to leading order, it is not surprising that it gives a different result compared to the leading order (LO) ones. Our numbers confirm the early results pointing out that at LO the COM has trouble explaining both data simultaneously.

As we mention in the Introduction, this anomaly may be cured by adding corrections due to the intrinsic transverse momentum of the partons [15] or higher-order (HO) QCD effects [14]. For small values of p_T, the multiple-gluon radiations from the initial and the final state at the Tevatron become sizeable. In the [22] these corrections were estimated by Monte Carlo simulation using PYTHIA [23] and the HO QCD could be parameterized as a K factor dependent on p_T [14]. In fact, such corrections produce

$$\begin{aligned} \langle {}^3S_1^{(8)} \rangle &= (0.47 \pm 0.09) \ \times 10^{-2} \text{GeV}^3 \\ M_3 &= (0.63 \pm 0.34) \ \times 10^{-2} \text{GeV}^3 \end{aligned}$$

for MRS (R2) PDF. This lower value of M_3 brings to a better agreement with the one extracted from HERA experiments.

In the Fig. 4 we display a parameter space for the color-octet NRQCD matrix elements. Although at 68% C.L. we still observe discrepancy between the bound for Tevatron HO QCD corrections and HERA, this picture changes dramatically at 95% C.L. We see that HERA favors much higher values for $\langle {}^3S_1^{(8)} \rangle$ than Tevatron, however they are not severely constrained. The reason is that the state ${}^3S_1^{(8)}$ contributes only through resolved photon processes in a region where $z < 0.4$, roughly speaking. In this intermediate region, the color-singlet contribution has a major role. Actually, this is clear if we consider a different c-quark mass value. For $m_c = 1.3$ GeV, the state ${}^3S_1^{(1)}$ has a bigger contribution, much closer to the experimental data. This means the CSM itself could describe the HERA data reasonable well, except in the region $z \to 1$.

The main conclusion from the Fig. 4 is that the introduction of HO QCD corrections brings a match between COM predictions at HERA and Tevatron, as already pointed in [14].

With the extraction of the NRQCD matrix elements, we are now in the position to discuss the implication of these results on the charmonium polarization predicted by the COM.

The quarkonium polarization can be measured from the angular dependence on $\psi \to \mu^+\mu^-$,

$$\frac{d\Gamma}{d\cos\theta} \propto 1 + \alpha \cos^2\theta, \tag{11}$$

with $\alpha = (1-3\xi)/(1+\xi)$, $\xi \equiv d\sigma^{\lambda=0} / \sum_\lambda d\sigma^\lambda$.

From the expressions in [11], it is possible to calculate the quarkonium cross section for each polarization through the Eqs. (3) and (4). Writing the polarization vector of quarkonium in the recoil (s-channel helicity) frame [24], we found the p_T dependence on α, as displaced in the Fig. 5. The polarizations were calculated for seven p_T bins, specified in [17].

At this point, we should be careful in comparing our results with the CDF preliminary analysis, once the data contain feed-down from χ_c, which contributes to $\sim 35\%$, and $\psi(2S)$ decay to J/ψ.

In order to avoid these extra contributions to the prompt charmonium production, we also performed analysis on $\psi(2S)$ productions, which do not receive feed-down contributions, neither decay of the higher states. Although from the theoretical point of view the $\psi(2S)$ state is simpler to analyze, the available data [2] are more limited statistically.

We extracted the NRQCD matrix elements in a similar way we have done for J/ψ. For the MRS (R2) PDF, we found $\langle {}^3S_1^{(8)} \rangle = (0.14 \pm 0.03) \times 10^{-2}$ GeV^3 and very small value for M_3, compatible with zero. Following [17], we calculated the $\psi(2S)$ polarization for three p_T bins, and once again the charmonium was found to be transverse polarized, as we can see from the Fig. 6.

The HO QCD corrections, that worked well to solve the Tevatron/HERA discrepancy, actually worsen the already poor LO predictions, as the Fig. 5 indicates.

In order to satisfy polarization data, the $\langle {}^3S_1^{(8)} \rangle$ contribution must vanish, since the M_3 brings to a almost unpolarized ψ production. Of course, there is a penalty doing just adjustment by hand. The nice fit, *e.g.*, Fig. 1 is no longer held. Besides, the M_3 value should be increased in order to have a better fit. From the Fig. 4, we see that this scenario will be disastrous if we compare with HERA bounds.

Although there are strong evidences that COM is not working well, before ruling out this model, we should investigate the possible solutions to solve the paradox of charmonium production:

- The complete QCD higher order corrections, which is not available so far, and contributions from higher $c\bar{c}$ states could in principle bring to a drastic change in the scenario.

- The emitted gluons, in order to produce the physical quarkonium, are not so soft. Therefore, the polarization of the $c\bar{c}$ system is not conserved during the evolution to non-perturbative regime.

On the other hand, although it is a strong statement, we could argue that the evolution of $Q\bar{Q}$ system to a physical vector boson is not well understood; the splitting between perturbative and non-perturbative regimes cannot be done trivially. This means that NRQCD is not appropriate to describe quarkonium production. Actually, if we remember that for the charmonium the perturbative expansion is based on $\mathcal{O}(m_c v_c)$, maybe m_c is not sufficiently small to allow such expansion. A closer examination on bottomonium states will be crucial to check if this state is held or not.

Although is still early to make any strong conclusions, it seems that the COM is once again in trouble. At least in leading order cannot explain simultaneously the Tevatron and HERA data. The existing solution, the implementation of HO QCD corrections, worsen the strong prediction of this model, the production of transverse polarized quarkonium.

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TABLES

	MRS (R2)	CTEQ 4L	GRV 94 LO
$\langle {}^3S_1^{(8)} \rangle$	0.70 ± 0.17 (45 ± 29)	0.54 ± 0.12 (25 ± 22)	0.57 ± 0.12 (24 ± 22)
$\langle {}^1S_0^{(8)} \rangle + \frac{3}{m_c^2} \langle {}^3P_J^{(8)} \rangle$	4.85 ± 0.95 (0.39 ± 0.18)	2.28 ± 0.55 (0.29 ± 0.14)	2.07 ± 0.53 (0.30 ± 0.14)

TABLE I. Leading-order Color-octet NRQCD matrix elements in units of 10^{-2} GeV^3 for direct J/ψ production at Tevatron (HERA).

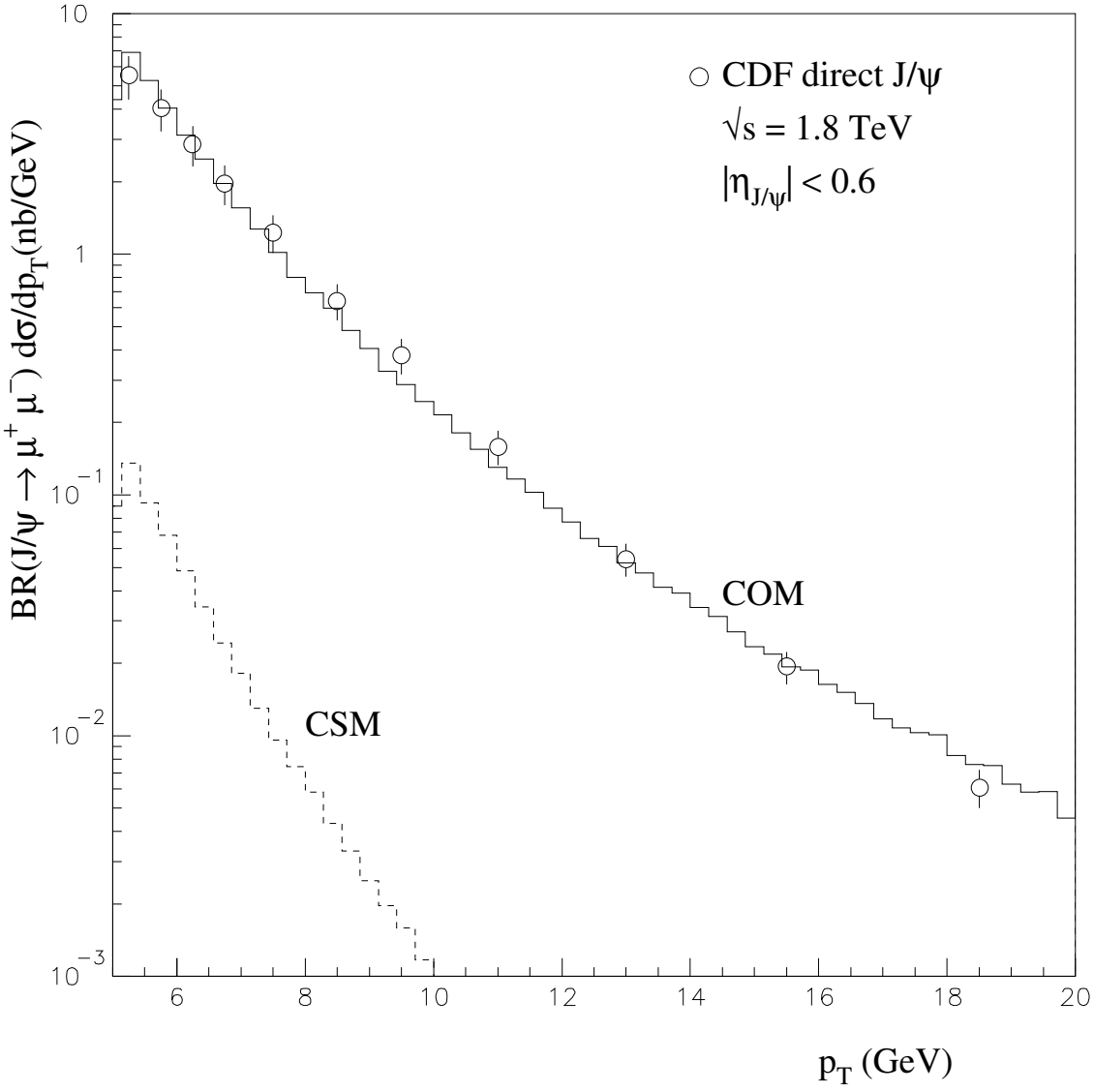

FIG. 1. The p_T distribution data (circles) for direct forward J/ψ production ($|\eta_{J/\psi}| < 0.6$) from the CDF Collaboration at $\sqrt{s} = 1.8$ TeV. The solid curve represents the COM prediction after choosing the values for the NRQCD matrix elements given in Table I for the CTEQ 4L parton distribution function. The dashed curve shows the color singlet contribution.

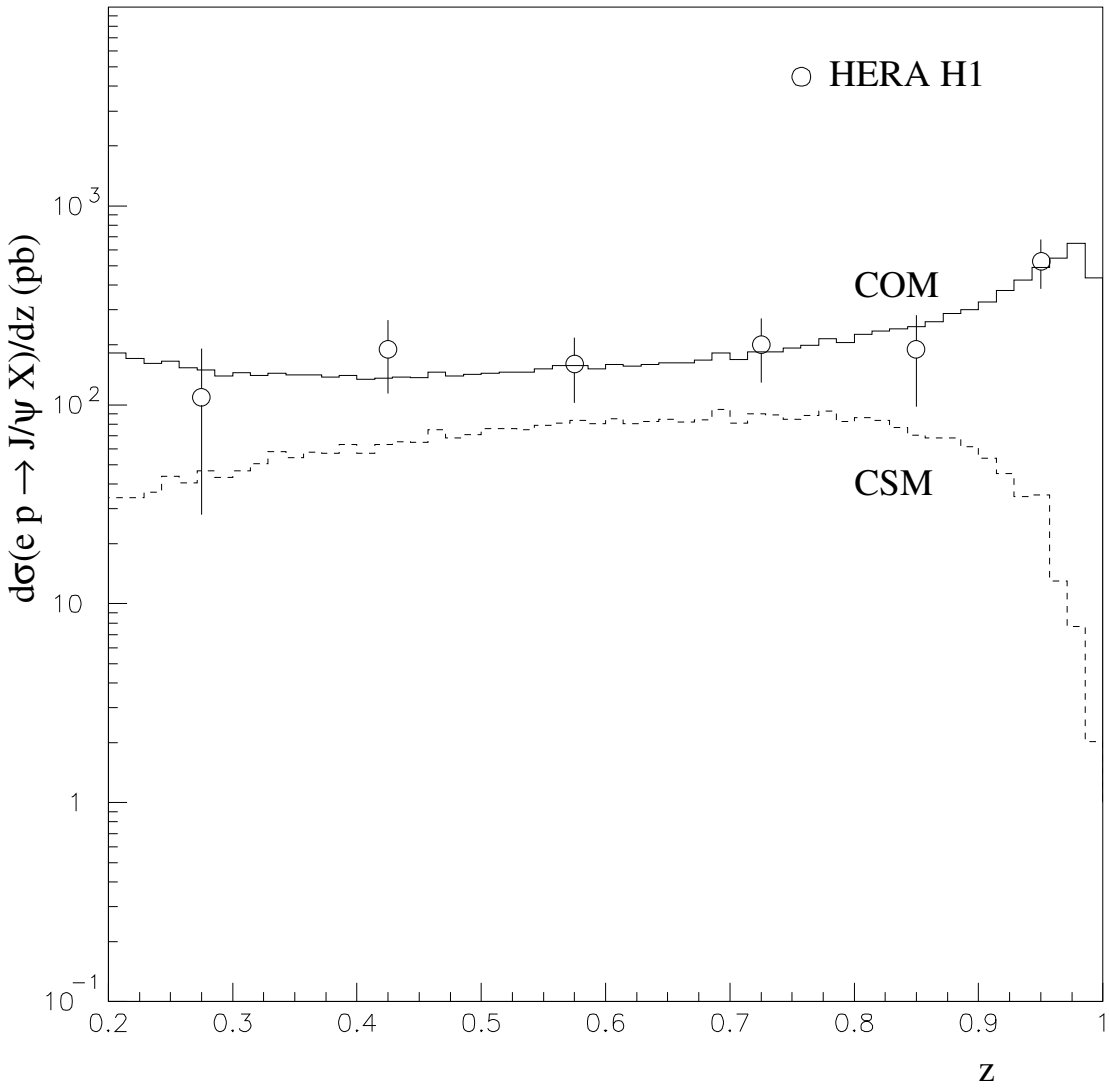

FIG. 2. The z distribution for the inelastic J/ψ production at HERA from H1 Collaboration in the kinematic region $4 < Q^2 < 80$ GeV^2, $p^2_{T,J/\psi} > 4$ GeV^2, $40 < W < 180$ GeV and $z > 0.2$. The solid curve represents the COM prediction after choosing appropriate values for the NRQCD matrix elements given in Table I for CTEQ 4L parton distribution function. The dashed curve shows the color singlet contribution.

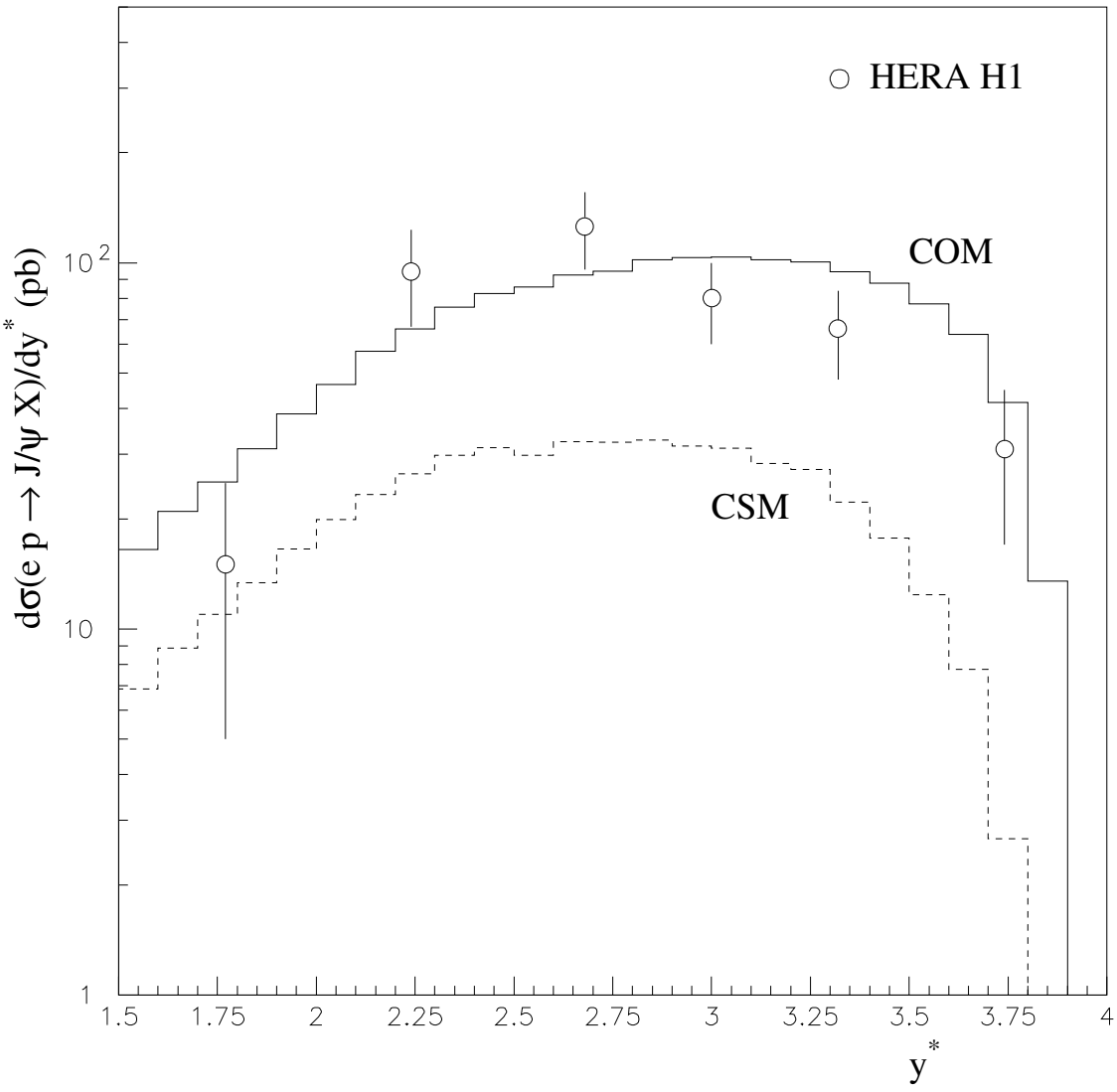

FIG. 3. Same as in Fig. 2 for the y^* (the rapidity of J/ψ in the $\gamma^* p$ center-of-mass frame) distribution.

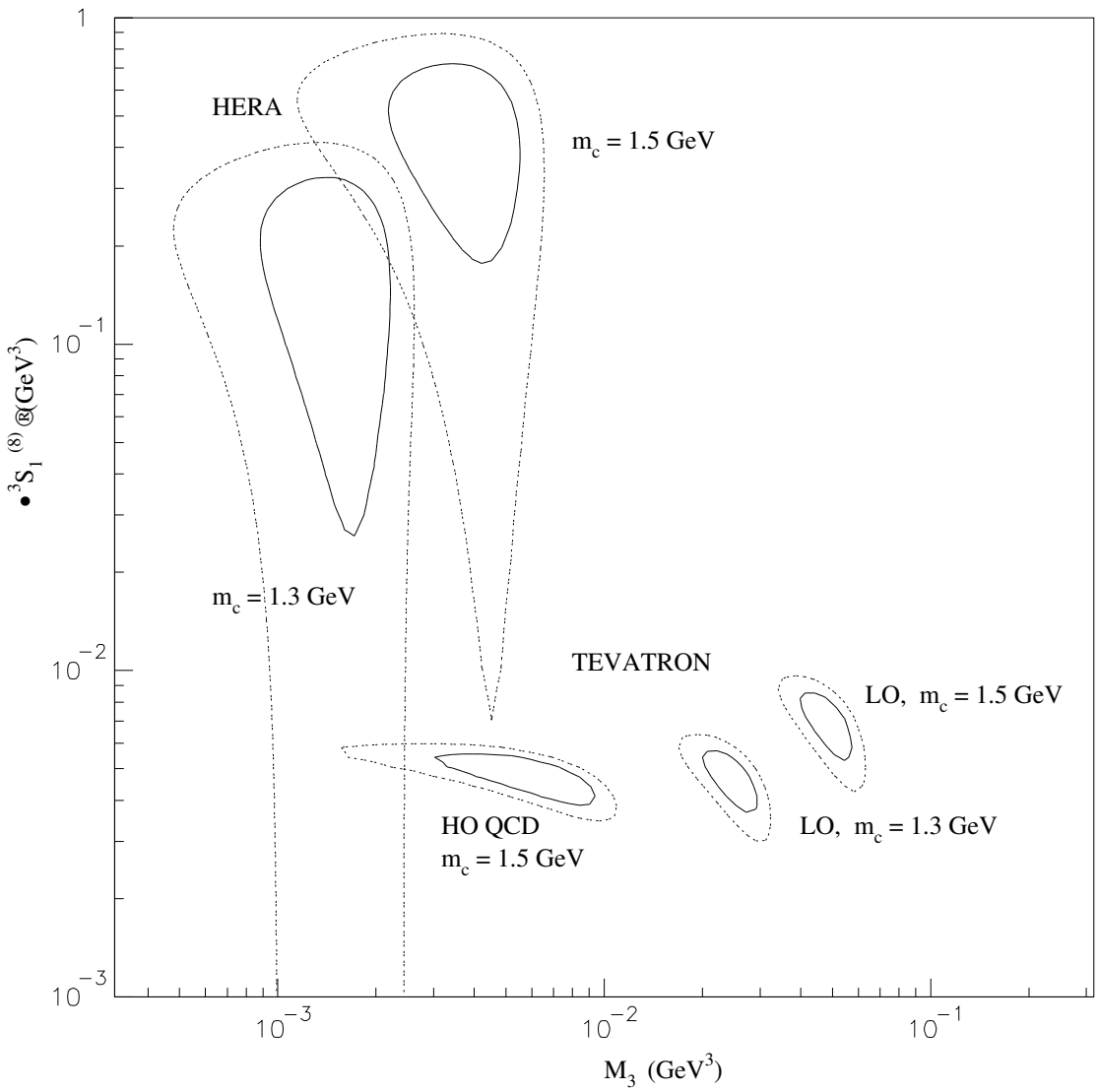

FIG. 4. Parameter space for the Color Octet NRQCD matrix elements. The bounds on $\langle {}^3S_1^{(8)} \rangle$ and M_3 for Tevatron and HERA are displaced at 68% C.L. (solid lines) and 95% C.L. (dashed lines). The results are for MRS (R2) parton distribution function.

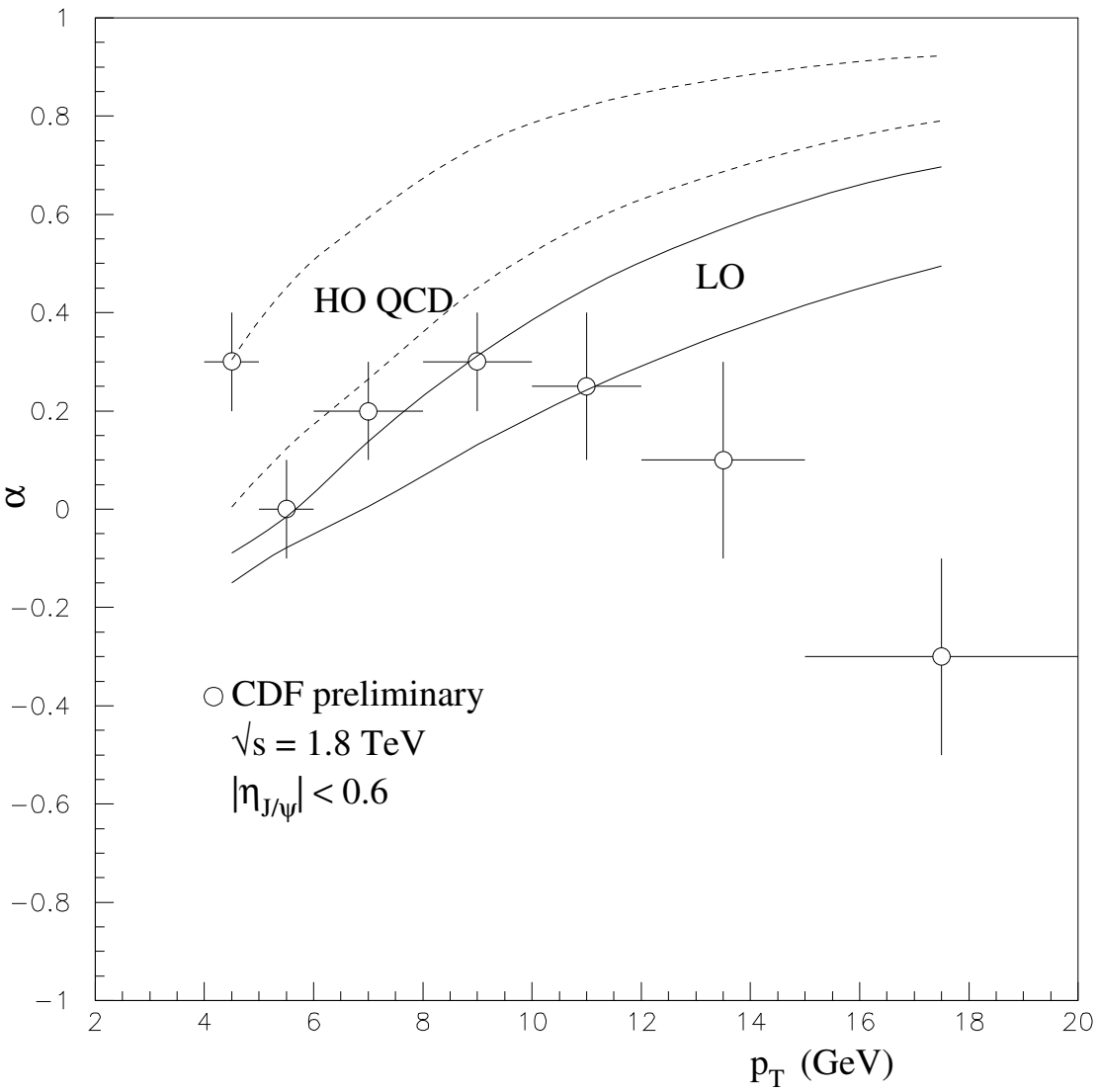

FIG. 5. The polarization parameter α as a function of p_T for the inclusive prompt J/ψ production at the Tevatron. The bounds at LO (solid lines) and HO QCD (dashed lines) are based on 68% C.L. including only errors due to the experimental data from CDF preliminary analysis for $|y_{J/\psi}| < 0.6$. The results are for MRS (R2) parton distribution function.

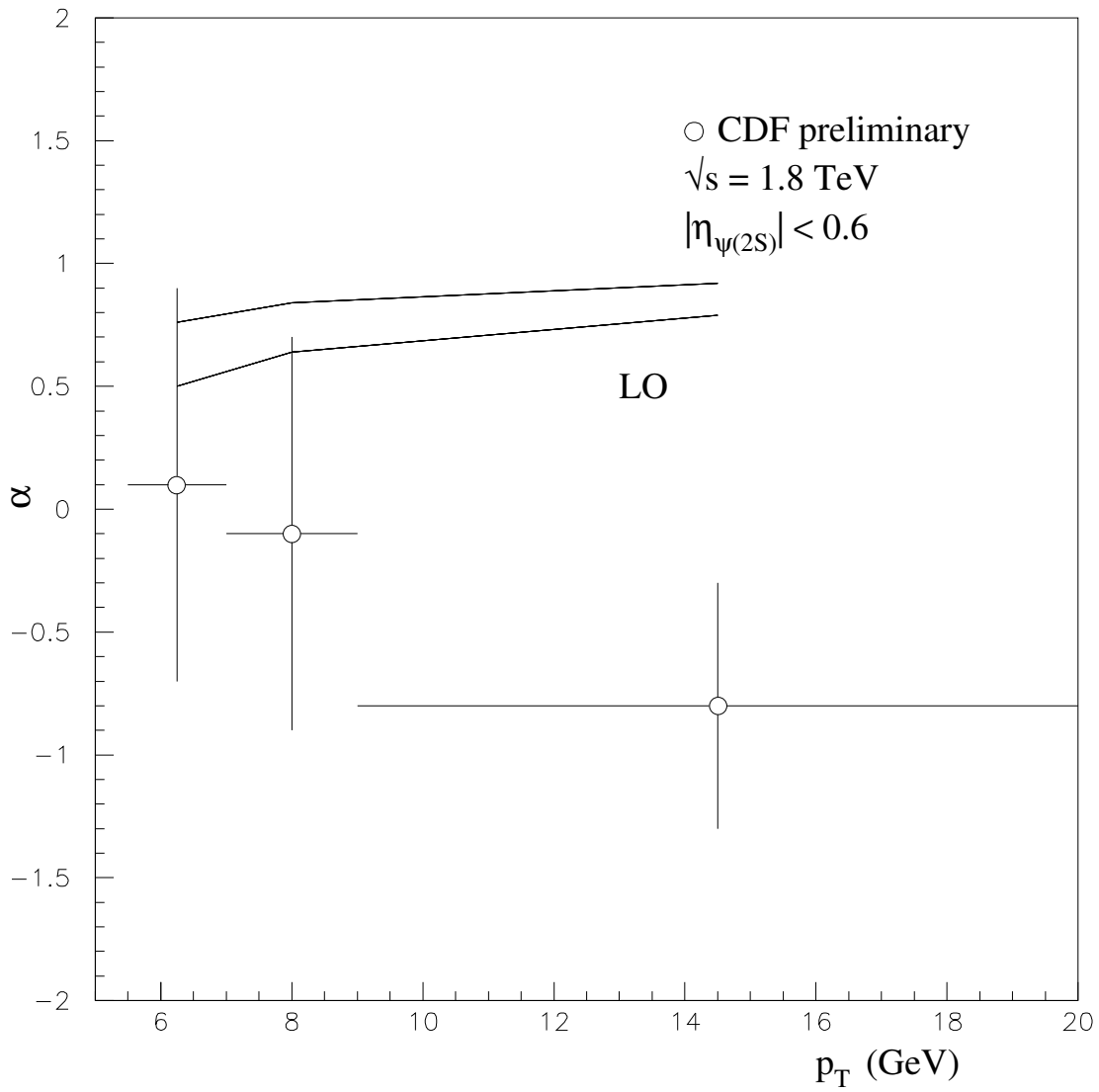

FIG. 6. Same as Fig. 5 for the $\psi(2S)$ production at the Tevatron.