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Model-independent information on γ from $B^\pm \to \pi K$ Decays

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Abstract:

Measurements of the rates for the hadronic decays $B^\pm \to \pi K$ can be used to derive information on the weak phase $\gamma = \arg(V_{ub}^*)$ in a largely model-independent way. Hadronic uncertainties can be reduced to the level of nonfactorizable contributions to the decay amplitudes that are power-suppressed in Λ/m_b and, in addition, either violate SU(3) flavor symmetry or are doubly Cabibbo suppressed. Various strategies to obtain bounds on γ and to extract its value with small theoretical uncertainty are described. The potential of $B^\pm \to \pi K$ decays for probing physics beyond the Standard Model is also discussed.

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Measurements of the rates for the hadronic decays $B^\pm \to \pi K$ can be used to derive information on the weak phase $\gamma = \arg(V_{ub}^*)$ in a largely model-independent way. Hadronic uncertainties can be reduced to the level of nonfactorizable contributions to the decay amplitudes that are power-suppressed in Λ/m_b and, in addition, either violate SU(3) flavor symmetry or are doubly Cabibbo suppressed. Various strategies to obtain bounds on γ and to extract its value with small theoretical uncertainty are described. The potential of $B^\pm \to \pi K$ decays for probing physics beyond the Standard Model is also discussed.

1. INTRODUCTION

The main objective of the B factories is to explore in detail the physics of CP violation, to determine many of the flavor parameters of the electroweak theory, and to probe for possible effects of physics beyond the Standard Model. This will test the Cabibbo–Kobayashi–Maskawa (CKM) mechanism, which predicts that all CP violation results from a single complex phase in the quark mixing matrix. Facing the announcement of evidence for a CP asymmetry in the decays $B \to J/\psi K_S$ by the CDF Collaboration [1], the confirmation of direct CP violation in $K \to \pi\pi$ decays by the KTeV and NA48 Collaborations [2,3], and the successful start of the asymmetric B factories at SLAC and KEK, the year 1999 has been an important step in achieving this goal.

The precise determination of the sides and angles of the "unitarity triangle" $V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$ plays a central role in the B-factory program [4]. With the standard phase conventions for the CKM matrix, only the two smallest elements in this relation, V_{ub}^* and V_{td}, have nonvanishing imaginary parts (to an excellent approximation). In the Standard Model the angle $\beta = -\arg(V_{td})$ can be determined in a theoretically clean way by measuring the time-dependent, mixing-induced CP asymmetry in the decays $B, \bar{B} \to J/\psi K_S$. The preliminary CDF result implies $\sin 2\beta = 0.79^{+0.41}_{-0.44}$ [1]. The angle $\gamma = \arg(V_{ub}^*)$, or equivalently the combination $\alpha = 180^\circ - \beta - \gamma$, is much harder to determine [4]. Recently, there has been significant progress in the theoretical understanding of the hadronic decays $B \to \pi K$, and methods have been developed to extract information on γ from rate measurements for these processes. Here we discuss the charged modes $B^\pm \to \pi K$, which are particularly clean from a theoretical perspective [5–7]. For applications involving the neutral decay modes the reader is referred to the literature [8,9].

In the Standard Model the main contributions to the decay amplitudes for the rare decays $B \to \pi K$ come from the penguin-induced flavor-changing neutral current (FCNC) transitions $\bar{b} \to \bar{s} q \bar{q}$, which exceed a small, Cabibbo-suppressed $\bar{b} \to \bar{u} u \bar{s}$ contribution from W-boson exchange. The weak phase γ enters through the interference of these two ("tree" and "penguin") contributions. Because of a fortunate interplay of isospin, Fierz and flavor symmetries, the theoretical description of the charged modes $B^\pm \to \pi K$ is very clean despite the fact that these are exclusive nonleptonic decays [5–7]. Without any dynamical assumption, the hadronic uncertainties in the description of the interference terms relevant to the determination of γ are of rel-

ative magnitude $O(\lambda^2)$ or $O(\epsilon_{\rm SU(3)}/N_c)$, where $\lambda = \sin\theta_C \approx 0.22$ is a measure of Cabibbo suppression, $\epsilon_{\rm SU(3)} \sim 20\%$ is the typical size of SU(3) flavor-symmetry breaking, and the factor $1/N_c$ indicates that the corresponding terms vanish in the factorization approximation. Factorizable SU(3) breaking can be accounted for in a straightforward way.

Recently, the accuracy of this description has been further increased, because it has been shown that nonleptonic B decays into two light mesons, such as $B \to \pi K$ and $B \to \pi\pi$, admit a heavy-quark expansion [10]. To leading order in Λ/m_b, and to all orders in perturbation theory, the decay amplitudes for these processes can be calculated from first principles, without recourse the phenomenological models. The QCD factorization theorem proved in [10] improves upon the phenomenological approach of "generalized factorization" [11], which emerges as the leading term in the heavy-quark limit. With the help of this theorem the irreducible theoretical uncertainty in the description of the $B^\pm \to \pi K$ decay amplitudes can be reduced by an extra factor of $O(\Lambda/m_b)$, rendering their analysis essentially model independent.

As a consequence of this fact, and because they are dominated by (hadronic) FCNC transitions, the decays $B^\pm \to \pi K$ offer a sensitive probe to physics beyond the Standard Model [7,12–15], much in the same way as the "classical" FCNC processes $B \to X_s\gamma$ or $B \to X_s\, l^+ l^-$. We will discuss how the bound on γ and the extraction of γ in the Standard Model could be affected by New Physics.

2. THEORY OF $B^\pm \to \pi K$ DECAYS

The hadronic decays $B \to \pi K$ are mediated by a low-energy effective weak Hamiltonian, whose operators allow for three distinct classes of flavor topologies: QCD penguins, trees, and electroweak penguins. In the Standard Model the weak couplings associated with these topologies are known. From the measured branching ratios for the various $B \to \pi K$ decay modes it follows that QCD penguins dominate the decay amplitudes [16], whereas trees and electroweak penguins are subleading and of a similar strength [17]. The theoretical description of the two charged modes $B^\pm \to \pi^\pm K^0$ and $B^\pm \to \pi^0 K^\pm$ exploits the fact that the amplitudes for these processes differ in a pure isospin amplitude $A_{3/2}$, defined as the matrix element of the isovector part of the effective Hamiltonian between a B meson and the πK isospin eigenstate with $I = \frac{3}{2}$. In the Standard Model the parameters of this amplitude are determined, up to an overall strong phase ϕ, in the limit of SU(3) flavor symmetry [5]. Using the QCD factorization theorem proved in [10], the SU(3)-breaking corrections can be calculated in a model-independent way up to nonfactorizable terms that are power-suppressed in Λ/m_b and vanish in the heavy-quark limit.

A convenient parameterization of the decay amplitudes $\mathcal{A}_{+0} \equiv \mathcal{A}(B^+ \to \pi^+ K^0)$ and $\mathcal{A}_{0+} \equiv -\sqrt{2}\,\mathcal{A}(B^+ \to \pi^0 K^+)$ is [7]

$$\begin{aligned} \mathcal{A}_{+0} &= P\,(1 - \varepsilon_a\, e^{i\gamma} e^{i\eta})\,, \\ \mathcal{A}_{0+} &= P\Big[1 - \varepsilon_a\, e^{i\gamma} e^{i\eta} - \varepsilon_{3/2}\, e^{i\phi}(e^{i\gamma} - \delta_{\rm EW})\Big]\,, \end{aligned} \tag{1}$$

where P is the dominant penguin amplitude defined as the sum of all terms in the $B^+ \to \pi^+ K^0$ amplitude not proportional to $e^{i\gamma}$, η and ϕ are strong phases, and ε_a, $\varepsilon_{3/2}$ and $\delta_{\rm EW}$ are real hadronic parameters. The weak phase γ changes sign under a CP transformation, whereas all other parameters stay invariant.

Let us discuss the various terms entering the decay amplitudes in detail. From a naive quark-diagram analysis one does not expect the $B^+ \to \pi^+ K^0$ amplitude to receive a contribution from $\bar b \to \bar u u \bar s$ tree topologies; however, such a contribution can be induced through final-state rescattering or annihilation contributions [18–23]. They are parameterized by $\varepsilon_a = O(\lambda^2)$. In the heavy-quark limit this parameter can be calculated and is found to be very small, $\varepsilon_a \approx -2\%$ [24]. In the future, it will be possible to put upper and lower bounds on ε_a by comparing the CP-averaged branching ratios for the decays $B^\pm \to \pi^\pm K^0$ and $B^\pm \to K^\pm \bar K^0$ [22]. Below we assume $|\varepsilon_a| \le 0.1$; however, our results will be almost insensitive to this assumption.

The terms proportional to $\varepsilon_{3/2}$ in (1) parameterize the isospin amplitude $A_{3/2}$. The contri-

bution proportional to $e^{i\gamma}$ comes from the tree process $\bar b \to \bar u u \bar s$, whereas the quantity $\delta_{\rm EW}$ describes the effects of electroweak penguins. The parameter $\varepsilon_{3/2}$ measures the relative strength of tree and QCD penguin contributions. Information about it can be derived by using SU(3) flavor symmetry to relate the tree contribution to the isospin amplitude $A_{3/2}$ to the corresponding contribution in the decay $B^+ \to \pi^+\pi^0$. Since the final state $\pi^+\pi^0$ has isospin $I = 2$ (because of Bose symmetry), the amplitude for this process does not receive any contribution from QCD penguins. Moreover, electroweak penguins in $\bar b \to \bar d q \bar q$ transitions are negligibly small. We define a related parameter $\bar\varepsilon_{3/2}$ by writing $\varepsilon_{3/2} = \bar\varepsilon_{3/2}\sqrt{1 - 2\varepsilon_a \cos\eta\cos\gamma + \varepsilon_a^2}$, so that the two quantities agree in the limit $\varepsilon_a \to 0$. In the SU(3) limit, this new parameter can be determined experimentally form the relation [5]

$$\bar\varepsilon_{3/2} = R_1 \left|\frac{V_{us}}{V_{ud}}\right| \left[\frac{2{\rm B}(B^\pm \to \pi^\pm\pi^0)}{{\rm B}(B^\pm \to \pi^\pm K^0)}\right]^{1/2} . \qquad (2)$$

SU(3)-breaking corrections are described by the factor $R_1 = 1.22 \pm 0.05$, which can be calculated in a model-independent way using the QCD factorization theorem for nonleptonic decays [24]. The quoted error is an estimate of the theoretical uncertainty due to uncontrollable corrections of $O(\frac{1}{N_c}\frac{m_s}{m_b})$. Using preliminary data reported by the CLEO Collaboration [25] to evaluate the ratio of CP-averaged branching ratios in (2) we obtain

$$\bar\varepsilon_{3/2} = 0.21 \pm 0.06_{\rm exp} \pm 0.01_{\rm th} . \qquad (3)$$

With a better measurement of the branching ratios the uncertainty in $\bar\varepsilon_{3/2}$ will be reduced significantly.

Finally, the parameter

$$\begin{aligned}\delta_{\rm EW} &= R_2 \left|\frac{V_{cb}^* V_{cs}}{V_{ub}^* V_{us}}\right| \frac{\alpha}{8\pi}\frac{x_t}{\sin^2\theta_W}\left(1 + \frac{3\ln x_t}{x_t - 1}\right) \\ &= (0.64 \pm 0.09) \times \frac{0.085}{|V_{ub}/V_{cb}|} , \end{aligned} \qquad (4)$$

with $x_t = (m_t/m_W)^2$, describes the ratio of electroweak penguin and tree contributions to the isospin amplitude $A_{3/2}$. In the SU(3) limit it is calculable in terms of Standard Model parameters [5,26]. SU(3)-breaking corrections are accounted for by the quantity $R_2 = 0.92 \pm 0.09$ [7,24]. The error quoted in (4) also includes the uncertainty in the top-quark mass.

Important observables in the study of the weak phase γ are the ratio of the CP-averaged branching ratios in the two $B^\pm \to \pi K$ decay modes,

$$R_* = \frac{{\rm B}(B^\pm \to \pi^\pm K^0)}{2{\rm B}(B^\pm \to \pi^0 K^\pm)} = 0.75 \pm 0.28 , \qquad (5)$$

and a particular combination of the direct CP asymmetries,

$$\begin{aligned}\widetilde A &= \frac{A_{\rm CP}(B^\pm \to \pi^0 K^\pm)}{R_*} - A_{\rm CP}(B^\pm \to \pi^\pm K^0) \\ &= -0.52 \pm 0.42 . \end{aligned} \qquad (6)$$

The experimental values of these quantities are derived from preliminary data reported by the CLEO Collaboration [25]. The theoretical expressions for R_* and $\widetilde A$ obtained using the parameterization in (1) are

$$\begin{aligned} R_*^{-1} &= 1 + 2\bar\varepsilon_{3/2}\cos\phi\,(\delta_{\rm EW} - \cos\gamma) \\ &\quad + \bar\varepsilon_{3/2}^2(1 - 2\delta_{\rm EW}\cos\gamma + \delta_{\rm EW}^2) + O(\bar\varepsilon_{3/2}\,\varepsilon_a) , \\ \widetilde A &= 2\bar\varepsilon_{3/2}\sin\gamma\sin\phi + O(\bar\varepsilon_{3/2}\,\varepsilon_a) . \end{aligned} \qquad (7)$$

Note that the rescattering effects described by ε_a are suppressed by a factor of $\bar\varepsilon_{3/2}$ and thus reduced to the percent level. Explicit expressions for these contributions can be found in [7].

3. LOWER BOUND ON γ AND CONSTRAINT IN THE $(\bar\rho, \bar\eta)$ PLANE

There are several strategies for exploiting the above relations. First, from a measurement of the ratio R_* alone a bound on $\cos\gamma$ can be derived, which implies a nontrivial constraint on the Wolfenstein parameters $\bar\rho$ and $\bar\eta$ defining the apex of the unitarity triangle [5]. Only CP-averaged branching ratios are needed for this purpose. Varying the strong phases ϕ and η independently we first obtain an upper bound on the inverse of R_*. Keeping terms of linear order in ε_a, we find [7]

$$\begin{aligned} R_*^{-1} &\le \left(1 + \bar\varepsilon_{3/2}\,|\delta_{\rm EW} - \cos\gamma|\right)^2 + \bar\varepsilon_{3/2}^2\sin^2\gamma \\ &\quad + 2\bar\varepsilon_{3/2}|\varepsilon_a|\sin^2\gamma . \end{aligned} \qquad (8)$$

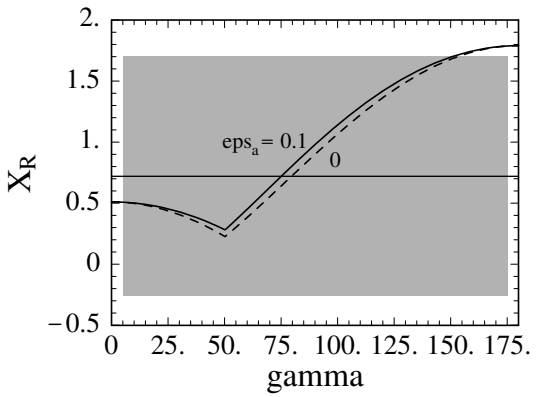

Figure 1. Theoretical upper bound on the ratio X_R versus $|\gamma|$ for $\varepsilon_a = 0.1$ (solid) and $\varepsilon_a = 0$ (dashed). The horizontal line and band show the current experimental value with its 1σ variation.

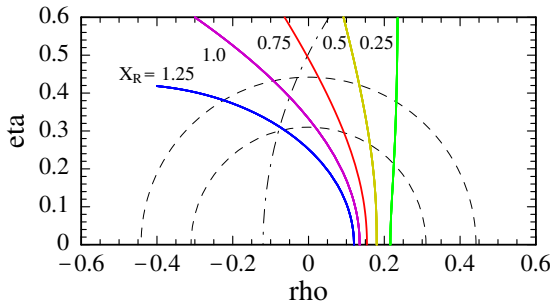

Figure 2. Theoretical constraint on the Wolfenstein parameters $(\bar\rho, \bar\eta)$ implied by a measurement of the ratio X_R in $B^\pm \to \pi K$ decays (solid lines), semileptonic B decays (dashed circles), and B_s–$\bar B_s$ mixing (dashed-dotted line).

Provided R_* is significantly smaller than 1, this bound implies an exclusion region for $\cos\gamma$, which becomes larger the smaller the values of R_* and $\bar\varepsilon_{3/2}$ are. It is convenient to consider instead of R_* the related quantity [15]

$$X_R = \frac{\sqrt{R_*^{-1}} - 1}{\bar\varepsilon_{3/2}} = 0.72 \pm 0.98_{\rm exp} \pm 0.03_{\rm th}\,. \quad (9)$$

Because of the theoretical factor R_1 entering the definition of $\bar\varepsilon_{3/2}$ in (2) this is, strictly speaking, not an observable. However, the theoretical uncertainty in X_R is so much smaller than the present experimental error that it is justified to treat this quantity as an observable. The advantage of presenting our results in terms of X_R rather than R_* is that the leading dependence on $\bar\varepsilon_{3/2}$ cancels out, leading to the simple bound $|X_R| \le |\delta_{\rm EW} - \cos\gamma| + O(\bar\varepsilon_{3/2}, \varepsilon_a)$.

In Figure 1 we show the upper bound on X_R as a function of $|\gamma|$, obtained by varying the input parameters in the intervals $0.15 \le \bar\varepsilon_{3/2} \le 0.27$ and $0.49 \le \delta_{\rm EW} \le 0.79$ (corresponding to using $|V_{ub}/V_{cb}| = 0.085 \pm 0.015$ in (4)). Note that the effect of the rescattering contribution parameterized by ε_a is very small. The gray band shows the current value of X_R, which clearly has too large an error to provide any useful information on γ.[2] The situation may change, however, once a more precise measurement of X_R will become available. For instance, if the current central value $X_R = 0.72$ were confirmed, it would imply the bound $|\gamma| > 75°$, which would mark a significant improvement over the limit $|\gamma| > 37°$ obtained from the global analysis of the unitarity triangle including information from K–$\bar K$ mixing [4].

So far, as in previous work, we have used the inequality (8) to derive a lower bound on $|\gamma|$. However, a large part of the uncertainty in the value of $\delta_{\rm EW}$, and thus in the resulting bound on $|\gamma|$, comes from the present large error on $|V_{ub}|$. Since this is not a hadronic uncertainty, it is more appropriate to separate it and turn (8) into a constraint on the Wolfenstein parameters $\bar\rho$ and $\bar\eta$. To this end, we use that $\cos\gamma = \bar\rho/\sqrt{\bar\rho^2 + \bar\eta^2}$ by definition, and $\delta_{\rm EW} = (0.24 \pm 0.03)/\sqrt{\bar\rho^2 + \bar\eta^2}$ from (4). The solid lines in Figure 2 show the resulting constraint in the $(\bar\rho, \bar\eta)$ plane obtained for the representative values $X_R = 0.25$, 0.5, 0.75, 1.0, 1.25 (from right to left), which for $\bar\varepsilon_{3/2} = 0.21$ would correspond to $R_* = 0.90$, 0.82, 0.75, 0.68, 0.63, respectively. Values to the right of these lines are excluded. For comparison, the dashed circles show the constraint arising from the mea-

[2] Unfortunately, the 2σ deviation from 1 indicated by the first preliminary CLEO result has not been confirmed by the present data.

surement of the ratio $|V_{ub}/V_{cb}| = 0.085 \pm 0.015$ in semileptonic B decays, and the dashed-dotted line shows the bound implied by the present experimental limit on the mass difference Δm_s in the B_s system [4]. Values to the left of this line are excluded. It is evident from the figure that the bound resulting from a measurement of the ratio X_R in $B^\pm \to \pi K$ decays may be very nontrivial and, in particular, may eliminate the possibility that $\gamma = 0$. The combination of this bound with information from semileptonic decays and B_s–$\bar{B}_s$ mixing alone would then determine the Wolfenstein parameters $\bar\rho$ and $\bar\eta$ within narrow ranges,[3] and in the context of the CKM model would prove the existence of direct CP violation in B decays.

4. EXTRACTION OF γ

Ultimately, the goal is of course not only to derive a bound on γ but to determine this parameter directly from the data. This requires to fix the strong phase ϕ in (7), which can be done either through the measurement of a CP asymmetry or with the help of theory. A strategy for an experimental determination of γ from $B^\pm \to \pi K$ decays has been suggested in [6]. It generalizes a method proposed by Gronau, Rosner and London [27] to include the effects of electroweak penguins. The approach has later been refined to account for rescattering contributions to the $B^\pm \to \pi^\pm K^0$ decay amplitudes [7]. Before discussing this method, we will first illustrate an easier strategy for a theory-guided determination of γ based on the QCD factorization theorem derived in [10]. This method does not require any measurement of a CP asymmetry.

4.1. Theory-guided determination

In the previous section the theoretical predictions for the nonleptonic $B \to \pi K$ decay amplitudes obtained using the QCD factorization theorem were used in a minimalistic way, i.e., only to calculate the size of the SU(3)-breaking effects parameterized by R_1 and R_2. The resulting bound on γ and the corresponding constraint in the $(\bar\rho, \bar\eta)$ plane are therefore theoretically very clean. However, they are only useful if the value of X_R is found to be larger than about 0.5 (see Figure 1), in which case values of $|\gamma|$ below 65° are excluded. If it would turn out that $X_R < 0.5$, then it is in principle possible to satisfy the inequality (8) also for small values of γ, however, at the price of having a very large value of the strong phase, $\phi \approx 180^\circ$. But this possibility can be discarded based on the model-independent prediction that [10]

$$\phi = O[\alpha_s(m_b), \Lambda/m_b]\,. \tag{10}$$

In fact, a direct calculation of this phase to leading power in Λ/m_b yields $\phi \approx -11^\circ$ [24]. Using the fact that ϕ is parametrically small, we can exploit a measurement of the ratio X_R to obtain a *determination* of $|\gamma|$ – corresponding to an allowed region in the $(\bar\rho, \bar\eta)$ plane – rather than just a bound. This determination is unique up to a sign. Note that for small values of ϕ the impact of the strong phase in the expression for R_* in (7) is a second-order effect, since $\cos\phi \approx 1 - \phi^2/2$. As long as $|\phi| \ll \sqrt{2\Delta\bar\varepsilon_{3/2}/\bar\varepsilon_{3/2}}$, the uncertainty in the value of $\cos\phi$ has a much smaller effect than the uncertainty in $\bar\varepsilon_{3/2}$. With the present value of $\bar\varepsilon_{3/2}$, this is the case as long as $|\phi| \ll 43^\circ$. We believe it is a safe assumption to take $|\phi| < 25^\circ$ (i.e., more than twice the value obtained to leading order in Λ/m_b), so that $\cos\phi > 0.9$.

Solving the equation for R_* in (7) for $\cos\gamma$, and including the corrections of $O(\varepsilon_a)$, we find

$$\cos\gamma = \delta_{\rm EW} - \frac{X_R + \frac{1}{2}\bar\varepsilon_{3/2}(X_R^2 - 1 + \delta_{\rm EW}^2)}{\cos\phi + \bar\varepsilon_{3/2}\delta_{\rm EW}} + \frac{\varepsilon_a \cos\eta \sin^2\gamma}{\cos\phi + \bar\varepsilon_{3/2}\delta_{\rm EW}}\,, \tag{11}$$

where we have set $\cos\phi = 1$ in the $O(\varepsilon_a)$ term. Using the QCD factorization theorem one finds that $\varepsilon_a \cos\eta \approx -0.02$ in the heavy-quark limit [24], and we assign a 100% uncertainty to this estimate. In evaluating the result (11) we scan the parameters in the ranges $0.15 \le \bar\varepsilon_{3/2} \le 0.27$, $0.55 \le \delta_{\rm EW} \le 0.73$, $-25^\circ \le \phi \le 25^\circ$, and $-0.04 \le \varepsilon_a \cos\eta \sin^2\gamma \le 0$. Figure 3 shows the allowed regions in the $(\bar\rho, \bar\eta)$ plane for the representative values $X_R = 0.25$, 0.75, and 1.25 (from

[3] An observation of CP violation, such as the measurement of ϵ_K in K–$\bar K$ mixing or $\sin 2\beta$ in $B \to J/\psi\, K_S$ decays, is however needed to fix the sign of $\bar\eta$.

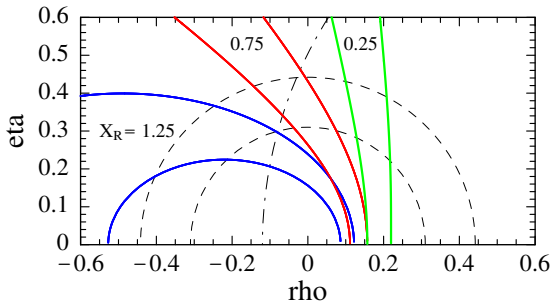

Figure 3. Allowed regions in the $(\bar\rho, \bar\eta)$ plane for fixed values of X_R, obtained by varying all theoretical parameters inside their respective ranges of uncertainty, as specified in the text. The sign of $\bar\eta$ is not determined.

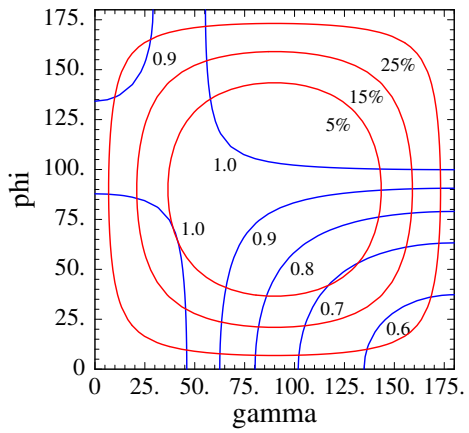

Figure 4. Contours of constant R_* ("hyperbolas") and constant $|\widetilde{A}|$ ("circles") in the $(|\gamma|, |\phi|)$ plane. The sign of the asymmetry $\widetilde{A}$ determines the sign of the product $\sin\gamma \sin\phi$. The contours for R_* refer to values from 0.6 to 1.0 in steps of 0.1, those for the asymmetry correspond to 5%, 15%, and 25%, as indicated.

right to left). We stress that with this method a useful constraint on the Wolfenstein parameters is obtained for *any* value of X_R.

4.2. Model-independent determination

It is important that, once more precise data on $B^\pm \to \pi K$ decays will become available, it will be possible to test the theoretical prediction of a small strong phase ϕ experimentally. To this end, one must determine the CP asymmetry $\widetilde{A}$ in addition to the ratio R_*. From (7) it follows that for fixed values of $\bar\varepsilon_{3/2}$ and $\delta_{\rm EW}$ the quantities R_* and $\widetilde{A}$ define contours in the (γ, ϕ) plane, whose intersections determine the two phases up to possible discrete ambiguities [6,7]. Figure 4 shows these contours for some representative values, assuming $\bar\varepsilon_{3/2} = 0.21$, $\delta_{\rm EW} = 0.64$, and $\varepsilon_a = 0$. In practice, including the uncertainties in the values of these parameters changes the contour lines into contour bands. Typically, the spread of the bands induces an error in the determination of γ of about 10° [7].[4] In the most general case there are up to eight discrete solutions for the two phases, four of which are related to the other four by the sign change $(\gamma, \phi) \to (-\gamma, -\phi)$. However, for typical values of R_* it turns out that often only four solutions exist, two of which are related to the other two by a change of signs. The theoretical prediction that ϕ is small implies that solutions should exist where the contours intersect close to the lower portion in the plot. Other solutions with large ϕ are strongly disfavored theoretically. Moreover, according to (7) the sign of the CP asymmetry $\widetilde{A}$ fixes the relative sign between the two phases γ and ϕ. If we trust the theoretical prediction that ϕ is negative [24], it follows that in most cases there remains only a unique solution for γ, i.e., *the CP-violating phase γ can be determined without any discrete ambiguity.*

As an example, consider the hypothetical case where $R_* = 0.8$ and $\widetilde{A} = -15\%$. Figure 4 then allows the four solutions where $(\gamma, \phi) \approx (\pm 82^\circ, \mp 21^\circ)$ or $(\pm 158^\circ, \mp 78^\circ)$. The second pair of solutions is strongly disfavored because of the large values of the strong phase ϕ. From the first pair of solutions, the one with $\phi \approx -21^\circ$ is closest to our theoretical expectation that $\phi \approx -11^\circ$, hence leaving $\gamma \approx 82^\circ$ as the unique solution.

[4] A precise determination of this error requires knowledge of the actual values of the observables. Gronau and Pirjol [28] find a larger error for the special case where the product $|\sin\gamma \sin\phi|$ is very close to 1, which however is highly disfavored because of the expected smallness of the strong phase ϕ.

5. SEARCH FOR NEW PHYSICS

In the presence of New Physics the theoretical description of $B^\pm \to \pi K$ decays becomes more complicated. In particular, new CP-violating contributions to the decay amplitudes may be induced. A detailed analysis has been presented in [15]. A convenient and completely general parameterization of the two amplitudes in (1) is obtained by replacing

$$\begin{aligned} &P \to P', \qquad \varepsilon_a\, e^{i\gamma} e^{i\eta} \to i\rho\, e^{i\phi_\rho}, \\ &\delta_{\rm EW} \to a\, e^{i\phi_a} + ib\, e^{i\phi_b}, \end{aligned} \tag{12}$$

where ρ, a, b are real hadronic parameters, and ϕ_ρ, ϕ_a, ϕ_b are strong phases. The terms $i\rho$ and ib change sign under a CP transformation. New Physics effects parameterized by P' and ρ are isospin conserving, while those described by a and b violate isospin. Note that the parameter P' cancels in all ratios of branching ratios and thus does not affect the quantities R_* and X_R as well as all CP asymmetries. Because the ratio R_* in (5) would be 1 in the isospin limit, it is particularly sensitive to isospin-violating New Physics contributions. The isospin-conserving effects parameterized by ρ enter only through interference with the isospin-violating terms proportional to $\varepsilon_{3/2}$ in (1) and hence are suppressed.

New Physics can affect the bound on γ derived from (8) as well as the value of γ extracted using the strategies discussed in the previous section. We will discuss these two possibilities in turn.

5.1. Effects on the bound on γ

The upper bound on R_*^{-1} in (8) and the corresponding bound on X_R shown in Figure 1 are model-independent results valid in the Standard Model. Note that the extremal value of R_*^{-1} is such that $|X_R| \le (1+\delta_{\rm EW})$ irrespective of γ. A value of $|X_R|$ exceeding this bound would be a clear signal for New Physics [7,12,15].

Consider first the case where New Physics may induce arbitrary CP-violating contributions to the $B \to \pi K$ decay amplitudes, while preserving isospin symmetry. Then the only change with respect to the Standard Model is that the parameter ρ may no longer be as small as $O(\varepsilon_a)$. Varying the strong phases ϕ and ϕ_ρ independently, and allowing for an *arbitrarily large* New Physics contribution to ρ, one can derive the bound [15]

$$|X_R| \le \sqrt{1 - 2\delta_{\rm EW}\cos\gamma + \delta_{\rm EW}^2} \le 1 + \delta_{\rm EW}\,. \tag{13}$$

Note that the extremal value is the same as in the Standard Model, i.e., isospin-conserving New Physics effects cannot lead to a value of $|X_R|$ exceeding $1+\delta_{\rm EW}$. For intermediate values of γ between 25° and 125° the Standard Model bound on X_R is weakened. But even for large values $\rho = O(1)$, corresponding to a significant New Physics contribution to the decay amplitudes, the effects are small.

If both isospin-violating and isospin-conserving New Physics effects are present and involve new CP-violating phases, the analysis becomes more complicated. Still, it is possible to derive model-independent bounds on X_R. Allowing for arbitrary values of ρ and all strong phases, one obtains [15]

$$\begin{aligned} |X_R| &\le \sqrt{(|a| + |\cos\gamma|)^2 + (|b| + |\sin\gamma|)^2} \\ &\le 1 + \sqrt{a^2+b^2} \le \frac{2}{\bar\varepsilon_{3/2}} + X_R\,, \end{aligned} \tag{14}$$

where the last inequality is relevant only in cases where $\sqrt{a^2+b^2} \gg 1$. The important point to note is that with isospin-violating New Physics contributions the value of $|X_R|$ can exceed the upper bound in the Standard Model by a potentially large amount. For instance, if $\sqrt{a^2+b^2}$ is twice as large as in the Standard Model, corresponding to a New Physics contribution to the decay amplitudes of only 10–15%, then $|X_R|$ could be as large as 2.6 as compared with the maximal value 1.8 allowed in the Standard Model. Also, in the most general case where b and ρ are nonzero, the maximal value $|X_R|$ can take is no longer restricted to occur at the endpoints $\gamma = 0°$ or 180°, which are disfavored by the global analysis of the unitarity triangle [4]. Rather, $|X_R|$ would take its maximal value if $|\tan\gamma| = |\rho| = |b/a|$.

The present experimental value of X_R in (9) has too large an error to determine whether there is any deviation from the Standard Model. If X_R turns out to be larger than 1 (i.e., one third of a standard deviation above its current central value), then an interpretation of this result in

the Standard Model would require a large value $|\gamma| > 91^\circ$ (see Figure 1), which may be difficult to accommodate. This may be taken as evidence for New Physics. If $X_R > 1.3$, one could go a step further and conclude that the New Physics must necessarily violate isospin [15].

5.2. Effects on the determination of γ

A value of the observable R_* violating the Standard Model bound (8) would be an exciting hint for New Physics. However, even if a more precise measurement will give a value that is consistent with the Standard Model bound, $B^\pm \to \pi K$ decays provide an excellent testing ground for physics beyond the Standard Model. This is so because New Physics may still cause a significant shift in the value of γ extracted from $B^\pm \to \pi K$ decays using the strategies discussed in Section 4. This may lead to inconsistencies when this value is compared with other determinations of γ.

A global fit of the unitarity triangle combining information from semileptonic B decays, B–$\bar{B}$ mixing, CP violation in the kaon system, and mixing-induced CP violation in $B \to J/\psi K_S$ decays provides information on γ, which in a few years will determine its value within a rather narrow range [4]. Such an indirect determination could be complemented by direct measurements of γ using, e.g., $B \to DK$ decays, or using the triangle relation $\gamma = 180^\circ - \alpha - \beta$ combined with a measurement of α in $B \to \pi\pi$ or $B \to \pi\rho$ decays. We will assume that a discrepancy of more than 25° between the "true" $\gamma = \arg(V_{ub}^*)$ and the value $\gamma_{\pi K}$ extracted in $B^\pm \to \pi K$ decays will be observable after a few years of operation at the B factories. This will set the benchmark for sensitivity to New Physics effects.

In order to illustrate how big an effect New Physics could have on the value of γ we consider the simplest case where there are no new CP-violating couplings. Then all New Physics contributions in (12) are parameterized by the single parameter $a \equiv \delta_{\rm EW} + a_{\rm NP}$. A more general discussion can be found in [15]. We also assume for simplicity that the strong phase ϕ is small, as suggested by (10). In this case the difference between the value $\gamma_{\pi K}$ extracted from $B^\pm \to \pi K$ decays and the "true" value of γ is to a good approximation given by

$$\cos\gamma_{\pi K} \simeq \cos\gamma - a_{\rm NP}\,. \qquad (15)$$

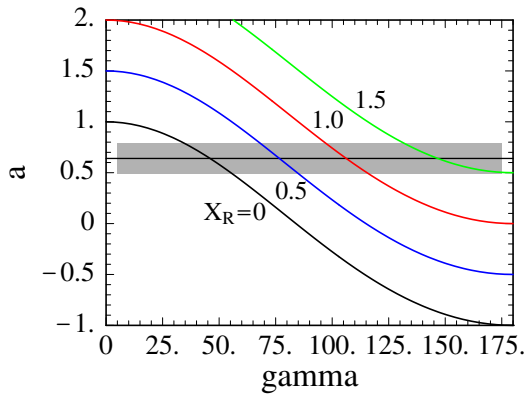

Figure 5. Contours of constant X_R versus γ and the parameter a, assuming $\gamma > 0$. The horizontal band shows the value of a in the Standard Model.

In Figure 5 we show contours of constant X_R versus γ and a, assuming without loss of generality that $\gamma > 0$. Obviously, even a moderate New Physics contribution to the parameter a can induce a large shift in γ. Note that the present central value of $X_R \approx 0.7$ is such that values of a less than the Standard Model result $a \approx 0.64$ are disfavored, since they would require values of γ exceeding 100°, in conflict with the global analysis of the unitarity triangle [4].

5.3. Survey of New Physics models

In [15], we have explored how physics beyond the Standard Model could affect purely hadronic FCNC transitions of the type $\bar{b} \to \bar{s}q\bar{q}$ focusing, in particular, on isospin violation. Unlike in the Standard Model, where isospin-violating effects in these processes are strongly suppressed by electroweak gauge couplings or small CKM matrix elements, in many New Physics scenarios these effects are not parametrically suppressed relative to isospin-conserving FCNC processes. In the language of effective weak Hamiltonians this implies that the Wilson coefficients of QCD and electroweak penguin operators are of a similar magnitude. For a large class of New Physics models we found that the coefficients of the electroweak

Table 1
Maximal contributions to $a_{\rm NP}$ in extensions of the Standard Model. Entries marked with a "*" are upper bounds derived using (14). For the case of supersymmetric models with R-parity the first (second) row corresponds to maximal right-handed (left-handed) strange–bottom squark mixing. For the two-Higgs–doublet models we take $m_{H^+} > 100\,\text{GeV}$ and $\tan\beta > 1$.

Model	$\lvert a_{\rm NP}\rvert$	$\lvert\gamma_{\pi K} - \gamma\rvert$
FCNC Z exchange	2.0	180°
extra Z' boson	14*	180°
SUSY without R-parity	14*	180°
SUSY with R-parity	0.4	25°
	1.3	180°
2HDM	0.15	10°
anom. gauge-boson coupl.	0.3	20°

penguin operators are, in fact, due to "trojan" penguins, which are neither related to penguin diagrams nor of electroweak origin.

Specifically, we have considered: (a) models with tree-level FCNC couplings of the Z boson, extended gauge models with an extra Z' boson, supersymmetric models with broken R-parity; (b) supersymmetric models with R-parity conservation; (c) two-Higgs–doublet models, and models with anomalous gauge-boson couplings. Some of these models have also been investigated in [13,14]. In case (a), the resulting electroweak penguin coefficients can be much larger than in the Standard Model because they are due to tree-level processes. In case (b), these coefficients can compete with the ones of the Standard Model because they arise from strong-interaction box diagrams, which scale relative to the Standard Model like $(\alpha_s/\alpha)(m_W^2/m_{\rm SUSY}^2)$. In models (c), on the other hand, isospin-violating New Physics effects are not parametrically enhanced and are generally smaller than in the Standard Model.

For each New Physics model we have explored which region of parameter space can be probed by the $B^\pm \to \pi K$ observables, and how big a departure from the Standard Model predictions one can expect under realistic circumstances, taking into account all constraints on the model parameters implied by other processes. Table 1 summarizes our estimates of the maximal isospin-violating contributions to the decay amplitudes, as parameterized by $|a_{\rm NP}|$. They are the potentially most important source of New Physics effects in $B^\pm \to \pi K$ decays. For comparison, we recall that in the Standard Model $a \approx 0.64$. Also shown are the corresponding maximal values of the difference $|\gamma_{\pi K} - \gamma|$. As noted above, in models with tree-level FCNC couplings New Physics effects can be dramatic, whereas in supersymmetric models with R-parity conservation isospin-violating loop effects can be competitive with the Standard Model. In the case of supersymmetric models with R-parity violation the bound (14) implies interesting limits on certain combinations of the trilinear couplings λ'_{ijk} and λ''_{ijk}, which are discussed in [15].

6. CONCLUSIONS

Measurements of the rates for the rare hadronic decays $B^\pm \to \pi K$ provide interesting information on the weak phase γ and on the Wolfenstein parameters $\bar\rho$ and $\bar\eta$. Using isospin, Fierz and flavor symmetries together with the fact that nonleptonic B decays into two light mesons admit a heavy-quark expansion, a largely model-independent description of these decays is obtained despite the fact that they are exclusive nonleptonic processes. In the future, a precise measurement of the $B^\pm \to \pi K$ decay amplitudes will provide an extraction of γ with a theoretical uncertainty of about 10°, and at the same time will allow for sensitive tests of physics beyond the Standard Model.

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