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Luminosity and disruption in e^-e^- linear colliders*

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Abstract

We present an empirical analytic approximation for the luminosity "de-enhancement" factor H_D in e^-e^- collisions, as a function of the disruption parameters D_x, D_y, the hour-glass parameters A_x, A_y, and the beam aspect ratio $R \equiv \sigma_x/\sigma_y$. We treat Gaussian beams with essentially arbitrary aspect ratio, assuming only that the vertical beam size is less than or equal to the horizontal beam size and that the vertical beta function is less than or equal to the horizontal beta function.

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LUMINOSITY AND DISRUPTION IN e^-e^- LINEAR COLLIDERS

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We present an empirical analytic approximation for the luminosity "de-enhancement" factor H_D in e^-e^- collisions, as a function of the disruption parameters D_x, D_y, the hour-glass parameters A_x, A_y, and the beam aspect ratio $R \equiv \sigma_x/\sigma_y$. We treat Gaussian beams with essentially arbitrary aspect ratio, assuming only that the vertical beam size is less than or equal to the horizontal beam size and that the vertical beta function is less than or equal to the horizontal beta function.

1 Introduction

The purpose of this note is to give simple analytic formulas for use in e^-e^- collider physics programs and design studies. Such formulas complement those previously given in the literature[1] for e^+e^-. We use simple physical arguments to guide us toward empirical fits to beam-beam simulations using the GUINEAPIG program[2].

The geometric luminosity per bunch, not taking account of disruption or hour-glass effect, is given by

$$\mathcal{L}_0 \equiv \frac{N^2}{4\pi\sigma_x\sigma_y} \quad , \tag{1}$$

where N is the number of particles per bunch and $\sigma_{x,y}$ are the horizontal and vertical beam sizes. We assume the beam distributions are Gaussian longitudinally and transversely.

The hour-glass effect reduces the undisrupted luminosity unless the parameters

$$A_{x,y} \equiv \frac{\sigma_z}{\beta^*_{x,y}} \tag{2}$$

are much less than 1. Here σ_z is the bunch length and $\beta^*_{x,y}$ are the horizontal and vertical betatron functions at the collision point.

The disruption parameters $D_{x,y}$ are defined by

$$D_{x,y} = \frac{2r_e\sigma_z N}{\gamma\sigma_{x,y}(\sigma_x + \sigma_y)} \quad . \tag{3}$$

We define the luminosity pinch enhancement factor by

$$H_D \equiv \frac{\mathcal{L}_D}{\mathcal{L}_0} \quad . \tag{4}$$

Here $\mathcal{L}_D$ denotes the actual luminosity with disruption and hour-glass effect taken into account. (Caution: H_D is sometimes defined as $\mathcal{L}_D/\mathcal{L}_A$ where $\mathcal{L}_A$ is the luminosity with hour-glass effect taken into account.) For e^+e^- collisions, $H_D > 1$ is greater than one, while for e^-e^- collisions, $H_D < 1$.

2 Simulation of $\mathbf{H_D}$ and analytic approximation for round beams

We begin by focusing on the case of round beams ($\sigma_x = \sigma_y$). We also assume $\beta_x = \beta_y$, so that $A_x = A_y$ (it would be possible to have different horizontal and transverse beta functions if the horizontal and transverse emittances also differed, but this is not generally the case for round beam designs).

First we checked that turning beamstrahlung on and off in simulations does not have a large effect on the luminosity. We found that even for fairly extreme parameter regimes (i.e., $H_D \approx 0.25$), simulation results with and without beamstrahlung turned on differ by only a few percent. We then went on to simulate a number of cases, varying A and D. The results are plotted as the solid curves in Figure 1.

The major features of these curves can be easily understood on physical grounds. For very small disruption, H_D asymptotically approaches the value expected from the hour-glass effect alone:

$$H_D \approx \eta_A \equiv \frac{2}{\sqrt{\pi}A}\int_0^\infty \frac{e^{-u^2/A^2}}{1+A^2u^2}du \quad , \tag{5}$$

For $0 < A < 1$, a reasonably good expansion is $\eta_A \approx 1 - A^2/4$.

One might try to factorize H_D as $H_D = F_D\eta_A$. Note, however, that for very large disruption the divergence of the beam due to the final focus system, represented by A, will be completely overwhelmed, explaining why the simulation curves for different A converge at large D. For $D \gg 1$, the beams disrupt each other away within a distance σ_z/D and the effective value of A becomes

$$\tilde{A} = A/D = \frac{2(\gamma\epsilon)}{r_e N} \quad , \tag{6}$$

depending only on inherent properties of the beam.

To smooth the transition between the regimes of D, we take

$$\eta(A,D) \approx 1 - \frac{1}{4}\left(\frac{A}{1+bD}\right)^2 \quad . \tag{7}$$

where b is an adjustable parameter (we can in addition adjust the parameter $1/4$ in front).

A derivation of F_D for round, Gaussian beams and small D^3 goes through for e^-e^- with only a change of sign from that for e^+e^-, and yields

$$F_D \approx 1 - \frac{2D}{3\sqrt{\pi}} \quad . \tag{8}$$

This is just the small-argument expansion of $\exp\left(-\frac{2D}{3\sqrt{\pi}}\right)$ so we might try matching onto that for larger D. One finds that the exponential drops off too quickly, but a modified Bessel function I_0 can be introduced to moderate this drop-off. We try

$$F_D = \exp\left(-\frac{2D}{3\sqrt{\pi}}\right) I_0\left(\frac{2D}{3\sqrt{\pi}}\right) \quad . \tag{9}$$

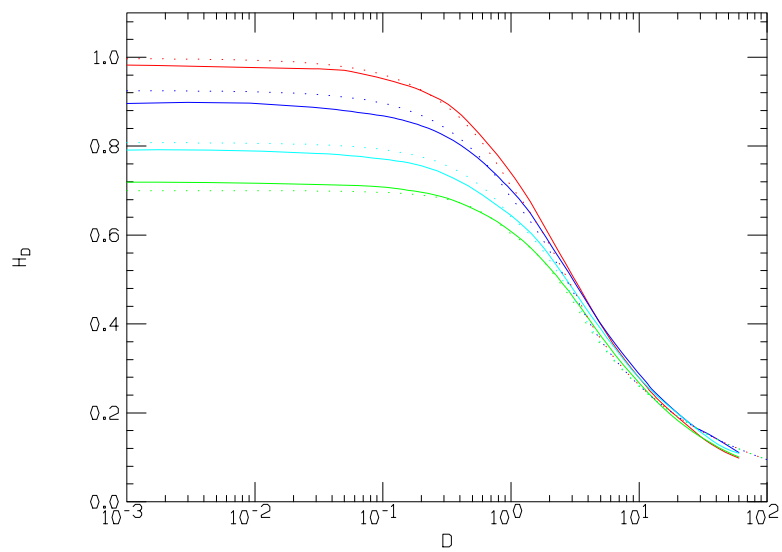

Figure 1: Disruption "de-enhancement" factor H_D as a function of D, for round beams. From top to bottom, the curves shown are for $A = 0.1$, $A = 0.5$, $A = 0.8$, $A = 1$. The solid curves are the simulation results and the dotted curves are the analytic approximation.

We could also adjust the coefficient $\frac{2}{3\sqrt{\pi}} \approx 0.376$ that appears in front of D in both the exponential and in I_0. But we did not gain a significant improvement by changing this coefficient in either or both of these places where it appears.

The modified Bessel function has expansions for large and small x that agree well for $x \sim 1$ and thus can be used to cover the entire range of D. These are given by:

$$I_0(x) = \begin{cases} 1 + \dfrac{x^2}{4} + \dfrac{x^4}{64} + \dfrac{x^6}{2304} & (x < 1) \\ \dfrac{e^x}{\sqrt{2\pi x}}\left[1 + \dfrac{1}{8x} + \dfrac{9}{128x^2}\right] & (x > 1) \end{cases} \quad . \tag{10}$$

Finally, to get a good fit for $1 \leq D \leq 100$ we needed to introduce a purely empirical fudge factor f_{ch} given by:

$$f_{ch}(D) = \begin{cases} 1 & (0 \leq D \leq 1) \\ 1 + 0.1 \ln D & (1 \leq D \leq 100) \end{cases} \quad . \tag{11}$$

Our final analytic approximation for H_D for round, Gaussian e^-e^- collisions, is:

$$H_D = \left[1 - 0.3\frac{A^2}{(1+0.4D)^2}\right] \exp\left(-\frac{2D}{3\sqrt{\pi}}\right) I_0\left(\frac{2D}{3\sqrt{\pi}}\right) f_{ch}(D) \quad , \tag{12}$$

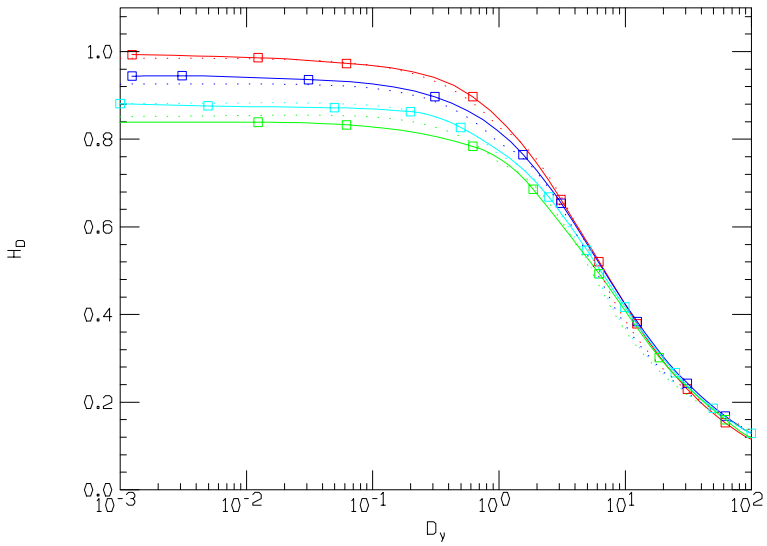

Figure 2: Disruption "de-enhancement" factor H_D as a function of D_y, for flat beams ($R = 100$). From top to bottom, the curves shown are for $A_y = 0.1$, $A_y = 0.5$, $A_y = 0.8$, $A_y = 1$. The solid curves are the simulation results and the dotted curves are the analytic approximation.

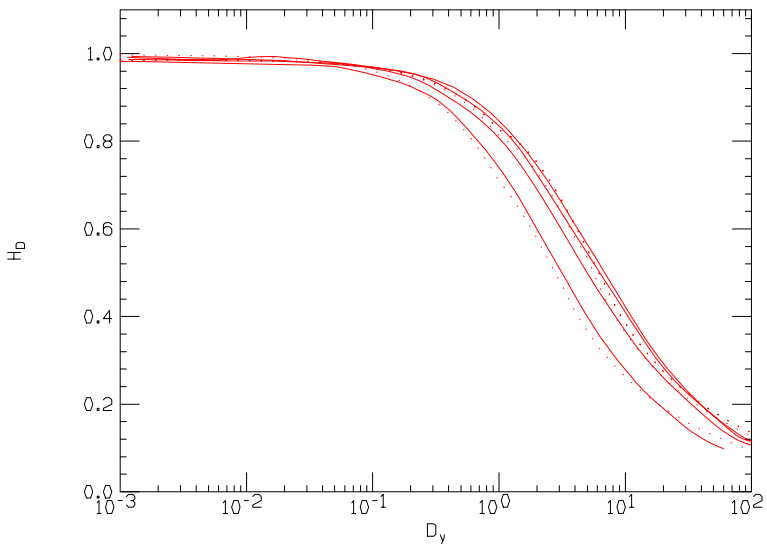

Figure 3: Disruption "de-enhancement" factor H_D as a function of D_y, for $A_y = 0.1$ and assuming $A_x \ll 1$. From top to bottom, the curves shown are for $R = 100$, $R = 10$, $R = 3$, $R = 1$. The solid curves are the simulation results and the dotted curves are the analytic approximation.

where we use the expansion for I_0 given above. This analytic approximation is shown as the dotted curves in Figure 1.

3 Generalization of $\mathbf{H_D}$ approximation to non-round beams

Next we generalize our results to non-round beams. Let the vertical dimension be the smaller dimension of the beam, and define the aspect ratio of the beam by $R \equiv \sigma_x/\sigma_y$.

Simulation results for the case of a very flat beam are plotted as the solid curves in Figure 2. The results shown are for the case $R = 100$ but are not very sensitive to R provided it is significantly greater than 1 — for example, we see from Figure 3, which shows H_D as a function of D for $A_y = 0.1$ and R varying from 1 to 100 (with $A_x \ll 1$), that the curves for $R = 10$ and $R = 100$ are not very different.

We look for an approximation to H_D that is a function of D_y, A_y, and R, and is valid for beams of any aspect ratio ($R = 1$ to ∞). We will assume A_x is small enough that the hour-glass effect is not significant in the horizontal plane unless the beam is close to round and A_y is near 1, i.e., if the beam is not round, we will assume $A_x \ll 1$, and if it is round, we will assume $A_x = A_y$. These assumptions generally hold in linear collider designs, since $\beta_x^\star > \beta_y^\star$ for flat-beam designs, $\beta_x^\star = \beta_y^\star$ for round-beam designs, and $A_y \leq 1$.

Let us define

$$f(R) \equiv 1 + \frac{1}{R^2} \quad . \tag{13}$$

Thus $f(R) = 2$ for round beams and $f(R) \approx 1$ for flat beams. We find that the following simple generalization of our round beam expression works very well for arbitrary $R \geq 1$:

$$H_D = \left[1 - 0.15 f(R) \left(\frac{A_y}{1 + 0.4 D_y}\right)^{f(R)}\right] \exp\left(-\frac{f(R) D_y}{3\sqrt{\pi}}\right) I_0\left(\frac{f(R) D_y}{3\sqrt{\pi}}\right) f_{ch}(D_y) \quad , \tag{14}$$

where the expansion for I_0 and the expression for f_{ch} are given in the previous section. This expression is identical to that in the previous section when $R = 1$.

Our analytic approximation for the flat beam case ($R \gg 1$) is shown as the dotted curves in Figure 2. We show also the result of this analytic approximation for R= 1, 3 ,10, and 100, and $A_y = 0.1$, as the dotted curves in Figure 3.

In summary, our expression for H_D agrees with simulations to within about 10% (as is the case for the e^+e^- expressions given in Reference 1) over the parameter ranges of interest for linear colliders. The agreement is even better over the most interesting parameter ranges, namely those where H_D is not too much less than 1.

References

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