SLAC-PUB-8047
CSUHEP 99/01
BaBar Note #481
hep-ph/9902313
February 1999

Discrete Ambiguities in the Measurement of the Weak Phase γ

Abner Soffer*

Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309, USA

Abstract

Several methods have been devised for measuring the phase γ of the Cabibbo-Kobayashi-Maskawa [1] unitarity triangle, using decays of the type $B \to DK$. It is shown that these and other direct CP-violation measurements suffer from discrete ambiguity which is at least 8-fold. Combining two measurement methods helps reduce the ambiguity and the experimental error. The measurement sensitivity and new physics discovery potential are estimated using a full Monte Carlo detector simulation with realistic background estimates, giving particular consideration to ambiguities.

PACS numbers: 11.30.E 14.40.N, 13.25.H,

Submitted to *Physics Letters B*

---

*Permanent address: Department of Physics, Colorado State University, Fort Collins, CO 80523.
Work supported by Department of Energy contracts DE-AC03-76SF00515 and DE-FG03-93ER40788, and by the National Science Foundation

I. INTRODUCTION

$B \to DK$ decays, recently observed by CLEO [2], provide several ways to measure the weak phase $\gamma = \arg(-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*)$. Since non-standard model effects are expected to be small in such decays, comparing these measurements with experiments which are more sensitive to new physics may be used to test the standard model [3]. Gronau and Wyler (GW) [4] have proposed to measure γ in the interference between the $\bar{b} \to \bar{c}u\bar{s}$ decay $B^+ \to \bar{D}^0 K^+$ and the color-suppressed, $\bar{b} \to \bar{u}c\bar{s}$ decay $B^+ \to D^0 K^+$. Interference occurs when the D is observed as one of the CP-eigenstates $D^0_{1,2} \equiv \frac{1}{\sqrt{2}}\left(D^0 \pm \bar{D}^0\right)$, which are identified by their decay products. The interference amplitude is

$$\sqrt{2}\, A(B^+ \to \bar{D}^0_{1,2} K^+) = \sqrt{\mathcal{B}(B^+ \to D^0 K^+)} e^{i(\delta_B + \gamma)} \pm \sqrt{\mathcal{B}(B^+ \to \bar{D}^0 K^+)}, \tag{1}$$

where δ_B is a CP-conserving phase. The value of γ is extracted from this triangle relation and its CP-conjugate, disregarding direct CP-violation in D^0 decays [5]. Several variations of the method have been developed [6,7].

In practice, measuring the branching fraction $\mathcal{B}(B^+ \to D^0 K^+)$ requires that the D^0 be identified in a hadronic final state, $f = K^-\pi^+(n\pi)^0$, since full reconstruction is impossible in semileptonic decays, resulting in unacceptably high background. Atwood, Dunietz and Soni (ADS) [8] pointed out that the decay chain $B^+ \to D^0 K^+$, $D^0 \to f$ results in the same final state as $B^+ \to \bar{D}^0 K^+$, $\bar{D}^0 \to f$, where the $\bar{D}^0$ undergoes doubly Cabibbo suppressed decay. Estimating the ratio between the interfering decay chains, they obtain

$$\left| \frac{A(B^+ \to \bar{D}^0 K^+)\ A(\bar{D}^0 \to f)}{A(B^+ \to D^0 K^+)\ A(D^0 \to f)} \right| \approx \left| \frac{V_{cb}^*}{V_{ub}^*}\ \frac{V_{us}}{V_{cs}}\ \frac{a_1}{a_2} \right| \sqrt{\frac{\mathcal{B}(\bar{D}^0 \to f)}{\mathcal{B}(D^0 \to f)}} \approx 0.9. \tag{2}$$

The numerical value in Equation (2) was obtained using $|V_{cb}^*/V_{ub}^*| = 1/0.08$ [9], $|V_{us}/V_{cs}| = 0.22$, $|a_1/a_2| = 1/0.26$ [10], and

$$\mathcal{B}(\bar{D}^0 \to f)/\mathcal{B}(D^0 \to f) = 0.0072, \tag{3}$$

which is the ratio measured for $f = K^-\pi^+$ [11]. Equation (2) implies that sizable interference makes it practically impossible to measure $\mathcal{B}(B^+ \to D^0 K^+)$, and the GW method fails.

ADS proposed to use the interference of Equation (2) to obtain γ from the decay rate asymmetries in $B^+ \to f_i K^+$, where $f_i,\ i = 1, 2$, are two D final states of the type $K^-\pi^+(n\pi)^0$. Measuring the four branching fractions, $\mathcal{B}(B^+ \to f_i K^+)$, $\mathcal{B}(B^- \to \bar{f}_i K^-)$, one calculates the four unknowns $\mathcal{B}(B^+ \to D^0 K^+)$, γ, and the two CP-conserving phases associated with the two decay modes. $\mathcal{B}(B^+ \to \bar{D}^0 K^+)$ and the D^0 decay branching fractions will have already been measured to high precision by the time the rare decays $B^+ \to f K^+$ are observed. In addition to the similar magnitudes of the interfering amplitudes, large CP-conserving phases are known to occur in D decays [12], making large decay rate asymmetries possible in this method.

Jang and Ko (JK) [13] and Gronau and Rosner [14] have developed a γ measurement method similar to the GW method, but in which $\mathcal{B}(B^+ \to D^0 K^+)$ is not measured directly. Rather, it is essentially inferred by using the larger branching fractions of the decays $B^0 \to D^- K^+$, $B^0 \to \bar{D}^0 K^0$ and $B^0 \to D_{1,2} K^0$, solving in principle the problem presented by Equation (2).

II. DISCRETE AMBIGUITIES

GW recognized that their method has a 4-fold discrete ambiguity in the determination of γ, due to the invariance of the $\cos(\delta_B \pm \gamma)$ terms in the decay widths under the two symmetry operations

$$
\begin{aligned}
S_{\rm sign} &: \gamma \to -\gamma, \quad \delta_B \to -\delta_B \\
S_{\rm exchange} &: \gamma \leftrightarrow \delta_B.
\end{aligned}
\tag{4}
$$

GW noted that application of $S_{\rm sign}$ to γ values within the currently allowed range,

$$
g \equiv \{50^\circ \lesssim \gamma \lesssim 150^\circ\},
\tag{5}
$$

yields values which are not within g, and are therefore inconsistent with the standard model. We note, however, the existence of a third symmetry,

$$S_\pi : \gamma \to \gamma + \pi, \quad \delta_B \to \delta_B - \pi, \tag{6}$$

which doubles the ambiguity to 8-fold. In addition, if $\gamma' = S_\pi S_{\rm sign}\gamma$ and $\gamma \in g$, then very often $\gamma' \in g$ as well. S_π thus causes the seemingly unphysical $S_{\rm sign}$ ambiguity to have serious implications for the ability to test the standard model using such measurements. These observations apply not only to the $B \to DK$ measurement methods discussed above, but to all CP-violation experiments in which the measurable widths depend only on trigonometric functions of the sum of a weak phase and a CP-conserving phase. Note that this includes decay rate asymmetries of the form $\sin\delta_B \sin\gamma$. The existence of multiple CP-conserving phases may break the S_π symmetry, but often does not, as in the case of the JK method.

When the magnitude of an amplitude is not known a-priori, such as $\mathcal{B}(B^+ \to D^0 K^+)$ in the ADS method, new ambiguities may exist in addition to those of Equations (4) and (6), for certain values of the unknowns. This is because the measured branching fractions may be satisfied, or almost satisfied, by several different values of $\mathcal{B}(B^+ \to D^0 K^+)$, and hence γ. Such accidental ambiguities may be resolved by using additional decay modes with different CP-conserving phases, or by constraints arising from improved theoretical understanding of color suppression in these decays.

III. COMBINING THE ADS AND THE GW METHODS

Since the $\bar{b} \to \bar{u}c\bar{s}$ amplitude in $B \to DK$ is very small and hard to detect, several methods will have to be combined in order to make best use of the limited data. Quantitative estimates of the resulting gain in sensitivity are rarely conducted, since they require realistic efficiency and background estimates, and depend on specific phase values. Here we undertake this task for the case of combining the ADS and GW methods (contributions of the JK method are commented on later). In this scheme, one obtains the unknown parameters

$$\xi \equiv \left\{\mathcal{B}(B^+ \to D^0 K^+),\ \gamma,\ \delta_B,\ \delta_D\right\}, \tag{7}$$

where $\delta_D = \arg[A(D^0 \to f) A(\bar{D}^0 \to f)^*]$, by minimizing the function

$$\chi^2(\xi) = \left(\frac{a(\xi) - a_m}{\Delta a_m}\right)^2 + \left(\frac{\bar{a}(\xi) - \bar{a}_m}{\Delta \bar{a}_m}\right)^2 + \left(\frac{b(\xi) - b_m}{\Delta b_m}\right)^2 + \left(\frac{\bar{b}(\xi) - \bar{b}_m}{\Delta \bar{b}_m}\right)^2 \tag{8}$$

with respect to the parameters ξ. In Equation (8) we use the symbols

$$\begin{aligned} a_m &\equiv \mathcal{B}(B^+ \to f K^+) \\ b_m &\equiv \mathcal{B}(B^+ \to D^0_{1,2} K^+) \end{aligned} \tag{9}$$

to denote the experimentally measured decay rates of interest, and

$$\begin{aligned} a(\xi) &\equiv \left| \sqrt{\mathcal{B}(B^+ \to \bar{D}^0 K^+)\ \mathcal{B}(\bar{D}^0 \to f)} + \sqrt{\mathcal{B}(B^+ \to D^0 K^+)\ \mathcal{B}(D^0 \to f)} e^{i(\delta_D + \delta_B + \gamma)} \right|^2 \\ b(\xi) &\equiv \frac{1}{2} \left| \pm \sqrt{\mathcal{B}(B^+ \to \bar{D}^0 K^+)} + \sqrt{\mathcal{B}(B^+ \to D^0 K^+)} e^{i(\delta_B + \gamma)} \right|^2 \end{aligned} \tag{10}$$

to denote the corresponding theoretical quantities. $\bar{a}_m$, $\bar{b}_m$, $\bar{a}(\xi)$ and $\bar{b}(\xi)$ are the CP-conjugates of a_m, b_m, $a(\xi)$ and $b(\xi)$, respectively. Δx_m represents the experimental error in the measurement of the quantity x_m.

Several gains over the individual methods are immediately apparent: In the ADS method, a D decay mode is "wasted" on measuring the uninteresting CP-conserving phases. By contrast, when combining the methods, knowledge of a_m, b_m, $\bar{a}_m$ and $\bar{b}_m$ in a single mode is in principle enough to determine the four unknowns, ξ, even if $\delta_D = \delta_B = 0$. In practice, adding the $D^0_{1,2}$ modes will decrease the statistical error of the measurement. In both the GW and the ADS methods, the ability to resolve the S_{exchange} ambiguity depends on the degree to which δ_B varies from one B^+ decay mode to the other. Experimental limits on CP-conserving phases in $B \to D\pi$, $D^*\pi$, $D\rho$ and $D^*\rho$ [15] suggest that δ_B may be small, making the S_{exchange} resolution difficult. When combining the methods, however, we note that $b(\xi)$ and $\bar{b}(\xi)$ are invariant under $\gamma \leftrightarrow \delta_B$, whereas $a(\xi)$ and $\bar{a}(\xi)$ are invariant under $\gamma \leftrightarrow \delta_B + \delta_D$. The S_{exchange} ambiguity is thus resolved in a single B^+ and D decay mode in which δ_D is far enough from 0 or π.

IV. SIGNAL AND BACKGROUND ESTIMATES

We proceed to estimate the sensitivity of the γ measurement combining the ADS and GW methods, at a future, symmetric e^+e^- B-factory, operating at the $\Upsilon(4S)$ resonance. The detector configuration is taken to be similar to that of CLEO-III [16]. The integrated luminosity is 600 fb^{-1}, corresponding to three years of running at the full luminosity of 3×10^{34} cm^{-2} s^{-1} [17] with an effective duty factor of 20%.

Crucial to evaluating the measurement sensitivity is a reasonably realistic estimate of the background rate in the measurement of a_m, whose statistical error dominates the γ measurement error, $\Delta\gamma$. We estimated the background by applying reconstruction criteria to Monte Carlo events generated using the full, GEANT-based [18] CLEO-II detector simulation. The event sample consisted of about 19×10^6 $e^+e^- \to B\bar{B}$ events and 14×10^6 continuum $e^+e^- \to q\bar{q}$ events, where q stands for a non-b quark. Since the full simulation did not include a silicon vertex detector or Čerenkov particle identification system, these systems were simulated using simple Gaussian smearing. The Čerenkov detector was taken to cover the polar region $\cos\theta < 0.71$.

D^0 candidates (reference to the charge conjugate modes is implied) were reconstructed in the final states $K^-\pi^+$, $K^-\pi^+\pi^0$, and $K^-\pi^+\pi^-\pi^+$. The π^0 and D^0 candidate invariant masses were required to be within 2.5 standard deviations (σ) of their nominal values. A Dalitz plot cut was applied in the $K^-\pi^+\pi^0$ mode to suppress combinatoric background. The B^+ candidate energy was required to be within 2.5σ of the beam energy. The beam-constrained mass, $\sqrt{E_b^2 - P_B^2}$, where E_b is the beam energy and P_B is the momentum of the B^+ candidate, was required to be within 2.5σ of the nominal B^+ mass. Since the K^+ and the D^0 fly back-to-back, all charged daughters of the B^+ candidate were required to be consistent with originating from the same vertex point. Continuum background was suppressed by applying cuts on the cosine of the angle between the the sphericity axis of the B^+ candidate and that of the rest of the event, and on the output of a Fischer discriminant [2]. In background events, the reconstructed K^+ and K^- come from two different D mesons, or are due to $s\bar{s}$

popping, while signal events often contain a third kaon, originating from the other B meson in the event. As a result, 90% of the background events are rejected by requiring that an additional K^- or K_S be found in the event and be inconsistent with originating from the B^+ candidate vertex.

With the above event selection criteria, we find that continuum events account for over 80% of the remaining background, with a rate of 7 events per 10^8 charged B mesons produced. This is comparable to the expected signal yield. Under such low signal, high background conditions, significant improvement is obtained by conducting a multi-variable maximum likelihood fit. In this technique, cuts on the continuous variables are greatly loosened, and the separation of signal from background is achieved by use of a probability density function, which describes the distribution of the data in these variables. As has been the case in several CLEO analyses of rare B decays, we assume that the effective background level in the likelihood analysis, B, as inferred from the signal statistical error, $\Delta S = \sqrt{S+B}$, will be similar to the level obtained with the Monte Carlo simulation. Signal efficiency will increase, however, due to the looser selection criteria.

The expected number of $B^+ \to fK^+$ signal events is

$$N_a = N_{B+}\ a(\xi)\ \epsilon(K^+ f), \tag{11}$$

where N_{B+} is the number of B^+ mesons produced, and $\epsilon(K^+ f)$ is the probability that the final state be detected and pass the loosened selection criteria of the likelihood analysis. For given values of δ_D, δ_B and γ, we calculate $a(\xi)$ using the $D^0 \to K^-\pi^+,\ K^-\pi^+\pi^0,\ K^-\pi^+\pi^-\pi^+$ branching fractions from [11], Equation (3), $\mathcal{B}(B^+ \to \bar{D}^0 K^+) = 2.57\times 10^{-4}$ [2], and $\mathcal{B}(B^+ \to D^0 K^+) = 2.3\times 10^{-6}$ (obtained from $\mathcal{B}(B^+ \to \bar{D}^0 K^+)$ and the values used in Equation (2)).

To estimate the efficiency $\epsilon(K^+ f)$, we start with the values in [2], 44% for the $K^-\pi^+$ mode, 17% for the $K^-\pi^+\pi^0$ mode, and 22% for the $K^-\pi^+\pi^-\pi^+$ mode. These are multiplied by the efficiency of finding the third kaon (45%), and the particle-ID efficiency (68%). The particle-ID efficiency is composed of the probability that a well-reconstructed K^+ be in the particle-ID system's fiducial region (83%), and that half the K^- daughters of the

D meson also be in the fiducial region. The momentum of the other half allows good identification using specific ionization, as does the momentum of the third kaon in most events. An additional efficiency loss of 10% is assumed due to non-Gaussian tails, Čerenkov ring overlaps, etc. The final efficiencies are 13% for the $K^-\pi^+$ mode, 5% for the $K^-\pi^+\pi^0$ mode, and 7% for the $K^-\pi^+\pi^-\pi^+$ mode.

Since $b_m \gg a_m$, suppression and accurate knowledge of the background in the measurement of b_m is much less critical. Starting from the continuum background level in [2] and applying vertex and particle-ID criteria, we arrive at a rate of 60 background events per 10^8 charged B mesons. The number of signal events observed in this channel is

$$N_b = N_{B^+}\, b(\xi)\, \epsilon(K^+) \sum_i \mathcal{B}(D^0 \to c_i)\, \epsilon(c_i), \tag{12}$$

where $\epsilon(K^+)$ is the efficiency for detecting the K^+ with the particle-ID criteria described above, and c_i are CP-eigenstate decay products of $D_{1,2}$. Using Table I, we obtain $\sum_i \mathcal{B}(D^0 \to c_i)\, \epsilon(c_i) = 0.011$.

V. MEASUREMENT SENSITIVITY

To estimate the measurement sensitivity for given values of the "true" parameters $\xi = \xi^0$, we compute the average numbers of observed signal events using Equations (11) and (12). An integrated luminosity of 600 fb^{-1} yields $N_{B^+} = 640 \times 10^6$. We assume that statistics will effectively triple if, in addition to $B^+ \to D^0 K^+$, one uses the modes $B^+ \to D^0 K^{*+}$, $B^+ \to D^{*0} K^+$, $B^+ \to D^{*0} K^{*+}$, $\bar{B}^0 \to D^0 K^{*0}$ and $\bar{B}^0 \to D^{*0} K^{*0}$. We therefore take $N_{B^+} = 1900 \times 10^6$. The resulting N_a, N_b and their CP-conjugates determine the experimental quantities a_m, b_m, $\bar{a}_m$ and $\bar{b}_m$ in the average experiment, ie., the experiment in which statistical fluctuations vanish. The minimization package MINUIT [19] is then used to find the parameters ξ, for which $\chi^2(\xi)$ is minimal in this experiment. Since the measurement is expected to be statistics-limited, only statistical errors are used to evaluate $\chi^2(\xi)$.

To demonstrate ambiguities, the trial value of γ is stepped between -180° and 180°, and δ_D, δ_B and $\mathcal{B}(B^+ \to D^0 K^+)$ are varied by MINUIT so as to minimize $\chi^2(\xi)$. Such γ scans are shown in Figure 1 for cases of particular interest. Evident from these scans is the fact that a large $\partial^2\chi^2(\xi)/\partial\gamma^2$ at the input value $\gamma = \gamma^0$ does not guarantee that $\chi^2(\xi)$ will obtain large values before dipping into a nearby ambiguity point. As a result, the quantity that meaningfully represents the measurement sensitivity is not $\Delta\gamma$, but $f_{\rm exc}$, the fraction of g which is excluded by the $B \to DK$ measurement, ie., for which $\chi^2(\xi) > 10$. The larger the value of $f_{\rm exc}$, the greater the a-priori likelihood that predictions of γ based on new physics-sensitive experiments will be inconsistent with the $B \to DK$ measurement, leading to the detection of new physics.

To evaluate $f_{\rm exc}$, 540 Monte Carlo experiments were generated, using randomly selected input values in the range $\gamma^0 \in g$, $-180^\circ < \delta_D^0 < 180^\circ$, $-180^\circ < \delta_B^0 < 180^\circ$ (Note that in reality, the CP-conserving phases will be different in the different decay modes). Depending on the input phases, the numbers of observed signal events varied between $700 < N_b < 1050$, $0 < N_a < 130$. For each set of phases, a γ scan was conducted in the range $\gamma \in g$, and $f_{\rm exc}$ was taken to be the fraction of the area of the scan for which $\chi^2(\xi) > 10$.

The $f_{\rm exc}$ distribution of the 540 random experiments is shown in Figure 2. Also shown is the distribution of the 93 experiments for which $|\sin(\delta_B)| < 0.25$. $f_{\rm exc}$ tends to be larger in this case, since small values of $\chi^2(\xi)$ associated with the $S_{\rm exchange}$ ambiguity (even if the ambiguity is resolved) are pushed away from the center of g. Since the distributions of phases used in the Monte Carlo experiments cannot be expected to represent the actual phases in nature, it is not meaningful to study the $f_{\rm exc}$ distribution in detail. Nevertheless, Figure 2 indicates that this measurement may reduce the allowed region of γ by as much as 70%.

VI. DISCUSSION AND CONCLUSIONS

We have studied in detail the measurement of γ using $B \to DK$ at a symmetric B factory. Use of this measurement to detect new physics effects is complicated by low statistics and an ambiguity which is at least 8-fold, not 4-fold as often stated. We show that combining the ADS and GW methods helps resolve the $S_{\rm exchange}$ ambiguity and decreases the statistical error, compared with the ADS method alone. The ambiguities associated with the $S_{\rm sign}$ and S_π symmetries are irremovable in measurements of this kind. Even when the $S_{\rm exchange}$ ambiguity is in principle resolved, in practice it still deteriorates the measurement by reducing $\chi^2(\xi)$ (or other experimental quantity of significance).

Being ambiguity-dominated, the sensitivity of future experiments should be evaluated in terms of the exclusion fraction $f_{\rm exc}$, rather than the weak phase error $\Delta\gamma$. With a luminosity of 600 fb^{-1}, we find that the $B \to DK$ measurement can exclude up to about $f_{\rm exc} \lesssim 0.7$ of the currently-allowed range of γ.

With 3×10^8 B mesons, 100% efficiency and no background, JK find $\Delta\gamma$ in their method to be between about 5° and 30° for $50^\circ < \gamma < 150^\circ$. Using more realistic estimates and noting out comments above, one would conclude that combining their method with the ADS and GW methods, while probably useful for the actual experiment, will not result in a dramatic change in the predictions of our analysis.

VII. ACKNOWLEDGMENTS

I am grateful to my colleagues at the CLEO collaboration for permitting the use of the excellent Monte Carlo sample which they have worked hard to tune and produce; to David Asner and Jeff Gronberg for sharing their knowledge of the performance of the CLEO silicon vertex detector; and to Michael Gronau and Yuval Grossman for discussions and useful suggestions. This work was supported by the U.S. Department of Energy under contracts DE-AC03-76SF00515 and DE-FG03-93ER40788, and by the National Science Foundation.

REFERENCES

[1] M. Kobayashi and K. Maskawa, Prog. Theor. Phys. **49** (1973), 652.

[2] CLEO Collaboration, M. Athanas *et al*, Phys. Rev. Lett. **80** (1998), 5493.

[3] See, for example, Y. Grossman, Y. Nir, and M.P. Worah, Phys. Lett. **B407** (1997), 307.

[4] M. Gronau and D. Wyler, Phys. Lett. **B265** (1991), 172.

[5] Y. Grossman and M.P. Worah, Phys. Lett. **B395** (1997), 241.

[6] I. Dunietz, Phys. Lett. **B270** (1991), 75.

[7] D. Atwood, G. Eilam, M. Gronau, and A. Soni, Phys. Lett. **B341** (1995), 372.

[8] D. Atwood, I. Dunietz, and A. Soni, Phys. Rev. Lett. **78** (1997), 3257.

[9] CLEO Collaboration (J.P. Alexander *et al.*), Phys. Rev. Lett. **77** (1996), 5000.

[10] CLEO Collaboration (B. Barish *et al.*), CLEO CONF 97-01, EPS 339.

[11] C. Caso *et al.*, (Particle Data Group), Eur. Phys. J. **C** (1997).

[12] See, for example, E687 Collaboration, P.I. Frabetti *et al*, Phys. Lett. **B331** (1994), 217.

[13] J.H. Jang and P. Ko, Phys. Rev. **D58** (1998), 111302.

[14] M. Gronau and J.L. Rosner, Phys. Lett. **B439** (1998), 171.

[15] H. Yamamoto , CBX 94-14, HUTP-94/A006; H.N. Nelson, private communication.

[16] S. Stone, Nucl. Instrum. Meth **A368** (1995), 68.

[17] D. Rubin, G. Dugan, A. Mikhailichenko, and J. Rogers, "Dual Aperture High Luminosity Collider at Cornell", to be published in Proc. 1997 Particle Accelerator Conference, Vancouver, B.C., Canada, May 12-16 (1997).

[18] R. Brun *et al.*, GEANT 3.15, CERN DD/EE/84-1.

t	$\mathcal{B}(D^0 \to t)$	$\epsilon(t)$	$\mathcal{B} \times \epsilon$
$K_S\ \pi^0$	0.011	0.22	0.0024
$K_S\ \eta(\to \gamma\gamma)$	0.0036	0.087	0.0003
$K_S\ \rho^0$	0.0061	0.28	0.0017
$K_S\ \omega(\to \pi^+\pi^-\pi^0)$	0.011	0.13	0.0014
$K_S\ \eta'(\to \pi^+\pi^-\eta)$	0.0086	0.062	0.0005
$K_S\ \eta'(\to \rho^0\gamma)$	0.0086	0.068	0.0006
$K_S\ \phi(\to K^+K^-)$	0.0043	0.14	0.0006
K^+K^-	0.0043	0.64	0.0028
$\pi^+\pi^-$	0.0015	0.64	0.0010
total			0.011

TABLE I. Branching fractions [11] of D^0 decays to CP-eigenstates, assumed reconstruction efficiencies, and their products. Efficiencies include sub-mode branching fractions, such as $K_S \to \pi^+\pi^-$, and are constructed assuming 80% track efficiency and 50% π^0 efficiency.

[19] F. James, MINUIT, CERN Program Library Long Writeup D506.

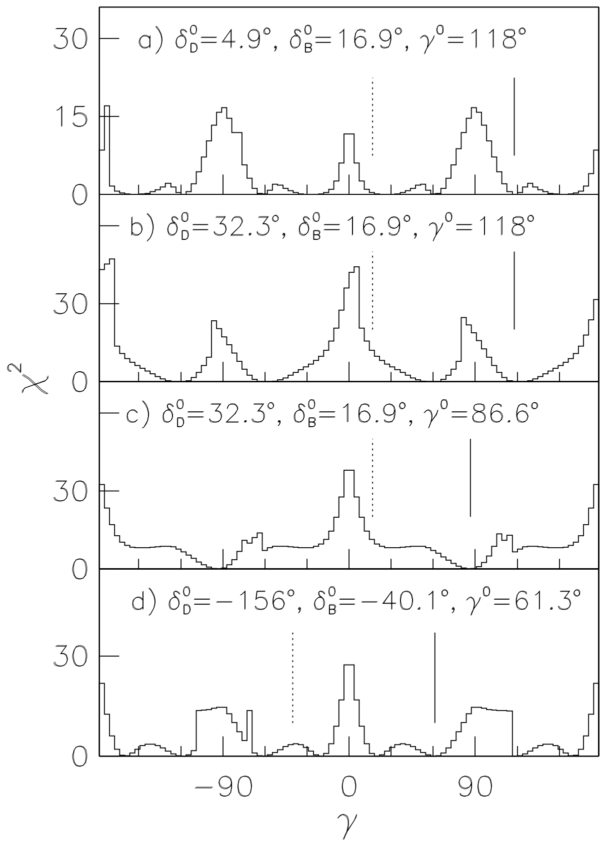

FIG. 1. $\chi^2(\xi)$ as a function of γ for different values of the actual phases, δ_D^0, δ_B^0, γ^0. For each value of γ, $\chi^2(\xi)$ is minimized with respect to $\mathcal{B}(B^+ \to D^0 K^+)$, δ_D and δ_B. The points $\gamma = \gamma^0$ and $\gamma = \delta_B^0$ are shown by a solid and a dotted line, respectively. Some asymmetry and noise are due to the dependence of the fit on the initial ξ values. **a)** The 8-fold ambiguity of Equations (4) and (6) is demonstrated for small δ_D^0. **b)** Increasing δ_D, the S_{exchange} ambiguity is resolved. **c)** With γ close to 90°, the S_π and S_{sign} ambiguities overlap. **d)** The S_{exchange} ambiguity is resolved, but an accidental ambiguity shows up at $\gamma \approx 17°$, with $\mathcal{B}(B^+ \to D^0 K^+)$ at approximately double its input value.

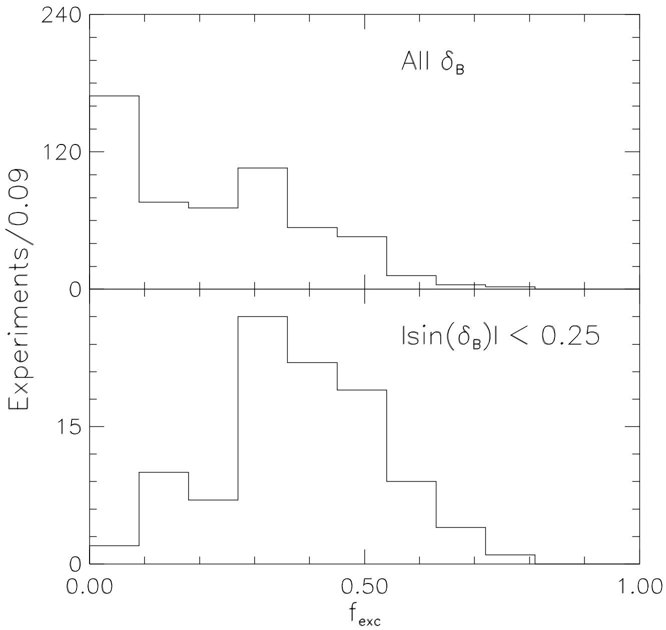

FIG. 2. The $f_{\rm exc}$ distribution of all Monte Carlo experiments conducted, and experiments with $|\sin(\delta_B)| < 0.25$.