A Generalised Area Law for Hadronic String Reinteractions[*]

Johan Rathsman

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94309, USA
e-mail: rathsman@slac.stanford.edu

Abstract

A new model for hadronic string reinteractions based on a generalised area law is presented. The model describes both the hadronic final states in e^+e^- annihilation and the diffractive structure function in deep inelastic scattering. The model also predicts a shift in the W-mass reconstructed from hadronic decays of W-pairs of the order 65 MeV.

Submitted to Physics Letters B

The problem addressed in this letter is how to translate a partonic final state consisting of quark and gluons, calculated in perturbative quantum chromo dynamics (QCD), into a final state of hadrons. Since this a process that takes place at low momentum transfer, perturbative methods cannot readily be applied. Instead one has to take resort to phenomenological models, like the Lund string model [1], to describe this non-perturbative transition. The string model is a semi-classical model which, when combined with perturbatively calculated partonic cross-sections, gives a good overall description of observed hadronic final states, especially in e^+e^- collisions.

One of the basic assumptions of the Lund string model is that the colour field between two colour charges forms a flux-tube with the dynamical properties of a one-dimensional relativistic string. In the simplest case with just one quark and one anti-quark connected by a string this gives the so called yo-yo model. Another assumption which is usually made in the application of the Lund string model is that two different strings will hadronise independently of each other even if they overlap. In other words there is no cross-talk or string reinteractions in a system consisting of more than one string.

This letter presents a new general model for taking string reinteractions into account in hadronic final states. String reinteractions as a way of understanding diffractive events was suggested a long time ago [2]. Similar ideas have also been considered in connections with W-pair production [3] and in the model for soft colour interactions (SCI) which originally was formulated for rapidity gaps in deep inelastic scattering [4,5]. The starting point for the present model will be the string configuration given by the colour flow in a reaction on partonic level where the parton configuration has been calculated perturbatively. On top of that interactions in the form of string rearrangements will be added. Thus, giving an altered colour configuration and thereby a different hadronic final state. The main difference compared to SCI is that the model presented here considers string reinteractions where the colour fields in the two strings interact whereas in SCI the perturbative partons interact with a hadronic background field. As examples of consequences of the model, rapidity gaps in e^+e^- annihilation and deep inelastic scattering as well as a shift in the W-mass reconstructed from hadronic decays of W-pairs will be considered in some detail.

The model. The Lund String model is based on the so called area law [6]. Simply put the area law means that configurations with a large area are exponentially suppressed. Within the Lund String model the probability for a configuration with area A is given by,

$$P \propto \exp(-bA), \tag{1}$$

where b is a phenomenological parameter of the order 0.6 GeV^{-2} if the area is calculated in energy-momentum coordinates. In the following, the area for a piece of string spanned between two partons p_i and p_j will be calculated as

$$A_{ij} = (p_i + p_j)^2 - (m_i + m_j)^2 = 2(p_i p_j - m_i m_j) \tag{2}$$

such that the area vanishes for two massive partons at rest. This way of defining the area reduces to the ordinary $A = p_i^+ p_j^- = (E + |p_z|)^2$, in the center of mass system with the partons along the z-axis, when the quark masses are neglected.

For a single string the only way to decrease the area is by "popping" quark pairs from the vacuum. However, for a system consisting of several strings it may also be possible to

decrease the area by doing a string rearrangement. The simplest example is given by the situation where there are two strings, each consisting of a quark anti-quark pair as illustrated in Fig. 1. Labeling the momenta of the partons in the two initial strings (p_1, p_2) and (p_3, p_4) respectively it may be possible that the system can decrease its area by making a string rearrangement into (p_1, p_4) and (p_2, p_3). The initial area is given by $A^{old} = A_{12} + A_{34}$ where as the alternative configuration has the area $A^{new} = A_{14} + A_{23}$.
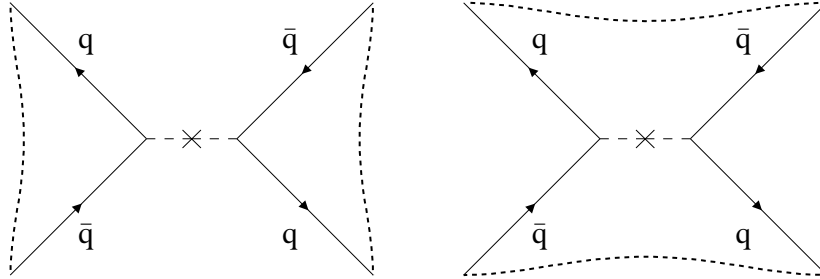

FIG. 1. Illustration of two different string configurations in W-pair production in the lowest order (the strings are indicated with dashed lines).

Generalising the area law to be applicable to a system of strings the probabilities for these two different configurations are given by $\exp(-bA^{old})$ and $\exp(-bA^{new})$ respectively times some overall factor depending on the parton momenta which is the same for both of them. In this paper it will be assumed that the probability for a string rearrangement is proportional to the normalised difference between the two configurations

$$P \propto \frac{\exp(-bA^{new}) - \exp(-bA^{old})}{\exp(-bA^{new})} = 1 - \exp(-b\Delta A) \tag{3}$$

where $\Delta A = A^{old} - A^{new}$ is the area difference between the old and the new configuration.

In addition there should also be a colour suppression factor of the order $\frac{1}{N_C^2}$ reflecting the required colour matching of the two strings. Since in this model it is the strings that are thought to interact it is the colour field in the string that is relevant and not the colour charges at the endpoints of the string. This should be compared with the fact that in a quark–gluon–anti-quark string the quark and the anti-quark are not in a colour singlet state. The string carries a octet charge and thus the colour factor required for matching the colour field in the two strings is of the order $\frac{1}{N_C^2}$.

The phenomenological ansatz of this paper will then be to have a string rearrangement between a pair of string pieces with the probability

$$P = R_0 \left[1 - \exp(-b\Delta A)\right] \tag{4}$$

where R_0 is a nonperturbative parameter of the order $\frac{1}{N_C^2}$. The parameter will be fixed to $R_0 = 0.1$ by comparing with data on rapidity gap events in deep inelastic scattering. This

formula for the probability should be compared with the SCI model where the probability is assumed to be a constant, i.e. without the suppression factor $[1 - \exp(-b\Delta A)]$[1]. The actual implementation of the string rearrangements in the model is very similar to the one used for the SCI model. In short there is a loop over all pairs of string pieces which have an alternative string configuration. A string rearrangement between the two string pieces is then made with the probability given by Eq. (4). In principle one has to worry about the fact that the order of the pairs in the loop can make a difference. However, since the probability to make a string rearrangement is small, this is a minor problem and the effects have been neglected.

For simplicity it will also be assumed that the hadronisation can be factored into two different phases, the first one being the possibility to decrease the area by string rearrangements and the second one being the ordinary string breakup into hadrons according to the Lund string model. On general grounds one would expect that the two ways of decreasing the string area are intertwined but as a first phenomenological ansatz it should be enough to consider the string rearrangements in one separate step.

The model is implemented in the Lund Monte Carlo (MC) framework[2]. In short a Lund MC contains the matrix-elements for the hard interaction to which additional QCD radiation is added using the parton shower approach down to a cut-off scale for perturbative QCD which normally is in the order of 1 GeV. The colour configurations considered in the parton shower is based on the planar diagrams. The perturbative partons are connected with strings according to these colour configurations and the strings are then hadronised using the Lund string model. The place in this chain where the new model enters is thus after the string drawing but before the hadronisation.

Electron-positron annihilation. Since in this model the string rearrangement is thought of as a general phenomenon and not connected with some hadronic background field as in [4,5] it has first of all to be retuned to data from e^+e^- annihilation at $\sqrt{s} = M_Z$. For this purpose the Jetset Monte Carlo version 7.4 [7] will be used together with the model. For easy reference most of the times the default version of Jetset will be used for comparison instead of data and differences smaller than a few percent will be considered satisfactory.

The model is tuned using the particle multiplicities and momentum distributions. By retuning the parameter b in the fragmentation function to $b = 0.45$ GeV^{-2} and the cut-off in the parton showers to $Q_0 = 2$ GeV, the mean multiplicity of charged particles ($<n_{ch}> = 20.9$) as well as the dispersion ($D = \sqrt{<n_{ch}^2> - <n_{ch}>^2} = 6.2$) are the same as in the default version. The resulting charged multiplicity distribution is shown in Fig. 2(a) together with the default version of Jetset and data from the ALEPH collaboration [8]. The multiplicities for individual mesons and baryons are typically within a percent of the default version.

Considering the momentum distributions the differences are typically of the order a few

---

[1]In an earlier version of the model the string-length was used to formulate a suppression factor. I would like to thank Gunnar Ingelman for suggesting to use the string-area instead when deriving the suppression factor

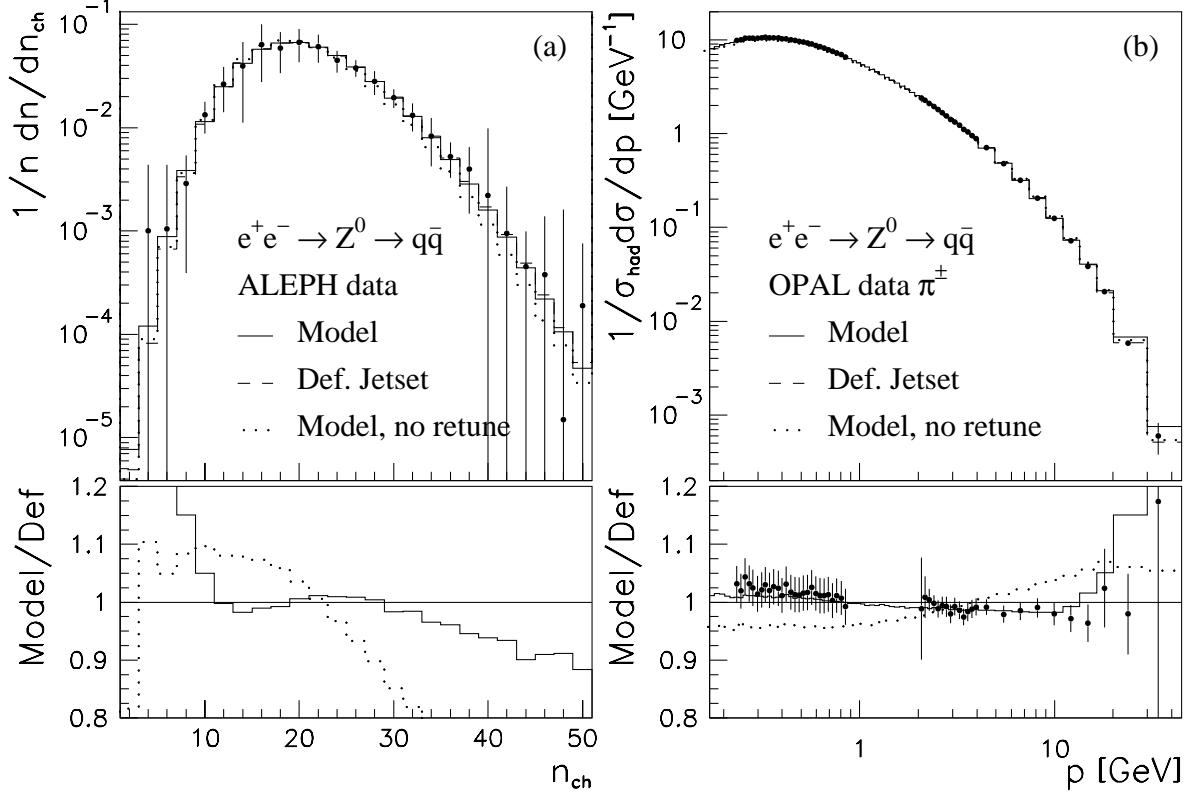[2]The code can be obtained from `http://www3.tsl.uu.se/thep/rathsman/gal/`

FIG. 2. The charged multiplicity distribution (a) and the momentum distribution for $\pi^{\pm}$ (b). The model (solid line) is compared with default Jetset (dashed line) and the model without retuning (dotted line). The data are from the ALEPH and OPAL collaborations respectively with the statistical and systematic errors added in quadrature. The ratio of the model, with (solid) and without (dotted) retuning, to default Jetset is shown below, (b) also shows the ratio of data to default Jetset.

percent which is about the size of the errors in the data. As an example Fig. 2(b) shows the $\pi^{\pm}$ momentum distribution for the new model with and without retuning compared to default Jetset and also compared with data from the OPAL collaboration [9]. As can be seen from the figure the new model is actually closer to the data than the default version and it should be possible to get a good agreement with data after a dedicated retuning.

The cut-off in the parton cascade obtained in the retuning, $Q_0 = 2$ GeV, is relatively large compared to the default $Q_0 = 1$ GeV. However, the range of validity of the perturbative description depends on what observable that is considered. For some inclusive quantities, like the energy distribution of hadrons, local parton hadron duality seems to be valid all the way down "to ... $k = $ NILL" [10]. For more exclusive final states like rapidity gaps considered here this need not be true. In this paper, the attitude taken is that the value of the cut-off Q_0 is a free parameter describing the boundary below which it is more fruitful to describe the fragmentation process in terms of strings instead of perturbative partons.

With the charged multiplicity and the momentum distributions used to retune the model, observables like the string effect, the thrust distribution as well as the rapidity and rapidity gap distribution will be used to check if the model is a viable alternative to the normal string model as implemented in Jetset.

The string effect [11–13] gives a measure of the colour structure in three-jet events in e^+e^- annihilation. In the the lowest order diagram $e^+e^- \rightarrow q\bar{q}g$ the string goes from the quark via the gluon to the anti-quark. In turn this means that there will be a depletion of particles produced between the two quark jets compared to the particle production between the gluon jet and either of the quark jets since there is no string between the quarks.

To study the string effect the JADE algorithm was used for jet reconstruction with the resolution $y_{cut} = 0.05$. The events giving three jets were analysed by projecting all particles on to the plane spanned by the two most energetic jets. Fig. 3(a) shows the particle flow in this plane as a function of the angle (ω) from the most energetic jet with ω defined such that the second jet has $\omega < \pi$. With the least energetic jet normally being the gluon jet this shows the relative depletion between the two quark jets. As can be seen from the figure the difference between the default version of Jetset and the model is very small (below a few percent as shown by the lower histogram giving the ratio of the two). For illustration the model without the area suppression factor is also shown.

Fig. 3(b) shows the thrust (T) distribution in the model compared with default Jetset. Thrust is an event shape observable which gives a measure of how isotropic an event is, $T = 1$ is an event with two back to back jets whereas $T = 0.5$ corresponds to an event with isotropic distribution of particles. The differences in the thrust distribution between the model and the default version of Jetset is very small, typically of the order of a few percent.

Another set of observables which are sensitive to the colour structure of an event are the rapidity and rapidity gap distributions. In e^+e^- annihilation the rapidity is usually defined along the thrust axis which will also be used here. The rapidity is thus defined as $y = \frac{1}{2}\ln\frac{E+p_z}{E-p_z}$, where the p_z is the momentum along the thrust axis and E is the energy. The rapidity and rapidity gap distributions have been measured by the SLD collaboration [14]. To be able to compare with these data the energy of a particle was calculated assuming the mass to be the $\pi^{\pm}$ mass and the rapidity calculated this way will be denoted y_π in the following.

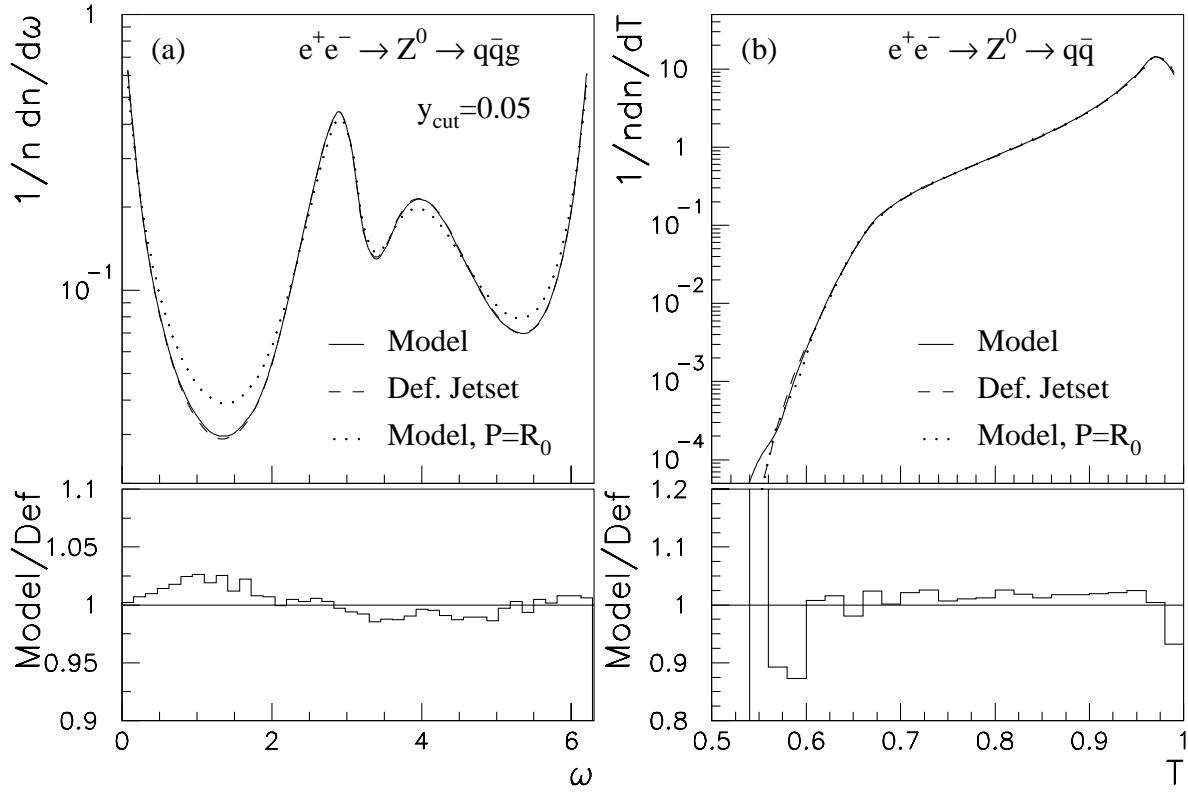Fig. 4 shows the rapidity distribution (a) as well as the distribution of the largest gap

6

FIG. 3. (a) Illustration of the string effect as explained in the text and (b) the thrust distribution. The model (solid line) is compared with default Jetset (dashed line) and the model without the area suppression factor (dotted line). The ratio of the model and default Jetset is shown below.
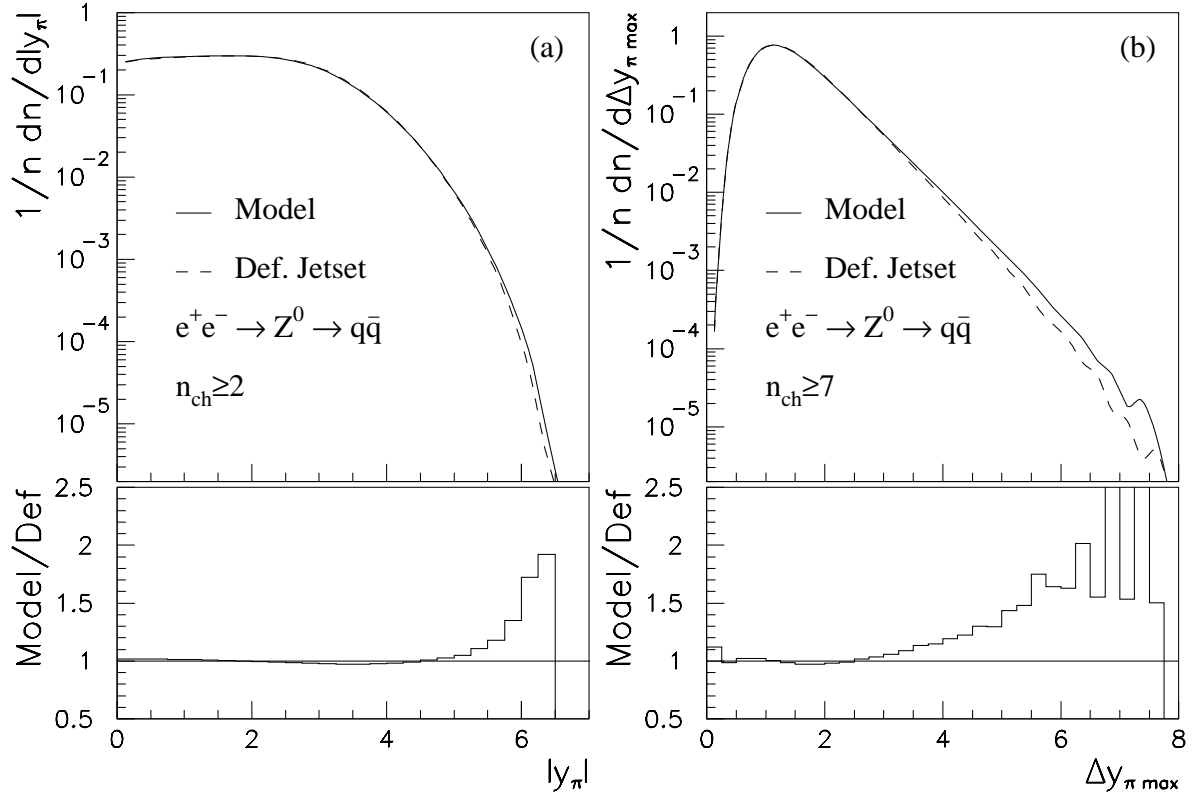
FIG. 4. The rapidity (a) and the largest rapidity gap (b) distribution in the model (full line) and default Jetset (dashed line). The ratio of the model and default Jetset is shown below.

in an event $\Delta y_{\pi \max}$ (b). As can be seen from the figure, the difference between the model and the default version of Jetset are small for $y_\pi < 5$ and $\Delta y_{\pi \max} < 4$ respectively but for larger values the difference grows and becomes as large as a factor 2 near the phase space limit $y_\pi = 6.5$ and for $\Delta y_{\pi \max} > 6$. However, judging by the figures presented in [14] the errors in the data points in this region is of the same size or even larger so at this point it is not possible to draw any conclusions from the existing measurement[3]. The important point to notice is that the model does not give any plateau in the rapidity gap distribution as one might expect but only a gradual decrease of the slope in the exponential suppression of large gaps. It should also be noted that the differences are quite sensitive to the specific cuts that are made and the specific gap observable under consideration. For example the SLD collaboration also presented data on the distribution in $y_{\pi 0}$ which is the size of a central gap with no particles in the region $|y_\pi| \leq y_{\pi 0}/2$. For this distribution the difference between the model and the default version of Jetset is much smaller and it only becomes larger than 10 % for $y_{\pi 0} > 6$ which is outside the experimentally measured region.

Diffractive deep inelastic scattering. As already mentioned the free parameter in the model $R_0 = 0.1$ is determined by calculating the diffractive structure function in deep inelastic scattering and comparing with data from the H1 collaboration [16]. The calculation is done using the Lepto Monte Carlo [17] version 6.5 with the CTEQ 4 leading order parton distributions [18] in a similar way that the diffractive structure function was calculated for the model of soft colour interactions in [4,5]. The only changes with respect to the default version of Lepto is to use the same values for the cut-offs in the initial and final state parton showers ($Q_0 = 2$ GeV) and the hadronisation parameter ($b = 0.45$ GeV^{-2}) as for e^+e^- annihilation given above[4]. To make sure that the cuts and other event selection procedures were the same as in the data analysis the diffractive structure function was evaluated using a subroutine from the HzTool package [19].

The agreement between the resulting diffractive structure function calculated from the model and the H1 data is very good as is shown in Fig. 5, especially if one takes into account that there is only one free parameter in the model. Both the so called Pomeron exchanges which are thought to dominate for small $x_{\rm I\!P}$, with $x_{\rm I\!P}$ being the longitudinal momentum fraction of the Pomeron with respect to the proton[5], as well as the other Regge exchanges which are important in the transition region $0.01 < x_{\rm I\!P} < 0.1$ are explained by the model. The model only fails for small masses of the diffractive system $M_X^2 = Q^2 \frac{1-\beta}{\beta}$ which are not included in the model because of the cut-off $M_X^2 > 4$ GeV^2 in the matrix-element.

W-pair production. The precise measurement of the W-mass is an important test of electroweak theory. At LEP2 W-pairs are produced close to threshold in e^+e^- annihilation.

---

[3]The SLD collaboration has more data on tape which is in the process of being analysed [15].

[4]In addition version 2 of the sea-quark treatment (see [5]) was used with the width of the mean virtuality set to 0.44 GeV. However, the result is not sensitive to this choice.

[5]With the mass of the diffractive system denoted by M_X, the photon virtuality Q^2 and W being the mass of the hadronic system, $x_{\rm I\!P} \simeq \frac{Q^2+M_X^2}{Q^2+W^2}$ and $\beta \simeq \frac{Q^2}{Q^2+M_X^2}$.
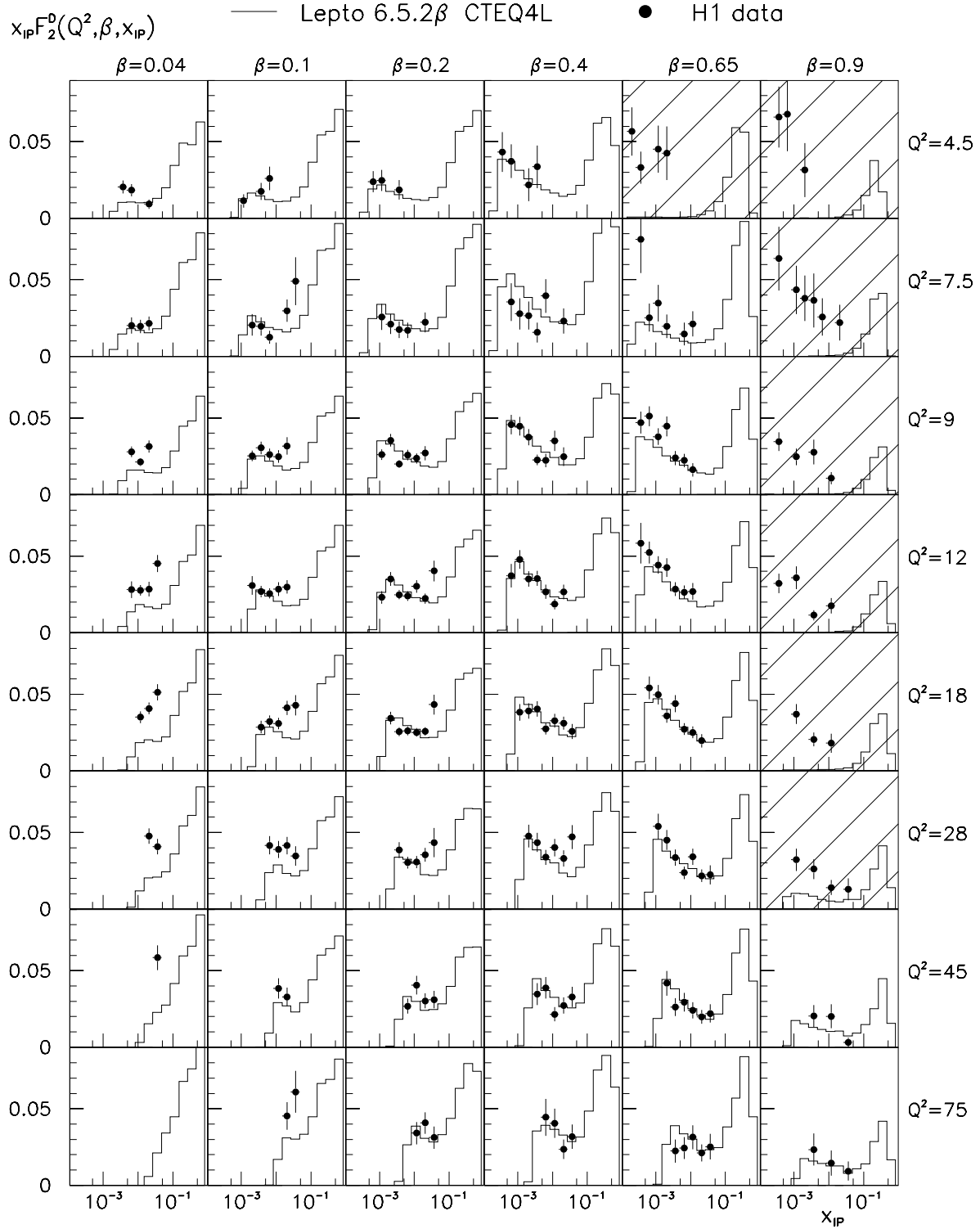
FIG. 5. The diffractive structure function obtained with the model applied to Lepto compared to data from the H1 collaboration. The hashed plots corresponds to kinematic points where the mass of the diffractive system M_X is smaller than 2 GeV which is the cut-off in the matrix element.

10

One way of measuring the W-mass directly is to reconstruct it from hadronic decays. When both of the W's decay hadronically one has to take into account the possibility that the two hadronic systems interfere with each other[6]. This was first studied in [3] whereas the effects on the W-mass was first considered in [20]. Later there have been several different models suggested for modeling the effects of colour reconnections on the reconstructed W-mass [21–27]. For a recent comparison of different models with data see [28].

In the present model, which has been implemented the Pythia Monte Carlo [7] version 5.7, similar effects are expected. Fig. 6 shows the dijet mass spectrum for W-pairs decaying hadronically using the JADE algorithm for jet reconstruction with $y_{cut} = 0.015$ and considering only those events giving four jets. Fitting a Breit-Wigner form plus a constant gives a reconstructed W-mass which is 65 ± 15 MeV larger in the model than in the default Pythia version (the error is statistical). This gives an estimate of how large mass-shift one can expect even though the precise number will depend on the analysis method used.
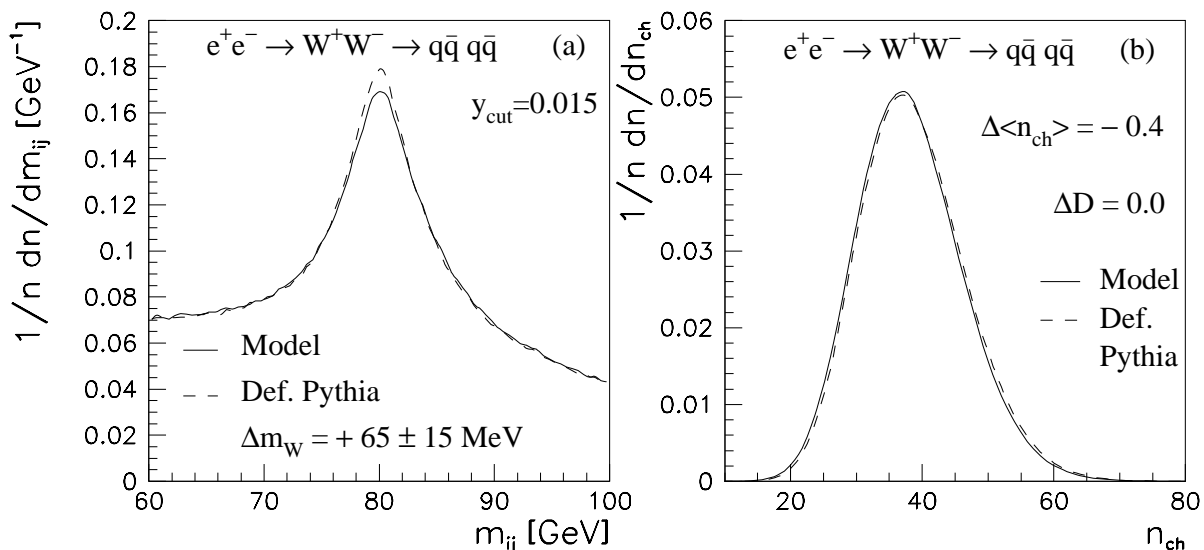

FIG. 6. The dijet mass spectrum (a) and the charged multiplicity (b) for W-pairs decaying hadronically produced in e^+e^- annihilation at $\sqrt{s} = 183$ GeV. The model (solid) is compared with default Pythia (dashed). As indicated, the reconstructed mass is shifted with $\Delta m_W = m_W^{Model} - m_W^{Def} = 65 \pm 15$ MeV, the mean multiplicity is shifted with $\Delta <n_{ch}> = <n_{ch}>^{Model} - <n_{ch}>^{Def} = -0.4$ whereas the dispersion $D = \sqrt{<n_{ch}^2> - <n_{ch}>^2}$ is unchanged.

The obtained mass-shift is approximately twice as big as the final statistical error for LEP2 predicted in [29]. On the one hand this could ruin the usefulness of the hadronic decays for an exact determination of the W-mass but on the other hand it could be used as

---

[6]The separation, at LEP2, between the two W's before they decay is small (~ 0.1 fm) on a hadronic scale.

a probe for non-perturbative dynamics. Other observables that have been used for comparing different models with data are the multiplicity and the thrust distribution (see e.g. [28]). In the present model the mean multiplicity is somewhat smaller than in default Pythia, $\Delta < n_{ch} > = -0.4$, whereas the dispersion is unchanged. The latter is also true for the thrust distribution.

Conclusions. A new general model for string rearrangements in hadronic final states has been presented. For hadronic final states in e^+e^- annihilation the model gives small differences compared to the Jetset Monte Carlo which in general describes data very well. Thus the model is a viable extension of the ordinary Lund string model. At the same time the model describes the diffractive structure function in deep inelastic scattering. The model also predicts an enhancement for large rapidity gaps in e^+e^- annihilation which in principle should be measurable with more statistics and a shift in the W-mass reconstructed from W-pairs decaying hadronically. With more data the model can be further tested and possibly provide a probe into nonperturbative QCD phenomena.

REFERENCES

[1] B. Andersson, G. Gustafson, G. Ingelman, T. Sjöstrand, Phys. Rep. **97** (1983) 31.

[2] X. Artru and G. Mennessier, Nucl. Phys. **B70** (1974) 93.

[3] G. Gustafson, U. Pettersson and P.M. Zerwas, Phys. Lett. **B209** (1988) 90.

[4] A. Edin, G. Ingelman, J. Rathsman, Phys. Lett. **B366** (1996) 371.

[5] A. Edin, G. Ingelman, J. Rathsman, Z. Phys. **C75** (1997) 57.

[6] K. G. Wilson, Phys. Rev. **D10** (1974) 2445.

[7] T. Sjöstrand, PYTHIA 5.7 and JETSET 7.4, CERN-TH.7112/93; Comp. Phys. Comm. 82, 74 (1994).

[8] ALEPH Collaboration, D. Buskulic *et al.*, Z. Phys. **C69** (1995) 15.

[9] OPAL Collaboration, R. Akers *et al.*, Z. Phys. **C63** (1994) 81.

[10] Y. Dokshitzer, Talk given at 29th International Conference on High-Energy Physics (ICHEP 98), Vancouver, Canada, 23-29 Jul 1998, hep-ph/9812252.

[11] B. Andersson, G. Gustafson and C. Peterson, Z. Phys. **C 1** (1980) 105.

[12] B. Andersson and G. Gustafson, Z. Phys. **C 3** (1980) 223.

[13] B. Andersson, G. Gustafson and C. Peterson, Phys. Lett. **B101** (1981) 129.

[14] SLD Collaboration, K. Abe *et al.*, Phys. Rev. Lett. **76** (1996) 4886.

[15] D. Muller, private communication.

[16] H1 Collaboration, C. Adloff *et al.*, Z. Phys. **C76** (1997) 613.

[17] G. Ingelman, A. Edin, J. Rathsman, LEPTO 6.5, Comp. Phys. Comm. 101, (1997) 108.

[18] CTEQ collaboration, MSUHEP-60416, CTEQ-604.

[19] N. Brook *et al.*, 'HzTool- A Package for Monte Carlo Generator - Data Comparisons at HERA (version 1.0)', `http://dice2.desy.de/~h01rtc/hztool.html`.

[20] T. Sjöstrand and V. A. Khoze, Phys. Rev. Lett. **72** (1994) 28.

[21] T. Sjöstrand and V. A. Khoze, Z. Phys. **C62** (1994) 281.

[22] G. Gustafson and J. Häkkinen, Z. Phys. **C64** (1994) 659.

[23] L. Lönnblad, Z. Phys. **C70** (1996) 107.

[24] J. Ellis and K. Geiger, Phys. Rev. **D54** (1996) 1967.

[25] C. Friberg, G. Gustafson and J. Häkkinen, N. Phys. **B490** (1997) 289.

[26] J. Ellis and K. Geiger, Phys. Lett. **B404** (1997) 230.

[27] A. Donnachie and P. V. Landshoff, Phys. Lett. **B421** (1998) 341.

[28] N. K. Watson, BHAM-HEP/98-01, hep-ph/9809564.

[29] W mass working group, Z. Kunszt *et al.*, in Physics at LEP2, eds. G. Altarelli, T. Sjöstrand and F. Zwirner,CERN report 96-01 (1996), Vol. 1, p.141, hep-ph/9602352.