SLAC–PUB–7987
November 1998

Nonlinear Fields and Dynamic Aperture near Low-Order Resonances at the KEK/ATF*

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We describe a scheme by which the nonlinear field contents of a storage ring can be estimated from the measured acceptance variation near low-order resonances. The method is applied to the KEK/ATF damping ring in an attempt to understand its small dynamic aperture.

Presented at the
1998 SPring-8 Beam Physics Study Meeting
Mihara, Mikazuki-cho, Hyogo, Japan, November 4–6, 1998

*Work supported in parts by the U.S. Department of Energy, contract DE-AC03-76SF00515.

NONLINEAR FIELDS AND DYNAMIC APERTURE NEAR LOW-ORDER RESONANCES AT THE KEK/ATF *

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Abstract

We describe a scheme by which the nonlinear field contents of a storage ring can be estimated from the measured acceptance variation near low-order resonances. The method is applied to the KEK/ATF damping ring in an attempt to understand its small dynamic aperture.

高エネルギー加速器研究機構 ATF における共鳴点近傍での非線形場および動力学的口径

共鳴点近傍での口径測定から円形加速器内の非線形場の描像を推察することができる。 本方法の適用により、高エネルギー加速器研究機構 ATF ダンピングリングの動力学的口径の 理解に努めた。

1 INTRODUCTION

The ATF damping ring is a prototype damping ring designed to develop the technologies and operational procedures required for a next-generation linear collider [1]. In order to produce beams with extremely small emittance the optics chosen is a FOBO structure, with combined function defocusing bending magnets. The phase advance per cell of 135° is close to the optimum value for minimum transverse emittance. Since the dispersion function is small and the chromaticity high, the sextupoles are strong. Therefore, the dynamic aperture of linear-collider damping rings has been a concern, and its study is one of the reasons for building the ATF.

In 1998, the ATF beam lifetime was limited primarily by elastic beam-gas scattering [2]. We have taken advantage of this and developed a procedure [3] to infer the effective transverse acceptance from simultaneous measurements of the time-dependent beam current and the average ring pressure. Various results were presented in Refs. [3, 4].

Figure 1 shows the measured acceptance A as a function of the horizontal tune Q_x, where the acceptance is defined by $A = r^2/\beta$, with r the effective dynamic aperture at a point with beta function β. The horizontal tune was varied by changing the strength of the main quadrupole family QF2R. This also caused some vertical tune variation, about 2 times smaller than the horizontal [3]. The beam could not be stored below a horizontal tune of 0.15, indicating a wide stop band at the integer resonance. Clearly visible are also the third and half integer resonances.

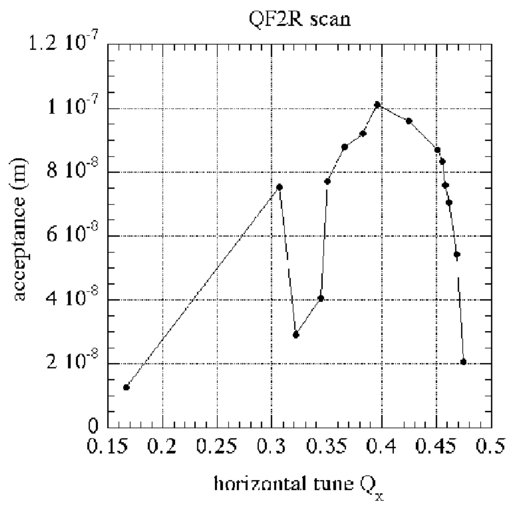

Figure 1: Measured transverse acceptance as a function of the fractional horizontal tune.

2 MODEL

The question arises if something can be learnt from the acceptance variation near different low-order resonances. To answer this question, we consider a simple model in $1\frac{1}{2}$ degrees of freedom, consisting of a single nonlinearity and a linear rotation. For example, if the nonlinearity is a sextupole, this map reads

$$\begin{pmatrix} x_f \\ x'_f \end{pmatrix} = \begin{pmatrix} \cos 2\pi Q & \beta \sin 2\pi Q \\ -\frac{1}{\beta}\sin 2\pi Q & \cos 2\pi Q \end{pmatrix} \times \times \begin{pmatrix} x_i \\ x'_i - \frac{1}{2}k_3 x_i^2 \end{pmatrix} \quad (1)$$

The subindices i and f refer to the initial and final phase-space coordinates, respectively, Q is the linear betatron tune, β the beta function at the sextupole, and k_3 the normalized sextupole strength. A general $2n$-pole would produce a kick $k_n x_i^{n-1}/(n-1)!$, where k_n is the normalized n-pole strength, related to a pole-tip field B_T at radius a as

$$k_n = \frac{(n-1)!\, l B_T}{(B\rho) a^{n-1}} \quad (2)$$

* Work supported by the U.S. Department of Energy under contract DE-AC03-76SF00515.

where $B\rho$ is the magnetic rigidity, and l the length of the magnet. The Hamiltonian that corresponds to Eq. (1) reads:

$$H_3(I,\phi,\theta) = QI + \frac{1}{2\pi}\frac{k_3}{6}x^3 \sum_{q=-\infty}^{\infty} e^{iq\theta} \tag{3}$$

where $x = \sqrt{2\beta I}\cos\phi$ with I and ϕ the action angle coordinates for the linear system, and θ the azimuthal angle around the ring, here functioning as the time coordinate. Keeping only the resonant term q for which $(3\phi - q\theta) \approx 0$ and transforming into a rotating frame, this simplifies to

$$H_3(I,\psi) \approx \Delta Q_3\ I + \frac{1}{2\pi}\frac{k_3}{6}(2\beta)^{3/2}\frac{1}{4}I^{3/2}\cos 3\psi \tag{4}$$

where $\psi = (\phi - q/3)$, and $\Delta Q_3 = (Q - q/3)$ the distance from the resonance. More generally the highest order resonance driven by an n pole, of the form $(Q - 1/n) \approx 0$, is described, for odd n, by the Hamiltonian

$$H_n(I,\psi) \approx \Delta Q_n\ I + \frac{1}{2\pi}\frac{k_n}{n!2^{n-1}}(2\beta)^{n/2}I^{n/2}\cos n\psi \tag{5}$$

and, for even n, by

$$H_n(I,\psi) \approx \Delta Q_n\ I + \frac{1}{2\pi}\frac{k_n}{n!}(2\beta)^{n/2}I^{n/2}\left(\frac{1}{2^{n-1}}\cos n\psi + \frac{(n-1)!!}{n!!}\right) \tag{6}$$

where $\Delta Q_n = (Q - 1/n)$ is the distance to the resonance, $(2k)!! = 2\cdot 4 \cdot \ldots 2k$, and $(2k-1)!! = 1\cdot 3\cdot \ldots (2k-1)$.

We can estimate the dynamic acceptance as that value of I for which the instantaneous tune $\partial H/\partial I$ experienced at either of the two extreme points, $\cos n\psi = +1$ or -1, is equal to 0. Solving for I, we can estimate the dynamic acceptance induced by an n-pole near the resonance $(Q - 1/n) \approx 0$. For odd n we find

$$I_{da} \approx \left(\frac{4\pi|\Delta Q_n|\ (n-1)!2^{n-1}}{(2\beta)^{n/2}k_n}\right)^{\frac{2}{n-2}} \tag{7}$$

and for even n

$$I_{da} \approx \left(\frac{4\pi|\Delta Q_n|\ (n-1)!}{(2\beta)^{n/2}k_n\left(\frac{1}{2^{n-1}} + \frac{(n-1)!!}{n!!}\right)}\right)^{\frac{2}{n-2}} \tag{8}$$

Here I_{da} is the value of the action variable at the limiting aperture.

For example, if the third integer resonance is driven by a single sextupole of strength k_3, in the vicinity of this resonance the acceptance should be a quadratic function of $\Delta Q_3 = (Q - 1/3)$, namely

$$I_{da} \approx \left(\frac{16\pi\ \Delta Q_3}{\sqrt{2}k_3\beta^{3/2}}\right)^2 \tag{9}$$

where β is the beta function at the sextupole.

Figure 2 shows that the analytical expression, Eq. (9) agrees well with the result of a tracking simulation. In the simulation, many particles were tracked over 5000 turns and the square of the maximum stable start amplitude was taken to be the dynamic acceptance. Simulation and analytical estimate agree well.

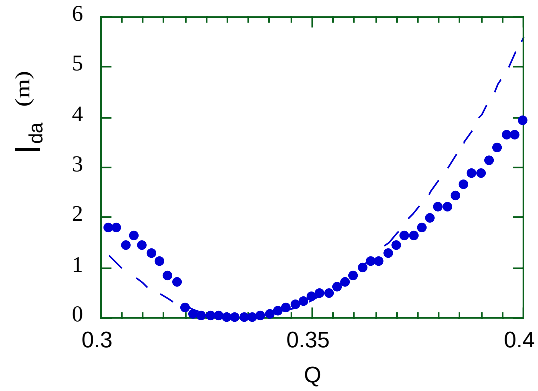

Figure 2: Dynamic acceptance due to a single sextupole as a function of distance from the 3rd order resonance, for $k_3 = 1\ \mathrm{m}^{-2}$ and $\beta = 1$ m; symbols: tracking simulation; dashed: the analytical estimate of Eq. (9).

An octupole magnet does not only drive the $1/4$ resonance, but also the $1/2$ resonance. The Hamiltonian for this case is

$$H_4(I,\psi) \approx \Delta Q_2\ I + \frac{1}{2\pi}\frac{k_4}{24}(2\beta)^2 I^2\left(\frac{\cos 2\psi}{2} + \frac{3}{8}\right) \tag{10}$$

where $\Delta Q_2 = (Q - 1/2)$. Again from the condition that the maximum or minimum value of the instantaneous tune exactly corresponds to the resonance, *e.g.*, $\partial H/\partial I = 0$ for $\cos 2\psi = 1$ or -1, we can crudely estimate the dynamic acceptance near $\Delta Q_2 = (Q - 1/2) \approx 0$, namely

$$I_{da} \approx \left(\frac{48\pi\ |\Delta Q_2|}{7k_4\beta^2}\right) \tag{11}$$

where β is the beta function at the octupole, and k_4 the octupole strength. Figure 3 compares the analytical expression, Eq. (11), with a tracking result. The agreement is reasonable.

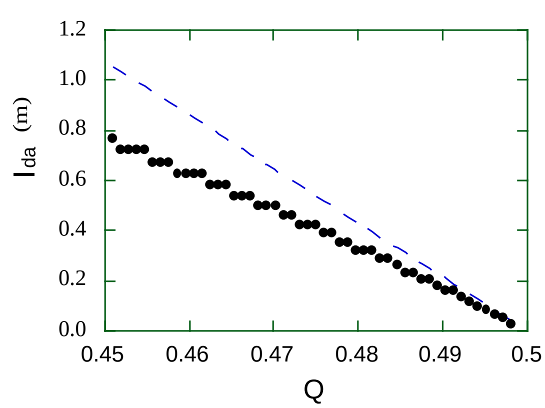

Figure 3: Dynamic acceptance due to a single octupole as a function of distance from the half-integer resonance, for $k_4 = 1\ \mathrm{m}^{-3}$ and $\beta = 1$ m; symbols: tracking simulation; dashed: the analytical estimate of Eq. (11).

3 ATF DATA

Figure 4 shows an enlarged view of the measured dynamic acceptance near $Q \approx 1/3$ resonance. The measured acceptance A defined above is equal to $2I_{da}$. Assuming that the acceptance near the 3rd order resonance is dominated by a single sextupolar field located somewhere in the ring, we can estimate the strength of this sextupole, by fitting the data to the expected quadratic dependence, Eq. (9). The fit result is

$$A = 2I_{da} \approx 3 \times 10^{-4}\ \mathrm{m}\ (\Delta Q_3)^2 \qquad (12)$$

By comparison with Eq. (9), and for a typical beta function $\beta \approx 5$ m, this corresponds to $k_3 \approx 130\ \mathrm{m}^{-2}$. Assuming a magnet length of 1 m, radius $a = 12$ mm, and beam energy 1.27 GeV, the equivalent pole-tip field is 0.4 kG, or 4% of the main dipole field. For a magnet length of 6 cm and 16 mm bore radius, the equivalent pole-tip field would be 9 kG, more than half the strength of a regular ATF sextupole.

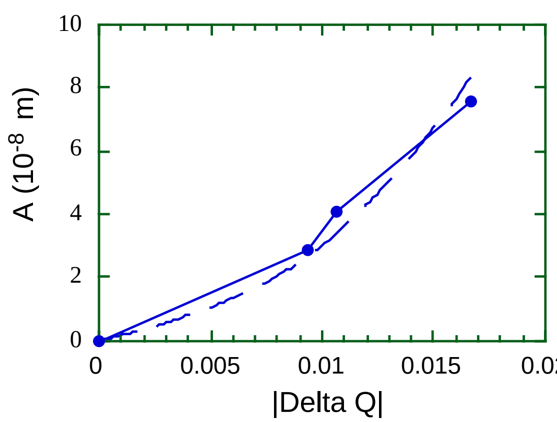

Figure 4: Measured dynamic acceptance of the ATF damping ring as a function of distance from the 3rd order resonance; symbols: measurement; dashed: fit to a quadratic function, according to Eq. (9).

In the same spirit we can fit the measured acceptance near $Q \approx 1/2$ to a linear function. The fit result is

$$A = 2I_{da} \approx 2 \times 10^{-6}\ \mathrm{m}\ |\Delta Q_2| \qquad (13)$$

Assuming the same values for β, l and a as before, we find $k_4 \approx 9 \times 10^5\ \mathrm{m}^{-3}$, which translates into a pole-tip field of 10 kG, comparable to the dipole field in the combined-function magnets. The deviation of the data from the linear fit appears too large to be explained by statistical measurement errors, which are smaller than 10% in case of the acceptance and below 0.001 for the tune. This could suggest that effects other than octupole fields contribute to the dynamic acceptance near the half integer, for example quadrupole gradient errors. However, given the small number of available data points, it is difficult to arrive at a definitive conclusion. In the future, we intend to repeat the measurement with better statistics.

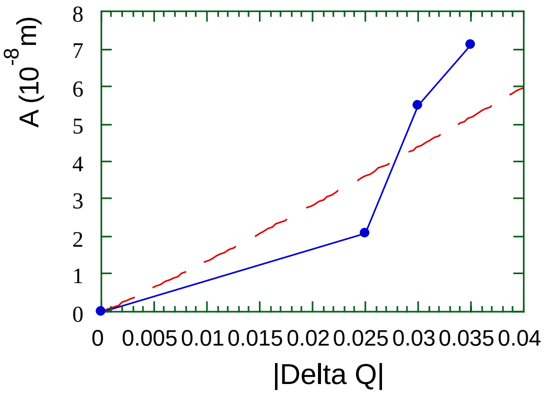

Figure 5: Measured dynamic acceptance of the ATF damping ring as a function of distance from the half-integer resonance; symbols: measurement; dashed: fit to a linear function, according to Eq. (11).

4 CONCLUSIONS

We have described a simple method for estimating the nonlinear field content of a storage ring from the measured variation of the dynamic acceptance near low-order resonances. The method was tested for a simple 2-dimensional map, where it gave promising results. Application to the ATF damping ring, using (sparse) measurements taken near the 1/3 and 1/2 resonance, indicates a large nonlinearity consistent with sextupole and octupole pole-tip fields of about 0.5 kG and 10 kG, respectively, over 1 m length.

5 ACKNOWLEDGEMENT

The authors acknowledge support by K. Takata and M. Ross. F. Zimmermann, M. Minty, J. Turner and M. Woodley thank the KEK ATF group for extraordinary hospitality. This work was supported in parts by the US Department of Energy under the contract no. DE–AC03–76SF00515.

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