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Tune Shift due to Asymmetry of the Resistive Beam Pipe

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1 Abstract

Resistive wall wake field in non-round beam pipes depends on the offset of trailing particle. Contributions of all bunches to this part of the wake are additive and give current dependent tune shift which may be a dominant for long trains.

2 Introduction

Measurements of the tune shift with current in the high energy ring (HER) of the PEP-II B-factory for a single bunch give

$$dQ_y/dI = -12.2\,10^{-4}\,1/mA; dQ_x/dI = -4.5\,10^{-4}\,1/mA. \quad (1)$$

This corresponds within a factor of two to the values predicted for these quantities from the impedance model of the ring.

The tune shift measured for a train of $n_b = 1656$ equidistant bunches for the total current within the range 10-50 mA is

$$dQ_y/dI_{beam} = -1.8\,10^{-5}\,1/mA; (-2.56\,10^{-5}\,1/mA) \quad (2)$$

$$dQ_x/dI_{beam} = 1.2\,10^{-5}\,1/mA; (2.05\,10^{-5}\,1/mA). \quad (3)$$

The numbers in brackets are measurements for the train of $n_b = 291$ equidistant bunches. Note the change of the sign of the slope in the horizontal plane compared to that of a single bunch.

Numerical calculations based on Wang's formalism [1] give the tune shift of individual modes. The maximum tune shift per bunch current obtained numerically is about the same for $n_b = 1650$ and $n_b = 291$ confirming that in this theory the tune shift per beam scales inversely proportional to n_b. The roll off of the tune variation with beam current in measurements is much slower.

Numerical calculations use the transverse impedance of resistive wall in the frequency domain which rolls off as $1/\sqrt{\omega}$. In this case, the single bunch frequency shift depends on the cut-off at high frequencies which is defined by the bunch length σ_l. The tune shift is

$$\frac{dQ}{dI_{beam}} \propto \frac{\Gamma[1/4]}{\sqrt{[Q_\beta]}} + \frac{1}{n_b}\sqrt{\frac{R}{\sigma_l}}, \quad (4)$$

where $\{Q_\beta\}$ is the fractional part of the tune. The first term, the multi-turn effect, is the dominant contribution for large number of bunches while the second term, the single bunch effect, gives dominant and large contribution for few bunches. Both terms are comparable when $n_b \propto \sqrt{R\{Q\}/\sigma}$. When the first term is large, $dQ_\perp/dI_{beam}$ is independent off n_b. For small n_b and large machines, the second term gives the main contribution, and $dQ_\perp/dI_{bunch}$ is constant with n_b while $dQ_\perp/dI_{beam}$

decreases as $1/n_b$. In this case, which is the case for PEP-II parameters, other mechanisms producing tune shift with current may become important.

Discrepancy of numerical calculations with experiment may indicate existence of an additional mechanisms for the tune shift. Additional to the machine impedance, dependence of the tune on current may come from tune dependence on the amplitude of transverse motion, and may be caused by ions. The amplitude dependence is too weak to explain the experimental results. Effect of ions will be described elsewhere. Here we try to relate the observation with the asymmetry of the resistive beam pipe.

The vacuum chamber in the HER of PEP-II is a stainless-steel round pipe in the straight sections and is almost rectangular copper chamber in the arcs. As it is well known, the transverse wake field of the round pipe in dipole approximation is proportional to the offset of the leading particle and is independent of the offset of the trailing particle. Such a wake gives bunch coupling and may lead to beam instability and split tune of the coupled bunch modes. For a rectangular (elliptical, etc.) beam pipe transverse wake depends also on the offsets of the trailing particle (for example, the paper [2] describes this dependence for a beam pipe of a general cross-section). The term of the dipole wake proportional to the offset of the trailing particle changes tune and leads to dependence of the tune on current. This part of the wake generated by all bunches is additive and may become noticeable for such a long train as train in the PEP-II. This effect was discussed before in relation with the design of linear colliders and reconsidered again recently by Burov and Danilov [3].

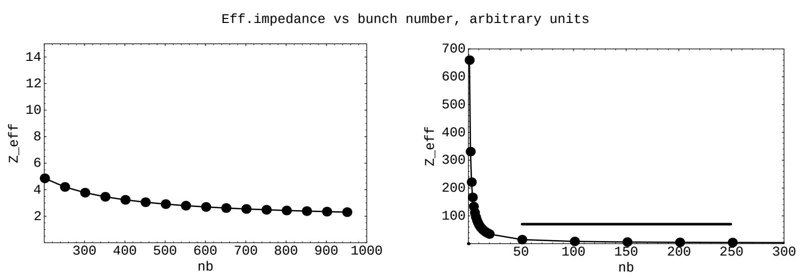

Figure 1: Effective impedance (or tune shift per beam current) vs bunch number. Figure in the left magnifies the tail at large n_b of the right figure. The solid line in the right figure shows effect of the ellipticity of the beam pipe (the last term in Eq.(21)

3 Impedances

The explicit form of the resistive wall impedance of a thick rectangular and elliptical beam pipes is given [4] for the case of equal offsets of the leading and trailing particles. The method used in this reference can be easily generalized for unequal offsets.

The longitudinal impedance per unit length takes form

$$Z_l^\omega(r_t, r_l) = -\frac{\zeta}{Z_0}\int dl(\vec{t}H_i^{*\omega}(r,r_t))(\vec{t}H_i^\omega(r,r_l)), \tag{5}$$

where the linear integral is taken over the circumference of the beam pipe cross-section.

The tangential component of magnetic field is equal to the normal component of electric field which is given by the scalar potential Φ of 2D Poisson equation with ideal metallic boundary conditions:

$$\vec{t}H_i^\omega(r,r_t) = \vec{n}E_i^\omega(r,r_t) = \vec{n}\nabla\Phi(r,r_t). \tag{6}$$

Explicit expression for Φ is given in reference [4] for elliptic and rectangular geometry. Potential Φ is real and independent of ω. Hence, ω dependence of the impedance (and dependence of the wake on distance between particles) is given by $\zeta(\omega)$.

Resistive wall impedance of a thick beam pipe of elliptical cross-section is given also by Piwinski [5]. The longitudinal impedance per unit length is

$$Z_l(\omega, u, \psi, u', \psi') = Z_0(\frac{1-i}{2})(\frac{\omega}{2\pi c})\frac{\delta}{b}F \tag{7}$$

where δ is skin depth, $\delta(-\omega) = -i\delta(\omega)$, and F is a form factor which can be presented as series over offsets of the leading and trailing particles,

$$F = 2(\frac{b}{a})[const + 2\frac{x_t^2 - y_t^2}{a^2+b^2}\Sigma_1 + 2\frac{x_l^2 - y_l^2}{a^2+b^2}\Sigma_1 + \frac{x_t x_l}{a^2}(\Sigma_0 + \Sigma_1) + \frac{y_t y_l}{b^2}(\Sigma_0 - \Sigma_1) + ...]. \tag{8}$$

Here

$$\Sigma_k = \sum_{l=k}\frac{[2l!]^2}{4^{2l}[l!]^2(l+k)!(l-k)!}(\frac{a^2-b^2}{a^2})^{2l}. \tag{9}$$

The transverse impedance can be obtained from Panofsky-Wenzel theorem:

$$Z_{tr}^\omega(r, r_l) = Z_0(\frac{1-i}{2\pi})\frac{\delta}{b}\nabla_\perp F \tag{10}$$

The main terms in the transverse impedances in the x- and y-planes, are linear in the offsets. Transverse wake fields have the same structure.

For the PEP-II vacuum 9×5 cm vacuum chamber ($b = 2.5$ cm), numerical calculations give for the form factors:

$$\frac{\partial F}{\partial x} = 1.068x_l - 1.142x_t + 0.579x_l y_l y + 0.222x_l^2 x - 0.222y_l^2 x + .. \tag{11}$$

$$\frac{\partial F}{\partial y} = 2.588y_l + 1.143y - 0.222x_l^2 y + 0.579x_l y_l x + 0.222y_l^2 y + .. \tag{12}$$

Neglecting octupole terms, the transverse wake can be written in the form

$$W_x(s) = W_\perp^0[x_l - 1.08x_t + ...], \tag{13}$$

$$W_y(s) = W_\perp^0[y_l + 0.44y_t + ...]. \tag{14}$$

Here, $W_\perp^0(s)$ is the transverse wake of a round beam pipe with radius b,

$$W_\perp^0(s) = \lambda_{RW}\sqrt{s_0/s}, \quad \lambda_{RW} = \frac{1}{\pi}\sqrt{\frac{2}{15}\frac{L}{b^3}}\sqrt{\frac{\rho}{s_0}}, \tag{15}$$

Here offsets x_l, y_l, x_t, y_t and the length L are in cm, resistivity ρ is in $Ohm\,cm$. For the HER parameters and $s_0 = 126$ cm, λ_{RW} of the arcs is $\lambda_{RW} = 0.114\ (1/cm)^2$.

4 Tune shift

Transverse dynamics is dominated by the resistive wall impedance. Neglecting other contributions to the wakes, the betatron oscillation of the k-th bunch centroid in a train of n_b bunches is described by the equation of motion

$$\frac{d^2x_k}{dt^2} + 2\gamma_d\frac{dx_k}{dt} + \omega_{0,x}^2 x_k(t) = \Lambda_0\lambda_{RW}\Sigma, \tag{16}$$

where

$$\Sigma = \sqrt{\frac{s_0}{s_b}}\sum_{p=0}^{\infty}\sum_{l=1}^{n_b}\frac{\theta(k-l+pn_t)}{\sqrt{k-l+pn_t}}[x_l(t-(k-l)\tau_b - pT_0) + Gx_k(t)]. \tag{17}$$

Here G is the parameter of the form factor, Eq. (11), Λ is proportional to number of particles per bunch, N_b,

$$\Lambda_0 = \frac{N_b r_e c^2}{2\pi R\gamma}, \tag{18}$$

$n_t = 2\pi R/s_b$, bunch spacing $s_b = c\tau_b$, ring circumference $cT_0 = 2\pi R = n_t s_b$, and θ is a step function. For the HER, $I_{bunch} = 0.6$ mA, $\Lambda_0/\omega_0^2 = 3.8310^{-3}(I_{bunch}/mA)\ cm^2$.

The leading particles are taken at the retired moments $t_l = t - (k-l)\tau_b - pT_0$ while the offset of the trailing particle is taken at the moment t. Hence, The last term in the RHS gives the tune shift

$$\Delta Q_x = -\frac{\Lambda_0\lambda_{RW}}{2Q_x\omega_0^2}G\sqrt{\frac{s_0}{s_b}}\sum_{p=0}^{\infty}\sum_{l=1}^{n_b}\frac{\theta(k-l+pn_t)}{\sqrt{k-l+pn_t}}. \tag{19}$$

For the HER, the coefficient $[\Lambda_0\lambda_{RW}/(2Q_x\omega_0^2)] = 0.93\,10^{-5}\,I_{bunch}/mA$.

The tune shift driven by ellipticity of the beam pipe has opposite sign in x- and y-planes, and is smaller in the vertical plane. Note also the sign of the tune shift in the horizontal plane is positive in agreement with the experiment.

Contributions proportional to the offset of the trailing particle from all bunches are additive. Numeric value of the tune shift depends on the magnitude of the sum. The sum in Eq. (17) diverges. This divergence reflects slow decay of the transverse resistive wall wake with time. If finite thickness w of the beam pipe wall is taken into account, the transverse wake rolls off with time as $1/\sqrt{(t)}$ only for time small compared to the time τ_{diff} of diffusion of magnetic field through the wall with resistivity ρ, $c\tau_{diff} = \frac{Z_0 wb}{2\rho}$. This defines the upper limit on the number of turns $p < p_m$, $p_m = \frac{Z_0 bw}{4\pi R\rho}$. For PEP-II parameters, $w = 6$ mm, $b = 2.5$ cm, $1/\rho_{Cu} = 6.10^5$ $Ohm^{-1} * cm^{-1}$, the maximum number of turns is large, $p_m = 308$. The sum in this case can be estimated as

$$\sqrt{\frac{s_0}{s_b}} \sum_{p=0}^{p_m} \sum_{l=1}^{n_b} \frac{\theta(k-l+pn_t)}{\sqrt{k-l+pn_t}} = 2n_b \sqrt{\frac{p_m s_0}{2\pi R}}. \tag{20}$$

Eq. (4) is modified to

$$\frac{dQ}{dI_{beam}} \propto \frac{\Gamma[1/4]}{\sqrt{\{Q_\beta\}}} + \frac{1}{n_b}\sqrt{\frac{R}{\sigma_l}} + 4\sqrt{p_m}, \tag{21}$$

Effectively, the tune shift for large n_b is a factor $4\sqrt{p_m/\{Q_\beta\}}$ larger than the multi-turn tune shift caused by leading particles. The ratio of the tune per beam current for large n_b to the single bunch tune shift is

$$\frac{4}{\Gamma[1/4]} \sqrt{\frac{p_m \sigma}{R}}. \tag{22}$$

For PEP-II, this factor is 0.10, by a factor two larger than the ratio of dQ_x/dI_{beam} given in Eqs. (2) and (1).

It is worth noting that elliptical shape of the beam pipe assumed here means that the thickness of the wall is different in horizontal and vertical planes contrary to the actuall constant thickness of the beam pipe. Therefore, the numbers here should be considered only as an estimate.

5 Conclusion

The tune shift with current caused by a resistivity of the walls of the PEP-II rectangular beam pipe gives tune shifts of opposite signs in horizontal and vertical planes in agreement with experiment. The tune shift per beam current is about the same for a train of 291 and 1654 bunches, again in agreement with experiment. The absolute magnitude of the effect is within a factor of two compared to what was measured in experiment.

Usually it is expected that if the transverse feeback can control the resistive wall instability for small number of bunches, it would control instability for large n_b as well. The reason for this is that both growth rate of instability and the the gain of feedback scale with n_b in the same way. Ellipticity of the beam pipe does not change this statement provided that parameter $\sqrt{p_m \sigma / R} < 1$ as it is for PEP-II.

6 Acknowledgement

I thank Cho Ng for MAFIA simulations. The simulations clearly confirm dependence of the wake in the elliptical beam pipe on the offset of the trailing particle.

References

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