SLAC-PUB-7973
October 1998

# A coupled channel unitary chiral approach to the meson-meson interaction and $\pi\pi$ scattering in a nuclear medium[1]

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**Abstract**

A new tool which combines chiral perturbation theory and unitarity in coupled channels is applied with success to the study of the meson-meson interaction, extending the theoretical predictions up to $\sqrt{s} = 1.2$ GeV, hence improving considerably the convergence radius of conventional chiral perturbation theory, $\chi PT$. The method is applied to obtain the meson-meson phase shifts and inelasticities as well as the isoscalar $\pi\pi$ scattering amplitude inside a nuclear medium.

Invited talk presented at
**Conference on Mesons and Light Nuclei '98**
Prouhonic, Prague, Czechoslovakia, August 31-September 4, 1998.

The starting point of the approach is unitarity in coupled channels, which in matrix form is most easily stated in terms of the real $K$ matrix as

$$T^{-1} = K^{-1} - i\,\sigma, \tag{1}$$

where $T$ is the scattering matrix and $\sigma$ is a diagonal matrix that measures the phase space available for the intermediate states

$$\sigma_{nn}(s) = -\frac{k_n}{8\pi\sqrt{s}}\,\theta\left(s - (m_{1n} + m_{2n})^2\right), \tag{2}$$

where $k_n$ is the on shell CM momentum of the meson in the intermediate state $n$ and $m_{1n}, m_{2n}$ the masses of the two mesons in the state $n$. The meson-meson states considered here are $K\bar{K}, \pi\pi, \eta\eta, \pi\eta, \pi K, \pi\bar{K}, \eta K$ and $\eta\bar{K}$.

[1]Research partially supported by the Department of Energy under contract DE-AC03-76SF00515 and Spanish CYCIT AEN-0776 and PB96-0753

From eq. (1) one immediately realizes that

$$K^{-1} = \mathrm{Re}\, T^{-1}, \tag{3}$$

and hence we have in matrix form

$$T = [\mathrm{Re}\, T^{-1} \; - i\,\sigma]^{-1}, \tag{4}$$

The next step is to make an expansion of $\mathrm{Re}\, T^{-1}$ in powers of $p^2$, like in $\chi PT$. Remarkably, the expansion of $\mathrm{Re}\, T^{-1}$ has better chances of convergence, since $T$ has poles (the "$\sigma$" in $I = 0$ appears around 500 MeV) and perturbation theory will necessarily break before that. Similarly one could not get the other meson-meson resonances like the $f_0(980)$ for $J = 0, I = 0$, or the $\rho$ and $K^*$ vector mesons, etc.... In contrast, where $T$ has poles $T^{-1}$ will have zeros, which, in principle, do not give any convergence problem, thus allowing us to obtain resonances.

The expansion of $\mathrm{Re}\, T^{-1}$ is also suggested by the analogy with the effective range formula in Quantum Mechanics, which states that (in an s-wave elastic channel, for simplicity)

$$K^{-1} = \sigma\, \mathrm{ctg}\, \delta \, \propto -\frac{1}{a} + \frac{1}{2}\, r_o k^2, \tag{5}$$

with $k$ the one particle momentum, $a$ the scattering length and $r_0$ the effective range.

In $\chi PT$, $T^{-1} \simeq T_2^{-1}(1 - T_4\, T_2^{-1}...)$, where $T_2$ is the lowest order, $O(p^2)$, amplitude and $T_4$ is just the $O(p^4)$ term. In order to avoid problems with the inversion of $T_2$, we multiply eq. (4) by $T_2 T_2^{-1}$ on the right and $T_2^{-1} T_2$ on the left. That is

$$T = T_2\, [T_2\, \mathrm{Re}\, T^{-1}\, T_2 - i\, T_2\, \sigma\, T_2]^{-1}\, T_2. \tag{6}$$

In addition, an ordinary expansion of $T_2\, \mathrm{Re}\, T^{-1}\, T_2$ in powers of $p^2$ gives

$$T_2\, \mathrm{Re}\, T^{-1}\, T_2 \simeq T_2 - \mathrm{Re}\, T_4 \cdots \tag{7}$$

Thus, recalling that in the physical region $\mathrm{Im}\, T_4 = T_2\, \sigma\, T_2$, our scheme provides, up to $O(p^4)$

$$T = T_2\, [T_2 - T_4]^{-1}\, T_2, \tag{8}$$

which is a generalization of the inverse amplitude method of [1] to coupled channels. The standard $O(p^2)$ and $O(p^4)$ Lagrangians of Gasser and Leutwyler [2] are used to evaluate $T_2$ and $T_4$. Those calculations are lengthy, but even if they are not complete, we can still use some approximations. We have several ways to proceed:

a) If a full calculation of $T_4$ with one loop in the $s$, $t$ and $u$ channels and tadpoles is avaliable, a straightforward application of eq. (8) is possible [3]. The method allows for a direct comparison of the $L_i$ coefficients of the $O(p^4)$ Lagrangians, which are fitted to the meson-meson data, with those of the standard $\chi PT$ expansion.

b) A simpler, equally successful scheme, is obtained by omitting crossed loops and tadpoles. Their effect is reabsorbed in the $L_i$ coefficients, which are also fitted to the data. In such case we can use the loop function (symmetric matrix) in the s-channel containing two meson propagators

$$G_{nn}(s) = i \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 - m_{1n} + i\epsilon} \frac{1}{(P-q)^2 - m_{2n}^2 + i\epsilon}, \tag{9}$$

with $P$ the total momentum of the two meson system. It satisfies

$$\mathrm{Im}\, G_{nn}(s) = \sigma_{nn}. \tag{10}$$

The real part of $G$ is obtained by means of a suitable cut off in $|\vec{q}|$ that makes the loop convergent. Dimensional regularization is equally suitable. Changes of the cut off revert in changes in the $L_i$ coefficients and the final solution for the meson-meson amplitudes is cut off independent. Thus, in this approach, we take

$$\mathrm{Re}\, T_4 = T_2\, \mathrm{Re}\, G\, T_2 + T_4^p, \tag{11}$$

where $T_4^p$ is a polynomial obtained from the tree level contribution of the $O(p^4)$ chiral Lagrangians.

This is the method followed in [4].

c) Finally a third option comes from assuming that for some particular cut off $T_2\, \mathrm{Re}\, G\, T_2$ in eq. (11) can make the $T_4^p$ contribution negligible. This is not possible for all the meson channels but it was shown to work in the scalar sector ($J = 0$) in [5]. In this latter case we would obtain

$$T = T_2\, [T_2 - T_2\, G\, T_2]^{-1}\, T_2 = [1 - T_2\, G]^{-1}\, T_2, \tag{12}$$

or, equivalently,

$$T = T_2 + T_2\, G\, T, \tag{13}$$

which is nothing but the Bethe-Salpeter equation, with $T_2$ playing the role of a potential, and where $T_2$ and $T$ are factorized outside the $d^4q$ integral in the $T_2\, G\, T$ term of eq. (13), a feature that was shown in [5] using different arguments. The cut off needed for a good description of the scalar data is around 1 GeV.

In fig. 1 we show some results which are obtained with the option b) discussed above. We fitted the values of the $L_i$ to the data, but $L_6$ and $L_8$ only appear in the combination $2L_6 + L_8$. Hence one has 7 parameters in the theory. The agreement with the data is quite good, and all the resonances (their masses, widths and associated poles) are well reproduced.

Next we discuss a subject of much interest in the interface between meson-meson interactions and nuclear physics, which is the $\pi\pi$ interaction in a nuclear medium.

The topic was initiated in [6], where, due to the interaction of the pions with the nucleus, the $J = 0$ $\pi\pi$ interaction developed some strength below threshold, leading to pairs of bound mesons, a kind of pion Cooper pairs. Further studies imposing minimal

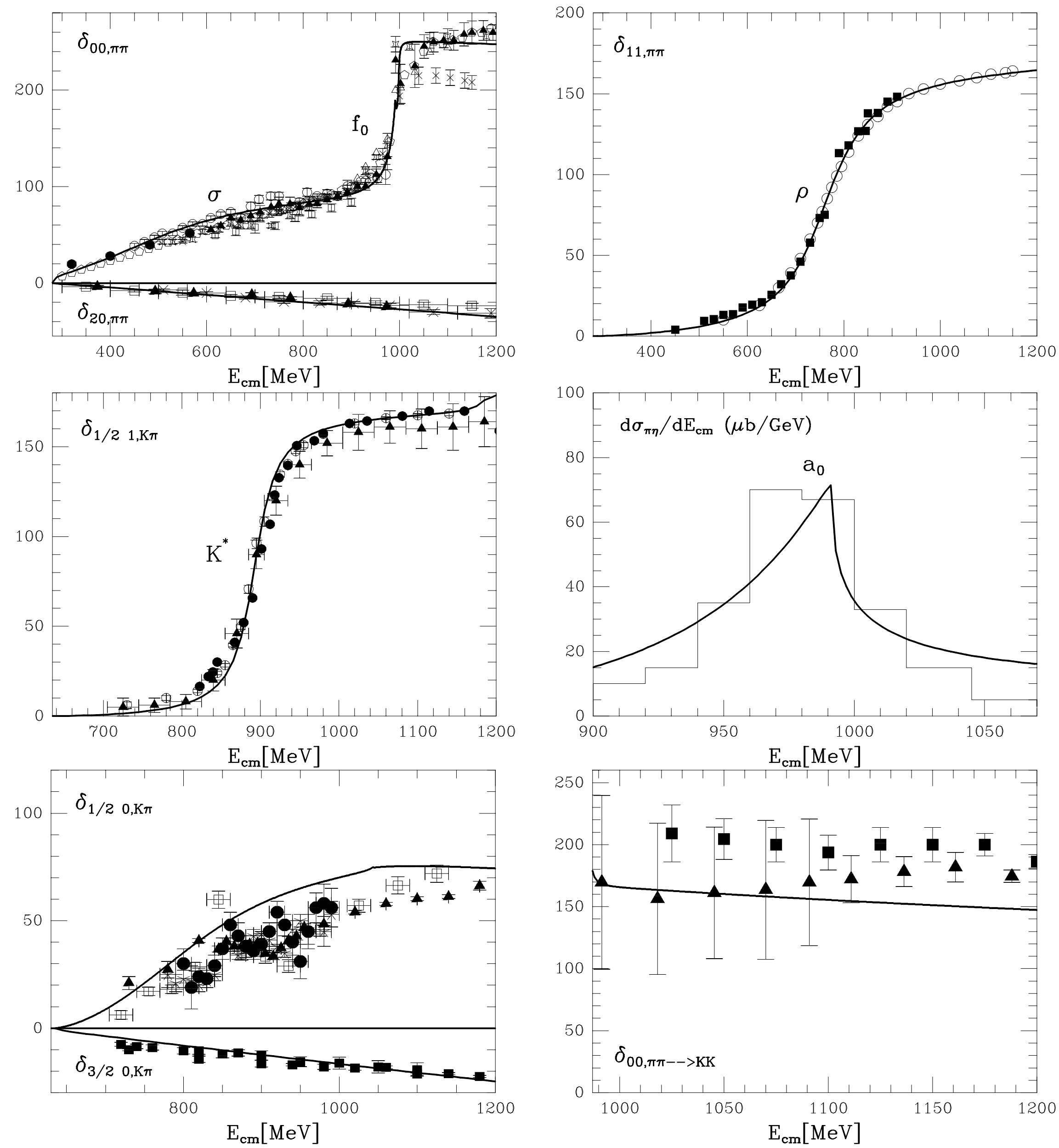


**Figure 1:**We display the results of method b) for the phase shifts of $\pi\pi$ scattering in the $(I, J) = (0, 0), (1, 1), (2, 0)$ channels, where the $\sigma$, $f_0$ and $\rho$ resonances appear, together with those of $\pi\pi \to K\bar{K}$, as well as the phase shifts of $\pi K$ scattering in the $(3/2, 0), (1/2, 0)$ and $(1/2, 1)$ channels, where we can see th appearance of the $K^*$ resonance. The results also include the $\pi^-\eta$ mass distribution for the $a_0$ resonance in the $(I, J) = (1, 0)$ channel from $K^-p \to \Sigma(1385)\pi^-\eta$. For reference to the data, see [4] and references therein.

chiral constraints in the amplitudes softened that strength and the singularities below threshold do not appear [7]. Yet, this accumulation of strength close to threshold, whereas the vacuum amplitude vanishes, could have some observable consequences. Indeed, in [8] the invariant mass distribution for $\pi^+\pi^-$ close to threshold was appreciably enhanced with respect to that of $\pi^+\pi^+$ in the study of $(\pi, 2\pi)$ reactions in nuclei.

We have performed a recent calculation using the previous method for the $\pi\pi$ interaction [9]. It is interesting to see that the off shell dependence of the $\pi\pi$ amplitude cancels out with some other diagrams and, hence, only the on shell $\pi\pi$ amplitudes in vacuum are needed as input [10].

In fig. 2 we show the results for $\mathrm{Im}\, T_{00}$ in $J = I = 0$ in $\pi\pi$ scattering for different values of the Fermi momentum, $k_F$. We can appreciate that as $k_F$ increases some strength accumulates at low energies around and below threshold which could explain the experimental increase [8] in the invariant mass distribution.

Summarizing, we have found a technique similar to that of the effective range in Quantum Mechanics, which improves the convergence of standard $\chi PT$. The approach yields all the meson resonances below 1.2 GeV, which cannot be obtained with standard $\chi PT$. The application of the method to the study of the scalar isoscalar $\pi\pi$ interaction in a nuclear medium leads to an enhanced strength around the two pion threshold which could explain present invariant $\pi^+\pi^-$ mass distribution measured in the $(\pi, 2\pi)$ reaction in nuclei.

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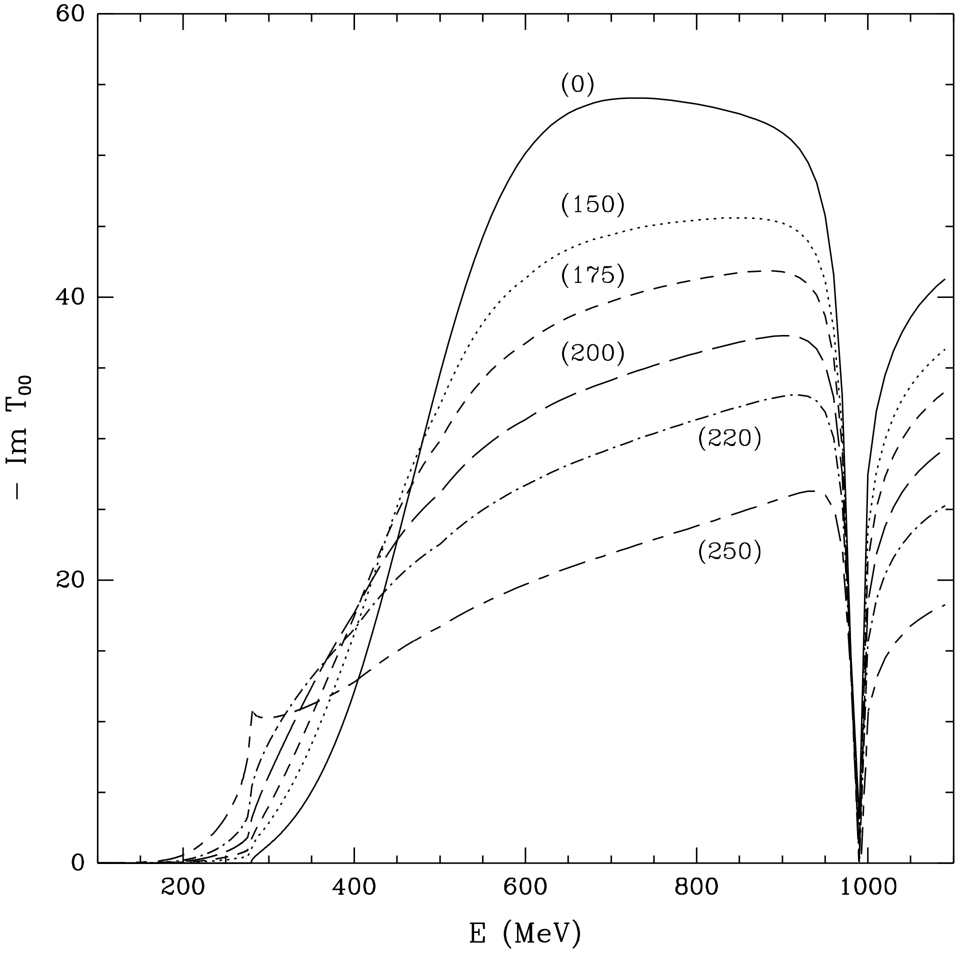


Fig. (7)

**Figure 2:** Im$T_{22}$ for $\pi\pi \rightarrow \pi\pi$ scattering in $J = I = 0$ ($T_{00}$ in the figure) in the nuclear medium for different values of $k_F$ versus the CM energy of the pion pair. The labels correspond to the values of $k_F$ in MeV.