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Possible Origin of Fermion Chirality and Gut Structure From Extra Dimensions *

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Abstract

The fundamental chiral nature of the observed quarks and leptons and the emergence of the gauge group itself are most puzzling aspects of the standard model. Starting from general considerations of topological properties of gauge field configurations in higher space-time dimensions, it is shown that the existence of non-trivial structures in ten dimensions would determine a class of models corresponding to a grand unified GUT structure with complex fermion representations with respect to $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. The discussion is carried out within the framework of string theories with characteristic energy scales below the Planck mass. Avoidance of topological obstructions upon continuous deformation of field configurations leads to global chiral symmetry breaking of the underlying fundamental theory, imposes rigorous restrictions on the structure of the vacuum and space-time itself and determines uniquely the gauge structure and matter content.

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From the perspective of theories in higher dimensions we observe zero modes of particle fields, which are protected by the symmetries of the theory from getting masses from a high energy scale or Kaluza-Klein excitations. Quarks and leptons of given helicity form a complex representation of the gauge group, and the gauge invariance of the theory forbids the fermions from acquiring masses at the tree level since there is no pairing of left and right-handed fermions with the same quantum numbers. Although a perplexing aspect of the standard model, the assignment of left-handed particles into doublets of the weak-isospin group and right-handed into singlets, is crucial for the occurrence of the fermion spectrum at low energies. A family of quarks and leptons u, d, e^-, ν form a complex representation of 15 fields, not equivalent to its complex conjugate, which transforms under $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ in a highly reducible representation $(3,2) \oplus (\bar{3},1) \oplus (\bar{3},1) \oplus (1,2) \oplus (1,1)$, with weak hypercharges $Y = 1/6, -2/3, 1/3, -1/2, 1$ respectively. This is the minimal set of fields which is free from chiral anomalies, a condition for the renormalizability of the theory.

In a grand unified theory [1], the different gauge interactions are embedded in a simple Lie-group whose symmetry is manifest at a larger energy scale, and the quantum number content of a given representation, which is arbitrary in the standard model, follows from the transformation properties of the unified gauge group [2]. In the minimal grand unified theory [1] each family of fermions is assigned to the 15-dimensional representation $\bar{\mathbf{5}} + \mathbf{10}$ of $SU(5)$. A family can also be assigned to an irreducible complex representation: the spinorial **16** of $SO(10)$ [3], or the fundamental **27** of E_6 [4]. Other groups which exhibit some attractive features, have only real representations with respect to $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ and no mechanism prevent the fermions from acquiring large masses. As an example, the spinor representations of the orthogonal series $SO(10+4N)$, $N > 0$, which otherwise would incorporate the unification of generations in a simple Lie group [5], are real with respect to $SO(10)$ and conjugate families of unobserved fermions appear. The same problem arises with E_7 and with E_8, the largest group of the exceptional series. Since the fundamental chiral nature of quarks and leptons could not depend on the details of the theory, but rather on general or qualitative properties of a class of theories, they should belong to a universality class [6]. Trying to understand the origin of chirality could thus amount to find the global or topological properties which characterizes such theories.

As a consequence of string dualities [7] the fundamental string scale is no longer necessarily tied to the Planck scale [8, 9], but could be anywhere between the Planck mass and the electroweak scale according to the value of the ten-dimensional dilaton field in the vacuum. In particular, heterotic strings at strong couplings are equivalent to weakly coupled Type I strings [10] with the possibility of lowering the string scale to energies as low as the electroweak scale, with large extra space-time dimensions and unique signatures [11]. Unification of gravity and gauge forces at the weak scale implies radical changes in the gravitational forces at short distances, avoid the gauge hierarchy problem altogether and even the need for low energy supersymmetry, which has been hitherto unobserved [12]. Large extra spacetime dimensions have also the remarkable consequence of changing the behavior of the running gauge coupling constants from logarithmic to power-law above $M_o = 1/R$, where R represents the size of the extra dimensions [13]. As a result, a four-dimensional GUT at scale of 10^{16} GeV, the perturbative unification scale, is replaced by a GUT in a higher dimensional space at a much lower energy scale. This new interpretation could also give a simple explanation of the fermion mass hierarchy [13], and could offer eventually a unified view of gauge and gravitational unification at a TeV scale within the context of Type I strings [14]. It should be noted, however, that two basically different approaches have been followed according to whether the extra dimensions are felt by gauge and gravitational interactions [11, 13] or by the gravitational forces only [12], according to different forms of incorporating the interactions in the underlying string theory.

In this paper we study the global chiral symmetry properties of the ground state of a class of theories with complex fermion representations with respect to $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. The discussion is carried out within the framework of reduced-scale strings where the extra dimensions are felt by all the gauge interactions, but the results are to a large extent independent of the particular value of the compactification scale or even the compactification mechanism, and depend rather on general or global properties of the underlying theory.

At energy scales well above M_o and up to the Plank scale M_P, where the theory is higher dimensional and space-time appears effectively flat, the existence of non-trivial topological structures in ten-dimensional space-time determines a class of models, which corresponds to topologically stable non-perturbative vacuum solutions that break the global chiral symmetry of the theory. This result is related to global gauge transformations by tunneling events between not equivalent gauge configurations which give rise generally to topological obstructions in the fermionic effective functional action.

The global transformations of the effective action in 10 dimensions are expressed in terms of the 11-dimensional spectral flow of zero modes. The restrictions imposed by the existence of topological obstructions leads to global chiral symmetry breaking of the underlying theory by 10-dimensional extended field configurations, and it is shown that those restrictions lead in fact to a unique solution. We further study the vacuum structure of ten-dimensional space-time upon global chiral symmetry breaking in terms of the particular embedding of the spin connection in the ten-dimensional manifold, arising from the maximal subalgebra decomposition of the original symmetry group. Poincaré invariance and breaking of CPT are important issues of the model.

In the usual compactification scheme with the string scale around the Planck scale M_P, the vacuum state is assumed to be a product of $M^4 \times K$ with M^4 the four-dimensional space and K a compact manifold with radius of order of $1/M_P$. In the scenario with the string scale around the electroweak scale, the physical system has strikingly different behavior according to whether the scale M is smaller or greater than M_o. If $M < M_o$, the effect of Kaluza-Klein states and additional dimensions are ignored, whereas for scales well above M_o, the effect of the Kaluza-Klein modes changes the scale-dependence of the running couplings from logarithmic to power-law thus lowering the scale of gauge coupling unification [13]. In this limit, the size of additional space-time dimensions appears as infinite with respect to the scale M, and thus for $M \gg M_o$ space-time appears effectively as a 10-dimensional flat space M^{10}, greatly simplifying the description of the vacuum. We will carry the discussion in terms of an effective field theory to describe the coupling of the fermions to background fields in the flat 10-dimensional space and ignore the Riemannian connection.

Before proceeding we briefly review some basic relations useful for our discussion. The Euclidean effective functional action $exp\,(-W[A])$ describing the coupling of gauge fields to fermions in a complex representation of a gauge group G in a 2n-dimensional Euclidean manifold M^{2n} is

$$exp(-W[A]) = \int [d\psi][d\overline{\psi}]\; exp\left(-\int d^{2n}x\; \overline{\psi} \not{D} \frac{1}{2}(1-\overline{\Gamma})\psi\right),$$

where $\not{D} = \Gamma^A D_A$, the Γ_A are 2n-dimensional gamma matrices $(A, B = 1...2n)$ and $\frac{1}{2}(1-\overline{\Gamma})$ the chirality projection operator, with $\overline{\Gamma} = \Gamma_1\, \Gamma_2 ... \Gamma_{2n}$. A finite, time independent gauge transformation is defined by the nontrivial wrapping of the map $g(x)$ around the gauge group G for the one-form gauge connection $A = A_B\; dx^B$ on M^{2n}: $A^g = g^{-1}A\; g + g^{-1}d\; g$, where $d = (\partial/\partial x^A)dx^A$, and $g(x) \to 1$ as $|x| \to \infty$, such that

$g(x)$ cannot be deformed to the identity. Nontrivial field configurations are determined by the mappings from S^{2n-1} into G and thus $\Pi_{2n-1}(G)$ classifies G-bundles over S^{2n}. In four dimensions, the mappings from S^3 into G are classified by $\Pi_3(G)$ the third homotopy group, which is equal to the group of integers **Z** for any simple Lie group. Nontrivial field configurations in four dimensions (instantons) are homotopically equivalent since according to a theorem due to Bott [15], any continuous mapping of S^3 into a simple Lie group can be continuously deformed in a mapping into an $SU(2)$ subgroup of G, equivalent to an $S^3 \to S^3$ mapping, and thus it is not possible to differentiate among topologically nontrivial configurations from different groups. On the other hand, theories in higher dimensions present a diversity of topological structure which is, from the point of view of the homotopy properties, absent in a four dimensional space.

The coupling of fermion zero modes to the background field strength $F = dA + A \wedge A = \frac{1}{2} F_{AB}\ dx^A \wedge dx^B$, is expressed in terms of the character-valued index of the Dirac operator [16] which is the difference of fermion zero modes of opposite chirality: $\psi^+ = \frac{1}{2}(1+\overline{\Gamma})\psi$ and $\psi^- = \frac{1}{2}(1-\overline{\Gamma})\psi$:

$$ind \not{D}_{2n} = n^+ - n^- = \int\ ch(F) = \int\ \frac{1}{n!}\ (\frac{i}{2\pi})^n\ Tr\ F^n,$$

where the integral is over a 2n-dimensional manifold and the trace is taken for states in a given fermion representation of G. The Chern character is given by $ch(F) = Tr\ e^{iF/2\pi}$. Since $Tr\ F^n$ is a closed 2n-form, $dTr\ F^n$ is locally exact and can be expressed as a $2n-1$ Chern-Simons form: $Tr\ F^n = d\omega_{2n-1}(A)$, with $\omega_{2n-1}(A) = n\ \int_0^1\ Tr[A\ F_t^{n-1}]\ dt$ and $F_t = t\ dA + t^2 A^2$. Using Stoke's theorem, and integrating by parts [16]

$$ind \not{D}_{2n} = \int_{S^{2n-1}} Q_{2n-1}(g^{-1}dg) = \frac{(n-1)!}{2^n \pi^n (2n-1)!}\ \int_{S^{2n-1}}\ Tr(g^{-1}dg)^{2n-1},$$

where $Q_{2n-1} = 1/n!\ (i/2\pi)^n\ \omega_{2n-1}$. The right hand side of the above expression is an integer representing the winding number or topological charge of the homotopy classes of $\Pi_{2n-1}(G)$, which depends only on the properties of $g(x)$.

In 2p-dimensions, the Chern class is given in terms of the 2p-form $d_{a_1 ... a_p}\ F^{a_1} \wedge ... \wedge F^{a_p}$, where $d_{a_1 ... a_p}$ is a totally symmetric invariant tensor. In terms of the representation matrices X of an irreducible representation of the generators of the Lie algebra of G, d is written as a symmetric trace denoted by STr, which is a trace over all the permutations of the product of p representation matrices: $d_{a_1 ... a_p} = STr(X_{a_1} ... X_{a_p})$. The operator

$$I_p = \frac{1}{p!} \sum_{a_1 ... a_p}\ d^{a_1 ... a_p}\ X_{a_1} ... X_{a_p},$$

commutes with all the elements of the algebra in a given representation, and is in fact a Casimir invariant of order p. If the Lie algebra has no Casimir invariant of order p ($d_{a_1 \ldots a_p} = 0$), the index theorem implies that there are no fermion zero modes of definite chirality.

Extended field configurations which carry a conserved quantum number are stable and the possible field configurations are determined by the condition that some functional of the fields is finite [17]. Such configurations are characterized by their global properties in terms of the elements of homotopy groups. To understand better the relation between a Casimir invariant and homotopy, as well as disposing of a simple tool for finding the properties of the homotopy groups, we express a compact connected Lie group as a product of spheres following Pontrjagin and Hopf. This useful expansion, allows us to determine the topological properties of simple Lie groups by reading the properties of the mappings between spheres [18]. The rank of G is equal to the number of Casimir invariants and the number of spheres in the expansion is equal to the rank of the group. A Lie group G of rank m behaves as the product of m odd-dimensional spheres S^{2p-1}, with p the order of each Casimir invariant in G. As an example the rank 2 group $SU(3)$ has Casimir invariants of order 2 and 3 and the exceptional rank 4 group F_4 of order 2, 6, 8 and 12. Thus, $SU(3)$ and F_4 are expressed as $SU(3) \sim S^3 \times S^5$ with dimension 8 and $F_4 \sim S^3 \times S^{11} \times S^{15} \times S^{23}$ with dimension 52. Since the mappings from $S^n \to S^n$ are classified by $\Pi_n(S^n) = \mathbf{Z}$, it follows that the existence of a Casimir of order p in the algebra of a compact group G is equivalent to the condition $\Pi_{2p-1}(G) = \mathbf{Z}$.

Consider a theory in a 2n-dimensional Euclidean manifold M^{2n} and the coupling of gauge fields to fermions in a representation of a gauge group J (not necessarily complex) described by the Euclidean effective functional action $exp(-W[A^J])$, such that the theory has trivial topology in $2n$ dimensions: $\Pi_{2n-1}(J) = 0$. We further assume that the underlying theory is free from perturbative anomalies, gauge and gravitational, before compactification or symmetry breaking. According to the discussion above, the operator $\not{D}(A^J)$ has no zero modes of definite chirality and furthermore we can set $A^J = 0$ in the vacuum. However the existence of nontrivial topological structures in higher dimensions [19], could imply that the vacuum state with all the gauge fields vanishing simultaneously is not a stable solution. If the theory has nontrivial field configurations for a subgroup G of J, $G \subset J$, $\Pi_{2n-1}(G) \neq 0$, the symmetry can be broken in the sector of the theory characterized by the non-trivial G-bundle. Equivalently, it can be stated that the theory is broken at the quantum level from the fermionic measure $[d\psi][d\overline{\psi}]$ in the effective Euclidean functional action.

To obtain the functional integral $exp(-W[A^G])$ from $exp(-W[A^J])$ we must consider the transition from A^J to A^G, which is equivalent to deform continuously one field configuration into the other, while keeping the effective action $W[A]$ finite. Since we are considering disconnected gauge transformations, the interpolation of configurations cannot be done along the connection fiber, but rather we should consider a one-parameter family A_t interpolating from A^J to A^G along a path defined in the space of gauge connections $\mathcal{C}$, which is contractible. We can take for example $A_t = A^J + t(A^G - A^J)$ with $0 \leq t \leq 1$, which corresponds to an instanton transition in the temporal gauge. Although $\mathcal{C}$ is contractible, topological obstructions will generally appear in the functional action which must be integrated over the gauge orbit space, $\mathcal{C}/\mathcal{G}$, the space of all gauge not equivalent configurations ($\mathcal{G}$ is the group of all gauge transformations). The space $\mathcal{C}/\mathcal{G}$ is in general non-contractible and gives rise to the non-trivial topology of the theory as we interpolate the effective action from $t = 0$ to $t = 1$ as a functional of A_t:

$$\Delta W = \int_0^1 \ dt \ \frac{d}{dt} W[A_t] = W[A^G] - W[A^J].$$

The change in the effective action ΔW is obtained by integrating out the fermionic fields and is given by a general formula derived by Witten in his study of global anomalies [20]

$$\Delta W = \frac{\pi i}{2} \eta \ \ (mod \ 2\pi i),$$

where η is the invariant of Atiyah, Patodi and Singer [21], which expresses the spectral asymmetry of the eigenvalues λ of the Hamiltonian

$$\eta = \lim_{\epsilon \to 0} \sum_{\lambda} \ sgn(\lambda) \ exp(-\epsilon |\lambda|),$$

on a $2n + 1$ dimensional manifold $M^{2n} \times S^1$.

It is very useful to represent the spectral asymmetry in $2n + 1$ dimensions as the continuous change in η, and express the spectral flow in terms of an index in $2n + 2$ dimensions [21, 22]. In terms of the gauge field A defined in the $2n$ manifold M^{2n} we can define a field in a $2n + 1$ dimensional manifold $M^{2n} \times S^1$ as $\mathcal{A} = (A, 0)$, as well as an interpolating $2n + 1$ field $\mathcal{A}_t$ by $\mathcal{A}_t = \mathcal{A}^J + t(\mathcal{A}^G - \mathcal{A}^J)$, $0 \leq t \leq 1$. Using the results of Ref. [22], we write the change in the effective functional action in terms of the difference between $2n+1$ Chern-Simons at the end-points of the path of integration along the variable t:

$$exp\left\{-(W[A^G] \ - \ W[A^J])\right\} = exp\ \left[-\frac{i}{2}\pi\int_{M^{2n}\times S^1} Q_{2n+1}(\mathcal{A}^G) \ - \ Q_{2n+1}(\mathcal{A}^J)\right].$$

Topological obstructions arise unless $Q_{2n+1}(\mathcal{A}^G)$ and $Q_{2n+1}(\mathcal{A}^J)$ vanish. In terms of homotopy this condition is equivalent to $\Pi_{2n+1}(G) = 0$ and $\Pi_{2n+1}(J) = 0$, since the $2n+1$ dimensional spectral flow and the $2n+2$ dimensional index are represented by the same invariant in the absence of gravity [22].

If the effective field theory at scales $M \gg M_o$ is embedded in a weak coupling Type I String, we could take advantage of the duality maps between strings to examine the strong coupling regime of the equivalent heterotic string with gauge group $E_8 \otimes E_8$ [23]. Since the topology of E_8 in ten dimensions is trivial, $\Pi_9(E_8) = 0$, the gauge fields associated with one of the E_8 groups in the vacuum could be set equal to zero and decouple from the fermion sector. As discussed above, the existence of non-trivial bundles in a sector of E_8 could imply that the non-perturbative vacuum structure cannot be a state with all the gauge fields vanishing simultaneously. We assume that the trivial vacuum is indeed broken by non-trivial topological structures given in terms of some finite functional of the fields. The possible field configurations are classified by their homotopy properties and are determined by the ninth homotopy group of the gauge group G, $\Pi_9(G)$, in ten dimensions.

In a survey of global properties of the compact connected simple Lie groups [24] it is found that:

$$\Pi_9(SU(5)) = \mathbf{Z}, \quad \Pi_9(SO(10)) = \mathbf{Z} + \mathbf{Z}_2, \quad \Pi_9(E_6) = \mathbf{Z},$$

which correspond precisely to the GUT theories with complex fermion representations with respect to $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ [25]. The higher dimensional groups $SU(N), \quad N \geq 6$, which also have complex fermions representations with respect to SU(5) belong to the same class. Π_9 is stabilized for the unitary groups at $N = 5$, which in fact is the lowest rank group that incorporates the strong and electromagnetic interactions. The groups $SO(10)$ and E_6, which are successful embedding the different integration subgroups into a larger rank group, belong to the same class. For all other Lie groups Π_9 has a finite number of elements or it is trivial [24]. In terms of representation theory, as discussed above, the condition $\Pi_9(G) = \mathbf{Z}$ is equivalent to find out among all compact Lie groups, which have a fifth-order Casimir invariant [26]. Note also the existence of fermion zero modes with definite chirality in ten dimensions for the class of GUT models with complex fermion representations, as consequence of the index theorem in the presence of non-trivial field configurations.

Let us finally examine the topological obstructions in the effective functional action from the global chiral symmetry breaking of the theory in ten dimensions, as we interpolate between states by continuously deforming the field configurations in the functional integral $exp(-W[A])$. As discussed above, the change in the effective functional action is expressed in terms of the 11-dimensional spectral flow of zero modes. The theory has no topological obstructions if the Chern-Simons form $Q_{11}(\mathcal{A}_t)$ vanishes for the initial and final configurations. Equivalently, it can be stated that the theory has no topological obstructions under global transformations between gauge non equivalent configurations if Π_{11} is zero for the initial and final gauge groups. Examining the global properties of the compact connected Lie groups we find the remarkable result that among an infinite number of possibilities Π_{11} is zero only for E_8 and $SU(5)$ [24]:

$$\Pi_{11}(E_8) = 0 \quad and \quad \Pi_{11}(SU(5)) = 0,$$

allowing a transition from E_8, the gauge group of the heterotic string, to $SU(5)$, the minimal grand unified theory that contains the observed quarks and leptons.

According to the CPT theorem the fermions of given chirality in ten dimensions transform under real representations of the gauge group. Supersymmetry requires that this representation is the adjoint. Both conditions are met by E_8, since its fundamental representation is its adjoint. The transition from E_8 to $SU(5)$, a group with complex fermion representations, breaks the chirality of the theory and furthermore supersymmetry is not preserved by the transformation. Note that CPT is also broken. Since E_8 has real fermionic representations, according to the character-valued index theorem of the Dirac operator, the appearance of fermion zero modes of definite chirality in ten dimensions signals the breaking of the global chiral symmetry of the theory.

Under the maximal subalgebra decomposition, E_8 breaks as $SU(3) \otimes E_6$, $SO(16) \supset SO(6) \otimes SO(10)$ and $SU(5) \otimes SU(5)$. In the usual description of string compactification of higher dimensions [27], the vacuum is assumed to be a product of the form $M^4 \times K$ to preserve four-dimensional Poincaré invariance and a vacuum expectation value is introduced on K to break E_8, otherwise there is no chiral asymmetry of fermions in four dimensions. This is accomplished by identifying the spin connection of K with background gauge fields on $SO(6)$ or $SU(3)$, which act as holonomy group of the manifold K, the group of rotations in the tangent space generated by parallel transport on closed loops on K.

In the present work, the E_8 symmetry is not broken by compactification but rather as consequence of the breaking of global chiral symmetry. As discussed above, this is only possible for a symmetry breaking along $SU(5) \otimes SU(5)$ since

$$\Pi_{11}(E_8) = 0 \quad and \quad \Pi_{11}(SU(5) \otimes SU(5)) = \Pi_{11}(SU(5)) \otimes \Pi_{11}(SU(5)) = 0.$$

Note that both $SU(5)$ groups have non-trivial configurations in ten dimensions. One SU(5) carries the GUT structure and its algebra the quantum numbers of the fermion representations, the other $SU(5)$ constitutes a natural embedding in a 10-dimensional manifold, where the spin connection is an $O(10)$ gauge field and the holonomy group is a subgroup of $O(10)$. If the spin connection w is identified with a background gauge field on the second $SU(5)$, acting as holonomy group, the configurations for the two-form field F and the Lie-algebra valued curvature two-form $\Omega = dw + w \wedge w$ are determined by the integer classes, $C_n = \int c_n$, corresponding to a fifth Chern class c_5 in a ten dimensional manifold:

$$\frac{i}{3840} \int_{M^{10}} F \wedge F \wedge F \wedge F \wedge F = C_5,$$

$$\frac{i}{3840} \int_{M^{10}} \Omega \wedge \Omega \wedge \Omega \wedge \Omega \wedge \Omega = C_5.$$

To describe the particular embedding of the second SU(5), it is convenient to introduce a complex manifold K^n, of dimension n, with holomorphic transition functions in a real manifold M^{2n}, of dimension $2n$ [28]. As an example, Euclidean 2n-dimensional space is identical to the n-dimensional complex manifold C^n with a complex metric. In a Kähler manifold, the vector representation of $SO(2N)$ is given in terms of the fundamental representations $N \oplus \overline{N}$ of the holonomy group $U(N)$, according to the real Kähler two-form harmonic metric $K = \frac{i}{2} g_{a\overline{b}} dz^a \wedge d\overline{z}^{\overline{b}}$, where

$$\int_{K^n} K \wedge K \wedge ... \wedge K \; > 0,$$

defines the natural orientation of the manifold. If a Kähler manifold K^n has a metric of $SU(N)$ holonomy, the first Chern class of the manifold c_1 is zero and the metric is

Calabi-Yau. Our particular embedding has a Calabi-Yau metric of SU(5) holonomy, and since a Calabi-Yau manifold is Ricci-flat it corresponds to a 10-dimensional manifold with cosmological constant zero. The Kähler form K can be expressed locally in terms of a zero form ϕ, the Kähler potential, as $K = \frac{i}{2}\, \partial\overline{\partial}\phi$. K is in fact the curvature of the manifold: $\Omega = 2iK$.

To obtain a physical theory in four dimensions Poincaré invariance should be recovered at large distances away from the extended field configurations or 10-dimensional instantons. In terms of the variables $z_1 = x + iy$, $z_2 = z + it$, z_3, z_4 and z_5 this implies that ϕ is flat along the z_1 and z_2 direction

$$\phi(z_1, ..., z_5)_{z_1,z_2\to\infty} \to \; |z_1|^2 + |z_2|^2,$$

and bounded by a distance R along the other complex coordinates. This configuration of space-time with extra dimensions is reminiscent of the domain wall interpretation given some time ago by Rubakov and Shaposhnikov [29] of the kink solution in a five dimensional space, leading to fermion zero modes in four dimensions. The actual solution in ten dimensions would follow from the 10-form equation given above in terms of the curvature Ω, with $c_1(\Omega) = 0$. The solution of the $SU(5)$ instanton F should be studied along similar lines, but without a restriction on the first Chern class, $c_1(F)$. Hopefully, only the diagonal group generators survive away from the gauge instanton, thus breaking $SU(5)$ and avoiding the proton decay problem. A thorough study of the ten-dimensional extended field configurations or instantons, including the stability of the solutions and the relation between the metric and field configurations, should be undertaken to confirm the theoretical viability of the ideas here exposed. In particular, the determination of the spectrum of fermion zero modes in four dimensional space-time is crucial. Experimentally, a signature of the global chiral symmetry breaking mechanism discussed in this paper include effects from the breaking of CPT and Poincaré invariance which are suppressed by a factor of order of M_o/M_P [30].

A major reason for the introduction of grand unified models as well as the motivation behind contemporary studies of Kaluza-Klein theories is the need to understand the quantum numbers of quarks and leptons and their fundamental chiral nature. We have shown in this paper how both approaches are related. We have used topological considerations as valuable tools for studying general properties of physics in higher dimensions due to the richness of nontrivial structures that are present. The existence of nontrivial topological structures give us an indication of the origin of the structure of grand unified theories and the observed spectrum of chiral fermions, which is protected from acquiring Kaluza-Klein excitations by the coupling of zero modes to nontrivial background field configurations. Breaking of the global chiral symmetry of the theory follows from the continuous deformation of field configurations in the fermionic functional action, leading to a unique solution imposed by the avoidance of topological obstructions. The solution found could also have profound implications in the structure of space-time itself as supported by extended field configurations with non trivial topology in the metric and the gauge fields and could constitute a viable alternative to compactification mechanisms.

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