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Nonlinear Resonant Collimation for Future Linear Colliders*

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Abstract

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Abstract. We present a scheme for collimating large amplitude particles in the main linacs of a linear collider, by adding octupoles to the FODO lattice of the linac. With this scheme the requirements on downstream collimation can be greatly reduced or perhaps even eliminated. An analytic estimate of the amplitude at which particles are lost is made by calculating the separatrix of the fourth order resonance, and is in good agreement with the results of simulations. Simulations of particle distributions in the beam core and halo are presented, as well as alignment tolerances for the octupoles.

INTRODUCTION

Present designs for future linear colliders, such as the NLC [1] have dedicated collimation systems several kilometers in length. The collimation systems [1] are designed to serve two different functions: to protect all downstream systems against bunch trains which enter with large betatron excursions or large energy errors, and to remove the beam halo, which otherwise would cause unwanted background in the detector. An additional requirement is that in this scheme each betatron phase and each plane must be collimated twice.

It would of course be desirable to prevent or reduce the formation of the halo in the first place. However, there are many (probably unavoidable) sources of beam halo: (1) beam-gas Coulomb scattering, (2) beam-gas bremsstrahlung, (3) Compton scattering on thermal photons, (4) linac wakefields, (5) the sources, damping rings, and bunch compressors. In the Stanford Linear Collider (SLC) collimation of the beam before it enters the final focus and detector area was found to be essential, and it is expected that this will also be true for future linear colliders such as the NLC.

The NLC collimation system length of several kilometers is determined by the condition that spoilers and absorbers should survive the impact of an entire bunch train (nearly 10^{12} electrons). This requires a minimum spot size, in order that the collimator surface does not fracture or that the collimator does not melt somewhere inside its volume. For the NLC parameters, fracture and melting conditions give rise to about the same spot-size limit (roughly $10^6/\mu\mathrm{m}^2$ for a copper absorber at 500 GeV [1]). While the surface fracture does not depend on the beam energy, the melting limit does, since the energy of an electromagnetic shower deposited per unit length increases in proportion to the beam energy. Therefore, the beam area at the absorbers must increase linearly with energy. Since, in addition, the emittances decrease inversely proportional to the energy, the beta functions must increase not linearly but quadratically. Assuming that the system length l scales in proportion to the maximum beta function at the absorbers, this results in a quadratic dependence [2], $l \propto \gamma^2$ ($\gamma \equiv E_{beam}/m_e c^2$). Counting both sides of the IP, the NLC collimation system is 5 km long. At 5 TeV the length of the collimation system could be 50 km. A schematic of such a conventional collimation system, as in the NLC design, is shown in Figure 1.

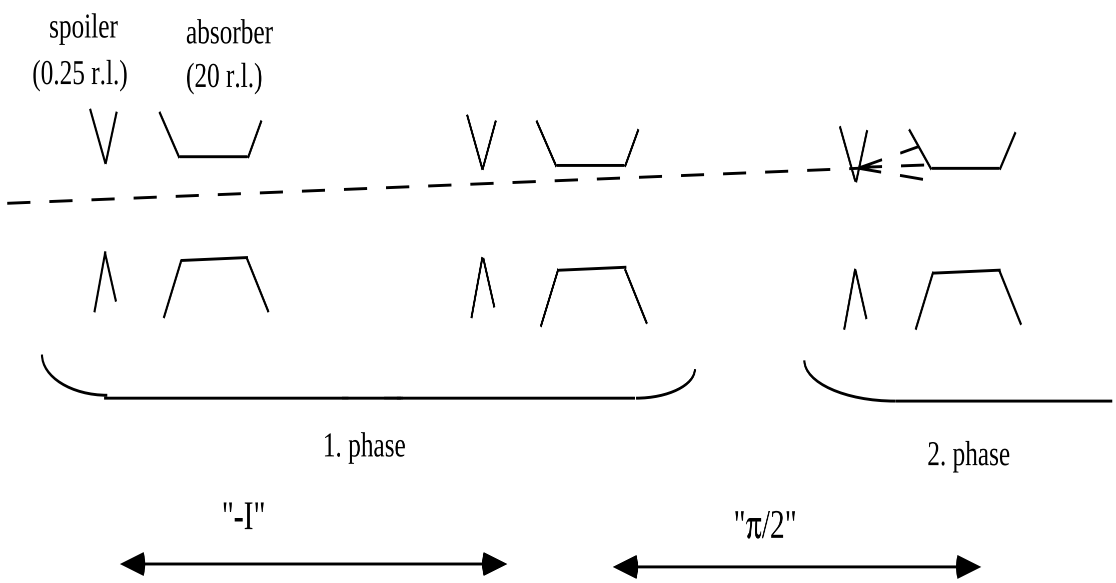

FIGURE 1. Schematic of a conventional collimation system, consisting of a series of spoilers and absorbers. The size of the spoilers and absorbers is approximately 1/4 and 20 radiation lengths (r.l.), respectively.

In this paper we present an alternative scheme for collimating large amplitude particles in the main linacs of a linear collider, by adding octupoles to the FODO lattice of the linac. The nonlinear fields of these octupoles are arranged in a configuration that resonantly destabilizes particles at large betatron amplitudes. The effective “dynamic aperture” of the linac can be controlled either through the octupole strength or through the phase advance per cell. Both halo particles and mis-steered beam pulses are dispersed by the nonlinear field, before they reach the end of the linac. Such a scheme could greatly reduce the requirements on a dedicated downstream collimation section, perhaps even eliminate it altogether.

The octupole magnets may be placed at every focusing quadrupole, where the horizontal beam size is largest, in order to remove horizontal beam tails. A similar beam line, with octupoles near the defocusing quadrupoles, may be employed for the vertical plane. Alternatively, an interleaved scheme which collimates both planes at the same time is conceivable.

In the NLC design, the required collimation depth is approximately 660 microns (corresponding to about $10\sigma_x$ at the defocusing quadrupoles). Note also that it may be acceptable to collimate further out by placing octupoles in front of the final focus quadrupole doublet; these have the effect of "folding in" residual beam tails before entering the doublet [3].

COLLIMATION IN ONE PLANE AT A TIME

We begin by considering the case of collimation in one plane at a time. As noted above, the octupoles are placed near every quadrupole that is focusing in this plane. First we give an analytic estimate of the collimating effect of such a system, and then we present simulations of the collimation, blow-up of the beam, and alignment tolerances.

Analytic estimate of collimation depth

For simplicity, in this section we describe the system by a smooth one-dimensional model and use a Hamiltonian approach to estimate the location of fourth-order separatrix introduced by the octupoles. A basic unit of the NLC octupole linac is assumed to consist of two FODO cells, with octupoles of effective integrated strength $k_1 = k_{oct,1}\beta^2$ and $k_2 = k_{oct,2}\beta^2$ (in the peculiar units of m^{-1}, where β is the beta function at the octupoles). Here $k_{oct,1}$, $k_{oct,2}$ are the conventional integrated octupole strengths in m^{-3}) near the two QF quadrupoles, i.e.

$$k_{oct} \equiv \frac{\partial^3 B_y/\partial x^3}{(B\rho)} \ell_{oct} \quad , \tag{1}$$

where ℓ_{oct} is the length of the octupole. This set of two FODO cells is then repeated periodically along the linac. The dynamics of this system is the same as for a storage ring with ring circumference equal to the length of the two FODO cells and with an effective tune equal to twice the phase advance per FODO cell divided by 2π. For the NLC [1], the effective tune is about 0.5, very appropriate for octupole collimation.

We choose as our canonical coordinates the action angle variables (I, ϕ) of the linear system. These are related to the physical transverse coordinates via

$$x(s) = \sqrt{2\beta(s)I}\cos\phi(s) \tag{2}$$

$$x'(s) = -\sqrt{2I/\beta(s)}\left(\sin\phi(s) + \alpha\cos\phi(s)\right), \tag{3}$$

where s is the position along this model linac. We will find it convenient to replace s by the azimuthal angle $\theta = 2\pi s/L$, where L denotes the circumference, and we will use θ as "time" variable.

The Hamiltonian describing this system then assumes the form

$$H(I,\phi,\theta) = QI + \frac{1}{4!}(2I)^2 \left[k_1 \cos^4\phi \sum_{q=-\infty}^{\infty} \delta(\theta + q2\pi) \right.$$
$$\left. + k_2 \cos^4\phi \sum_{q=-\infty}^{\infty} \delta(\theta + \pi + q2\pi)\right] \tag{4}$$

where $\theta = 0$ is the location of the first octupole. We can expand the trigonometric functions and the delta functions, and then find

$$k_1 \cos^4\phi \sum_{q=\infty}^{\infty} \delta(\theta + q2\pi) = \frac{k_1}{16}\left[e^{i4\phi} + e^{-i4\phi} + 4e^{i2\phi} + 4e^{-i2\phi} + 6\right] \frac{1}{2\pi} \sum_p e^{i\theta p} \tag{5}$$

$$k_2 \cos^4\phi \sum_{q=-\infty}^{\infty} \delta(\theta + \pi + q2\pi) =$$
$$\frac{k_2}{16}\left[e^{i4\phi} + e^{-i4\phi} + 4e^{i2\phi} + 4e^{-i2\phi} + 6\right] \frac{1}{2\pi} \sum_p e^{i(\theta+\pi)p} \tag{6}$$

with $\sum_p$ a sum over all integer numbers p. There are infinitely many terms. Of relevance are only the resonant terms, which are not rapidly oscillating. For a linac phase advance of 90 degree per cell, we have $2\phi(\theta) \approx \theta$ (note that the "ring" comprises two FODO cells). Hence, we must keep the resonant terms $\pm(2\phi - \theta)$ and $\pm(4\phi - 2\theta)$, as well as the secular term.

If we choose $k_1 = -k_2$, the driving term of the $\pm(4\phi - 2\theta)$ resonance and the secular term cancel. Changing variables to $\psi = \phi - \theta/2$ and defining $\Delta Q = Q - 1/2$, the total Hamiltonian in the "rotating" frame is

$$\hat{H}(I,\psi)_{k_1=-k_2} = \Delta Q I + \frac{1}{12\pi} I^2 \; k_1 \cos(2\psi) \tag{7}$$

If, on the other hand, we use equal-sign octupoles, the Hamiltonian becomes:

$$\hat{H}(I,\psi)_{k_1=k_2} = \Delta Q I + \frac{1}{48\pi} I^2 \; k_1 \cos(4\psi) + \frac{1}{16\pi} I^2 \; k_1 \tag{8}$$

The odd-sign configuration of Eq. (7) is preferable, since the coefficient in front of the resonance driving term is four times larger and there is no amplitude-dependent tune, which could drive large-amplitude particles away from the resonance.

Let us then examine the odd-octupole configuration more closely. The Hamiltonian of Eq. (7), suggests that we might attempt to estimate the dynamic aperture limit A by setting the instantaneous tune where the particle spends most of its time ($\cos 2\psi \approx \pm 1$) to the resonant tune of 1/2:

$$\left[\frac{\partial \hat{H}}{\partial I}\right]_{I=A,\ \cos^2 2\psi=\pm 1} = \frac{\partial}{\partial I}\left[\Delta Q I \pm \frac{1}{12\pi} I^2\ k_1\right]_{I=A} = 0 \tag{9}$$

or

$$A \approx \left|\frac{6\pi \Delta Q}{k_1}\right| \quad , \tag{10}$$

indicating a strong dependence on the tune difference ΔQ. Figure 2 compares this estimate with a simulation result for a two-dimensional map consisting of linear rotations and octupole kicks.

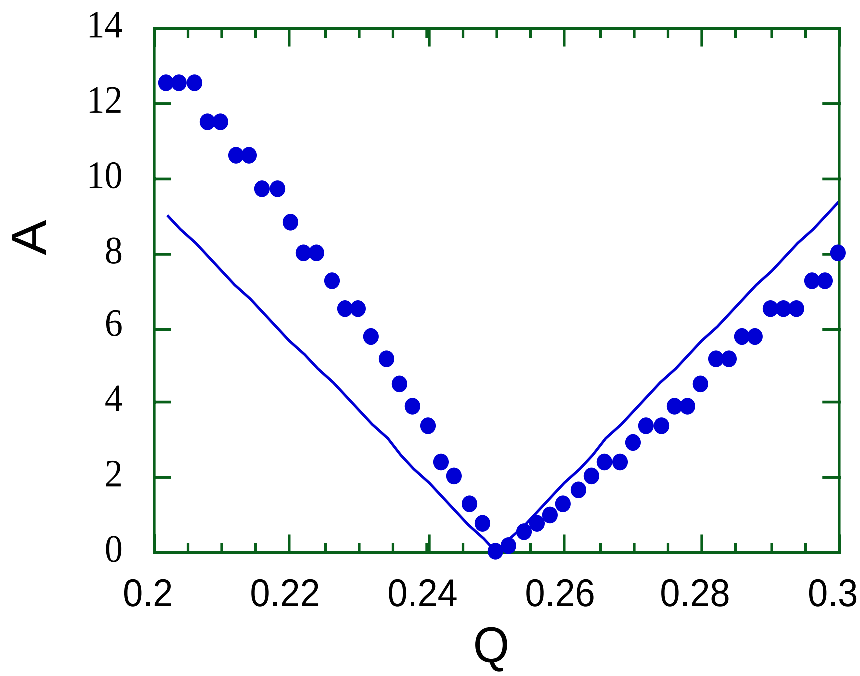

FIGURE 2. Dynamic aperture vs. the phase advance per cell $Q/2$: simulation result (dotted) and the crude analytical estimate of Eq. (10) (solid). For this example we have taken $k_1 = 1$.

Simulation results

In the following we present a series of simulation results for a realistic example, which approximates the situation over the first few kilometers in the NLC main linac. We assume normalized emittances of the order $\gamma\epsilon_x = 3 \times 10^{-6}$ m, and $\gamma\epsilon_y = 3 \times 10^{-8}$ m. The vertical emittance is slightly smaller than the NLC design emittance, and thus represents a worst case for the purpose of this study. The beam energy at the linac entrance is 10 GeV. The real linac accelerates the beam of course, and the spacing and strength of the quadrupoles increase along its length. We may, however, consider an "equivalent" linac with beam energy equal to that of the incoming beam in the real linac (10 GeV), without acceleration, and with strength and spacing of the quadrupoles equal to that of the magnets at the beginning of the real linac. The pole tip fields of the octupoles in the real linac would scale approximately as γ, if they are taken as constant in this “equivalent” linac.

The linac section considered consists of FODO cells with a length of 12.5 m, and a horizontal phase advance per cell close to 90 degree. Octupoles with an integrated strength of 4000 m^{-3} are placed at the focusing quadrupoles. At a beam energy of 10 GeV, this strength corresponds to a pole-tip field of 0.3 kG, for a 10 cm long octupole with 5 mm bore radius. The sign of the octupole field alternates from cell to cell, in accordance with our considerations above. For these parameters, collimation in the horizontal plane takes place at about 10–12 σ_x. This horizontal collimation section must be followed by an equivalent linac segment with octupoles at the defocusing quadrupoles, and with a vertical phase advance near 90 degree per cell. One advantage of separating the horizontal and vertical octupole sections is that the resonant collimation is most effective near a 90° phase advance. This phase advance can be established only in one plane at a time, because the NLC design foresees a 10° per cell phase-advance difference between the two planes, in order to reduce the sensitivity to skew quadrupole errors and to ions.

We present simulation results for collimation in the horizontal plane. Fig. 3 and Fig. 4 depict the emittance growth and the beam loss experienced by a beam that is injected into the linac with a betatron oscillation of varying amplitude. For injection oscillations less than about $10\sigma_x$ there is no significant emittance growth due to the octupoles. For larger oscillations, the beam size blows up quickly, while at the same time the transmission drops rapidly from 100% to roughly 0. For a $15\sigma_x$ incoming oscillation all particles are lost in the linac. The collimation amplitude is thus sharply defined.

Figure 5 illustrates that for larger oscillations the beam is lost completely in the linac. The positions of particle losses are spread out over a large region, as long as the betatron oscillation is less than about $40\sigma_x$. This is important in order to reduce the heat load for dedicated absorbers. At large oscillations the entire beam is lost within a few FODO cells after injection. Oscillations of this magnitude must be prevented by a machine protection system.

Alignment tolerances

We have also performed simulations to quantify the effects of octupole misalignments. The misalignments are assumed to be randomly distributed according to a Gaussian truncated at either $\pm 2\sigma_{misal}$ or $\pm 3\sigma_{misal}$. The value of σ_{misal} is assumed to be the same for both x and y. In Figure 6 we show the emittance blow-up factors in the two transverse planes as a function of σ_{misal}, for the cases of truncation at $2\sigma_{misal}$ or $3\sigma_{misal}$.

We see that to keep the emittance growth down to a few percent, the required tolerance is a standard deviation of about 200 μm. The octupoles must be tied to

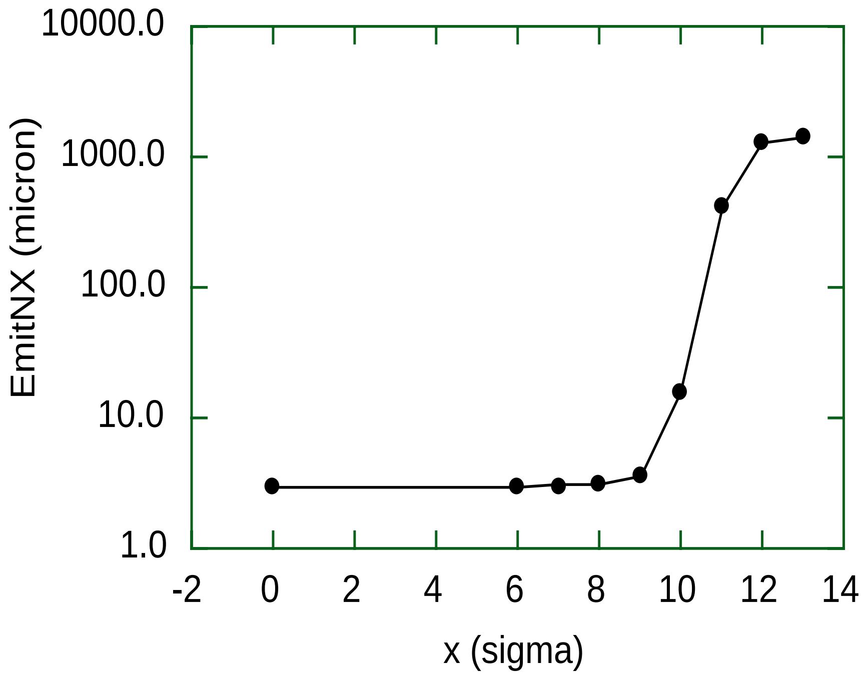

FIGURE 3. Emittance growth due to an incoming betatron oscillation: Shown is the horizontal emittance growth after about 160 FODO cells, as a function of the initial horizontal betatron amplitude in units of the rms beam size.

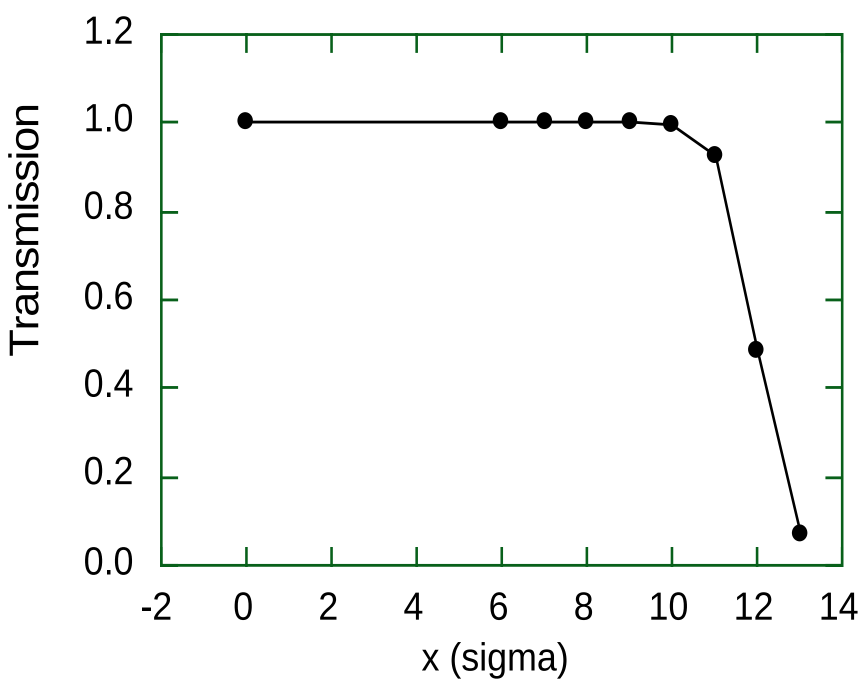

FIGURE 4. Beam loss due to an incoming betatron oscillation: Shown is the fraction of beam lost after about 160 FODO cells, as a function of the initial betatron amplitude in units of the rms beam size.

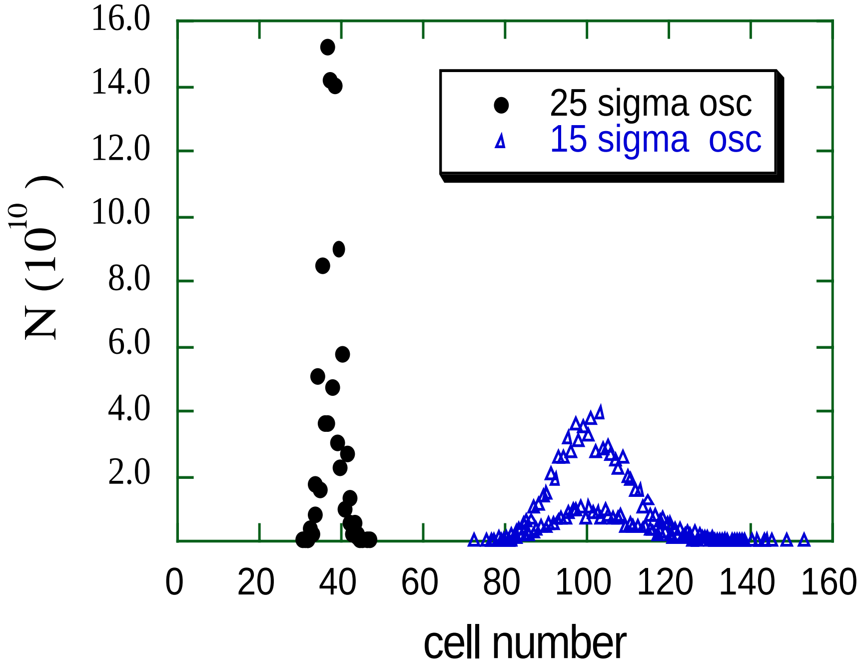

FIGURE 5. Distribution of lost particles along the NLC linac, considering incoming horizontal oscillations, with an amplitude of $15\sigma_x$ and $25\sigma_x$, respectively.

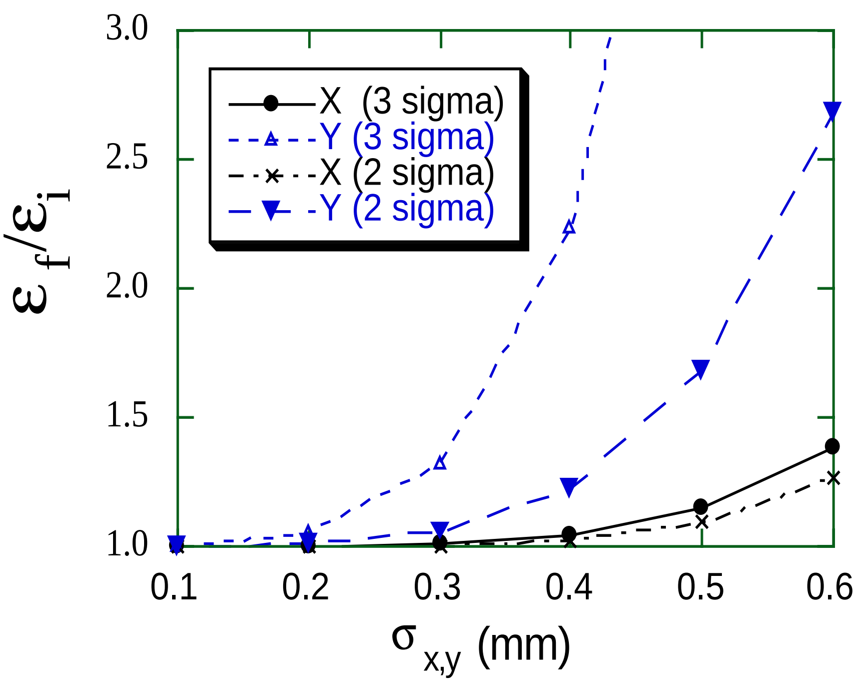

FIGURE 6. Emittance blow-up factor due to octupole misalignments

the quadrupoles at this level of precision. The quadrupoles themselves are aligned with respect to the beam to within 1–2 μm. At the SLC it was attempted to align the final-focus sextupoles with respect to the next quadrupoles with a precision of 50 μm, which proved to be too difficult a tolerance to maintain in the actual tunnel. However 150 μm is routinely achieved.

SIMULTANEOUS COLLIMATION IN BOTH TRANSVERSE PLANES

In this section, we present simulation results using an octupole configuration that allows simultaneous collimation in both x and y planes. The advantage of this scheme is that the halo is collimated in both transverse plane at once, which may save space or increase the collimation efficiency. There is an octupole at every quadrupole (defocusing and focusing), and there is a phase advance per FODO cell close to 90 degrees in both transverse planes. The basic linac cell for our proposed scheme is shown in Figure 7. The sign of k_{oct} alternates from FODO cell to FODO cell, where the kicks in x and y are related to k_{oct} according to:

$$\Delta x' = -\frac{1}{6} k_{oct}(x^3 - 3xy^2) \ , \tag{11}$$

$$\Delta y' = -\frac{1}{6} k_{oct}(y^3 - 3x^2 y) \ . \tag{12}$$

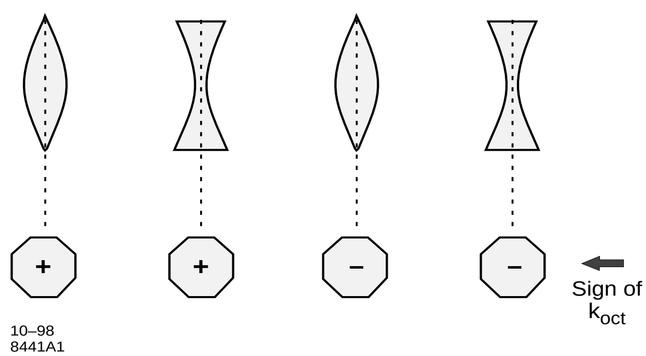

FIGURE 7. Basic linac cell for octupole resonant collimation scheme. The significance of the sign of k_{oct} is discussed in the text.

As discussed previously, we consider an "equivalent" linac with beam energy 10 GeV. This constant FODO cell length is taken to be 12.4 m. We use stronger octupoles than in the preceding section, namely 40000 m^{-3} (which for a beam energy of 10 GeV, an inner bore radius of 5 mm and an octupole length of 10-cm, yields a 3-kG pole tip field). The beam sizes are $\sigma_x = 66\mu$m (27μm) and $\sigma_y = 3.3\mu$m (8.0μm) for the quadrupoles that are focusing (defocusing) in x.

To visualize the effects of collimation, we consider initial distributions consisting of rings in the $x - y$ plane; the distribution is taken to be uniform in such a ring and also in a rectangle in the $x' - y'$ plane that is cut off at $\pm n_x \sigma'_x$ and $\pm n_y \sigma'_y$, where $n_x(n_y)$ is the radius of the ring in units of $\sigma_x(\sigma_y)$. In Figure 8, we show the fate of the particles in such a halo ring after 350 FODO cells (the equivalent of the full length of the linac), as a function of halo ring radius. The solid curve (with o's) shows the percentage of particles that leave the halo ring and are collimated. The dashed curve (with x's) shows the percentage of particles still remaining in the halo ring. The dotted curve (with +'s) shows the percentage of particles that leave the halo ring, but do not reach the collimation radius of 4 mm. We see that all particles beyond 8-10σ are collimated. It may be possible to reduce the number of particles that are removed from the halo rings between approximately 2σ and 6σ but do not reach the collimation radius, by progressively weakening the octupoles along the linac.

In Figure 9, we show the pattern of loss of the particles in three different halo rings in the $x - y$ plane: (a) a ring from $4\sigma_x$ to $6\sigma_x$, (b) a ring from $6\sigma_x$ to $8\sigma_x$, (a) a ring from $8\sigma_x$ to $10\sigma_x$, as the particles travel down the linac. The losses are distributed over a substantial fraction of the linac length in all cases.

Blow-up of mis-steered beam

The beam density of a mis-steered beam that hits an absorber somewhere along the linac has to be sufficiently low to guarantee absorber survival; as noted earlier, a particle density of about $10^6 \mu\text{m}^{-2}$ suffices throughout the linac. Near the beginning of the linac, the nominal particle density in a full train of n_b bunches with N particles per bunch hitting perpendicular to an absorber surface would be about $n_b N/\sigma_x \sigma_y \sim 10^9\ \mu\text{m}^{-2}$. In our scheme, only a fraction f of the train particles are lost on a given absorber (see Figure 10 below; at any given magnet f is at most a few percent). Thus, a nominal-emittance NLC bunch train needs to be blown up in transverse area by a factor of about $10^3 f$.

In Figure 10 we show the blow-up of the beam area for a beam that is mis-steered and lost somewhere along the linac, as a function of the location where it is lost. The phase advance per cell is 90° and k_{oct}=40000 m^{-3}. The beam is initially offset in x by $10\sigma_x$, and at the initial (focusing in x) quadrupole, the beam σ_x=66 μm and σ_y=3.3 μm. The vertical axis is the product of the standard deviations of the particle distributions in the two transverse directions. Note that the losses occur near the peaks of the beam area curve since these are also where the orbit excursions are largest. The beam area at the location of the losses blows up by a factor of over 100 after about 40-50 FODO cells (80-100 magnets on horizontal axis), and the fraction of beam particles lost at a time is not more than a few percent. Thus the above criterion for the blow-up factor is met, and so a mis-steered beam will not destroy the absorbers or accelerator structures.

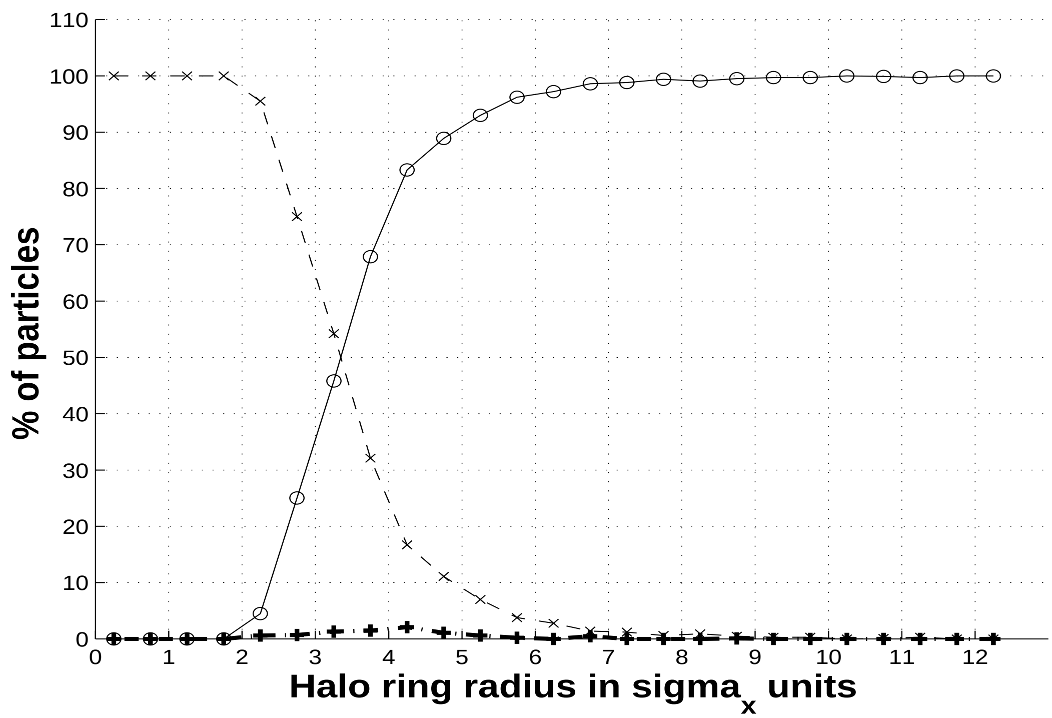

FIGURE 8. As a function of halo ring radius: Percent of particles remaining in halo ring after 350 FODO cells (dashed curve with x's), percent of particles that leave halo ring but do not reach collimation radius of 4 mm (dotted curve with +'s), percent of particles in halo ring that have been collimated (solid curve with o's)

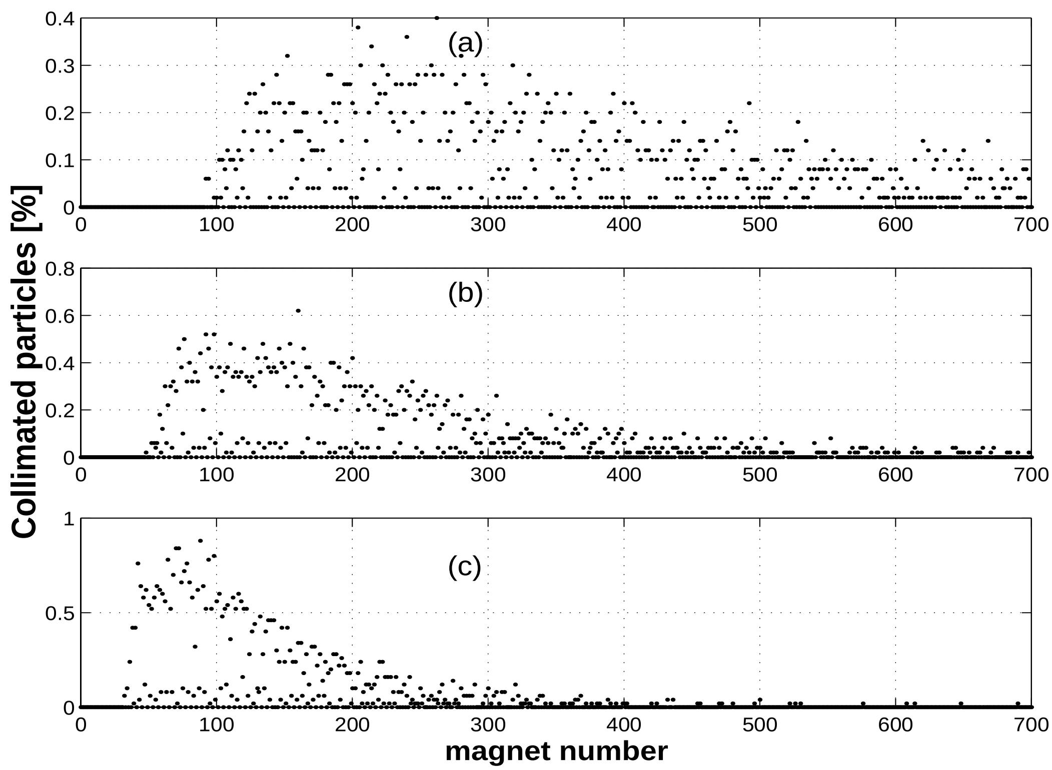

FIGURE 9. Pattern of loss of particles in three different halo rings, as a function of distance along the linac, for a collimation radius of 4 mm. (a) For halo ring from $4\sigma_x$ to $6\sigma_x$. (b) For halo ring from $6\sigma_x$ to $8\sigma_x$. (c) For halo ring from $8\sigma_x$ to $10\sigma_x$.

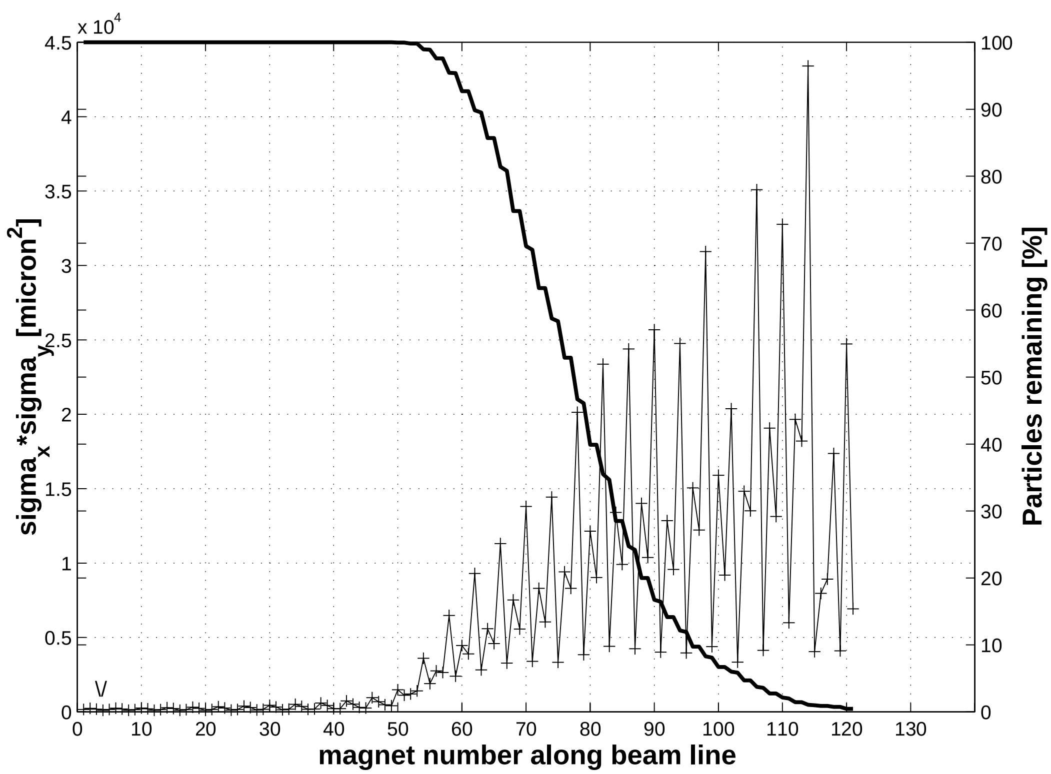

FIGURE 10. Beam blow-up (dotted curve) and loss (solid curve) as a function of quadrupole number along the linac, for a beam offset by $10\sigma_x$ (= 660 μm) in x.

CONCLUSIONS AND FUTURE WORK

The scheme presented here seems quite promising; it appears that it can both collimate the beam at the required collimation depth with reasonable octupole strengths, and it can blow up a mis-steered, nominal emittance beam by the required factor of 100 or more before hitting an absorber. This scheme can be integrated into the main linac and thus may allow removing or at least shortening dedicated collimation sections after the linac, which in the present NLC design are several kilometers in length. Further study is needed to determine the optimum distribution of x and y collimation in the linac. Another natural topic for further study is the use of dodecupoles instead of or in addition to octupoles, in which case one would expect a sharper collimation of the halo and a better preservation of the core.

A number of other issues remain to be looked into. This scheme inherently involves reducing the dynamic aperture of the linac; the implications of this for linac operations needs to be fully explored. There are concerns about spreading the lost particles over large regions, in particular the possibility of significant radioactivation. Another issue is the specification of linac absorbers, and whether these can be placed close to the beam without diluting the emittance. Finally, the issue of halo regeneration from particles in the core, including the effects of wakefields and beam-gas scattering, needs to be looked at more closely.

REFERENCES

1. C. Adolphsen et al., "Zeroth Order Design Report for the Next Linear Collider," *SLAC-Report* **474** (1996).
2. F. Zimmermann, "New Final Focus Concepts at 5 TeV and Beyond," Eighth Advanced Accelerator Concepts Workshop, Baltimore, 5-11 July 1998; SLAC-PUB-7883.
3. N. Tsoupas, R. Lankshear, C.L. Snead,Jr., T.E. Ward, M. Zucker, H.A. Enge, "Uniform Beam Distributions Using Octupoles", Proceedings of IEEE Particle Accelerator Conference, San Francisco (1991).